Probabilistic Reasoning by First-Order Model Counting

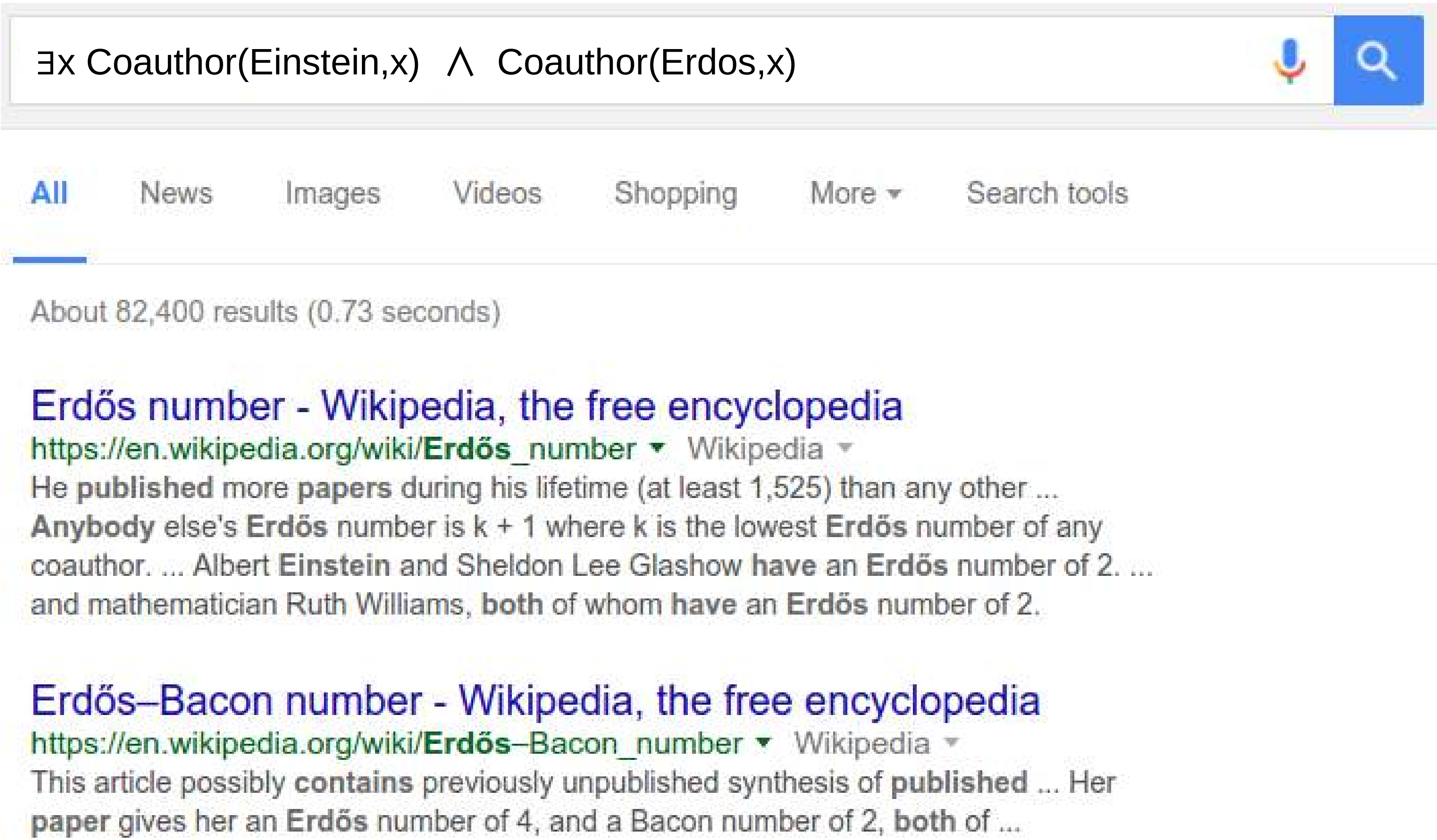
Guy Van den Broeck

UCLA

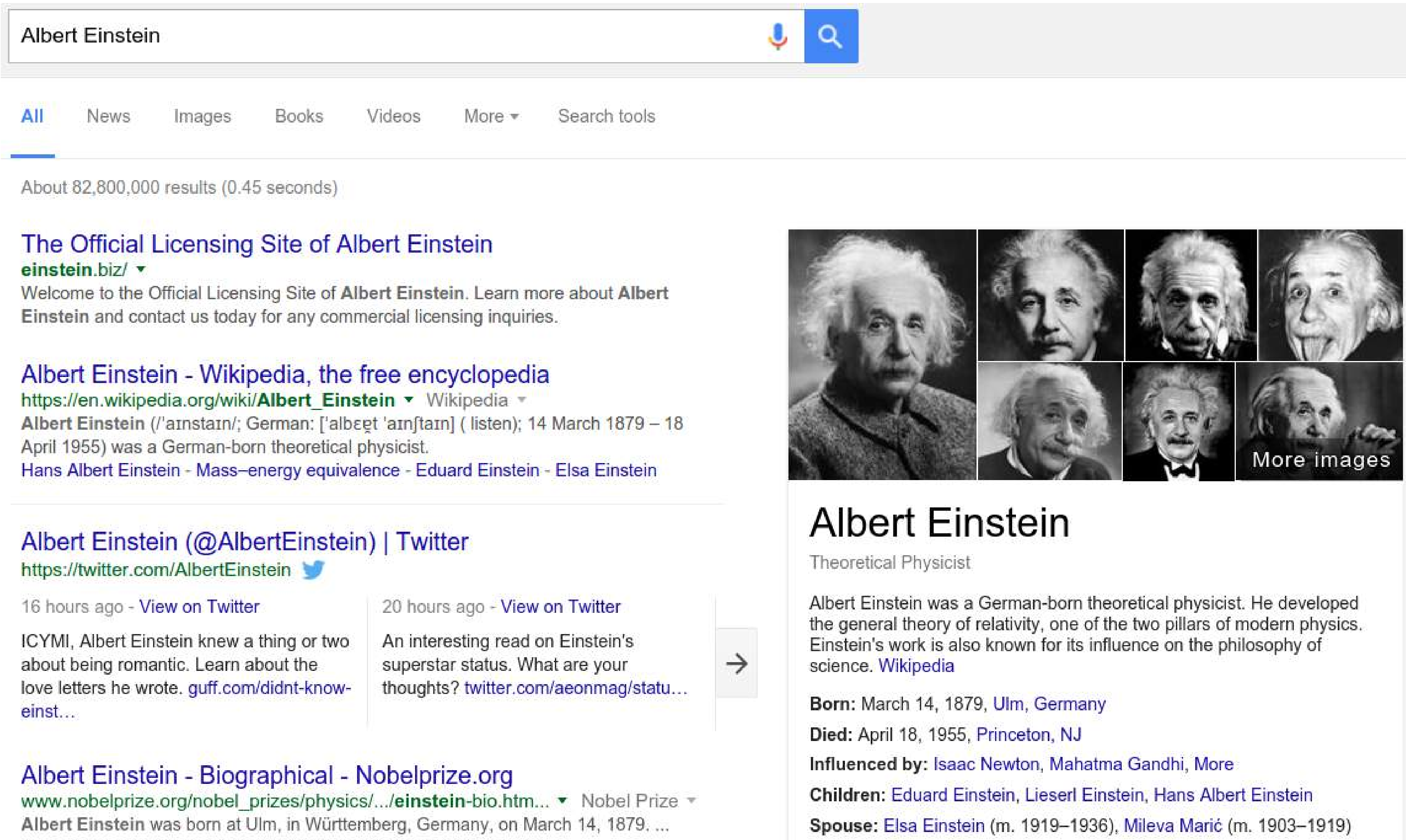
Simons Workshop on Uncertainty in Computation
Oct 5, 2016

What is this good for?

What we'd like to do…



Einstein is in the Knowledge Graph



Erdős is in the Knowledge Graph

Paul Erdos

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Paul Erdős was a Hungarian Jewish mathematician. He was one of the most prolific mathematicians of the 20th century. He was known both for his social ...
Fan Chung - Ronald Graham - Béla Bollobás - Category:Paul Erdős

The Man Who Loved Only Numbers - The New York Times
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Paul Erdős was one of those very special geniuses, the kind who comes along only once in a very long while yet he chose, quite consciously I am sure, to share ...

Paul Erdos | Hungarian mathematician | Britannica.com
www.britannica.com/biography/**Paul-Erdos** ▾ Encyclopaedia Britannica ▾
Paul Erdős, (born March 26, 1913, Budapest, Hungary—died September 20, 1996, Warsaw, Poland), Hungarian "freelance" mathematician (known for his work ...

Paul Erdős - University of St Andrews
www-groups.dcs.st-and.ac.uk/~history/Biographies/**Erdos**.html ▾
Paul Erdős came from a Jewish family (the original family name being Engländer) although neither of his parents observed the Jewish religion. Paul's father ...

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by J Hill - 2004 - Related articles

Paul Erdős

Mathematician

Paul Erdős was a Hungarian Jewish mathematician. He was one of the most prolific mathematicians of the 20th century. He was known both for his social practice of mathematics and for his eccentric lifestyle.
Wikipedia

Born: March 26, 1913, Budapest, Hungary

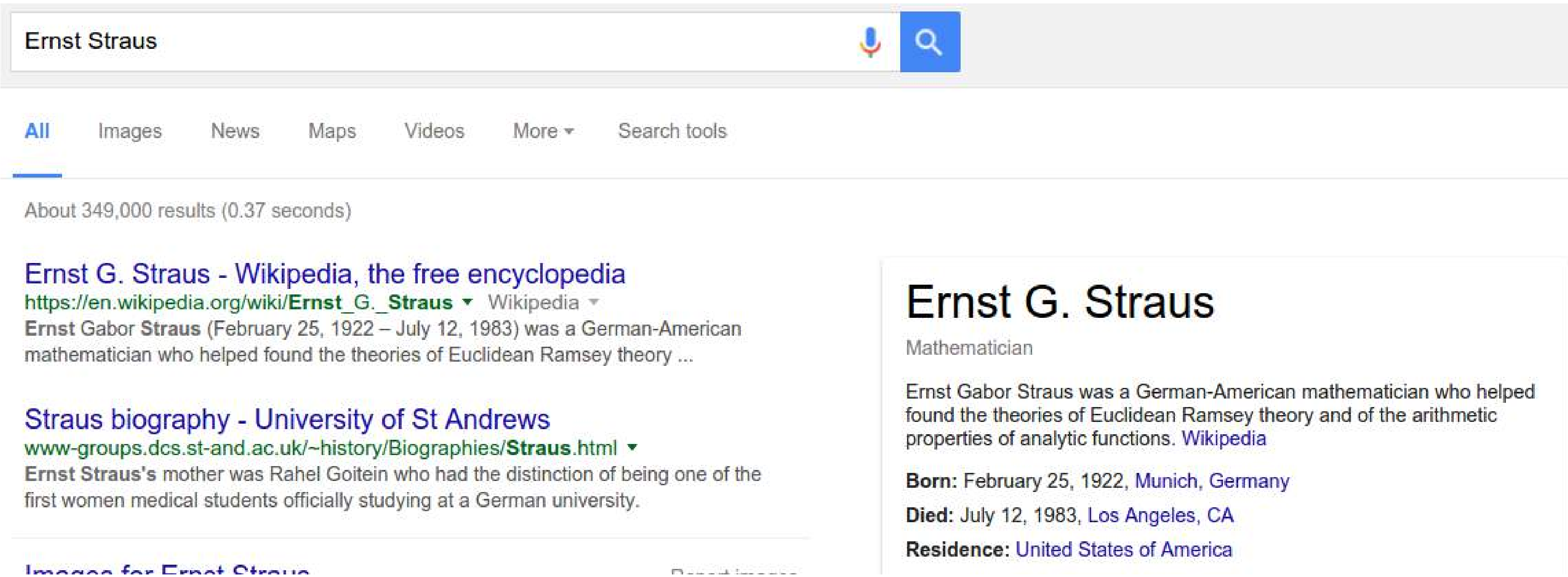
Died: September 20, 1996, Warsaw, Poland

Education: Eötvös Loránd University (1934)

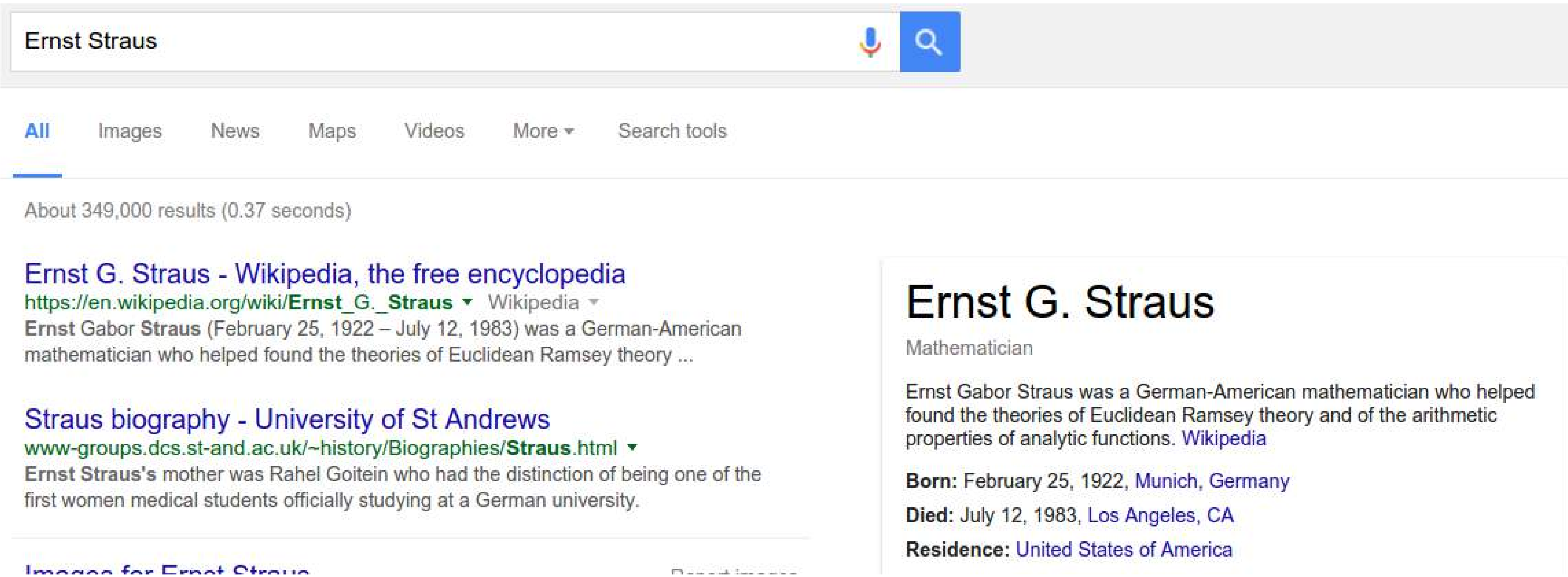
Books: Probabilistic Methods in Combinatorics, More

Notable students: Béla Bollobás, Alexander Soifer, George B. Purdy, Joseph Kruskal

This guy is in the Knowledge Graph



This guy is in the Knowledge Graph



… and he published with both Einstein and Erdos!

