The Trials and Tribulations of Tractably Tabulating Triangles

C. Seshadhri
(Sandia National Labs, Livermore)

Joint work with Madhav Jha and Ali Pinar (Sandia National Labs, Livermore)

Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000."

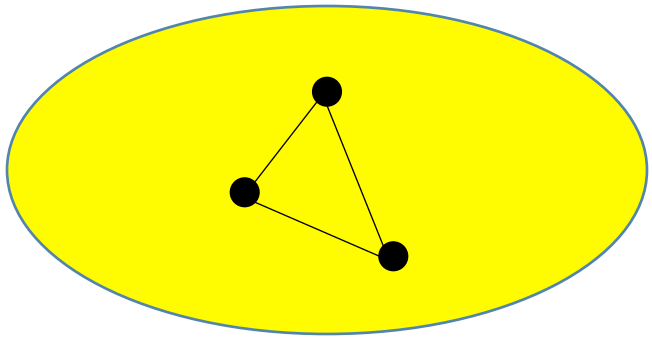
Theory and practice

Theory

Practice

There and back again

We love triangles



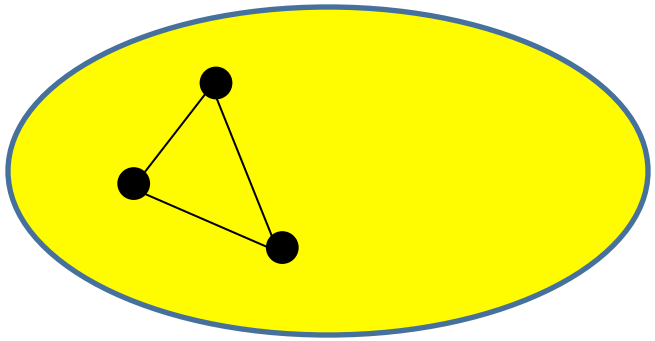
- The building block of communities. The sign of social networks. The transitivity of relationships
- One subgraph to rule them all

Social sciences: [Holland-Leinhardt70] [Coleman88] [Skvoretz90] [Portes98] [Burt04] [Welles etal10][Faust10] [Szell-Thurner10]

Physical sciences: [Watts-Strogatz98] [Eckmann-Moses02] [Fagiolo07] [Milo etal10] [Son etal10] [Leskovec etal10] [Winkler-Reichardt13]

Algorithmics: [Becchetti etal08][Berry etal11] [Gleich-S12] [Rohe-Qin13]

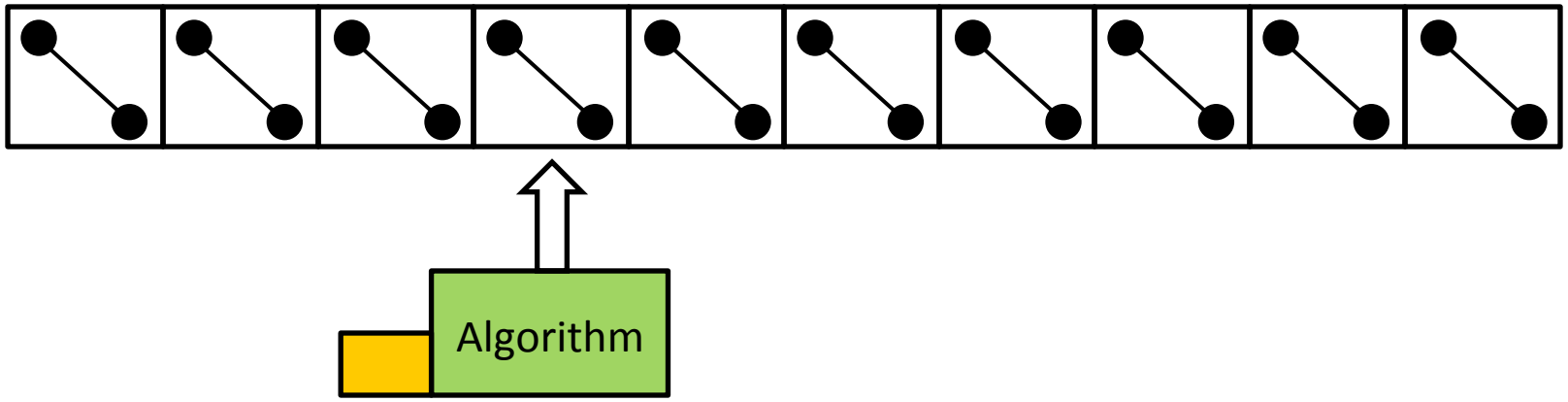
Counting triangles



Graph [SNAP, LAW]	n	m	T
web-BerkStan	700K	6.6M	64M
flickr	1.8M	15M	550M
livejournal	5.2M	48M	300M
uk-union	132M	4B	450B

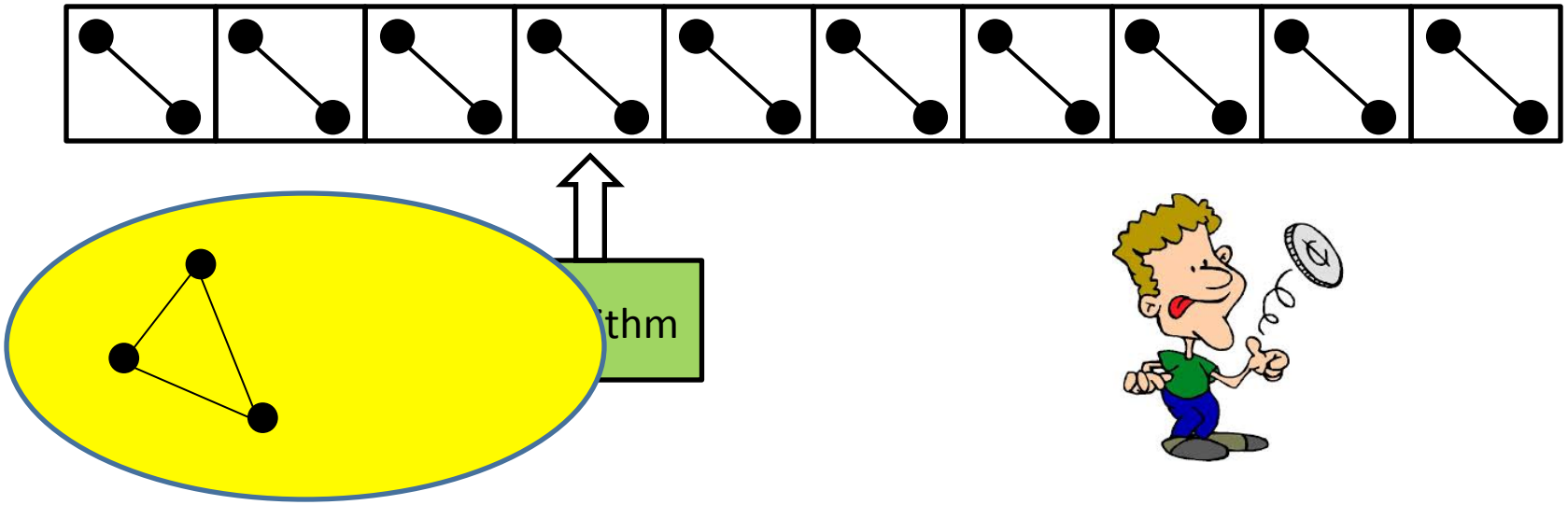
- How to accurately count/estimate T, or related “triangle measures”?
- We’re looking for efficient, scalable algorithms
- Even clever enumeration won’t work

