Improved Approximation Algorithms for Non-Preemptive Scheduling Problems via Time-Indexed LP Relaxations

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Outline

1. Introduction
2. 1.8786-Approximation for $R|r_j|\sum_j w_j C_j$
3. $(2 + 2\ln 2)$-Approximation for $P|\text{prec}|\sum_j w_j C_j$

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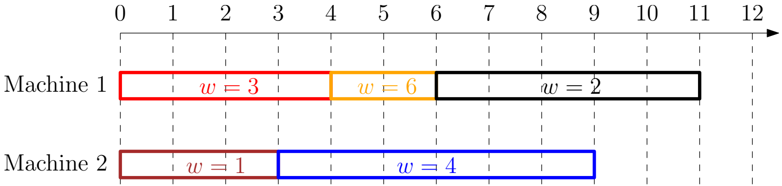

$$\sum_{j \in J} w_j C_j = 3 \times 4 + 6 \times 6 + 2 \times 11 + 1 \times 3 + 4 \times 9 = 109$$

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- in this talk it is always $\sum_j w_j C_j$

List of Our Results

Problems	Prev. approx. ratio	Our approx. ratio
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$P\|\text{prec}, p_j=1\|\sum_j w_jC_j$	3 [1]	$1+\sqrt{2}$
$Q\|\text{prec}\|\sum_j w_jC_j$	$O(\log m)$ [2]	$O(\frac{\log m}{\log\log m})$
$R\|\|\sum_j w_jC_j$	$1.5-\epsilon$ [3]	$1.5-\frac{1}{6000}$
$R\|r_j\|\sum_j w_jC_j$	2 [4]	1.8786

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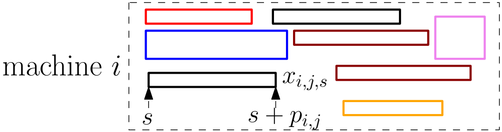

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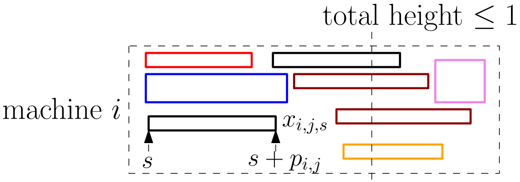

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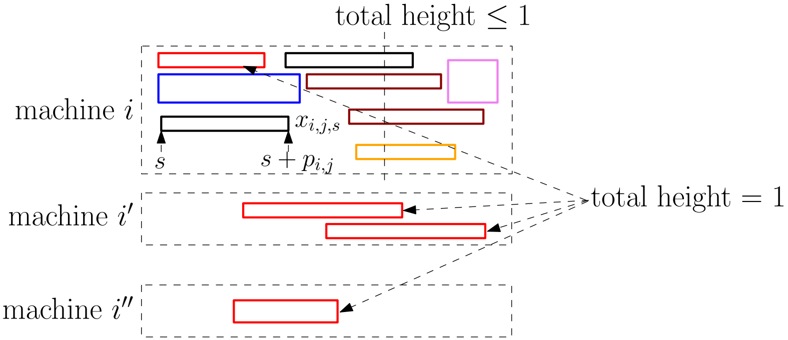

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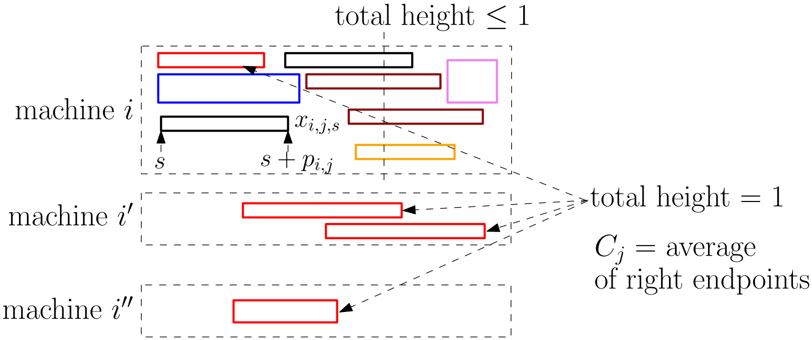