Desired Query Answer

Ernst Straus

Barack Obama, …

Justin Bieber, …

Desired Query Answer

Ernst Straus

Barack Obama, …

Justin Bieber, …

1. Fuse uncertain information from web

⇒ **Embrace probability!**

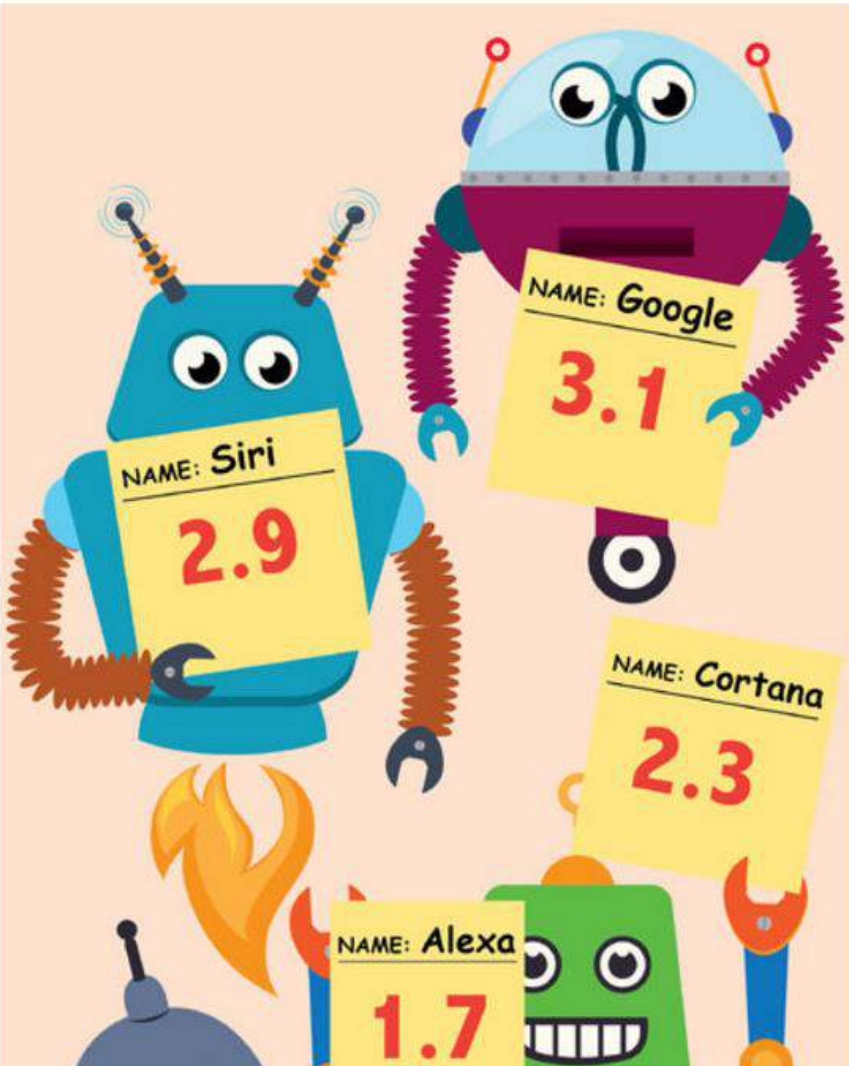
2. Cannot come from labeled data

⇒ **Embrace query eval!**

Siri, Alexa and Other Virtual Assistants Put to the Test

Tech Fix

By BRIAN X. CHEN JAN. 27, 2016



WHEN I asked Alexa earlier this week who was playing in the Super Bowl, she responded, somewhat monotonously, "Super Bowl 49's winner is New England Patriots."

"Come on, that's last year's Super Bowl," I said. "Even I can do better than that."

At the time, I was actually alone in my living room. I was talking to the virtual companion inside Amazon's wireless speaker, Echo, which was released last June. Known as Alexa, she has gained raves from Silicon Valley's tech-obsessed digerati and has become one of the newest members of the virtual assistants club.

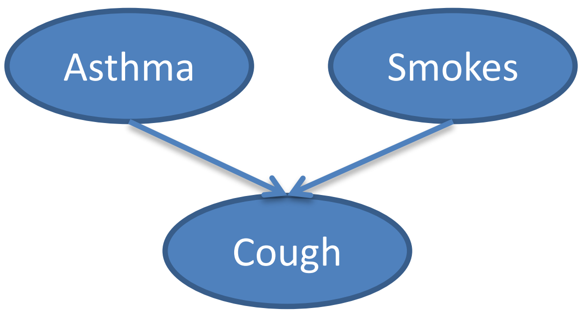
All the so-called Frightful Five tech

[Chen'16]
(NYTimes)

Statistical Relational Learning

Augment graphical model with relations between entities (rows).

<u>Intuition</u>



+ Friends have similar smoking habits

+ Asthma can be hereditary

<u>Markov Logic</u>

2.1 $\text{Asthma}(x) \Rightarrow \text{Cough}(x)$

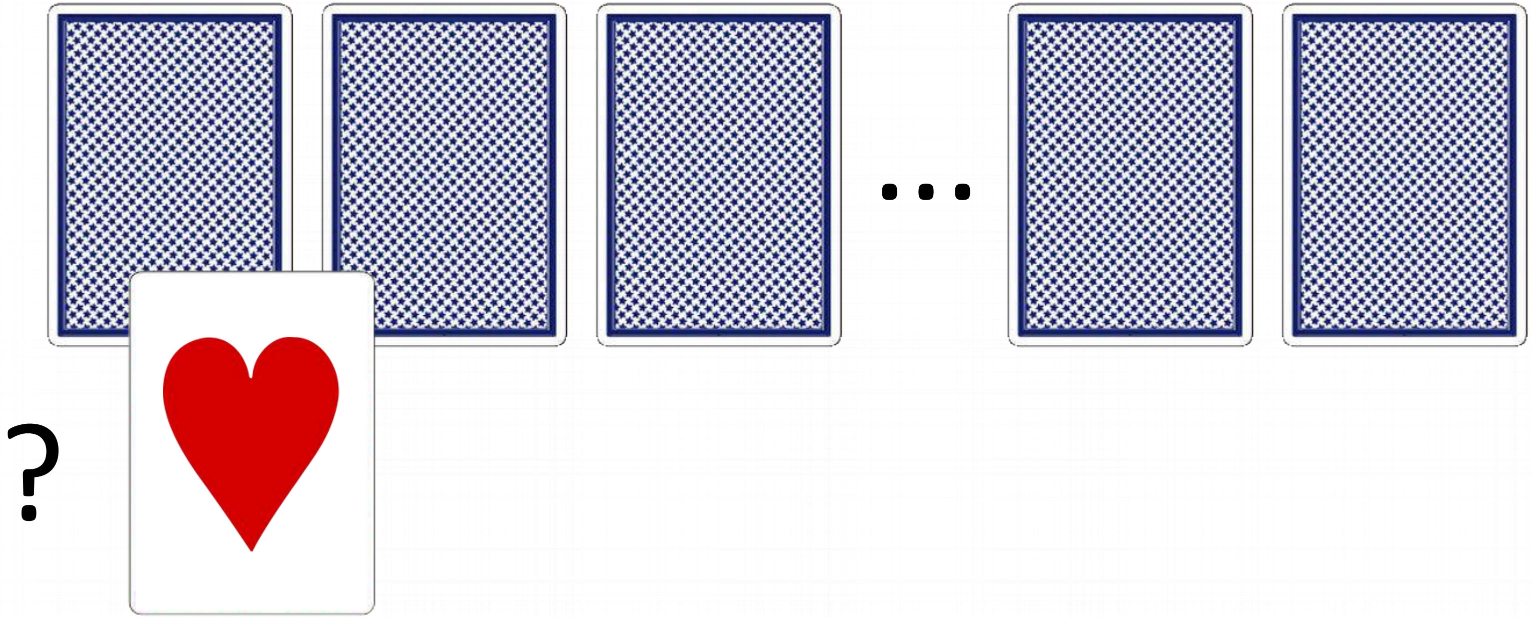
3.5 $\text{Smokes}(x) \Rightarrow \text{Cough}(x)$

1.9 $\text{Smokes}(x) \land \text{Friends}(x,y) \Rightarrow \text{Smokes}(y)$

1.5 $\text{Asthma}(x) \land \text{Family}(x,y) \Rightarrow \text{Asthma}(y)$

Why do we need first-order probabilistic ***reasoning****?*

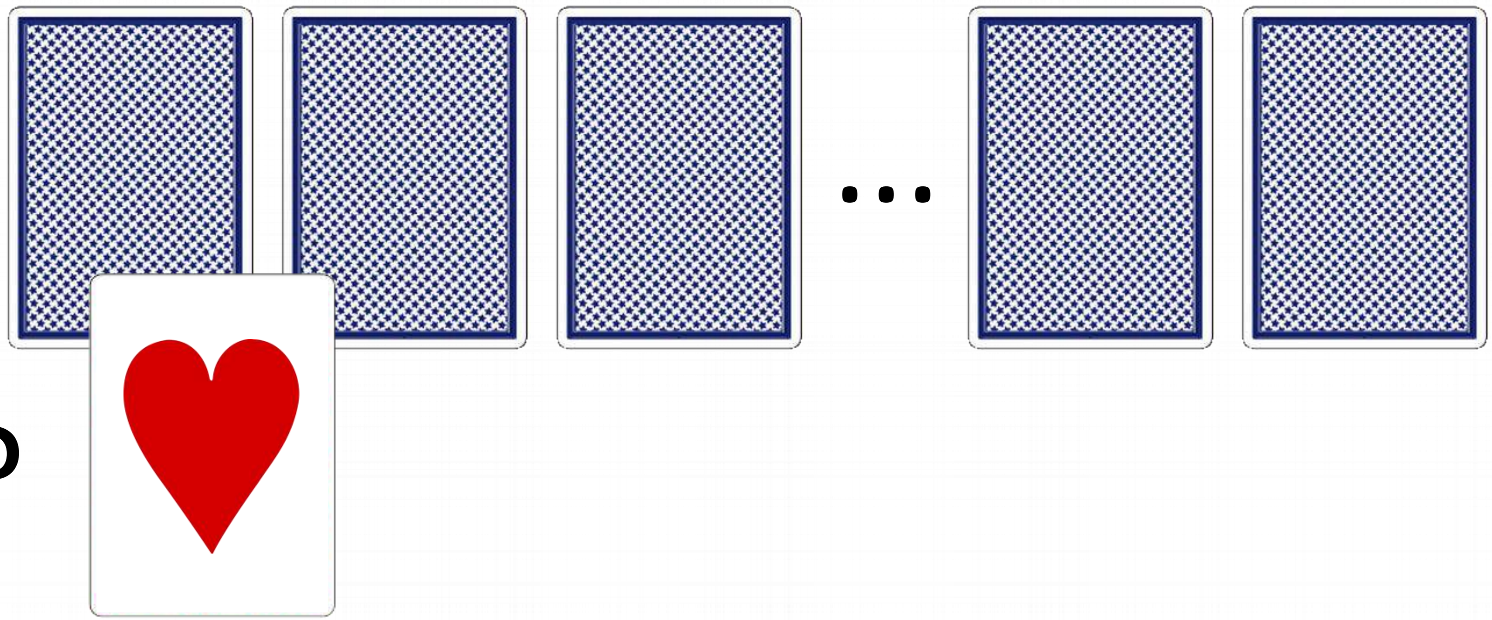
A Simple Reasoning Problem



Probability that Card1 is Hearts?

[Van den Broeck; AAAI-KRR'15]

A Simple Reasoning Problem

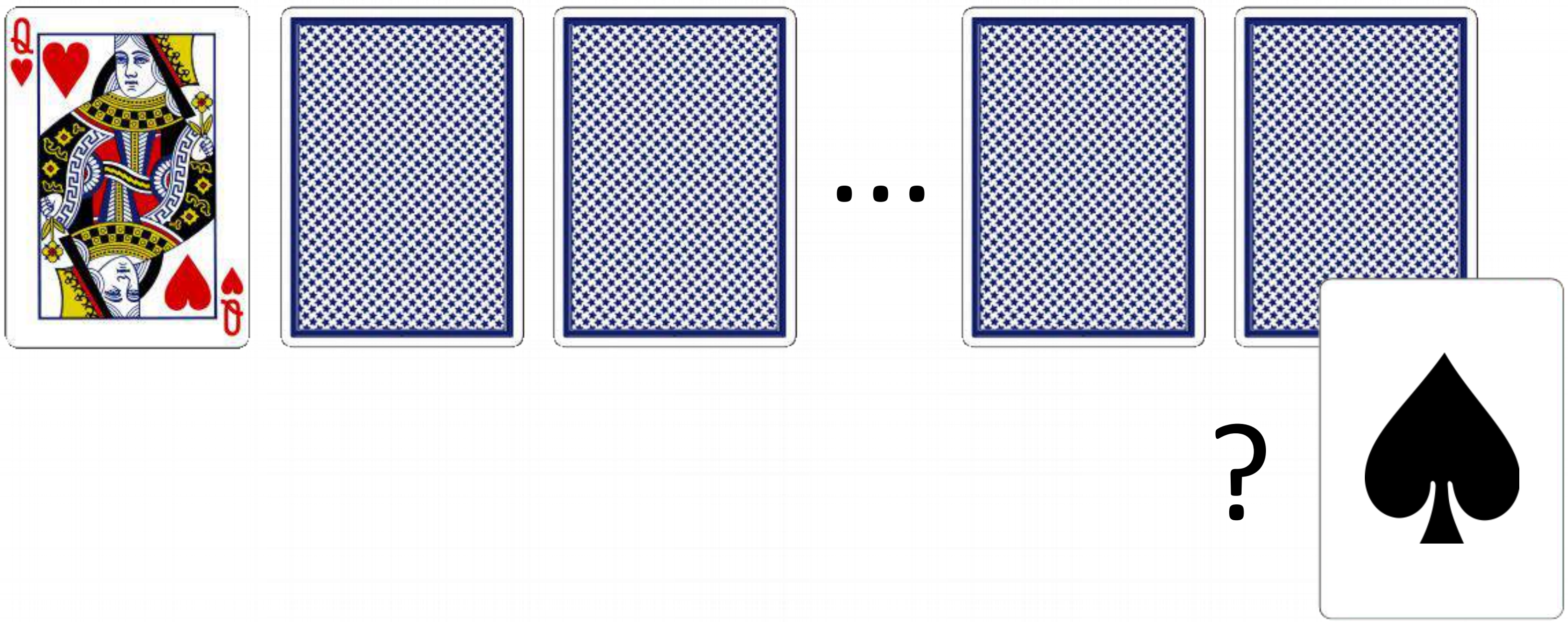


?

Probability that Card1 is Hearts? 1/4

[Van den Broeck; AAAI-KRR'15]

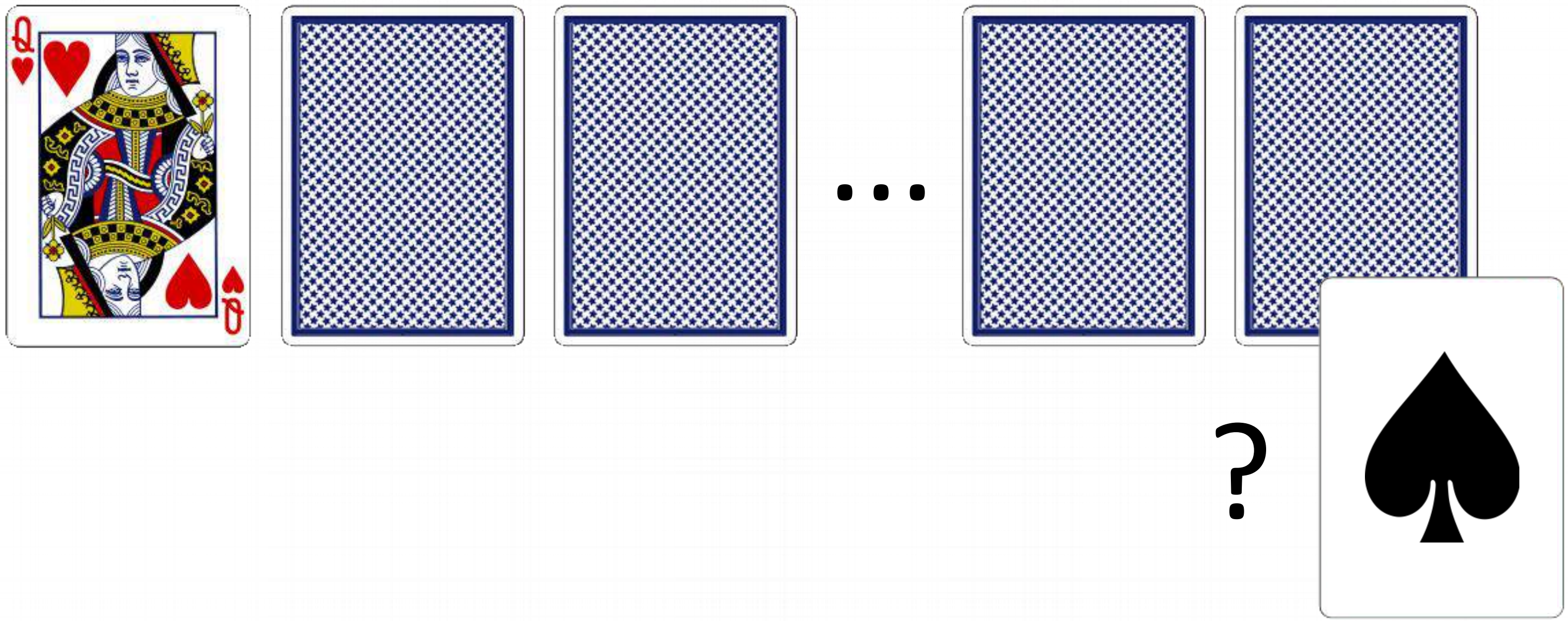
A Simple Reasoning Problem



Probability that Card52 is Spades given that Card1 is QH?

[Van den Broeck; AAAI-KRR'15]

A Simple Reasoning Problem



Probability that Card52 is Spades given that Card1 is QH?