The streaming setting



- Crucial for real-time analytics of temporal graph
  - Memory orders of magnitude smaller than stream
- No assumption on graph or ordering of edges
- Single pass over data. If algorithm forgets something, too bad

The streaming setting



- Algorithm should output (estimate) number of triangles at the end
- Output with high probability (> 0.99) over coin flips, NOT over the input

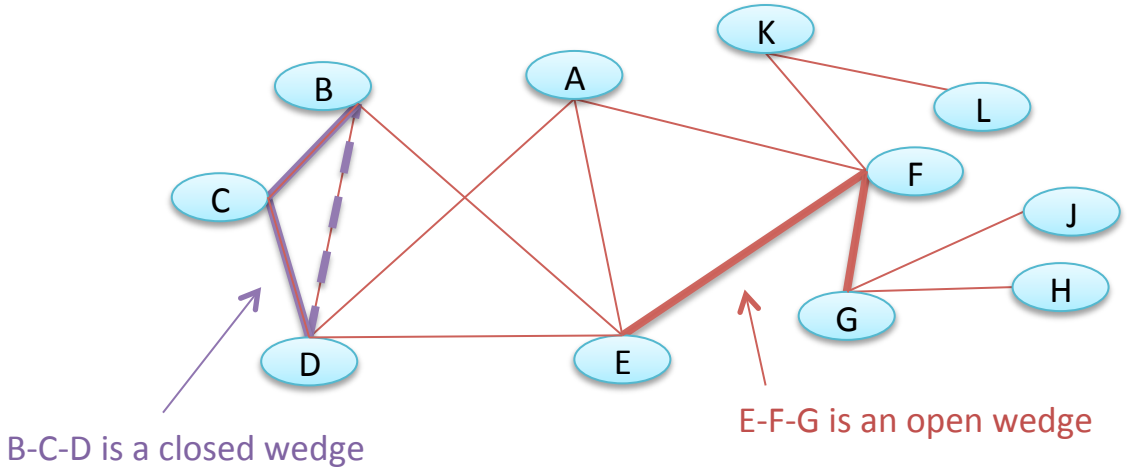
History

[Bar-Yossef etal02]
[Jowhari-Ghodsi05]
[Ahn-Guha-McGregor12]
[Pagh-Tsourakakis12]
[Kane etal12]
[Braverman-Ostrovsky-Vilenchik13]

[Buriol etal06]

(Breaks down at million edge graph.)

Notation



- n = no. of vertices
- m = no. of edges
- W = no. of wedges (paths of length 2)
  - "Center" of wedge is middle vertex
- T = no. of triangles
- Transitivity = $\tau = 3T/W$ = fraction of closed wedges
- Amen

Result

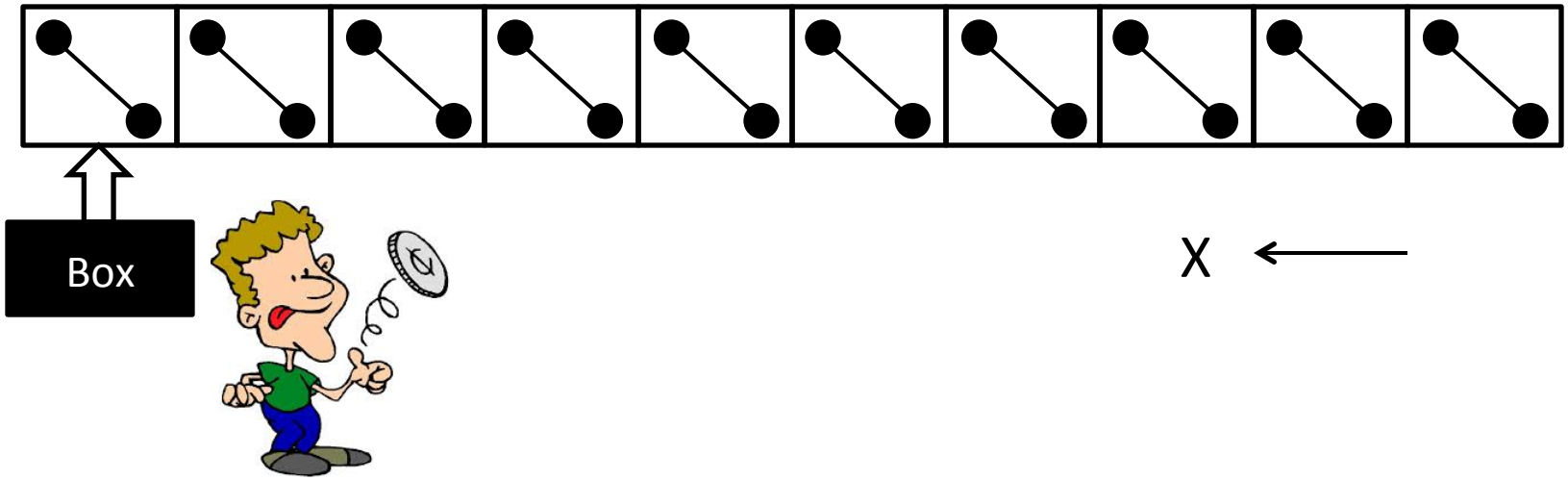
- [Jha-S-Pinar13] Fix $\epsilon > 0$. Algorithm's features:
  - $\mathcal{O}(\sqrt{n})$ space
  - Output τ' such that $|\tau - \tau'| < \epsilon$ w.h.p.
  - Works terrible in practice