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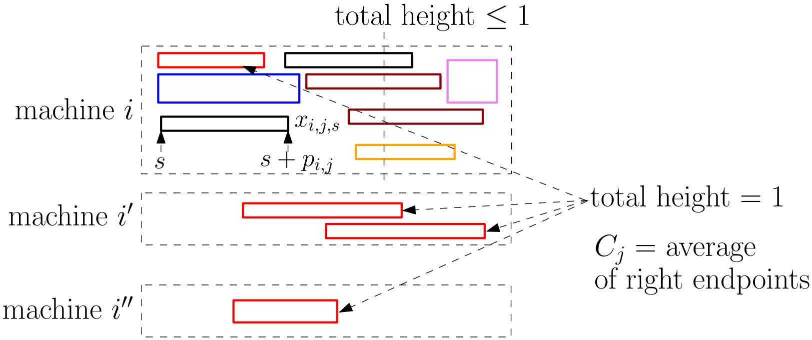

- to capture $j \prec j'$: $C_{j'} \geq C_j + p_{j'}$

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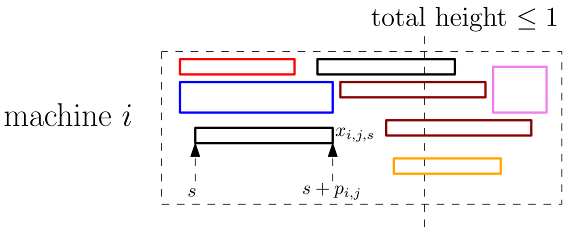

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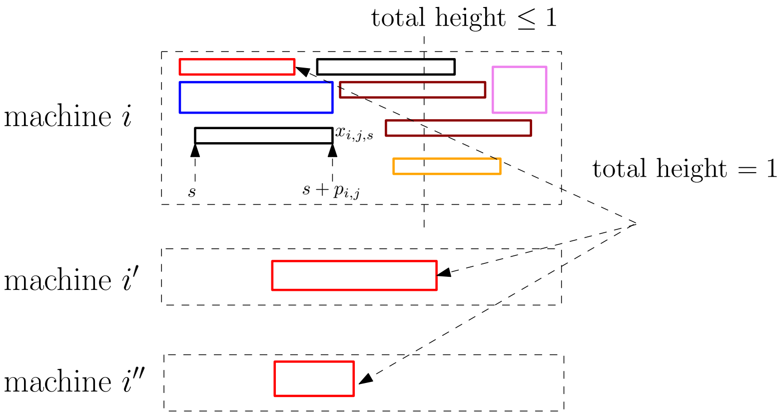

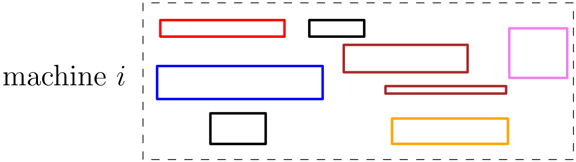

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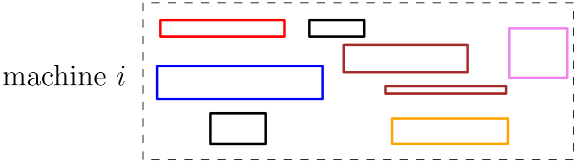
1. solve time-indexed LP relaxation to obtain $\{x_{i,j,s}\}$



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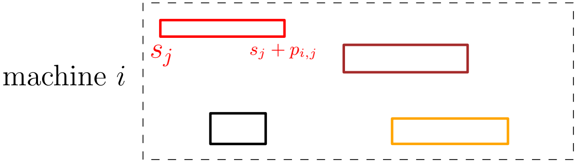

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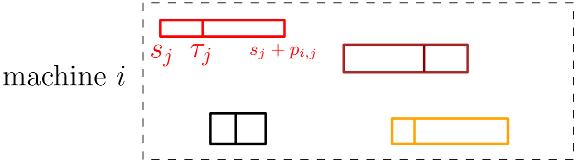

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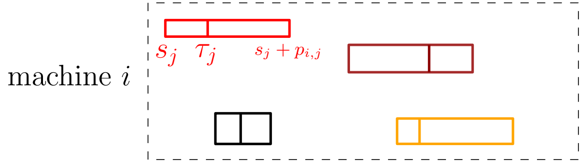

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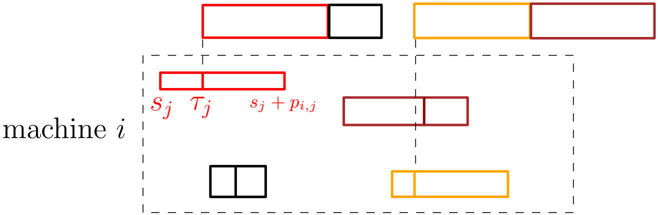