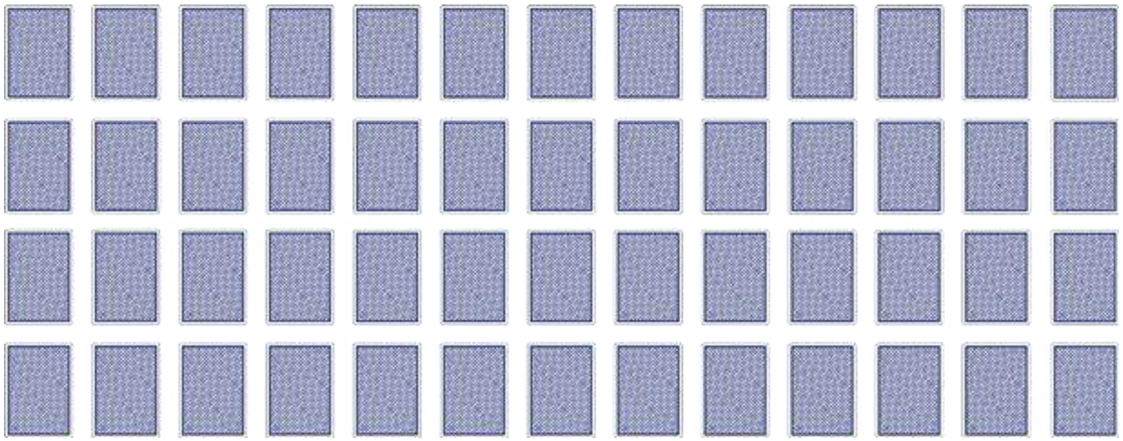
13/51

[Van den Broeck; AAAI-KRR'15]

Automated Reasoning

Let us automate this:

1. Probabilistic graphical model (e.g., factor graph)



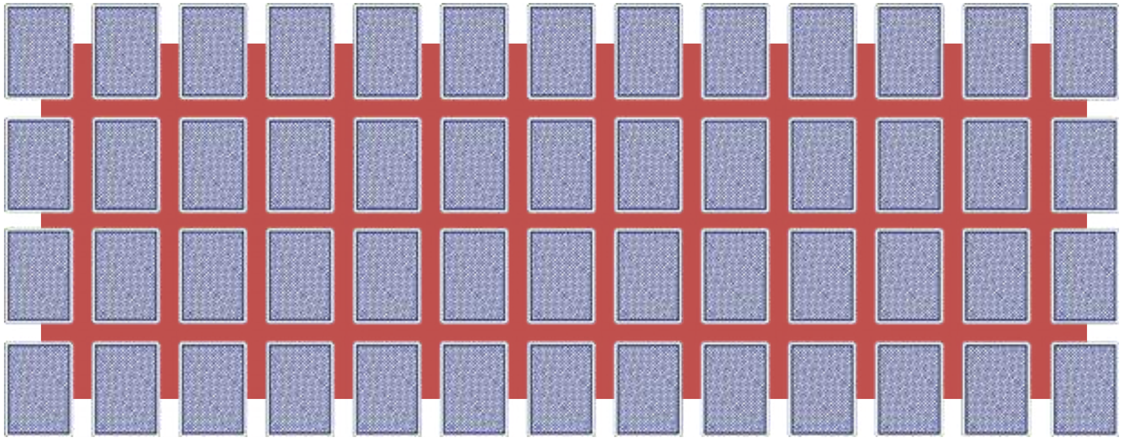
2. Probabilistic inference algorithm (e.g., variable elimination or junction tree)

[Van den Broeck; AAAI-KRR'15]

Automated Reasoning

Let us automate this:

1. Probabilistic graphical model (e.g., factor graph) is fully connected!



(artist's impression)

2. Probabilistic inference algorithm (e.g., variable elimination or junction tree) builds a table with $\mathbf{52^{52}}$ rows

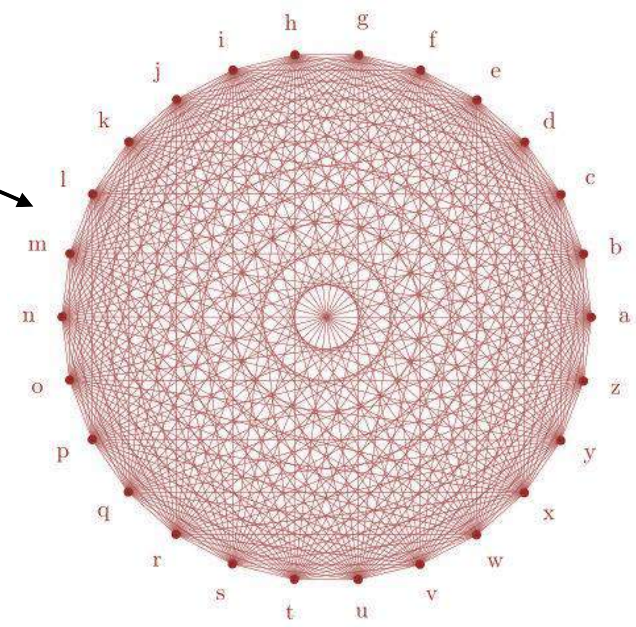
[Van den Broeck; AAAI-KRR’15]

Statistical Relational Learning

- Statistical relational model (e.g., MLN)

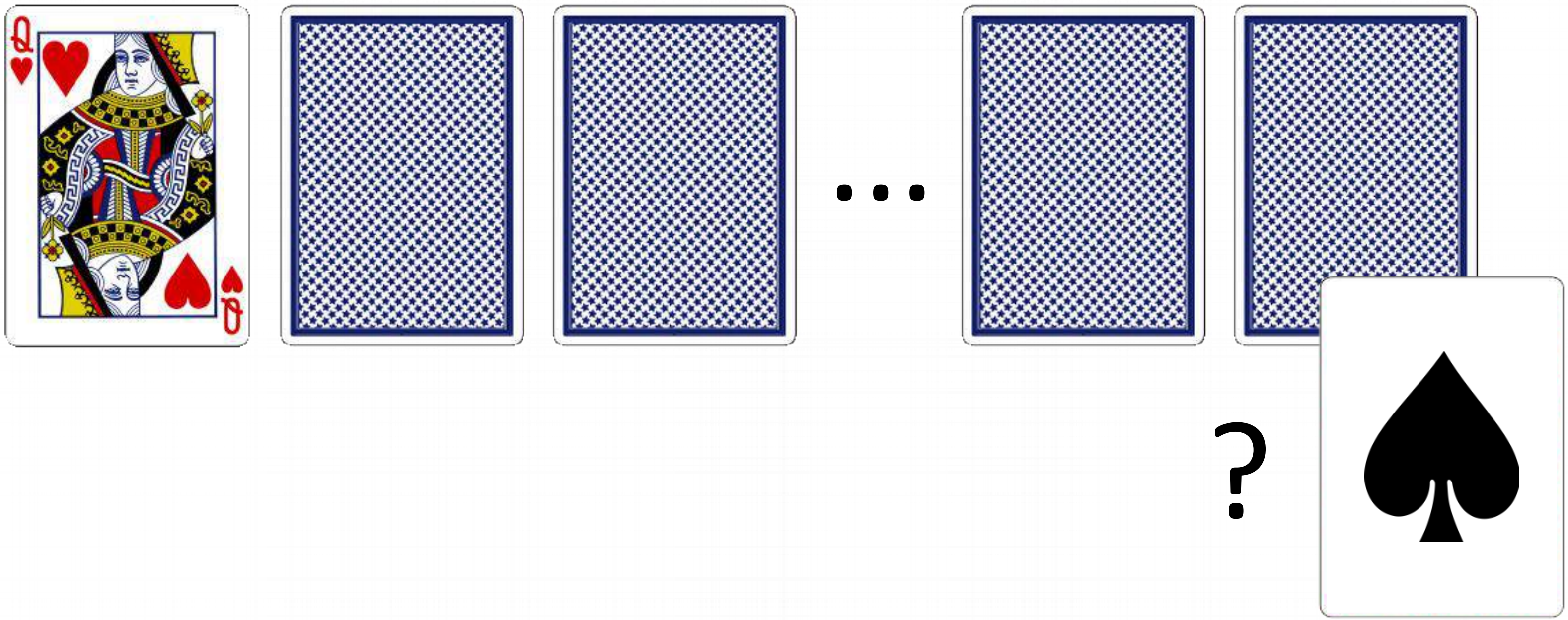
  3.14 FacultyPage(x) ∧ Linked(x,y) ⇒ CoursePage(y)

- As a probabilistic graphical model:
  - 26 pages; 728 variables; 676 factors
  - 1000 pages; 1,002,000 variables; 1,000,000 factors



- Highly intractable?

What's Going On Here?



Probability that Card52 is Spades given that Card1 is QH?

[Van den Broeck; AAAI-KRR'15]

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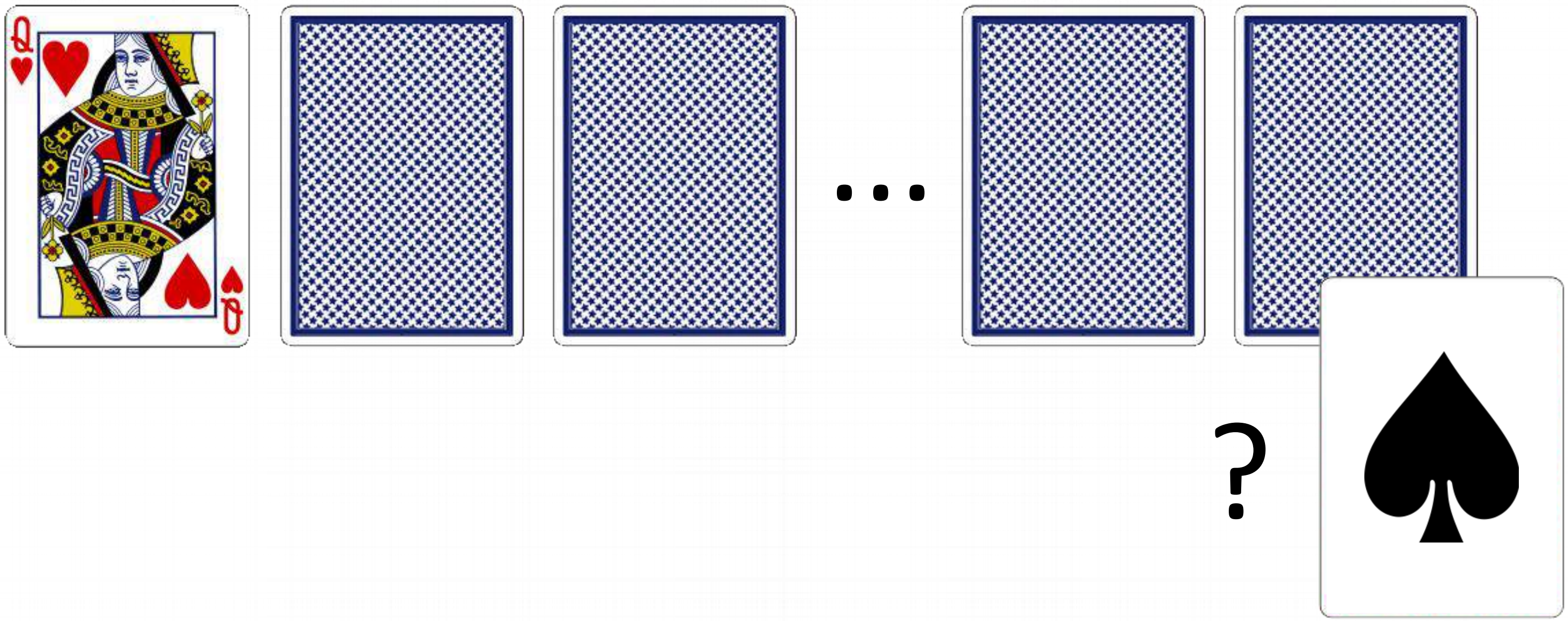
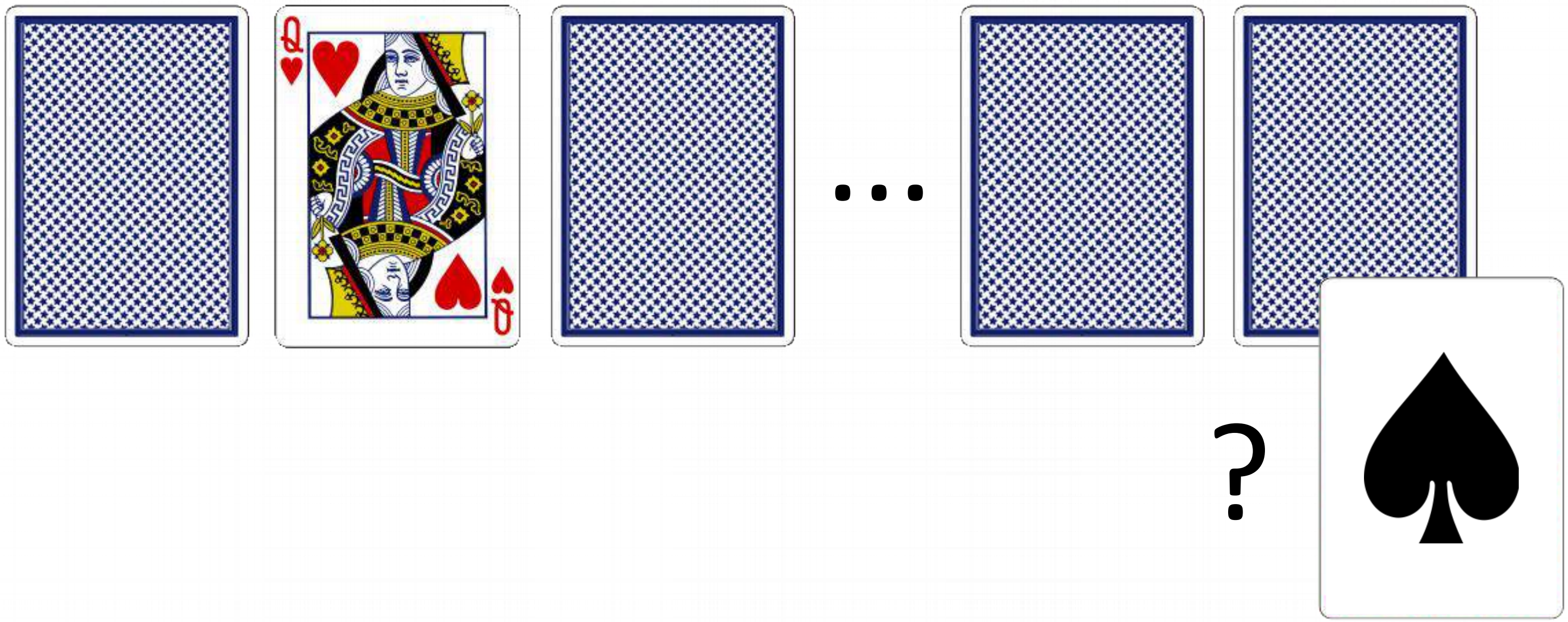


Probability that Card52 is Spades given that Card1 is QH?

13/51

[Van den Broeck; AAAI-KRR'15]

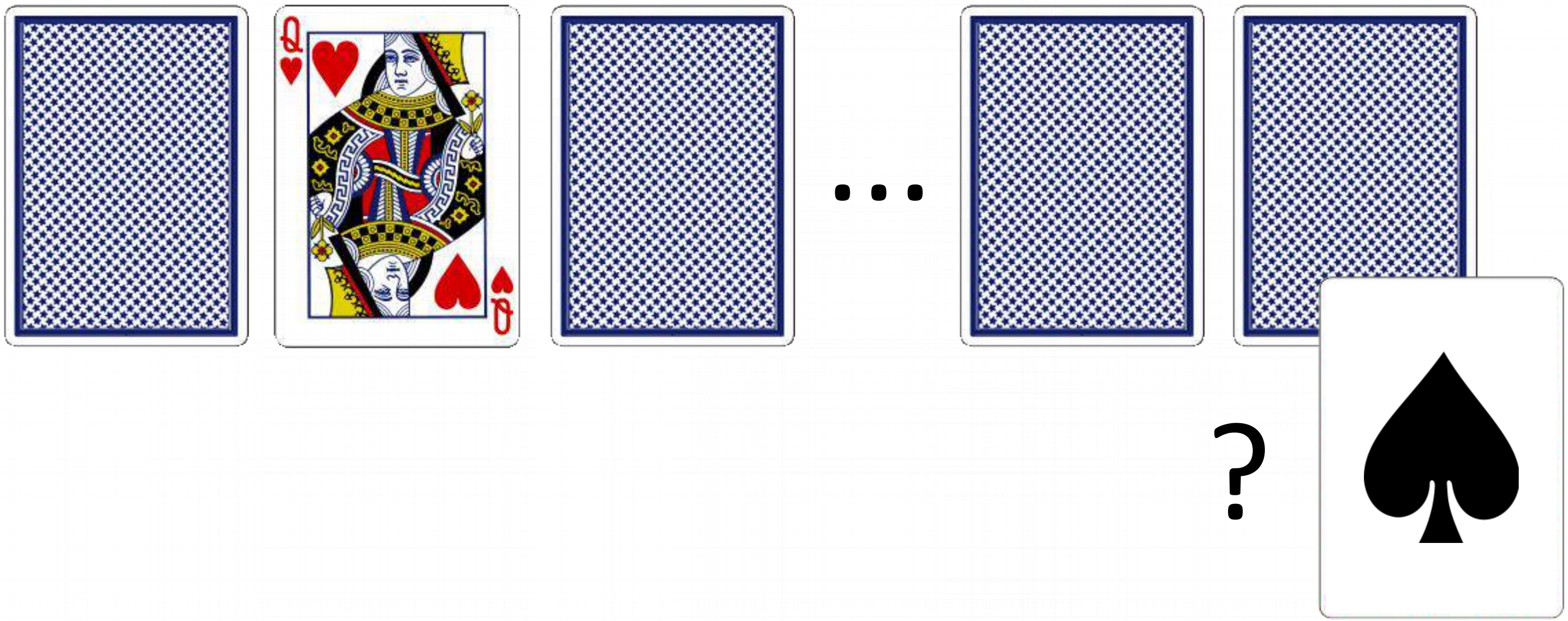
What's Going On Here?