- Heuristic "fix"
  - Somewhat principled, I think
  - Works well in practice
  - Estimates τ to within 0.01 error

  with 40K edges, for graph with > 200M edges

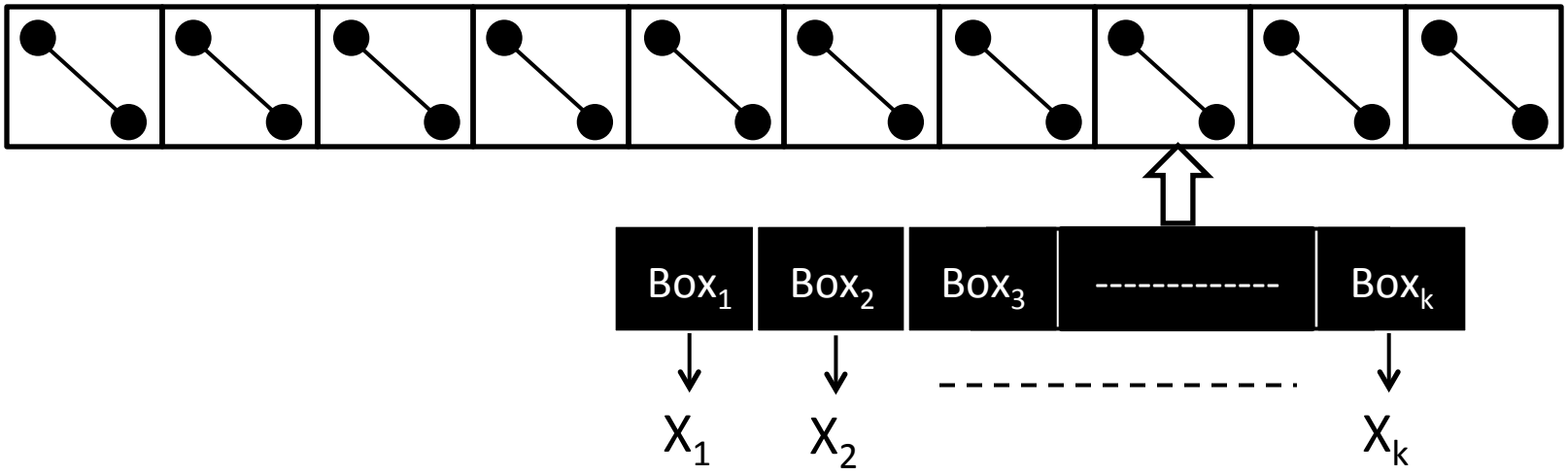
- [Pavan-Tangwongsan-Tirthapura-Wu13] Alternate approach, works well in practice

The scheme



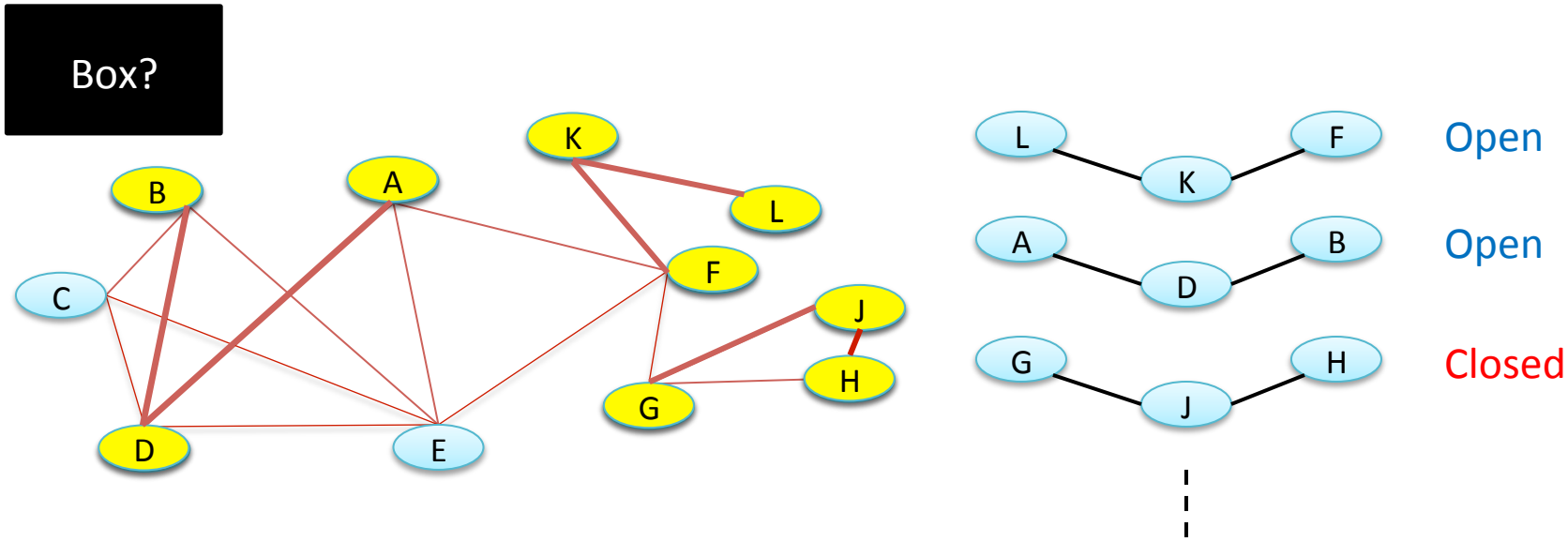
- Fix the graph stream
- Construct small space box that outputs Bernoulli r.v. X such that E[X] = τ

The scheme



- Generate i.i.d. $X_1, X_2, \ldots, X_k$
  - Output fraction of these that are 1 as τ'
- [Chernoff] If $k \sim 1/\epsilon^2$, $|\tau - \tau'| < \epsilon$ w.h.p.
- Total space = $|\text{Box}|/\epsilon^2$

