The expressiveness of recognizability by orbite finite nominal monoids

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{Ley, Puppis}

{symmetry, logic, computation}
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Fondamental theorems

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Fondamental theorem of regular languages:

monadic second-order logic $\overset{=}{\text{eff}}$ recognizability by monoids $\overset{=}{\text{eff}}$ deterministic automata $\overset{=}{\text{eff}}$ NDA = Rat = altA = …

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languages of words of length ω

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What happens in the **nominal world**?

Regular data languages?

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- many notions of recognizers now have distinct expressiveness
  DRA ≠ **NRA** ≠ **NRA'** ≠ **Monoid** ≠ **Alternating automata** …
- many problems usually decidable, are not anymore
  universality of **NFA** / SAT of **FO** / SAT of **MSO**, …

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There is no convincing notion of regular language of data words.

In this talk

{C., Ley, Puppis}2011

For languages of data words:

definable in rigid monadic second-order logic	$=_{\text{eff}}$	recognizability by orbit finite nominal monoids
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What is an orbit finite nominal monoid?

What are logics for data words?

What is rigidity?

Other consequences?

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Examples: $\mathbf{D}$, $\mathbf{D}\times\mathbf{D}$, $\mathbf{D}^*$, sets.

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An **equivariant** set/relation/function is one that is invariant under permutations of **D**.

Equivalently, it is definable using only the equality between **D** objects.

Languages recognized by an orbit finite nominal monoid

A is an **orbit finite alphabet** (**D** or B×**D**)

(M, ▪ ,1) is a **monoid** where M is orbit finite
and 1 and ▪ : M×M → M are equivariant.

h : A* → **M** is an equivariant **monoid morphism**.

F⊆M is an equivariant **set of accepting elements**.

The **recognized language** is L(M,h,F) = { u∈A* : f(u)∈F }

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Extremities: **E** = 'words such that the first and the last datas coincide'

elements	1, (d,e) for d,e datas (three orbits)
product	(d,e) · (d',e')=(d,e')
morphism	h(d) = (d,d) for all datas d (and generated)
accepting set	F = { (d,d) : d∈**D** }

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Sdistinct: **SD** = ‘words such that every consecutive datas are different’

elements	1, 0, (d,e) for d,e datas (four orbits)
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Counting: **C(k)** = 'words that contain exactly k distinct data values'

elements	sets of values of cardinality at most k, and 0
product	union, 0 when size k is exceeded
morphism	$h(d) = \{ d \}$
accepting set	$F = \{ X : \|X\|=k\}$

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Distinct: **D** = 'words such that all datas are distinct'

Impossible: the Myhill-Nerode congruence for this language has infinitely many orbits…

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Word (a,1) (b,124) (a,3) (b,2) (a,5) (a,124)

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First-order logic FO[<,~]:

- (first-order) variables range over positions in the word
- predicates are
  - **x<y** = ‘position x is to the left of position y’
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Automatically, definable languages are equivariant

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Over the alphabet $\{a,b\}\times\mathbf{D}$, finite graphs can be interpreted in a data word:

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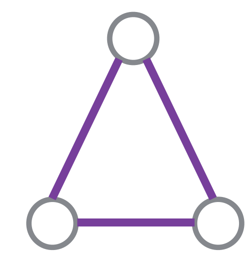

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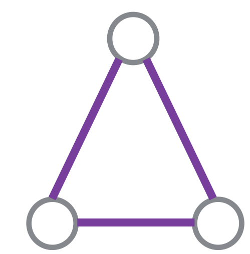

Fact: For every graph, there is a data-word in which it is interpreted.

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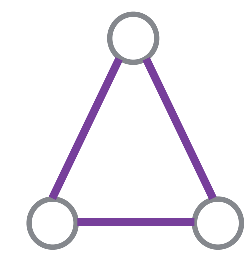

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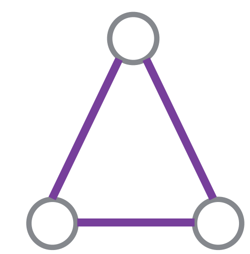

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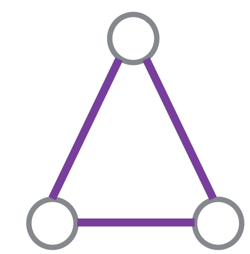

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Use a syntactic restrictions specially tailored for the expressiveness of recognizability by orbit finite monoids → **rigidity**

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{Bojanczyk}2010:
Every data language recognized by an orbit finite monoid is MSO[<,~] definable.