Probability that Card52 is Spades given that Card2 is QH?

[Van den Broeck; AAAI-KRR’15]

What's Going On Here?

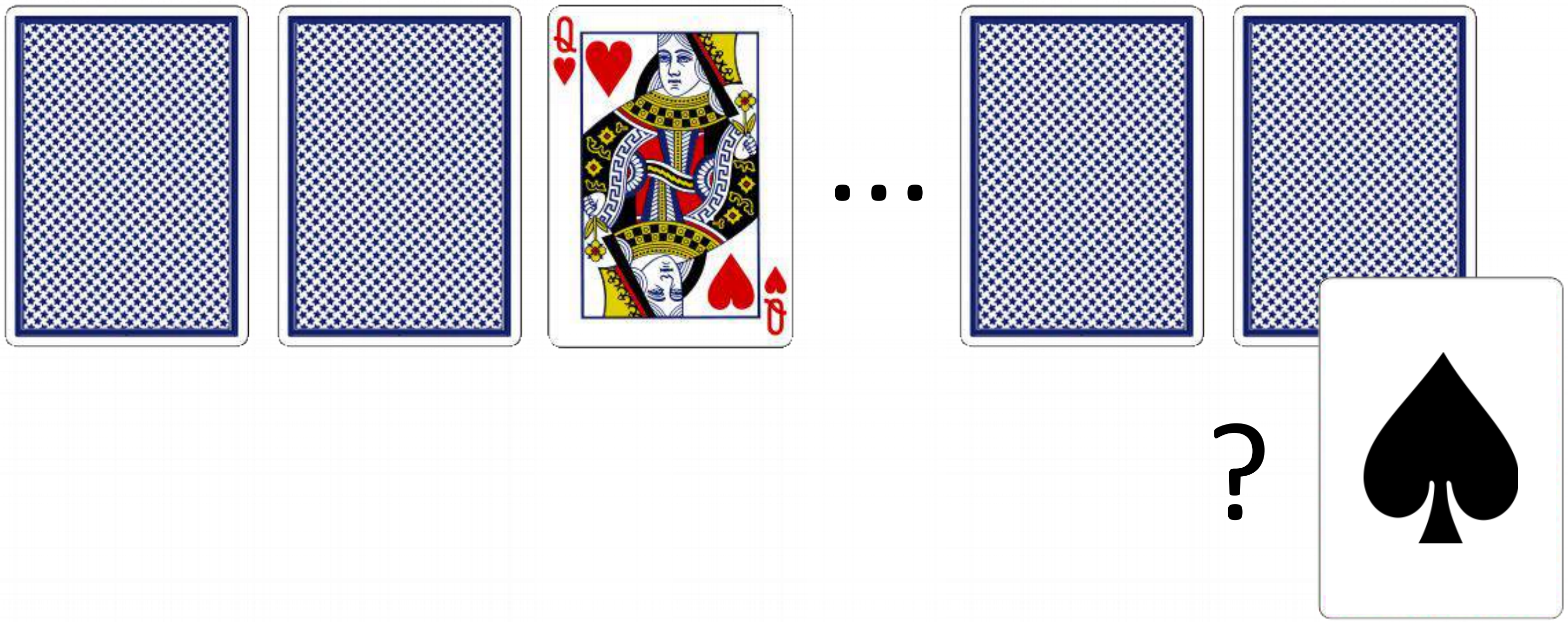


Probability that Card52 is Spades given that Card2 is QH?

13/51

[Van den Broeck; AAAI-KRR'15]

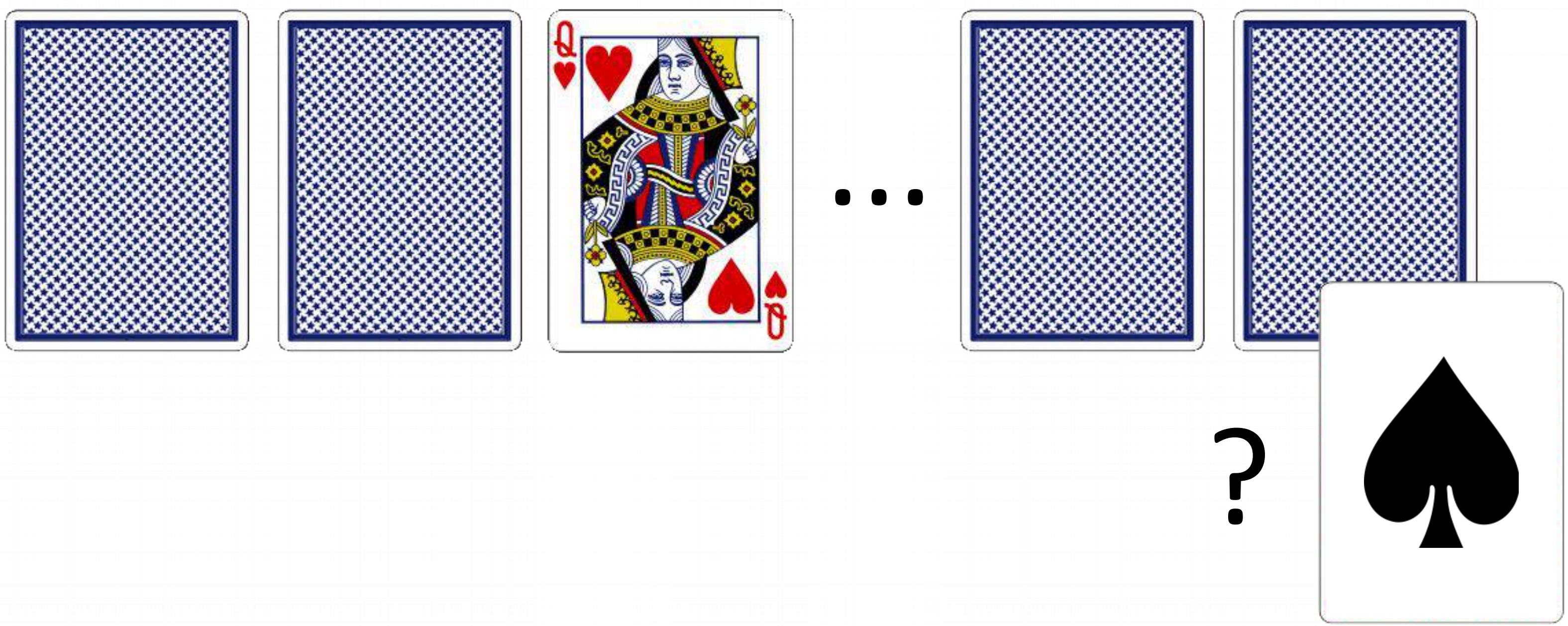
What's Going On Here?



Probability that Card52 is Spades given that Card3 is QH?

[Van den Broeck; AAAI-KRR'15]

What's Going On Here?

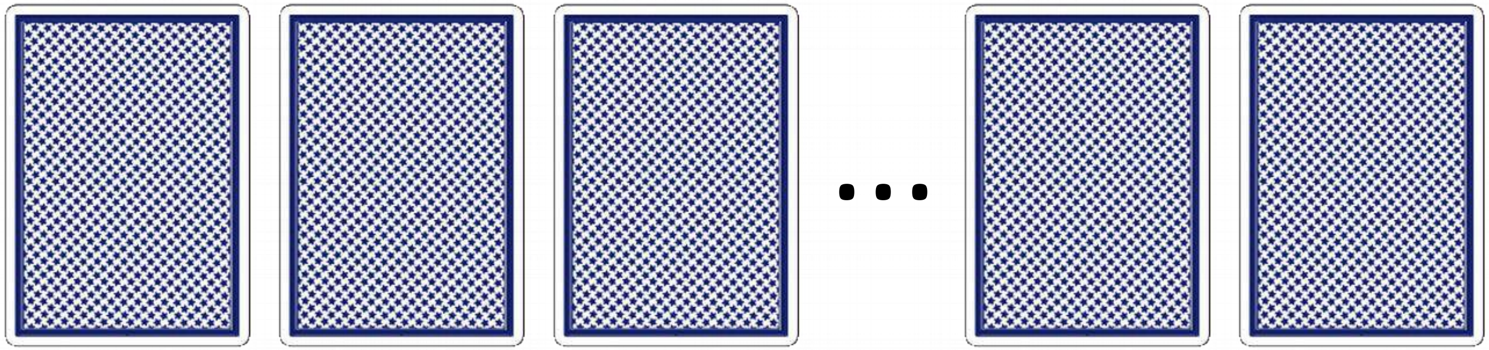


Probability that Card52 is Spades given that Card3 is QH?

13/51

[Van den Broeck; AAAI-KRR'15]

Tractable Reasoning

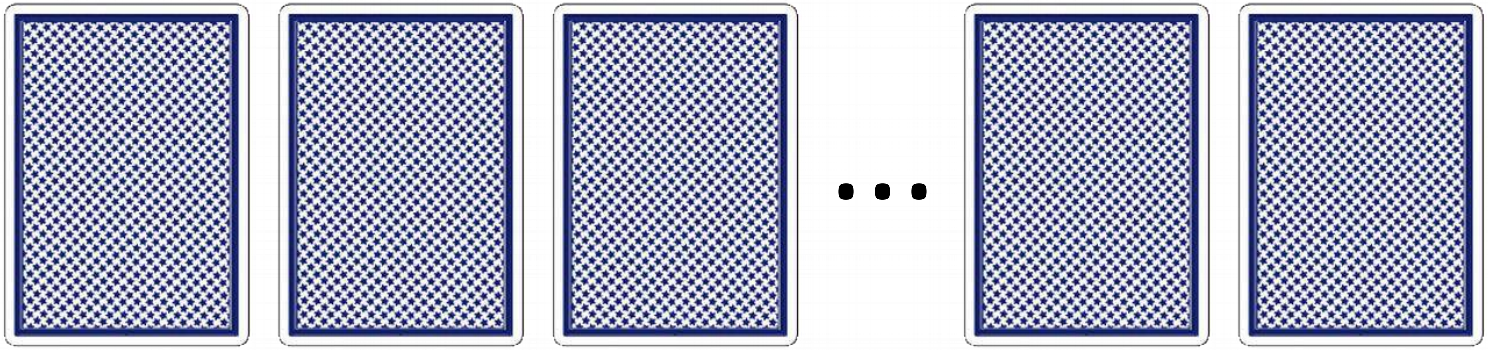


What's going on here?

Which property makes reasoning tractable?

[Niepert and Van den Broeck, AAAI' 14], [Van den Broeck, AAAI-KRR'15]

Tractable Reasoning



What's going on here?

Which property makes reasoning tractable?

- High-level (first-order) reasoning
- Relational structure
- Symmetry
- Exchangeability

⇒ **Lifted Inference**

[Niepert and Van den Broeck, AAAI' 14], [Van den Broeck, AAAI-KRR'15]

How does lifted inference ***work****?*

First-Order Model Counting

Model = solution to first-order logic formula Δ

$\Delta = \forall d\ (\text{Rain}(d) \Rightarrow \text{Cloudy}(d))$

Days = {Monday}

First-Order Model Counting

Model = solution to first-order logic formula Δ

Δ = ∀d (Rain(d) ⇒ Cloudy(d))

Days = {Monday}

Rain(M)	Cloudy(M)	Model?
T	T	Yes
T	F	No
F	T	Yes
F	F	Yes

+

FOMC = 3

First-Order Model Counting

Model = solution to first-order logic formula Δ

Δ = ∀d (Rain(d)
⇒ Cloudy(d))

Days = {Monday
Tuesday}

First-Order Model Counting

Model = solution to first-order logic formula Δ

Δ = ∀d (Rain(d) ⇒ Cloudy(d))

Days = {Monday **Tuesday**}

Rain(M)	Cloudy(M)	Rain(T)	Cloudy(T)	Model?
T	T	T	T	Yes
T	F	T	T	No
F	T	T	T	Yes
F	F	T	T	Yes
T	T	T	F	No
T	F	T	F	No
F	T	T	F	No
F	F	T	F	No
T	T	F	T	Yes
T	F	F	T	No
F	T	F	T	Yes
F	F	F	T	Yes
T	T	F	F	Yes
T	F	F	F	No
F	T	F	F	Yes
F	F	F	F	Yes

\+ ———

#SAT = 9

Weighted First-Order Model Counting

Model = solution to first-order logic formula Δ

Δ = ∀d (Rain(d) ⇒ Cloudy(d))

Days = {Monday Tuesday}

Rain

d	w(R(d))	w(¬R(d))
M	1	2
T	4	1

Cloudy

d	w(C(d))	w(¬C(d))
M	3	5
T	6	2

Rain(M)	Cloudy(M)	Rain(T)	Cloudy(T)	Model?
T	T	T	T	Yes
T	F	T	T	No
F	T	T	T	Yes
F	F	T	T	Yes
T	T	T	F	No
T	F	T	F	No
F	T	T	F	No
F	F	T	F	No
T	T	F	T	Yes
T	F	F	T	No
F	T	F	T	Yes
F	F	F	T	Yes
T	T	F	F	Yes
T	F	F	F	No
F	T	F	F	Yes
F	F	F	F	Yes

+ ———

#SAT = 9

[VdB'11, Gogate'11]

Weighted First-Order Model Counting

Model = solution to first-order logic formula Δ

Δ = ∀d (Rain(d) ⇒ Cloudy(d))

Days = {Monday **Tuesday**}

Rain

d	w(R(d))	w(¬R(d))
M	1	2
T	4	1

Cloudy

d	w(C(d))	w(¬C(d))
M	3	5
T	6	2

Rain(M)	**Cloudy(M)**	**Rain(T)**	**Cloudy(T)**	**Model?**	**Weight**
T	T	T	T	Yes	1 * 3 * 4 * 6 = 72
T	F	T	T	No	0
F	T	T	T	Yes	2 * 3 * 4 * 6 = 144
F	F	T	T	Yes	2 * 5 * 4 * 6 = 240
T	T	T	F	No	0
T	F	T	F	No	0
F	T	T	F	No	0
F	F	T	F	No	0
T	T	F	T	Yes	1 * 3 * 1 * 6 = 18
T	F	F	T	No	0
F	T	F	T	Yes	2 * 3 * 1 * 6 = 36
F	F	F	T	Yes	2 * 5 * 1 * 6 = 60
T	T	F	F	Yes	1 * 3 * 1 * 2 = 6
T	F	F	F	No	0
F	T	F	F	Yes	2 * 3 * 1 * 2 = 12
F	F	F	F	Yes	2 * 5 * 1 * 2 = 20

+ ———

#SAT = 9

[VdB'11, Gogate'11]

Weighted First-Order Model Counting

Model = solution to first-order logic formula Δ

Δ = ∀d (Rain(d) ⇒ Cloudy(d))

Days = {Monday **Tuesday**}

Rain

d	w(R(d))	w(¬R(d))
M	1	2
T	4	1

Cloudy

d	w(C(d))	w(¬C(d))
M	3	5
T	6	2

Rain(M)	Cloudy(M)	Rain(T)	Cloudy(T)	Model?	Weight
T	T	T	T	Yes	1 * 3 * 4 * 6 = 72
T	F	T	T	No	0
F	T	T	T	Yes	2 * 3 * 4 * 6 = 144
F	F	T	T	Yes	2 * 5 * 4 * 6 = 240
T	T	T	F	No	0
T	F	T	F	No	0
F	T	T	F	No	0
F	F	T	F	No	0
T	T	F	T	Yes	1 * 3 * 1 * 6 = 18
T	F	F	T	No	0
F	T	F	T	Yes	2 * 3 * 1 * 6 = 36
F	F	F	T	Yes	2 * 5 * 1 * 6 = 60
T	T	F	F	Yes	1 * 3 * 1 * 2 = 6
T	F	F	F	No	0
F	T	F	F	Yes	2 * 3 * 1 * 2 = 12
F	F	F	F	Yes	2 * 5 * 1 * 2 = 20

+ ———— + ————

#SAT = 9 **WFOMC = 608**

[VdB'11, Gogate'11]