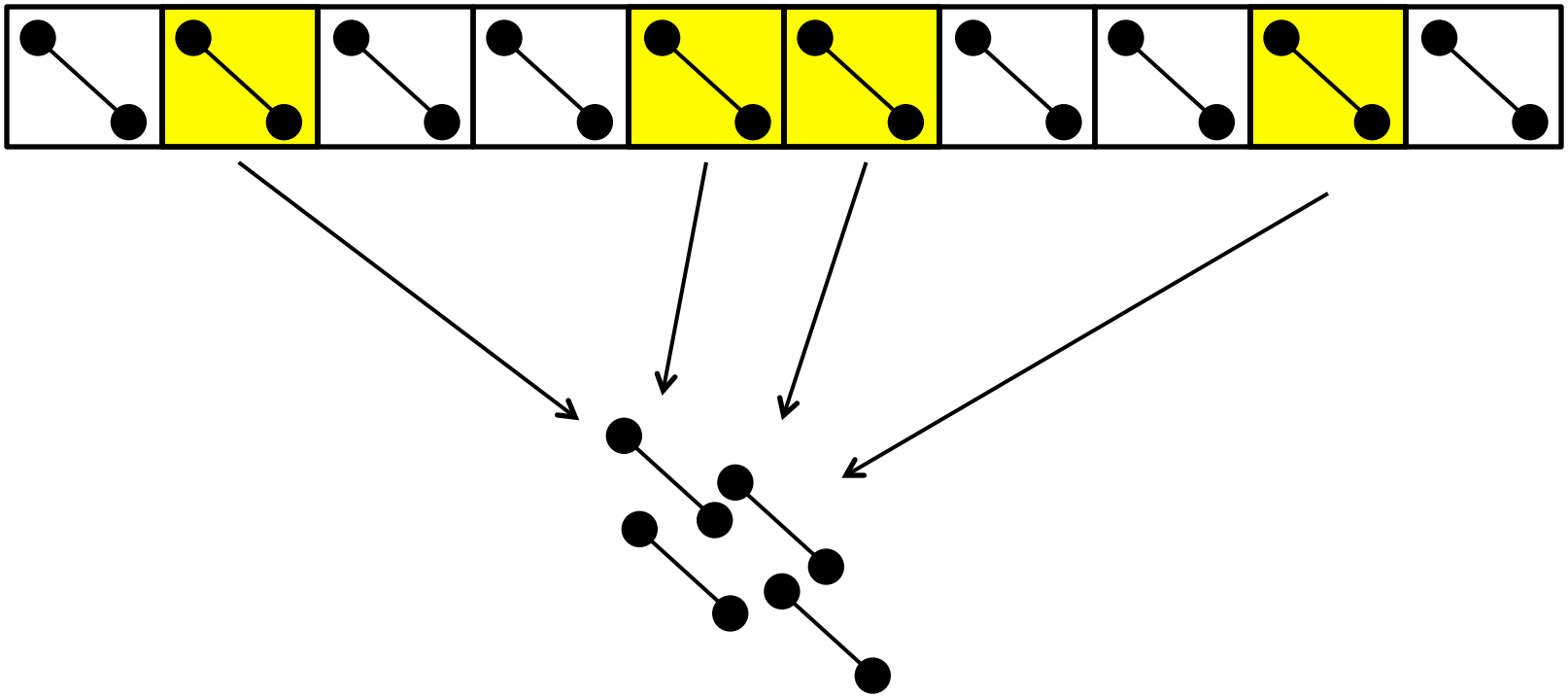
What has it got in its pocketses?



- τ= 3T/W = fraction of closed wedges
- Sample uniform at random (u.a.r.) wedge.

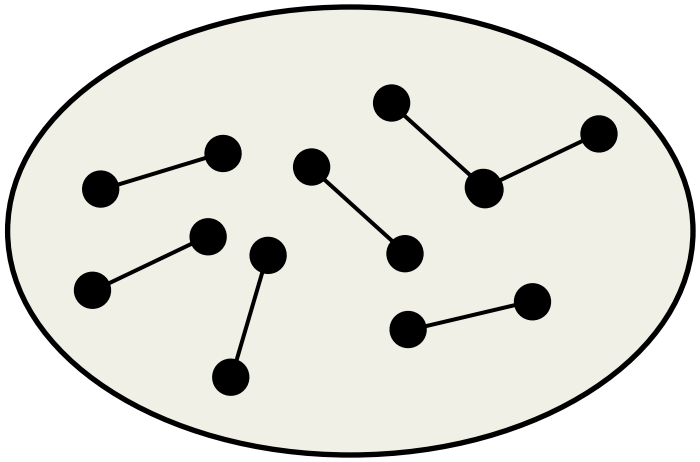
  X = 1 if closed and X = 0 otherwise
- How to sample u.a.r. wedge from edge stream?

Getting a wedge



- Picking random edges from stream is easy
  - [Vitter85] Reservoir sampling
- How many to pick before you get a wedge?

Getting a wedge

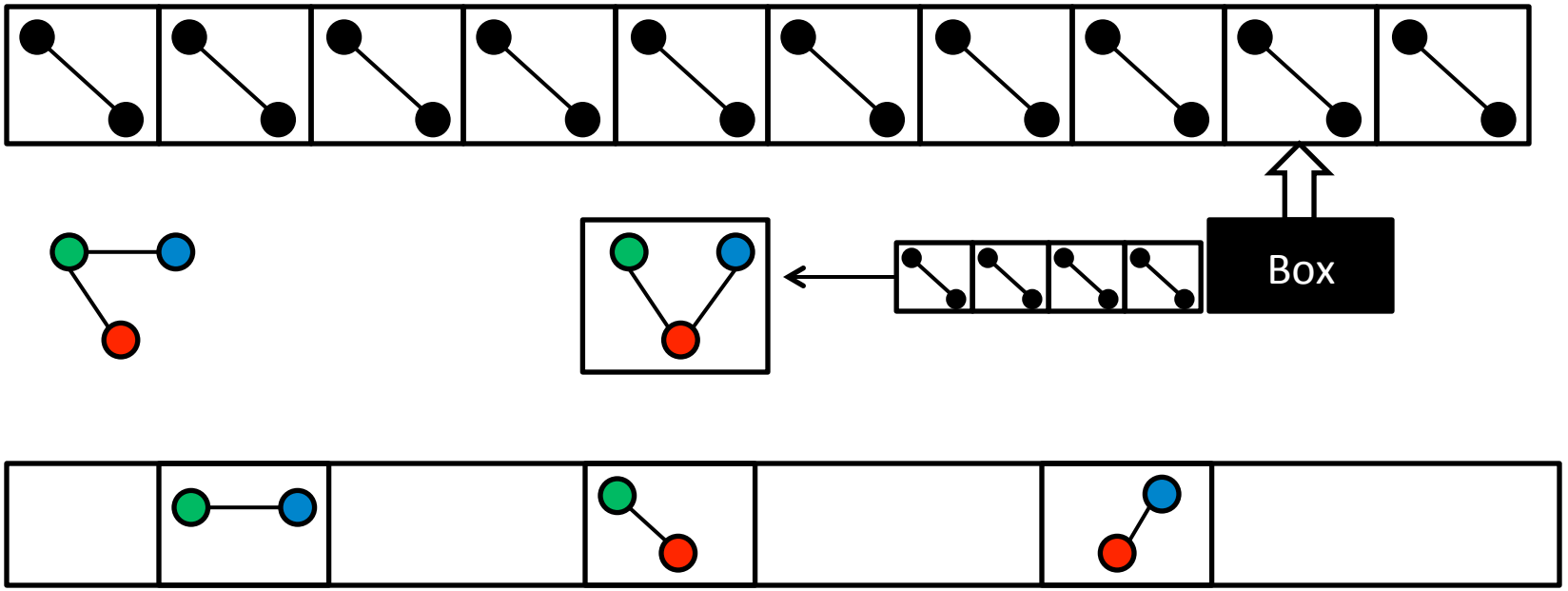


1 2 3 ---------- n

- Birthday paradox calculation!
- $O(m/\sqrt{W}\,)=O(\sqrt{n}\,)$ u.a.r. edges suff