Every FO[<,~] definable data languages is recognized by an aperiodic syntactic monoid (not orbit finite in general).

If a data language is recognizable by an orbit finite aperiodic monoid, then it is FO[<,~] definable.

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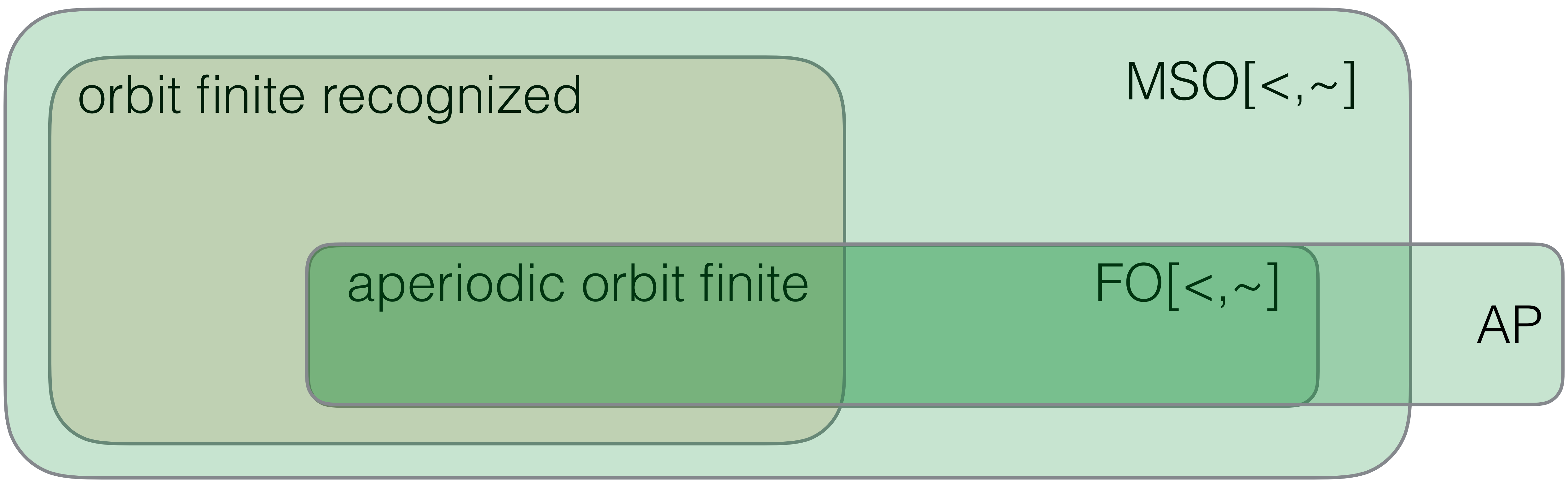

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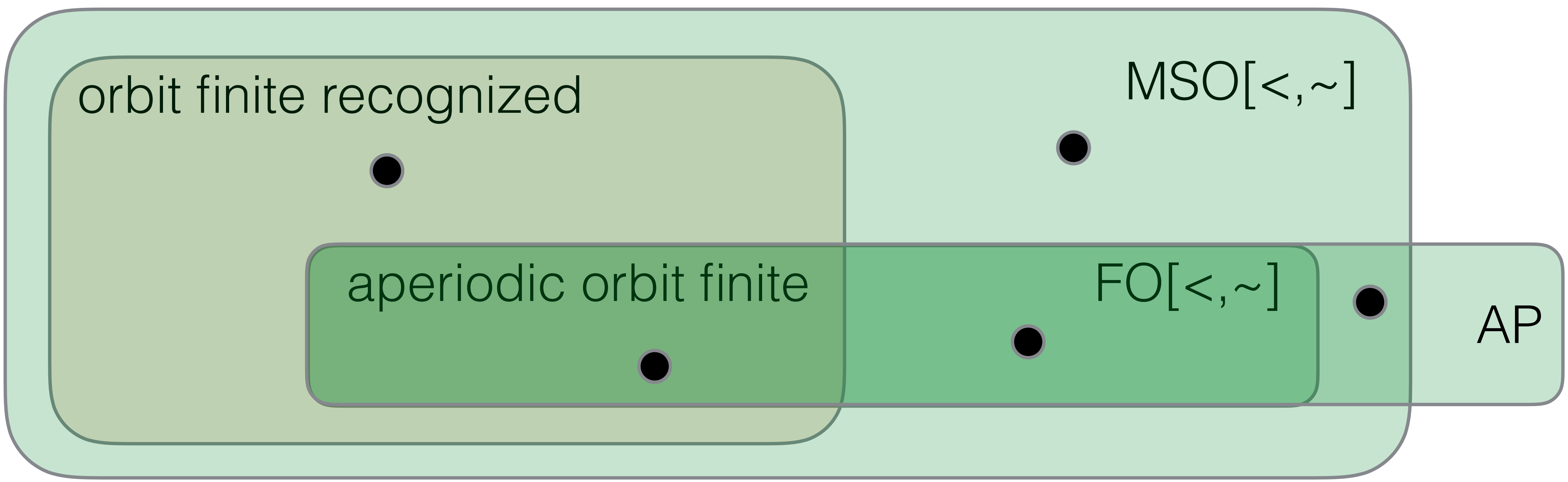