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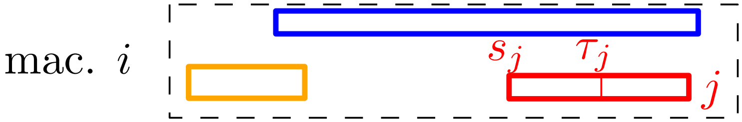

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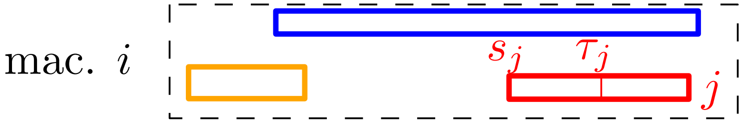

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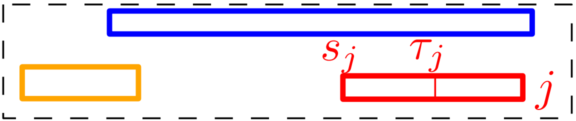

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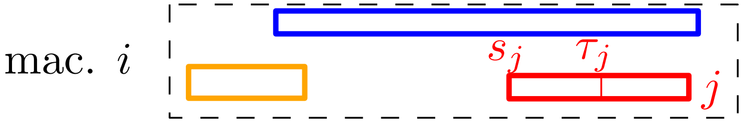

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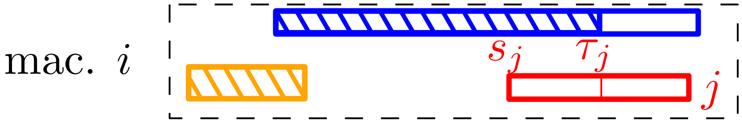

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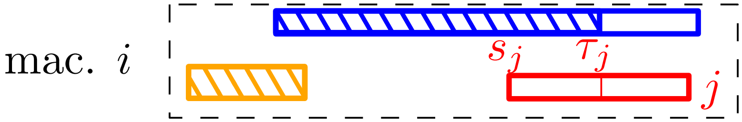

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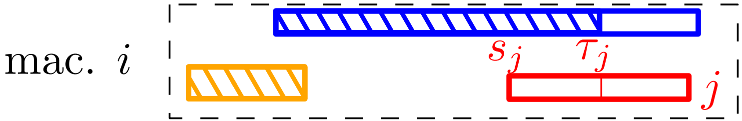

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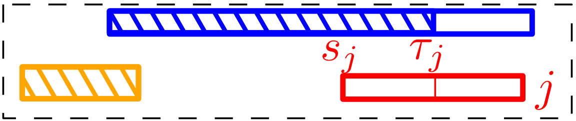

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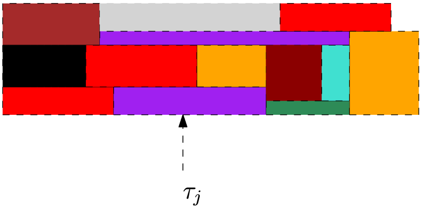

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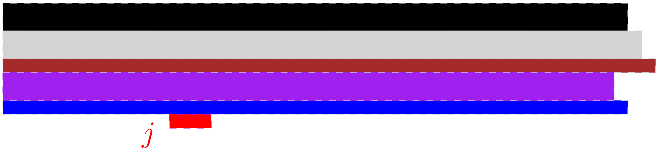
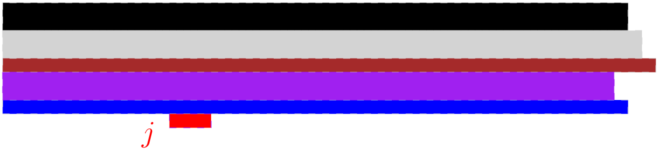
- The only bad case: most rectangles have length $>> \tau_j$





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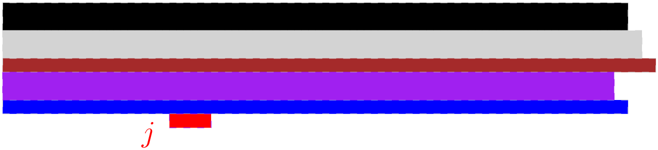