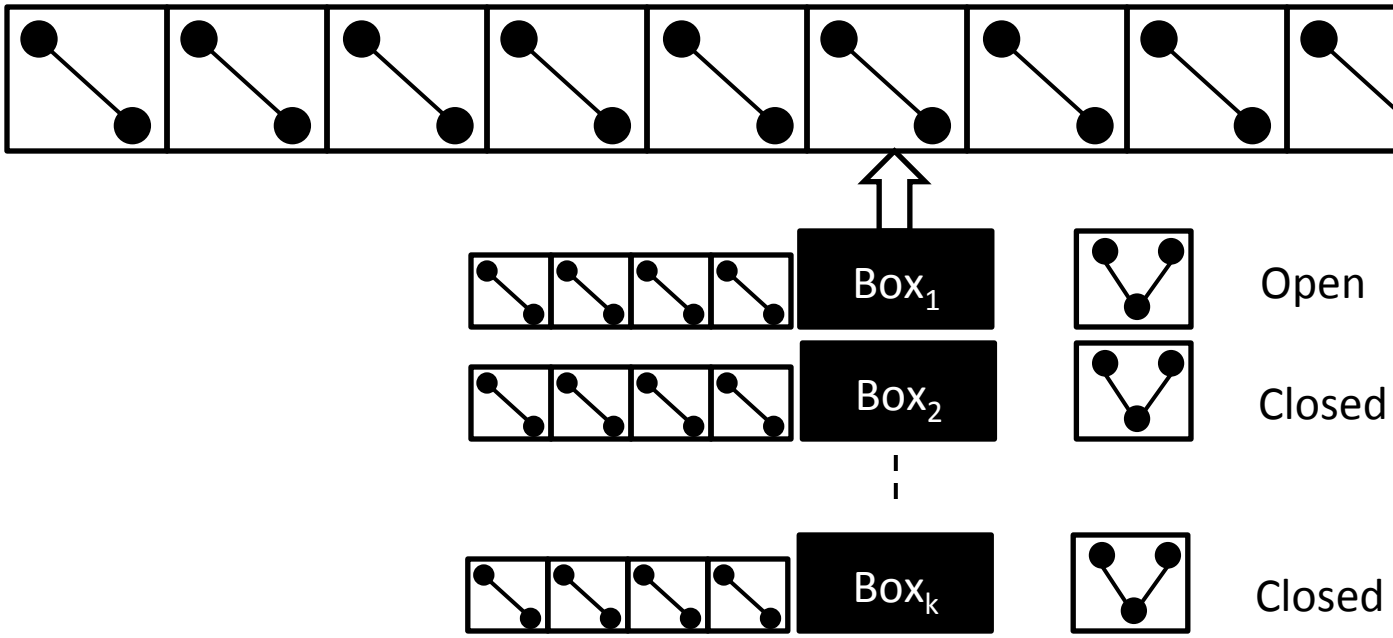
to get wedge w.h.p.

The box



- Each triangle has one future-closed wedge
- Fraction of future-closed wedge $= \tau/3$
- We can estimate this fraction instead, scale by 3

All together



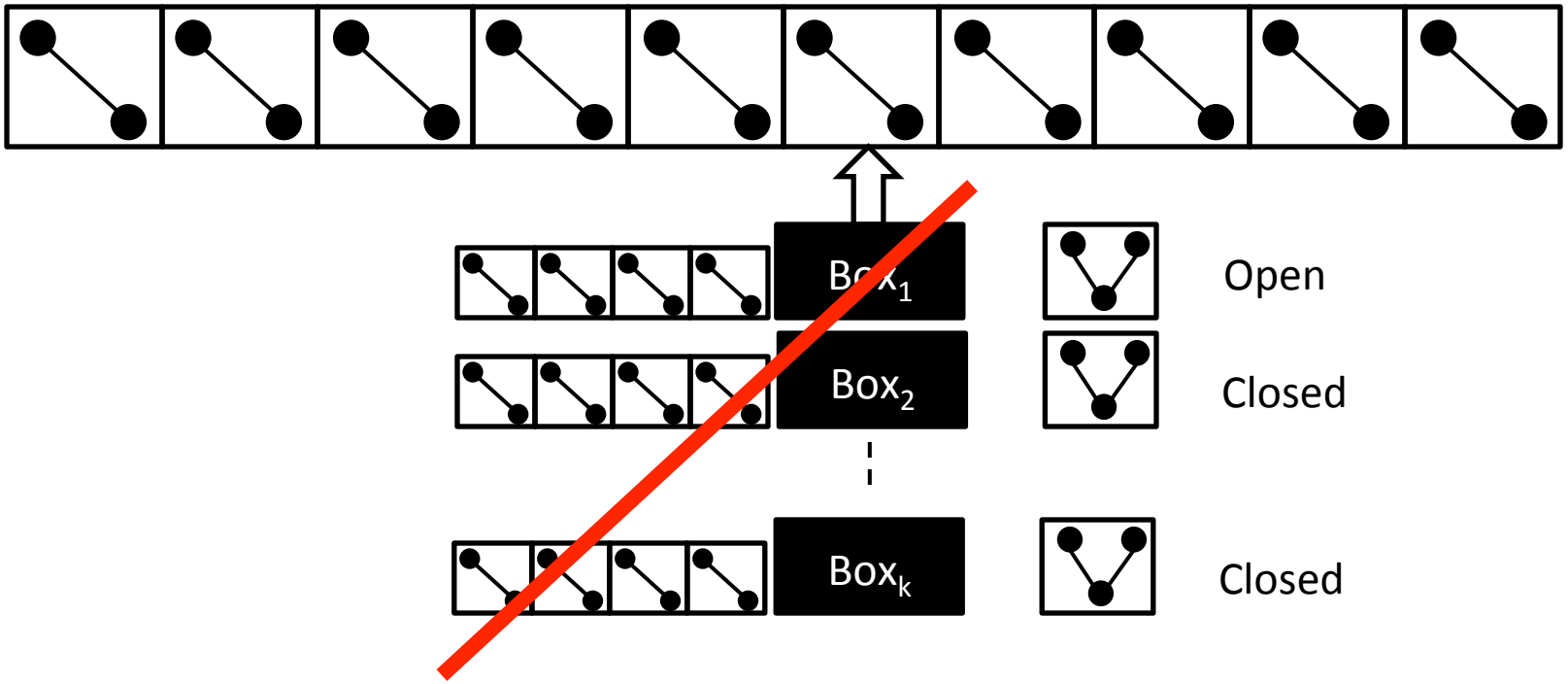
- $O(\sqrt{n})$ space for constant error ϵ
- Result looks nice, cute arguments, we're done!