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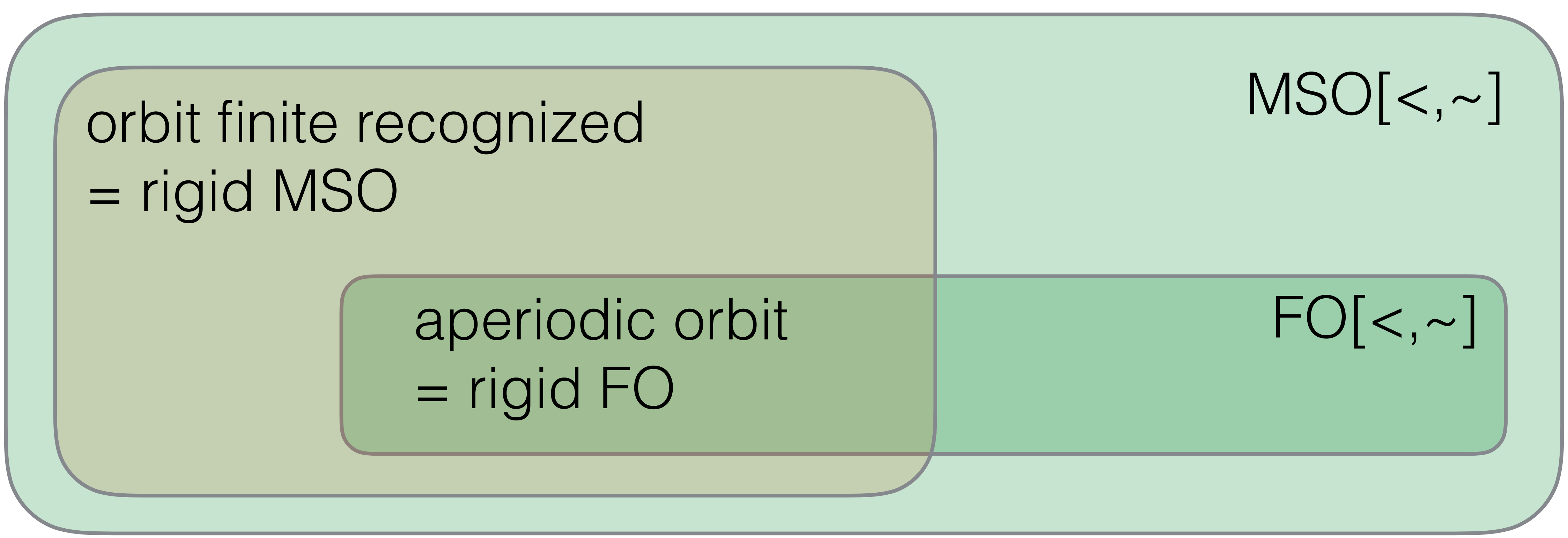

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Remark: Rigidity can be first-order enforced as follows

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- Every odd position letters carry the same value
- There are two consecutive letters that carry the same value
- non-example: some value appears twice

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How to translate MSO into monoids?

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$a \longmapsto \{\ h(\mathbf{t},a),\ h(\mathbf{f},a)\ \}$

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Problem: The powerset does not preserve orbit finite sets.

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nominal

nominal orbit finite

A equivariant map $h : A{\times}B \to C$ is **projectable** (w.r.t A) if whenever $a,a' \in A$,

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Rigidly guarded test operation produces projectable monoids.

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➔ The nominal monoid to rigid logic translation cannot be as ‘flat’ as usual for word languages.

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Uses the same technique, and requires to understand Green's relations in a nominal orbit finite monoid (even more for MSO!).

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A consequence is that every orbit finite monoid is the quotient of one that has a permutation free presentation.

Thank you!

Thanks to the organizers of the workshop.

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Thanks to Simons Institute.