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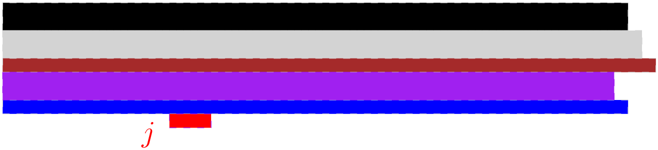

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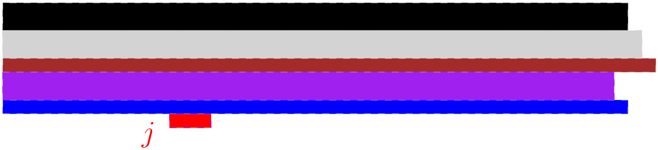

Improving the Factor of 2

- Choose each τ_j from a different distribution (instead of uniformly between s_j and $s_j + p_{i_j,j}$)
  - Shift mass from left to center: long jobs are less likely to delay j
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Choosing each τ_j uniformly from $[s_j + \delta p_{i,j}, s_j + (1-\delta)p_{i,j}]$ for small $\delta > 0$ can already improve the ratio 2!

Analyze Effect of Using Non-Uniform Distributions

- Let $\boldsymbol{\Theta}$ be distribution over $[0,1]$ for shifting parameters
- i.e, $\tau_j = s_j + \theta_j p_{i,j}$, where $\theta_j \sim \boldsymbol{\Theta}$

$$C_j = \sum_{j' \neq j : i_{j'} = i, \tau_{j'} < \tau_j} p_{i,j'} + \left(|\text{idle time}| \text{ before } \tau_j\right) + p_{i,j}$$

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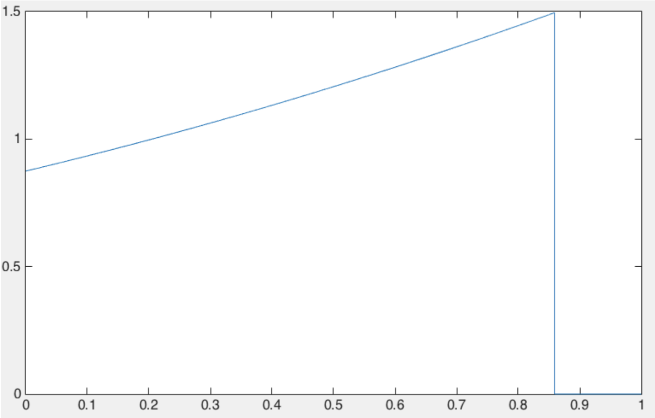
- Purely analytical proof for approximation ratio

Probability Density Function for Θ



$$f(\theta) = \begin{cases} 0.1702\theta^2 + 0.5768\theta + 0.8746 & \text{if } 0 \leq \theta \leq 0.85897 \\ 0 & \text{if } 85897 < \theta \leq 1 \end{cases}$$

Summary for $R|r_j|\sum_{j\in J} w_jC_j$

Thm There is a 1.8786-approx. for $R|r_j|\sum_{j\in J} w_jC_j$.

Algorithm for $R|r_j|\sum_{j\in J} w_jC_j$

1. solve time-indexed LP relaxation to obtain $\{x_{i,j,s}\}$
2. for each $j \in J$, independently do:
3. let $(i_j, s_j) = (i, s)$ with probability $x_{i,j,s}$
4. choose θ_j from $\mathbf{\Theta}$, and let $\tau_j = s_j + \theta_j p_{i_j,j}$
5. for each $i \in M$, schedule jobs assigned to i in increasing order of τ_j, pretending τ_j's are releasing times

Outline

1. Introduction

2. 1.8786-Approximation for $R|r_j|\sum_j w_j C_j$

3. $(2 + 2\ln 2)$-Approximation for $P|\text{prec}|\sum_j w_j C_j$

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- P: identical machines (instead of unrelated machines)
- prec: we have precedence constraints

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- $x_{j,s} \in [0,1]$: j scheduled on interval $(s, s+p_j]$?