It's awful in practice

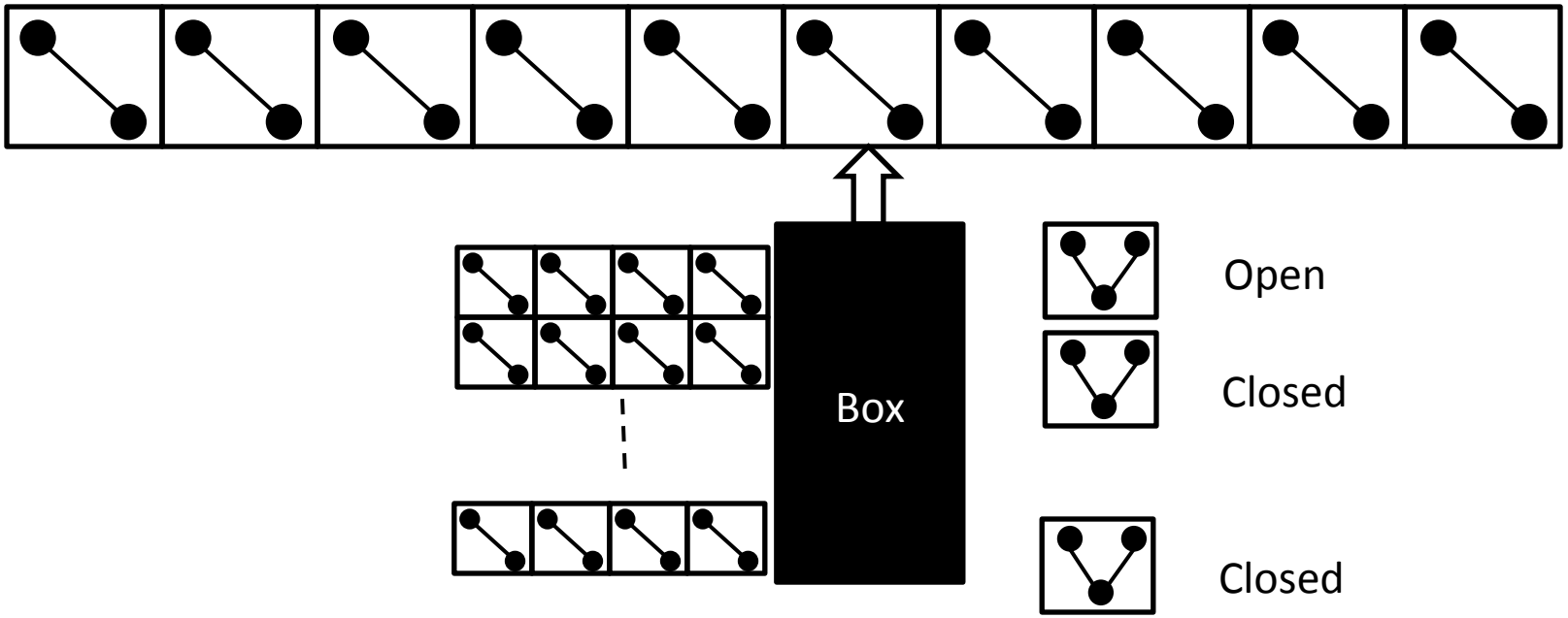
$$\sqrt{n}/\epsilon^2 \approx \sqrt{10^6}/(0.01)^2 = 10^7$$