Probabilistic Database Rules

$$P(\neg Q) = 1 - P(Q)$$

Negation

$$P(Q1 \wedge Q2) = P(Q1)\, P(Q2)$$
$$P(Q1 \vee Q2) = 1 - (1 - P(Q1))\,(1 - P(Q2))$$

Decomposable ∧,∨

$$P(\exists z\, Q) = 1 - \Pi_{A\, \in Domain}\, (1 - P(Q[A/z]))$$
$$P(\forall z\, Q) = \Pi_{A\, \in Domain}\, P(Q[A/z])$$

Decomposable ∃,∀

$$P(Q1 \wedge Q2) = P(Q1) + P(Q2) - P(Q1 \vee Q2)$$
$$P(Q1 \vee Q2) = P(Q1) + P(Q2) - P(Q1 \wedge Q2)$$

Inclusion/ exclusion

[Suciu'11]

Asymmetric WFOMC Rules

Normalization constants Z
(easy to compute)

$\mathrm{WMC}(\neg\Delta) = Z - \mathrm{WMC}(\Delta)$

Negation

$\mathrm{WMC}(\Delta_1 \wedge \Delta_2) = \mathrm{WMC}(\Delta_1) * \mathrm{WMC}(\Delta_2)$
$\mathrm{WMC}(\Delta_1 \vee \Delta_2) = Z - (Z_1 - \mathrm{WMC}(\Delta_1)) * (Z_2 - \mathrm{WMC}(\Delta_2))$

Decomposable ∧,∨

$\mathrm{WMC}(\exists z\, \Delta) = Z - \prod_{C \in Domain} (Z_C - \mathrm{WMC}(\Delta[C/z]))$
$\mathrm{WMC}(\forall z\, \Delta) = \prod_{C \in Domain} \mathrm{WMC}(\Delta[C/z])$

Decomposable ∃,∀

$\mathrm{WMC}(\Delta_1 \wedge \Delta_2) = \mathrm{WMC}(\Delta_1) + \mathrm{WMC}(\Delta_2) - \mathrm{WMC}(\Delta_1 \vee \Delta_2)$
$\mathrm{WMC}(\Delta_1 \vee \Delta_2) = \mathrm{WMC}(\Delta_1) + \mathrm{WMC}(\Delta_2) - \mathrm{WMC}(\Delta_1 \wedge \Delta_2)$

Inclusion/
exclusion

Example MLN to WFOMC

3.75   Smoker(x) $\wedge$ Friend(x,y) $\Rightarrow$ Smoker(y)

1. **Formula Δ**

2. **Weight function w(.)**

Example MLN to WFOMC

3.75   Smoker(x) ∧ Friend(x,y) ⇒ Smoker(y)

1. **Formula Δ**

Δ = ∀x∀y (F(x,y) ⇔ [Smoker(x) ∧ Friend(x,y) ⇒ Smoker(y)])

2. **Weight function w(.)**

Example MLN to WFOMC

3.75 Smoker(x) ∧ Friend(x,y) ⇒ Smoker(y)

1. Formula Δ

Δ = ∀x∀y (F(x,y) ⇔ [Smoker(x) ∧ Friend(x,y) ⇒ Smoker(y)])

2. Weight function w(.)

F

x	y	w(F(x,y))	w(¬F(x,y))
A	A	exp(3.75)	1
A	B	exp(3.75)	1
A	C	exp(3.75)	1
B	A	exp(3.75)	1
	. . .	. . .	

Example MLN to WFOMC

$3.75 \quad Smoker(x) \wedge Friend(x,y) \Rightarrow Smoker(y)$

1. **Formula Δ**

$\Delta = \forall x \forall y\ (F(x,y) \Leftrightarrow [Smoker(x) \wedge Friend(x,y) \Rightarrow Smoker(y)])$

2. **Weight function w(.)**

F

x	y	w(F(x,y))	w(¬F(x,y))
A	A	exp(3.75)	1
A	B	exp(3.75)	1
A	C	exp(3.75)	1
B	A	exp(3.75)	1
	. . .	. . .	

$Z = WFOMC(\Delta)$

Symmetric WFOMC Rules

- Simplification to decomposable ∃,∀ rules:

If $\Delta[C_1/x]$, $\Delta[C_2/x]$, … are independent

$$WMC(\exists z\ \Delta) = Z - (Z_{C_1} - WMC(\Delta[C_1/z])^{|Domain|}$$

$$WMC(\forall z\ \Delta) = WMC(\Delta[C_1/z])^{|Domain|}$$

[VdB'11]

Symmetric WFOMC Rules

- Simplification to decomposable ∃,∀ rules:

  If $\Delta[C_1/x]$, $\Delta[C_2/x]$, … are independent

  $WMC(\exists z\ \Delta) = Z - (Z_{C_1} - WMC(\Delta[C_1/z]))^{|Domain|}$

  $WMC(\forall z\ \Delta) = WMC(\Delta[C_1/z])^{|Domain|}$

- A powerful new inference rule: *atom counting*
  Only possible with symmetric weights
  Intuition: **Remove unary relations**

The workhorse of Symmetric WFOMC

[VdB'11]

FOMC Inference

$\Delta = \forall x,y, (Smokes(x) \land Friends(x,y) \Rightarrow Smokes(y))$

Domain = {n people}

FOMC Inference

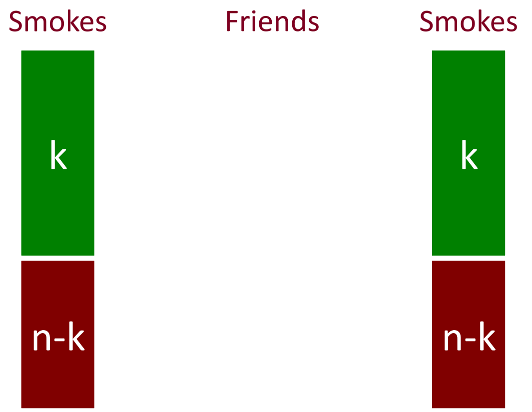
Δ = ∀x,y, (Smokes(x) ∧ Friends(x,y) ⇒ Smokes(y))

Domain = {n people}

- If we know precisely who smokes, and there are *k* smokers?

Database:

Smokes(Alice) = 1
Smokes(Bob) = 0
Smokes(Charlie) = 0
Smokes(Dave) = 1
Smokes(Eve) = 0
...



FOMC Inference

Δ = ∀x,y, (Smokes(x) ∧ Friends(x,y) ⇒ Smokes(y))

Domain = {n people}

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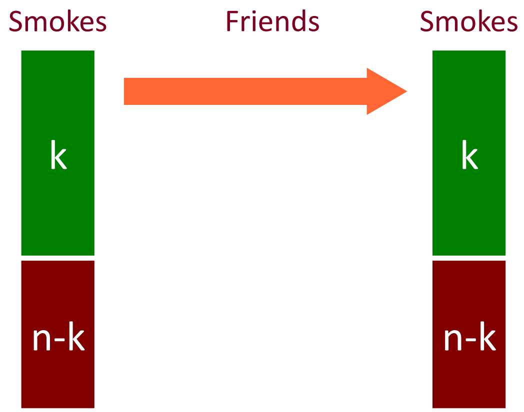

FOMC Inference

$\Delta = \forall x,y, (Smokes(x) \wedge Friends(x,y) \Rightarrow Smokes(y))$

Domain = {n people}

- If we know precisely who smokes, and there are *k* smokers?

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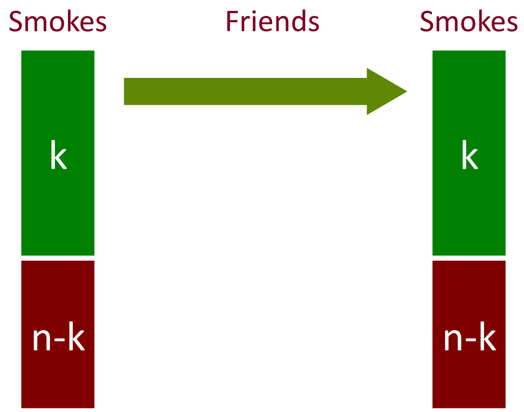

FOMC Inference

Δ = ∀x,y, (Smokes(x) ∧ Friends(x,y) ⇒ Smokes(y))

Domain = {n people}

- If we know precisely who smokes, and there are *k* smokers?

Database:

Smokes(Alice) = 1
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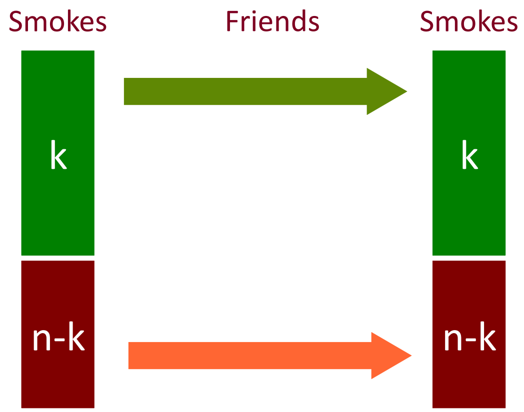

FOMC Inference

Δ = ∀x,y, (Smokes(x) ∧ Friends(x,y) ⇒ Smokes(y))

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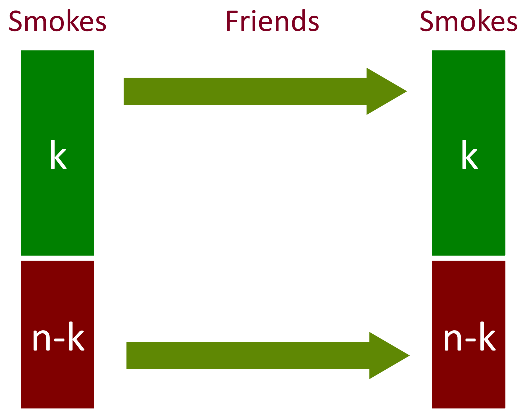

FOMC Inference

$\Delta = \forall x,y, (Smokes(x) \land Friends(x,y) \Rightarrow Smokes(y))$

Domain = {n people}

- If we know precisely who smokes, and there are *k* smokers?

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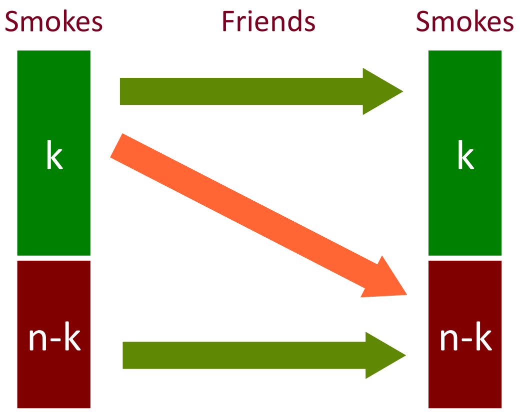

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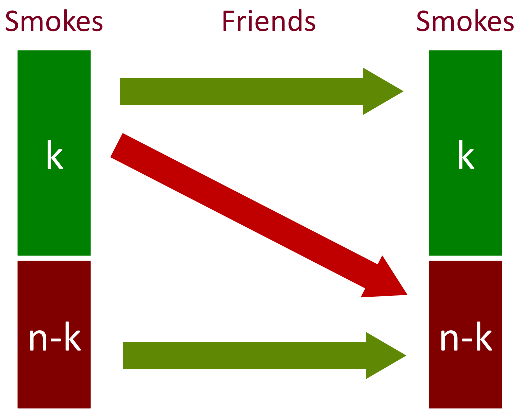

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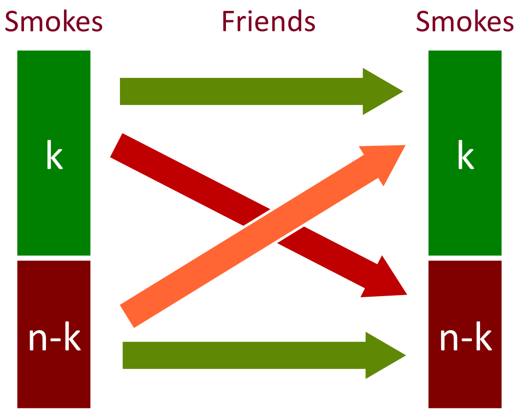

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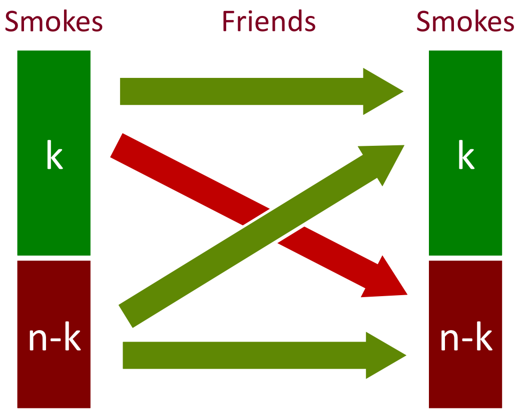

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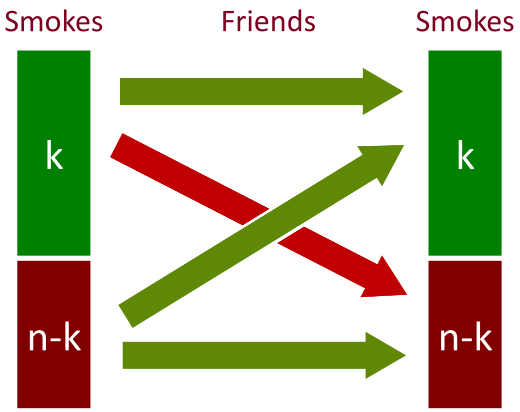
→ $2^{n^2 - k(n-k)}$ models



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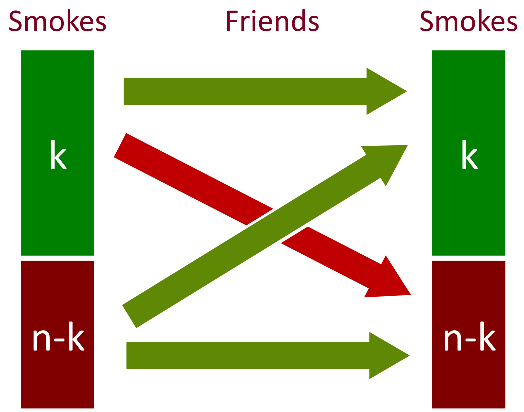

- If we know that there are *k* smokers?

FOMC Inference

Δ = ∀x,y, (Smokes(x) ∧ Friends(x,y) ⇒ Smokes(y))

Domain = {n people}

- If we know precisely who smokes, and there are *k* smokers?

Database:

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Smokes(Bob) = 0
Smokes(Charlie) = 0
Smokes(Dave) = 1
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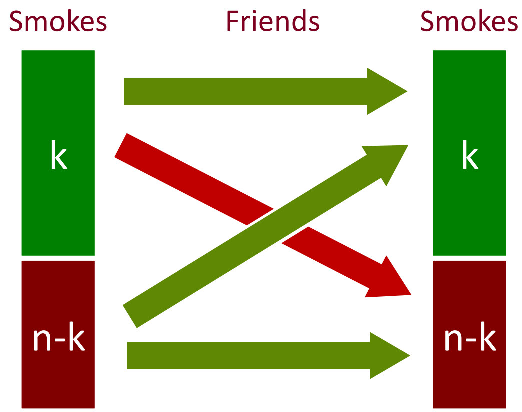
...

→ $2^{n^2-k(n-k)}$ models



- If we know that there are *k* smokers?

→ $\binom{n}{k} 2^{n^2-k(n-k)}$ models

FOMC Inference

Δ = ∀x,y, (Smokes(x) ∧ Friends(x,y) ⇒ Smokes(y))

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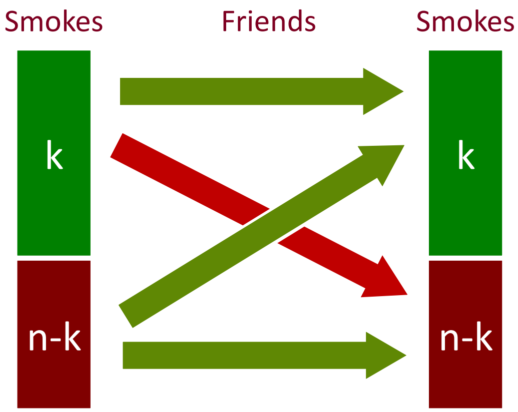
→ $2^{n^2-k(n-k)}$ models



- If we know that there are *k* smokers? → $\binom{n}{k} 2^{n^2-k(n-k)}$ models

- In total…

FOMC Inference

Δ = ∀x,y, (Smokes(x) ∧ Friends(x,y) ⇒ Smokes(y))

Domain = {n people}

- If we know precisely who smokes, and there are *k* smokers?