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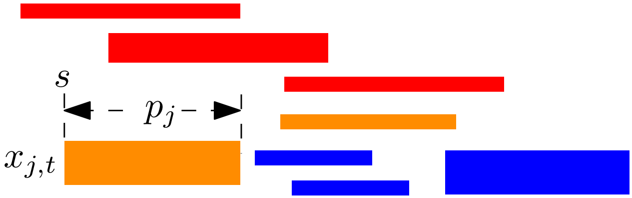

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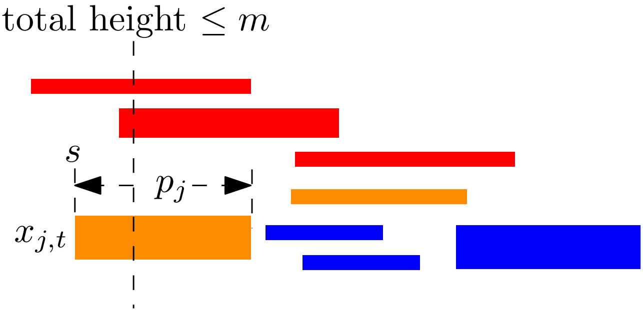

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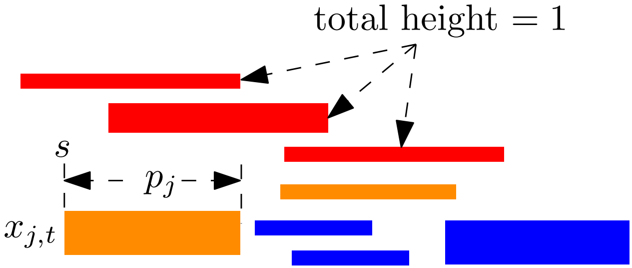

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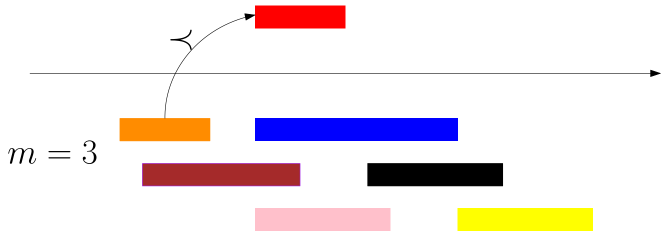

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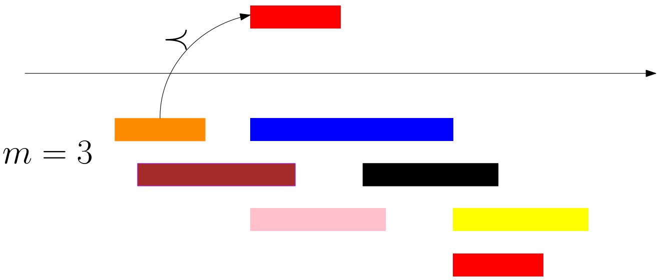

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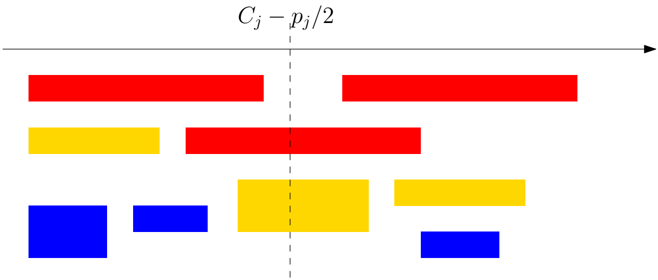

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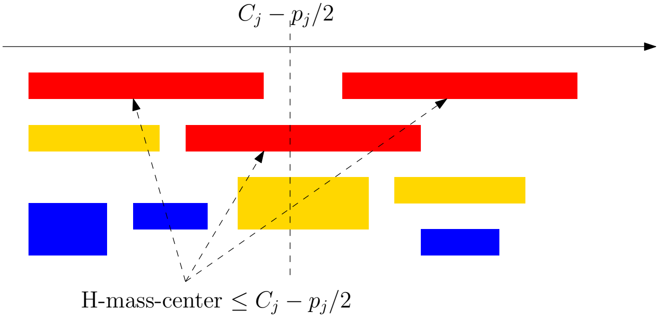

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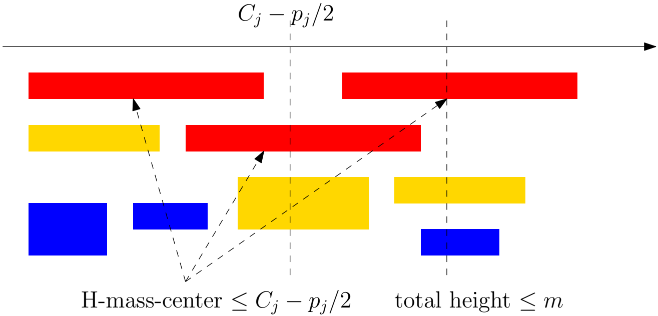