Graph with million vertices, desired error of 0.01

Heuristica

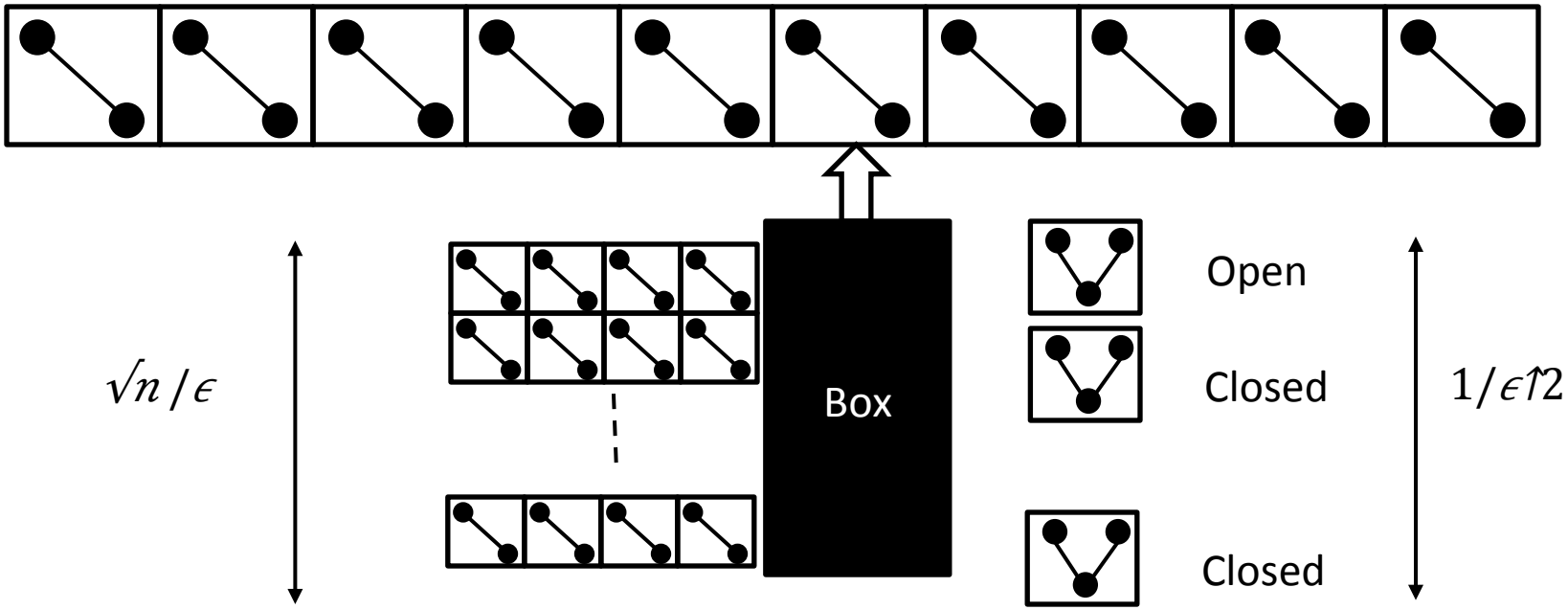


Heuristica

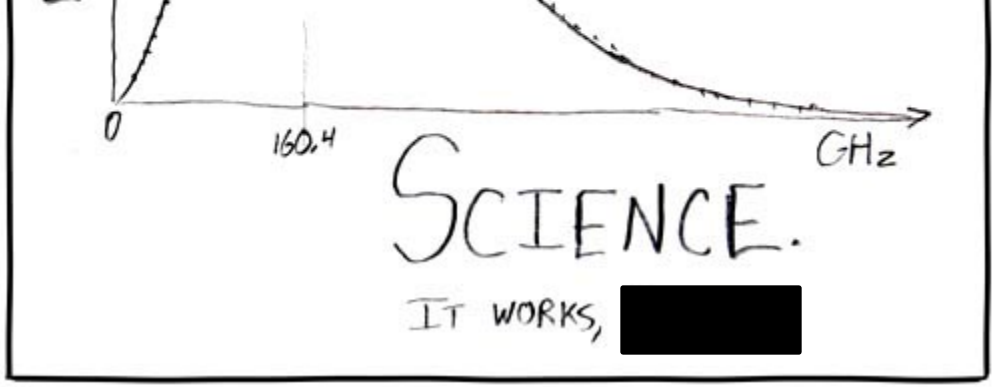


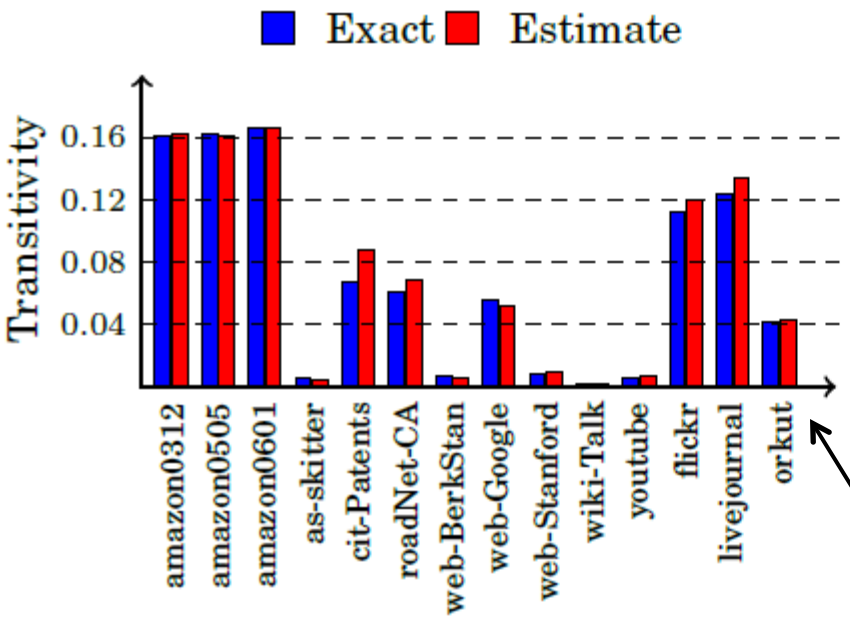
- Chernoff: "To get ϵ error, need $1/\epsilon^2$ wedges"
- Birthday paradox: "If you select $k\sqrt{n}$ u.a.r. edges, you get k^2 u.a.r. wedges w.h.p"
- Chernoff needs independent wedges, birthday gives dependent wedges

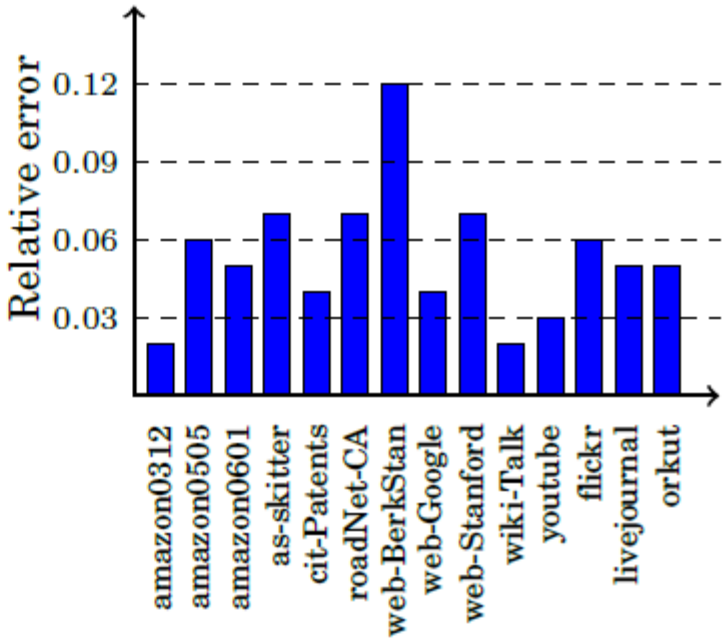
Crossing fingers



- Assume wedges are sufficiently de-correlated
- $\sqrt{n}/\epsilon$ edges suffice. Huge savings!