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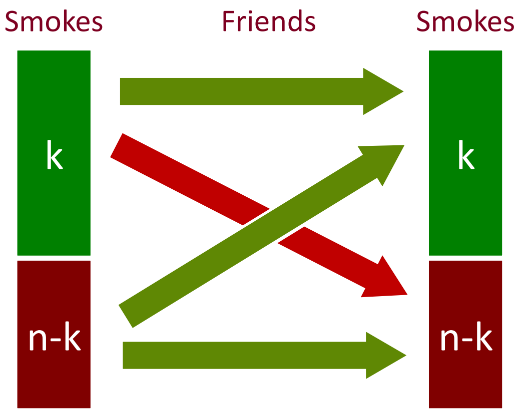
...

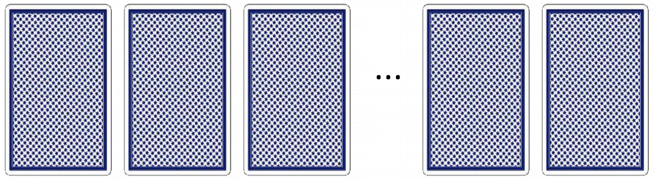
$\rightarrow 2^{n^2 - k(n-k)}$ models



- If we know that there are *k* smokers? $\rightarrow \binom{n}{k} 2^{n^2 - k(n-k)}$ models

- In total… $\rightarrow \sum_{k=0}^{n} \binom{n}{k} 2^{n^2 - k(n-k)}$ models

Playing Cards Revisited



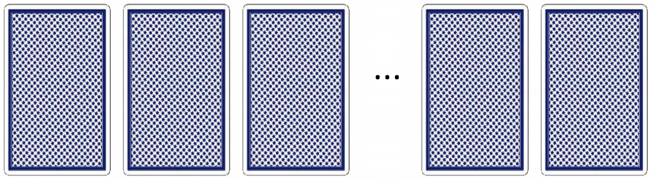
$$\forall p, \exists c, Card(p,c)$$

$$\forall c, \exists p, Card(p,c)$$

$$\forall p, \forall c, \forall c', Card(p,c) \wedge Card(p,c') \Rightarrow c = c'$$

[Van den Broeck.; AAAI-KR'15]

Playing Cards Revisited



∀p, ∃c, Card(p,c)

∀c, ∃p, Card(p,c)

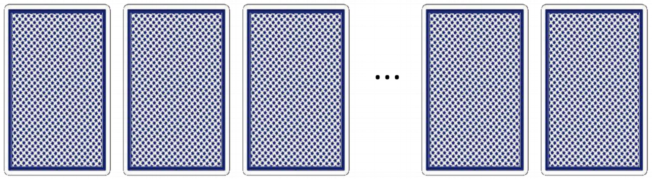
∀p, ∀c, ∀c', Card(p,c) ∧ Card(p,c') ⇒ c = c'

↓

$$\#SAT = \sum_{k=0}^{n} \binom{n}{k} \sum_{l=0}^{n} \binom{n}{l} (l+1)^k (-1)^{2n-k-l} = n!$$

[Van den Broeck.; AAAI-KR'15]

Playing Cards Revisited



∀p, ∃c, Card(p,c)
∀c, ∃p, Card(p,c)
∀p, ∀c, ∀c', Card(p,c) ∧ Card(p,c') ⇒ c = c'

↓

$$\#SAT = \sum_{k=0}^{n} \binom{n}{k} \sum_{l=0}^{n} \binom{n}{l} (l+1)^k (-1)^{2n-k-l} = n!$$

Computed in time polynomial in n

[Van den Broeck.; AAAI-KR'15]

When does it work?

Why not do propositional WMC?

Reduce to propositional model counting:

[VdB'15]

Why not do propositional WMC?

Reduce to propositional model counting:

$$
\begin{aligned}
\Delta = \;& \text{Card}(A\heartsuit,p_1) \vee \ldots \vee \text{Card}(2\clubsuit,p_1) \\
& \text{Card}(A\heartsuit,p_2) \vee \ldots \vee \text{Card}(2\clubsuit,p_2) \\
& \ldots \\
& \text{Card}(A\heartsuit,p_1) \vee \ldots \vee \text{Card}(A\heartsuit,p_{52}) \\
& \text{Card}(K\heartsuit,p_1) \vee \ldots \vee \text{Card}(K\heartsuit,p_{52}) \\
& \ldots \\
& \neg\text{Card}(A\heartsuit,p_1) \vee \neg\text{Card}(A\heartsuit,p_2) \\
& \neg\text{Card}(A\heartsuit,p_1) \vee \neg\text{Card}(A\heartsuit,p_3) \\
& \ldots
\end{aligned}
$$

[VdB'15]

Why not do propositional WMC?

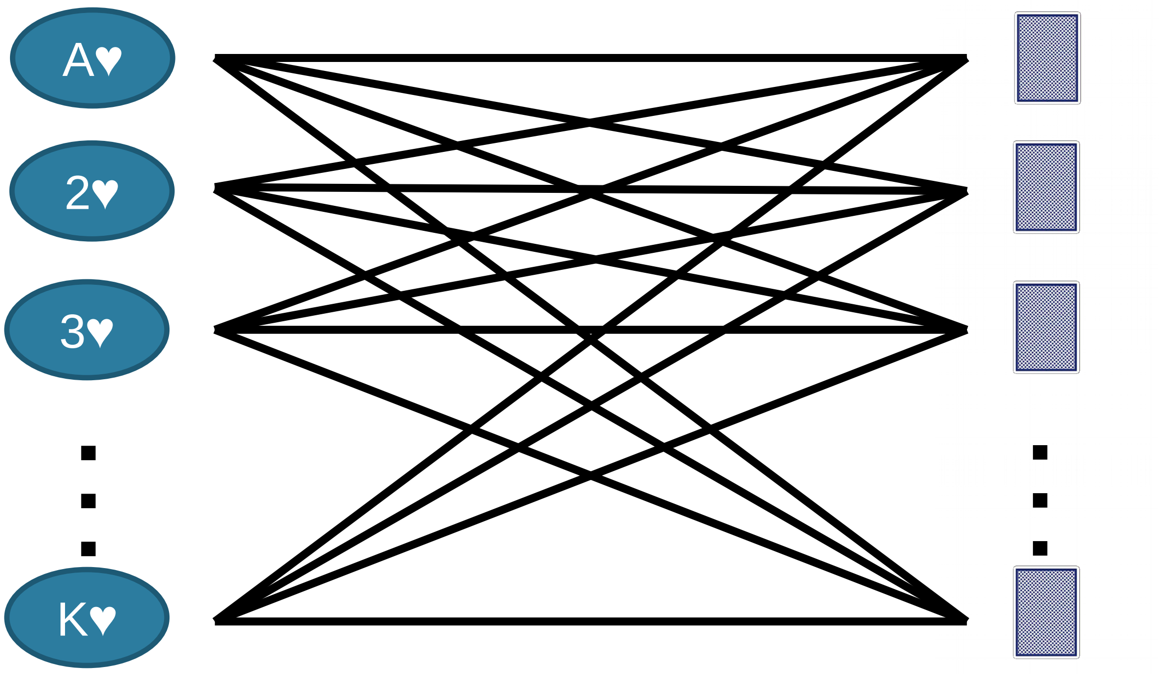
Reduce to propositional model counting:

Δ = Card(A♥,p_1) v … v Card(2♣,p_1)
Card(A♥,p_2) v … v Card(2♣,p_2)
…
Card(A♥,p_1) v … v Card(A♥,p_{52})
Card(K♥,p_1) v … v Card(K♥,p_{52})
…
¬Card(A♥,p_1) v ¬Card(A♥,p_2)
¬Card(A♥,p_1) v ¬Card(A♥,p_3)
…

What will happen?

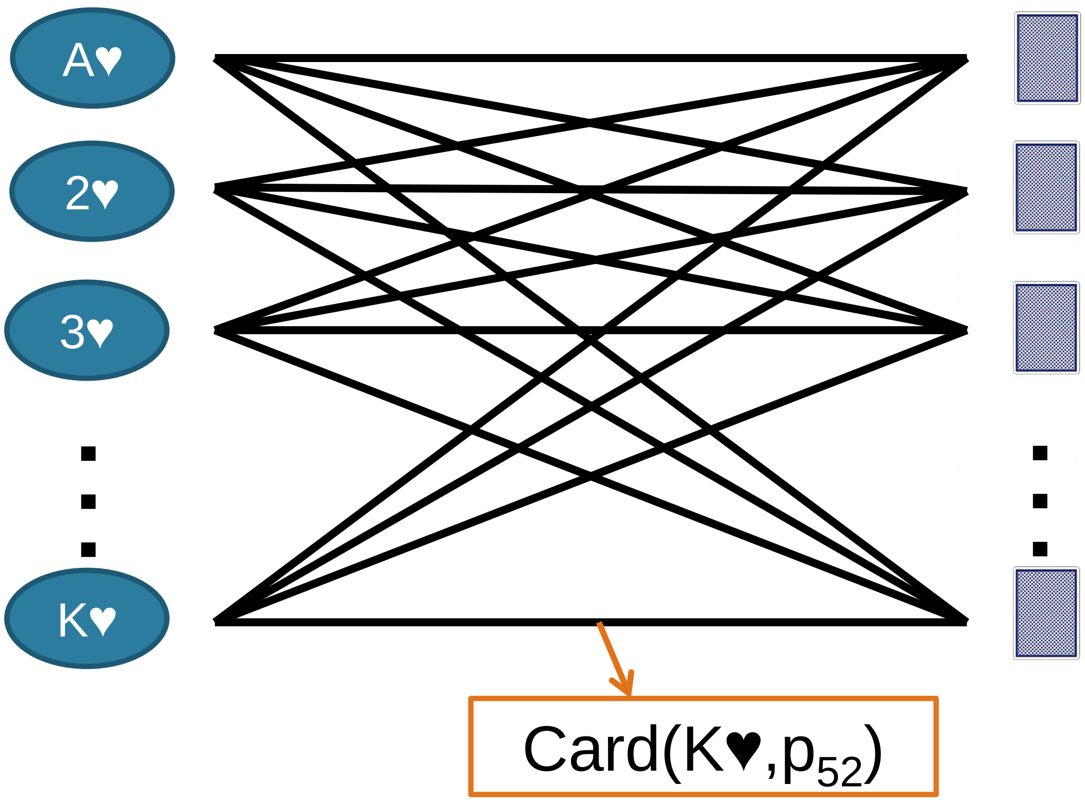
[VdB'15]

Deck of Cards Graphically



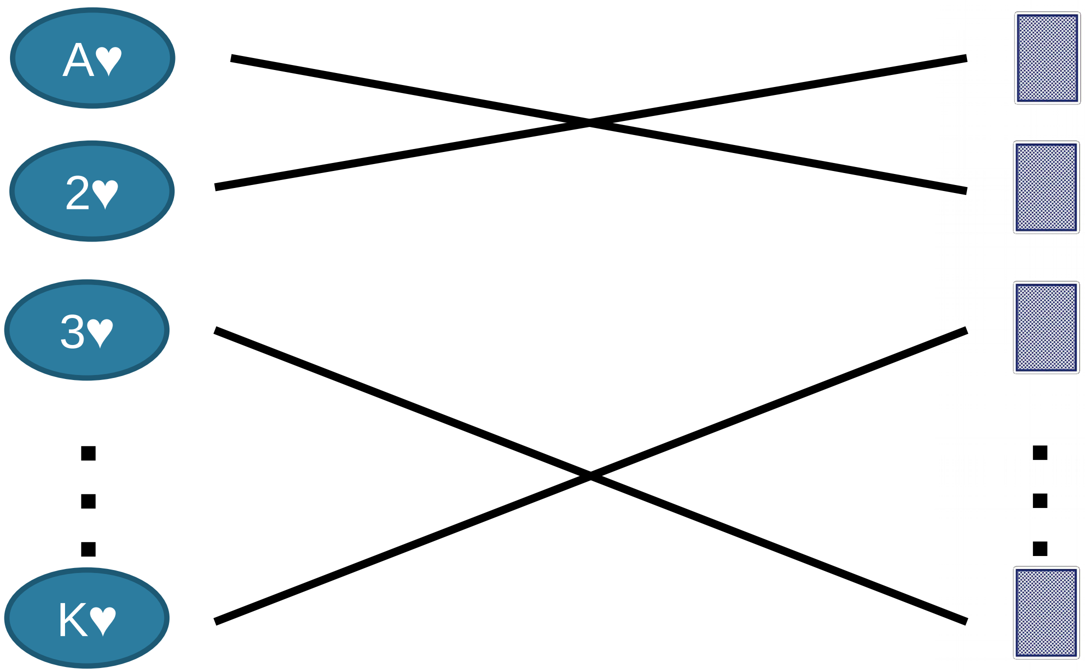
[VdB'15]

Deck of Cards Graphically



[VdB'15]

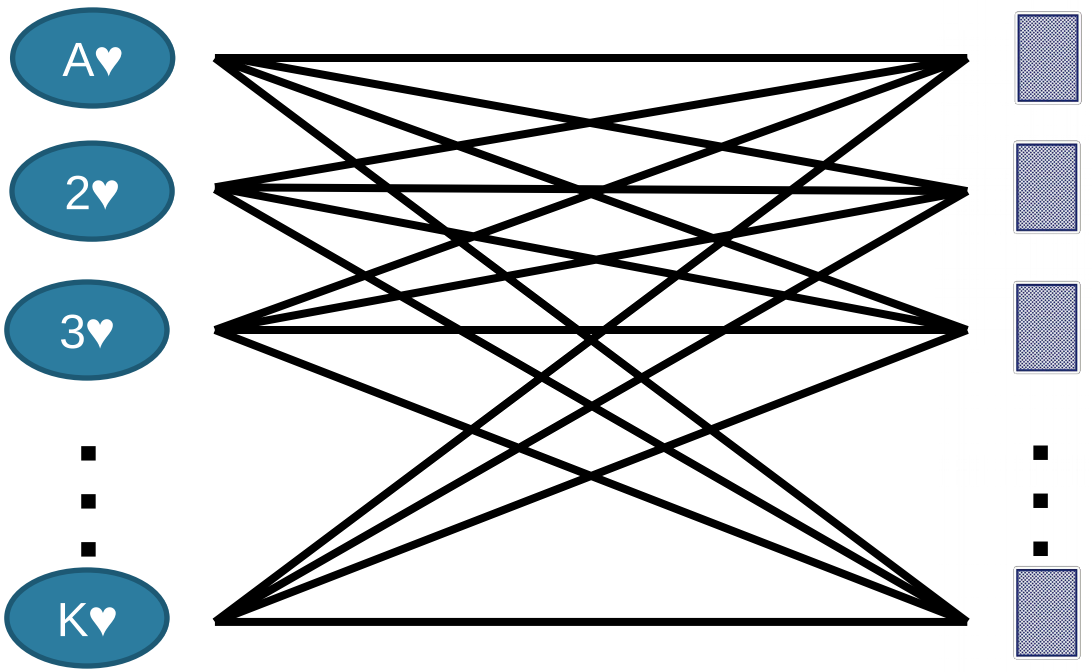
Deck of Cards Graphically



One model/*perfect matching*

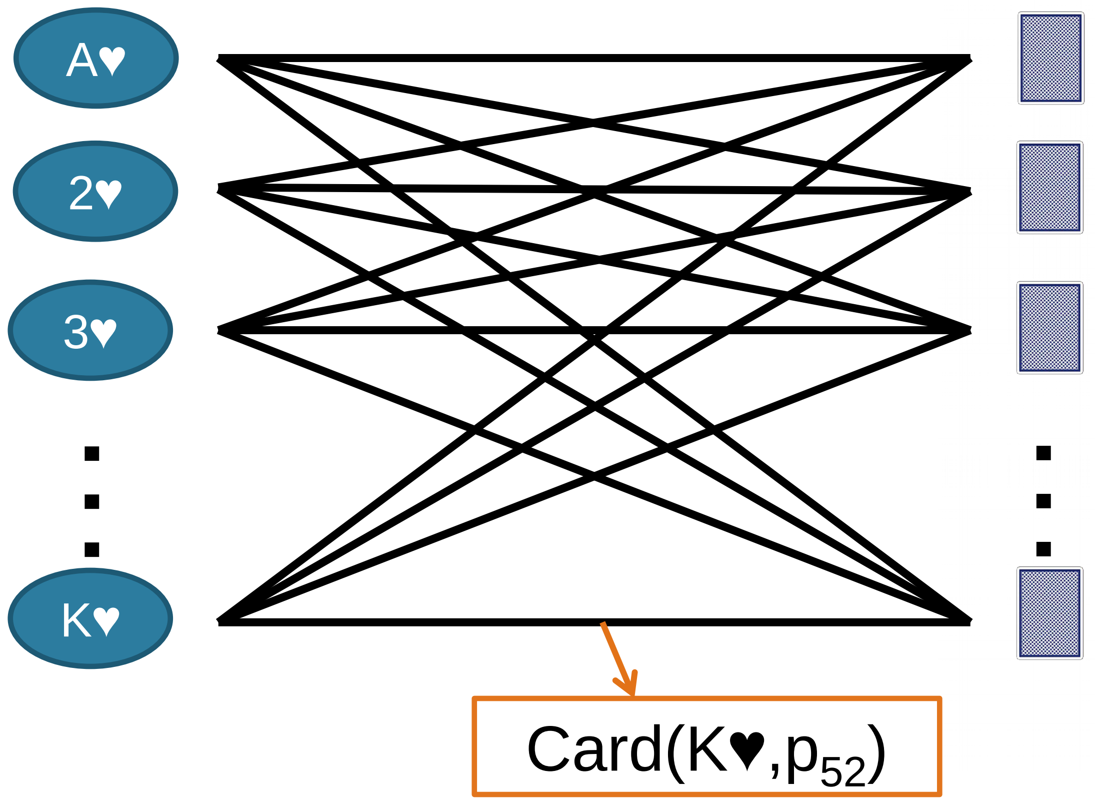
[VdB’15]