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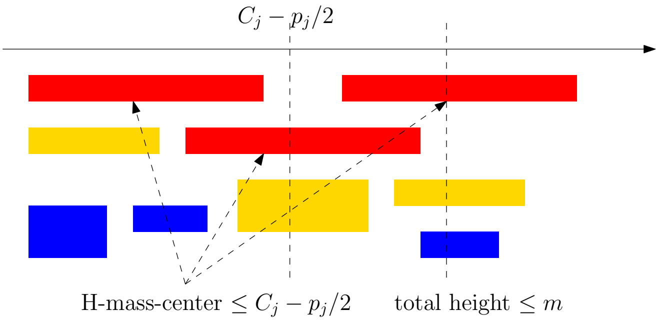

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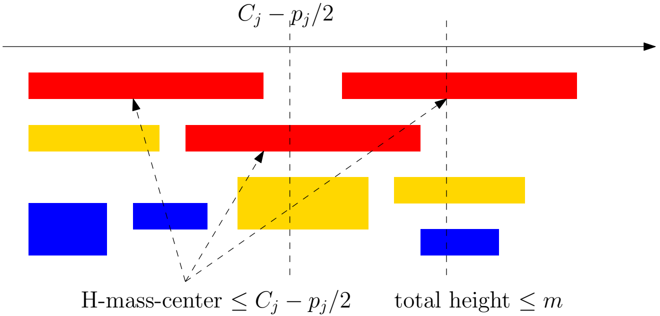

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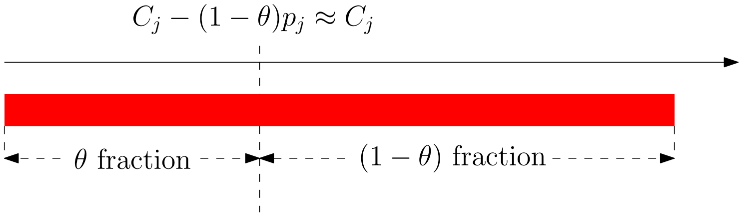

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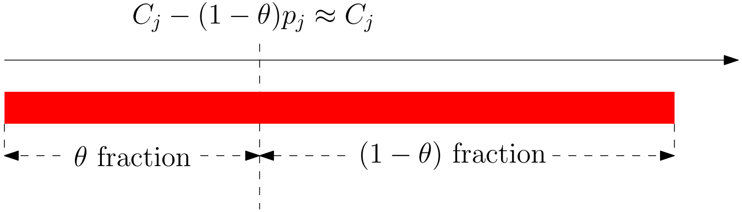

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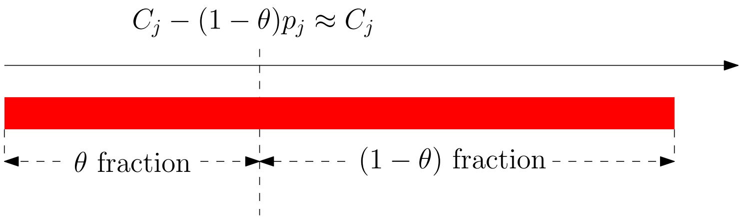

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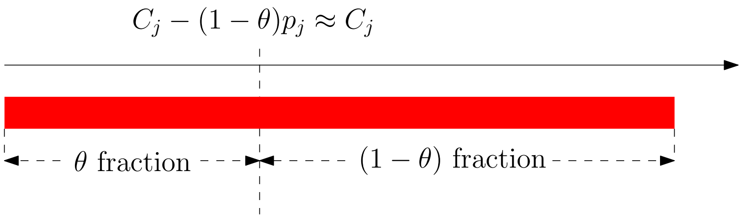

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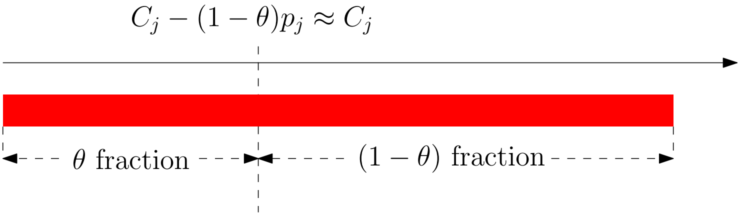

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