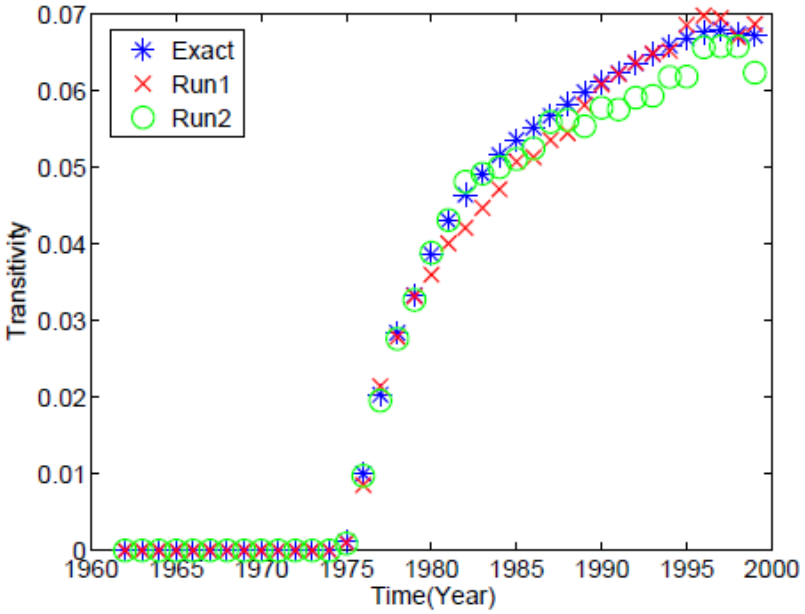




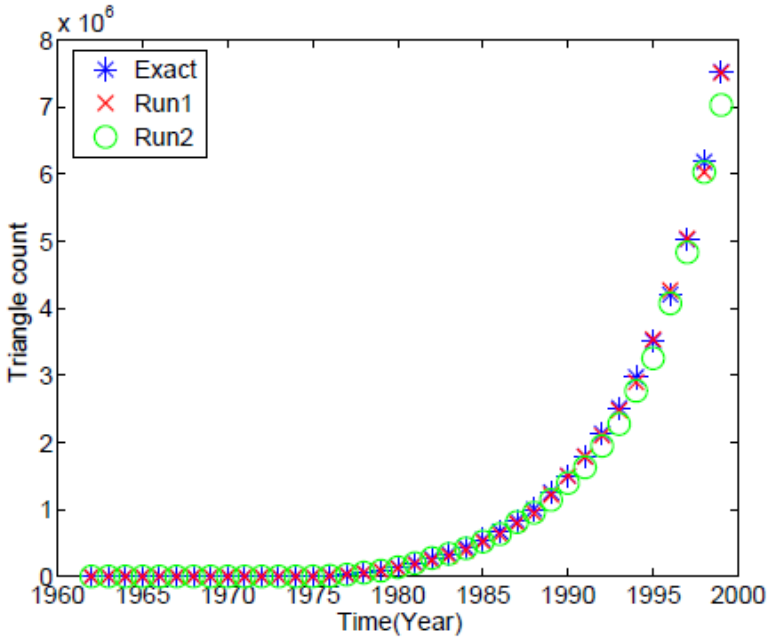
200M edges

Total edge storage = 40K

Something that isn't boring



(a) Transitivity



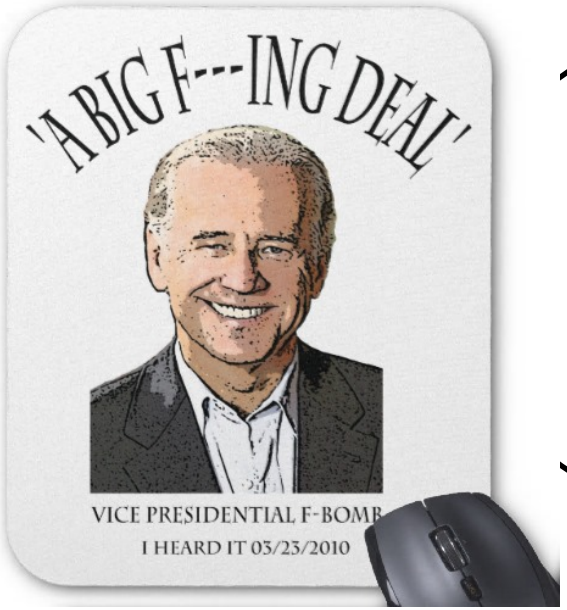
(b) Triangle count

cit-Patents: 16M edges, 100K edge storage

A dialogue

This is really cool! Some theory and it works!

Darn.



Real graph streams are multi-graphs. And repeated edges will kill your algorithm. And removing repeated edges from a stream is expensive.

- [Jha-S-Pinar13] Extension to multi-graphs

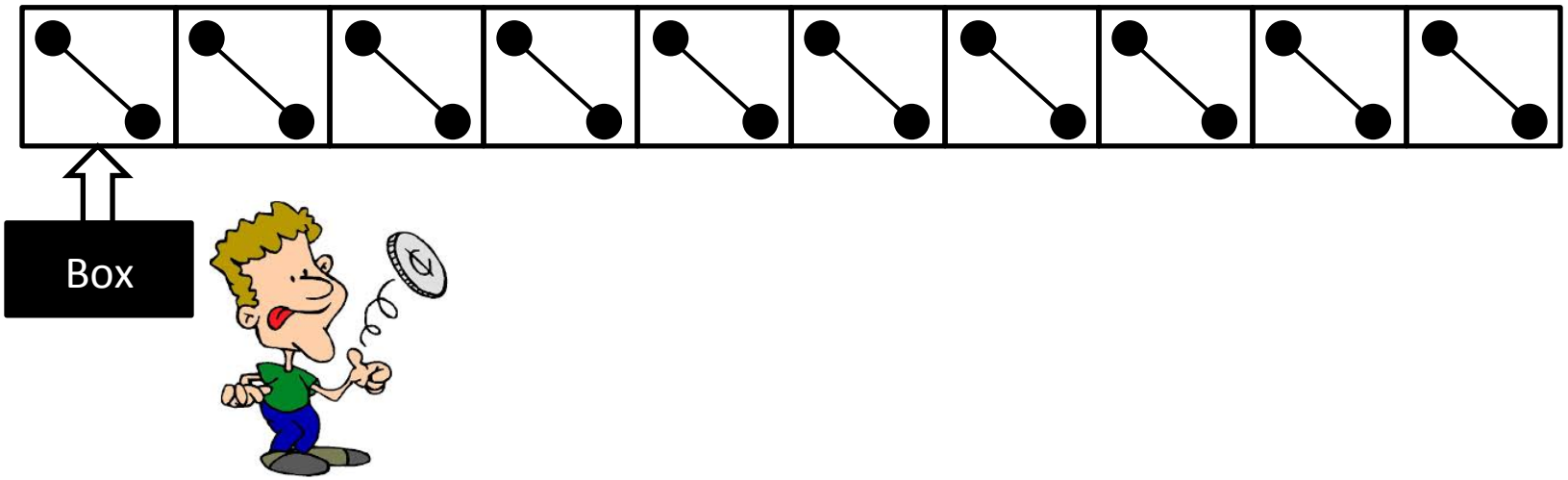
Who cares?

- Nobody.

  No one really “cares”

- We don’t want simply the transitivity, or the number of triangles

More complex quantities



- Estimate T_v , for high degree v?
- Estimate T_d , triangles incident to deg. d?
- Sudden jumps in clustering coefficient?
- More vertex/edge based triangle numbers, not global count

Get stronger results

- What we discussed was baby stuff

for streaming algorithms theory

- How to use more powerful tools?
  - Andrew's talk…
- [Alon-Matias-Szegedy] type results?
- $\ell \downarrow p$ -sampling anyone?

Bigger pictures?

- [Ahmed-Neville-Kompella13] How to get a representative subsample for streamed graph?
  - That preserves "most structure"

- Our sample only counts triangles. Can you do more with it?
- Our sample has flavor of induced edge sampling…?

Thanks!