Deck of Cards Graphically



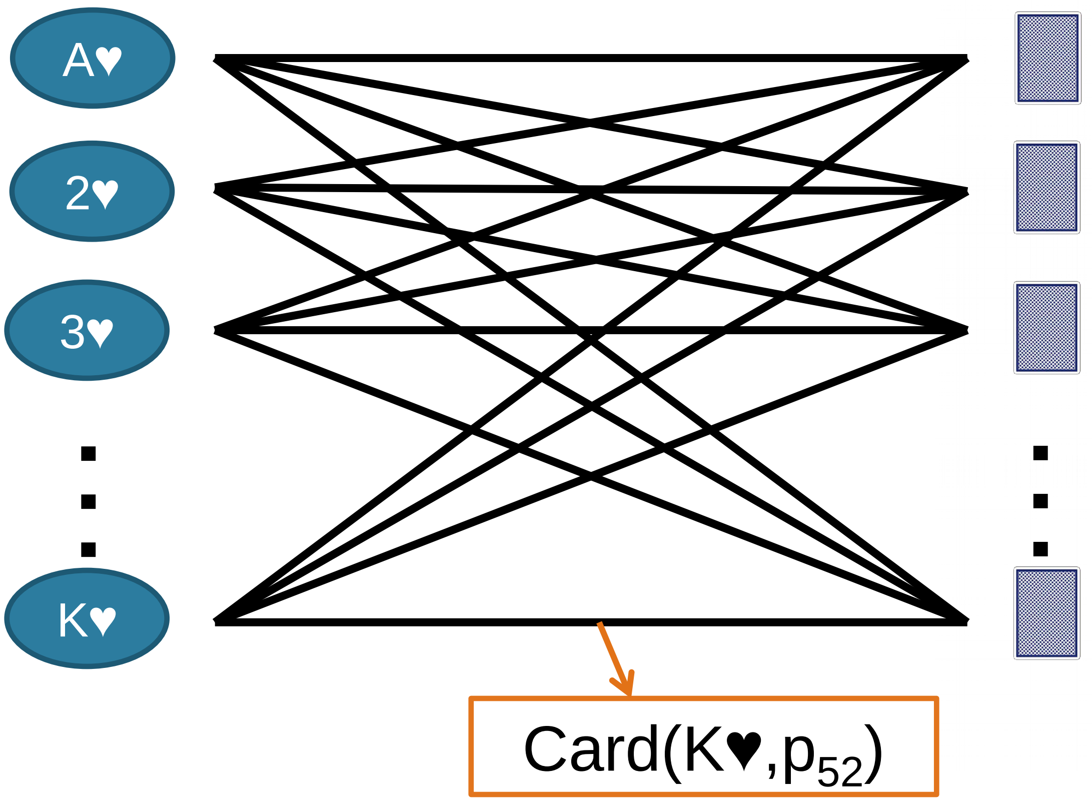
[VdB'15]

Deck of Cards Graphically



[VdB'15]

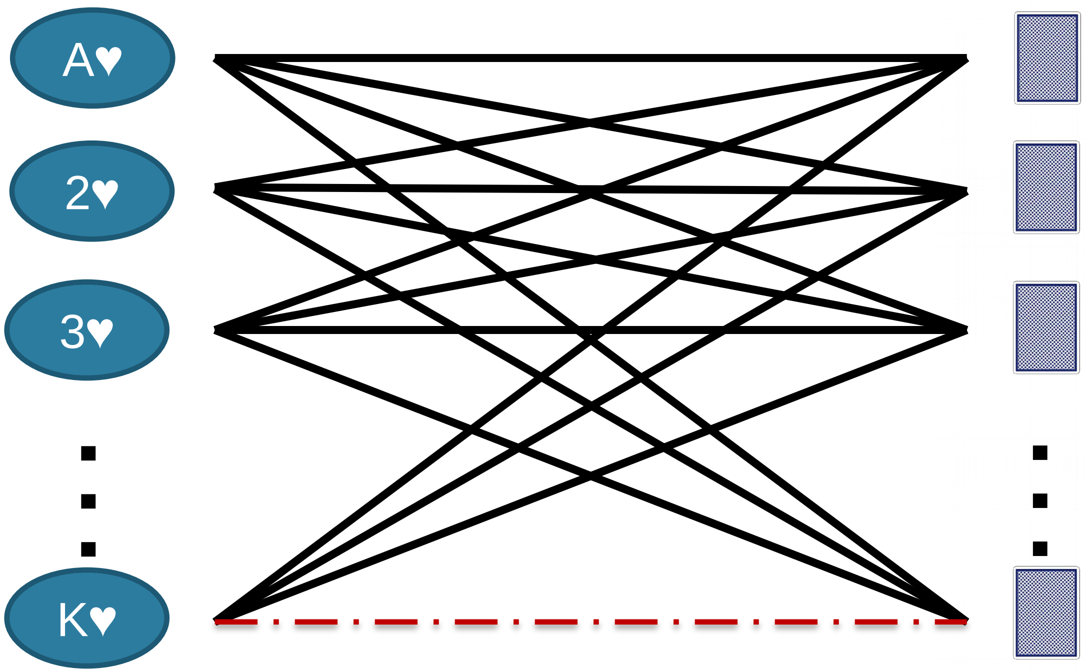
Deck of Cards Graphically



Model counting: How many *perfect matchings*?

[VdB'15]

Deck of Cards Graphically

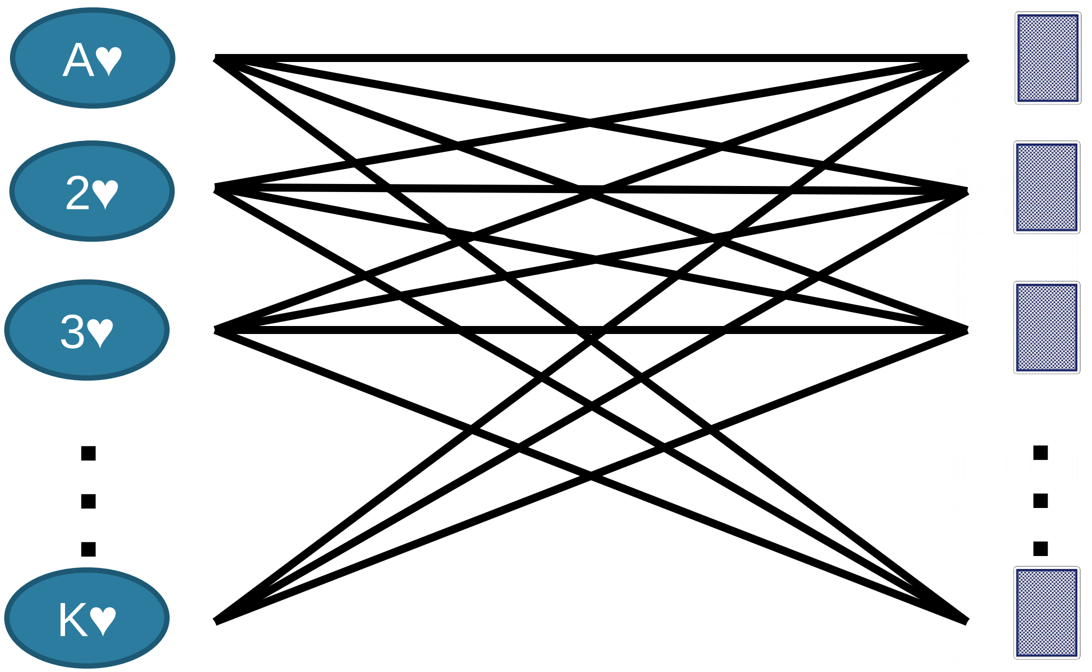


What if I set

w(Card(K♥,p_{52})) = 0?

[VdB'15]

Deck of Cards Graphically



What if I set

w(Card(K♥,p_{52})) = 0?

[VdB'15]

Observations

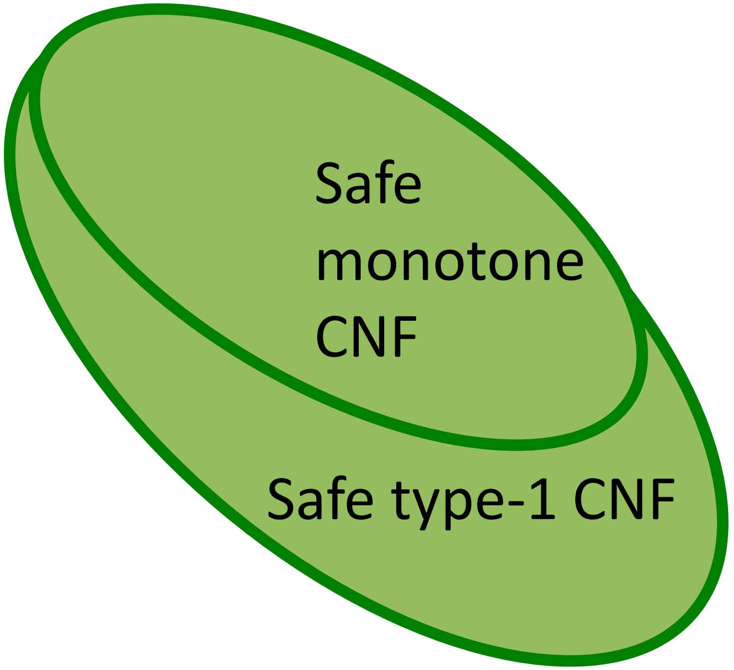
- Asymmetric weight function = bigraph

- # models = #perfect matchings
- Problem is #P-complete! ☹

- All non-lifted WMC solvers efficiently handle asymmetric weights
- No solver does cards problem efficiently!

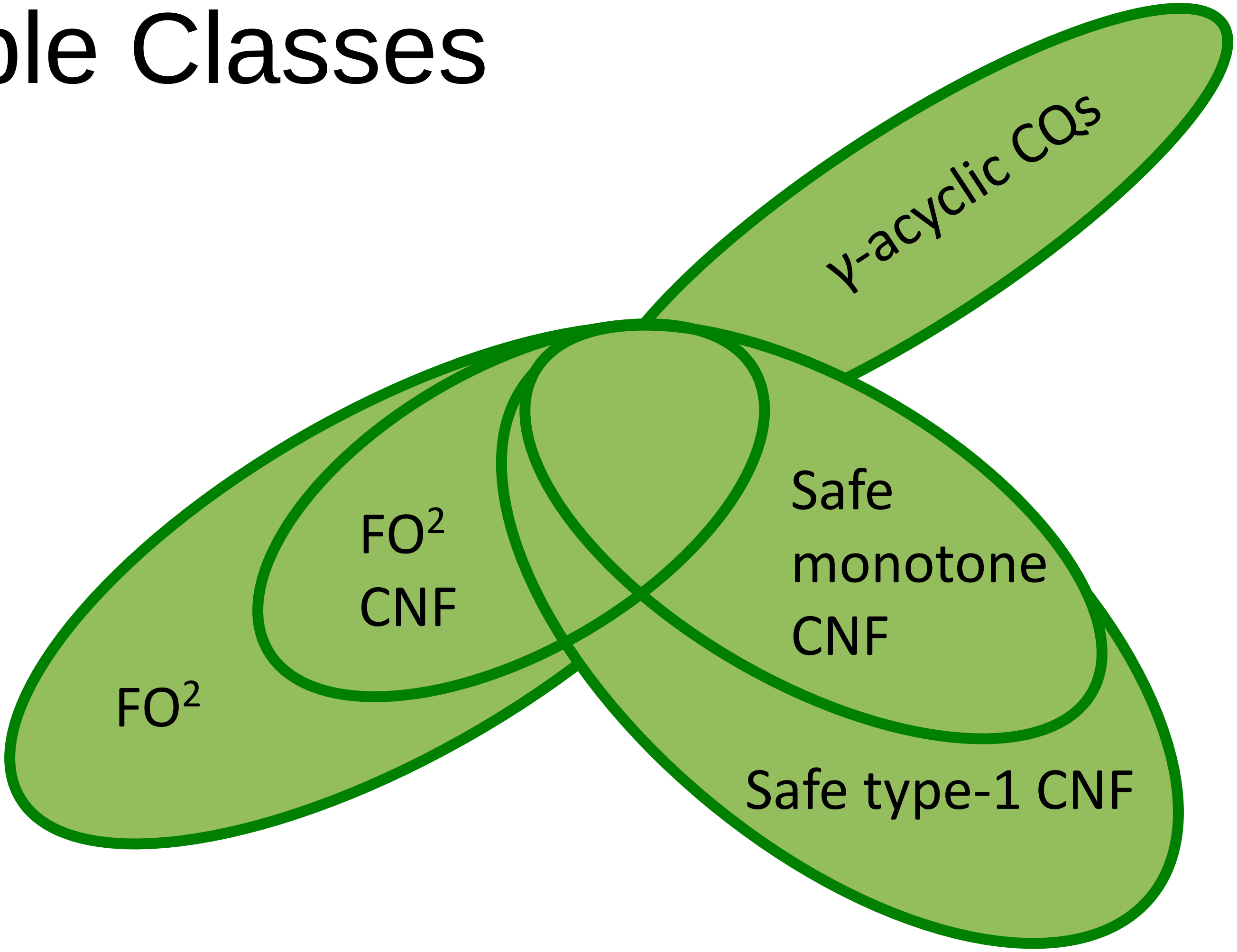
[VdB'15]

Tractable Classes



[VdB; NIPS'11], [VdB et al.; KR'14], [Gribkoff, VdB, Suciu; UAI'15], [Beame, VdB, Gribkoff, Suciu; PODS'15], etc.

Tractable Classes



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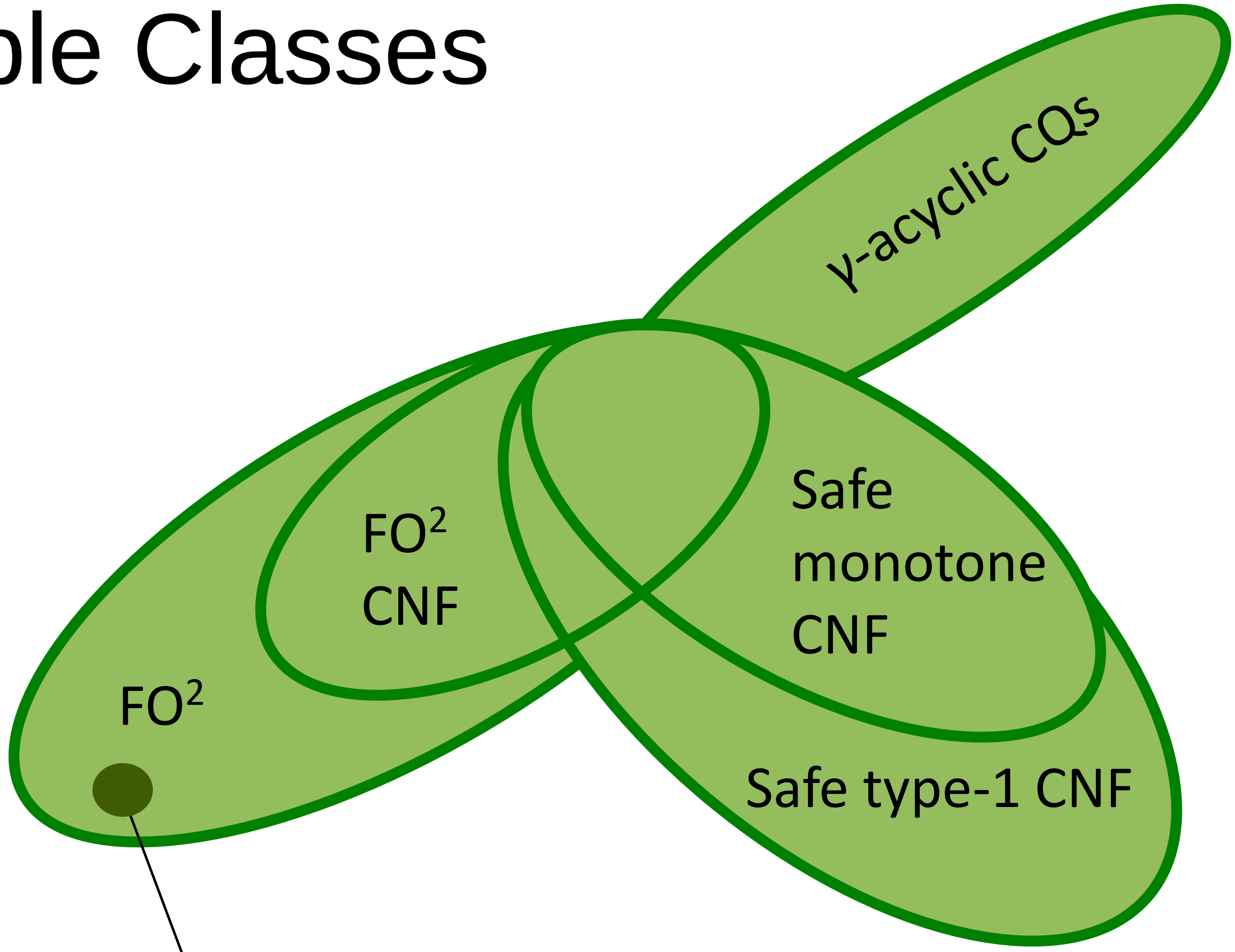

$\Delta = \exists x,y,\ R(x) \land S(x,y) \land T(y)$

[VdB; NIPS'11], [VdB et al.; KR'14], [Gribkoff, VdB, Suciu; UAI'15], [Beame, VdB, Gribkoff, Suciu; PODS'15], etc.

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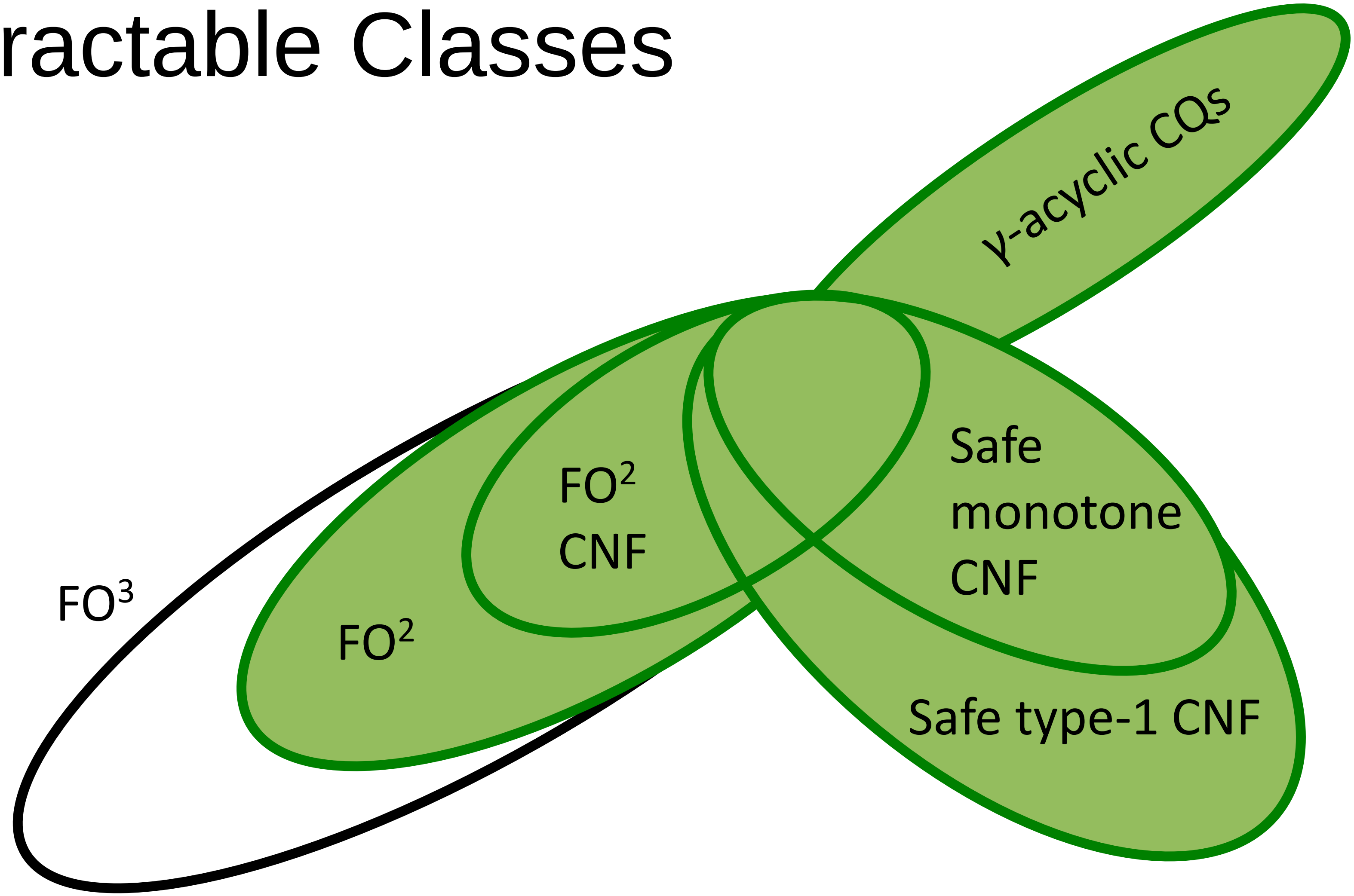

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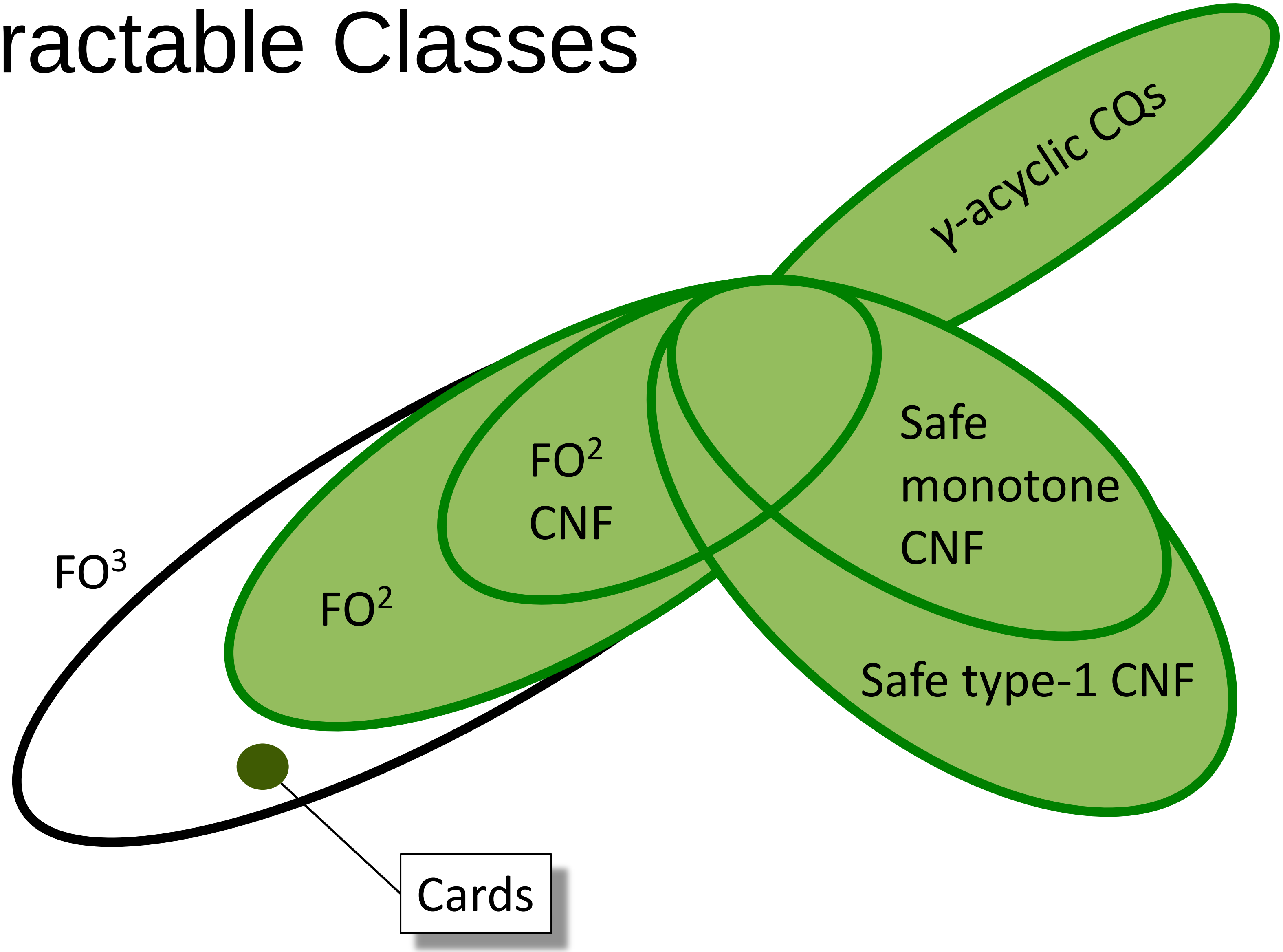

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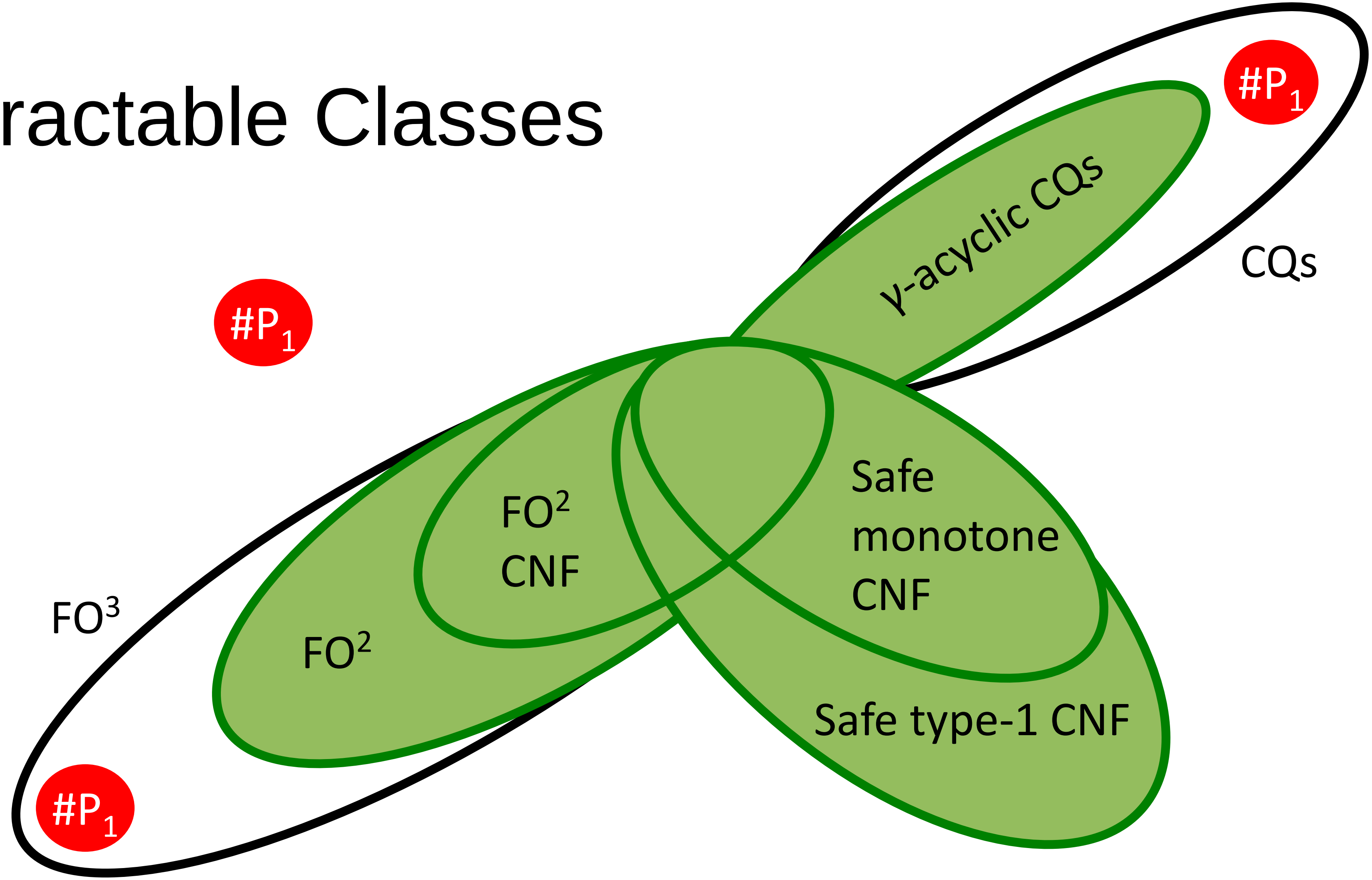

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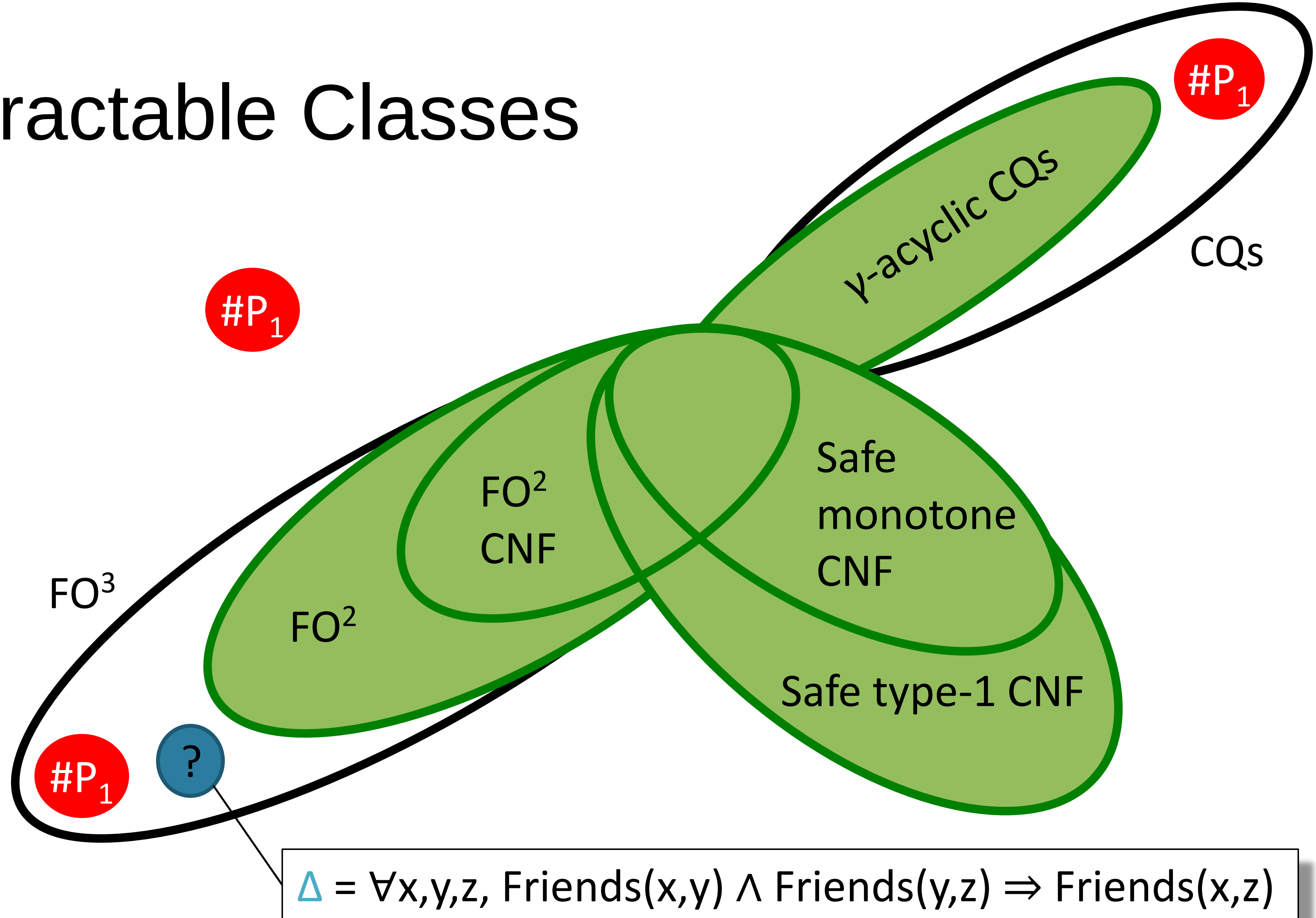

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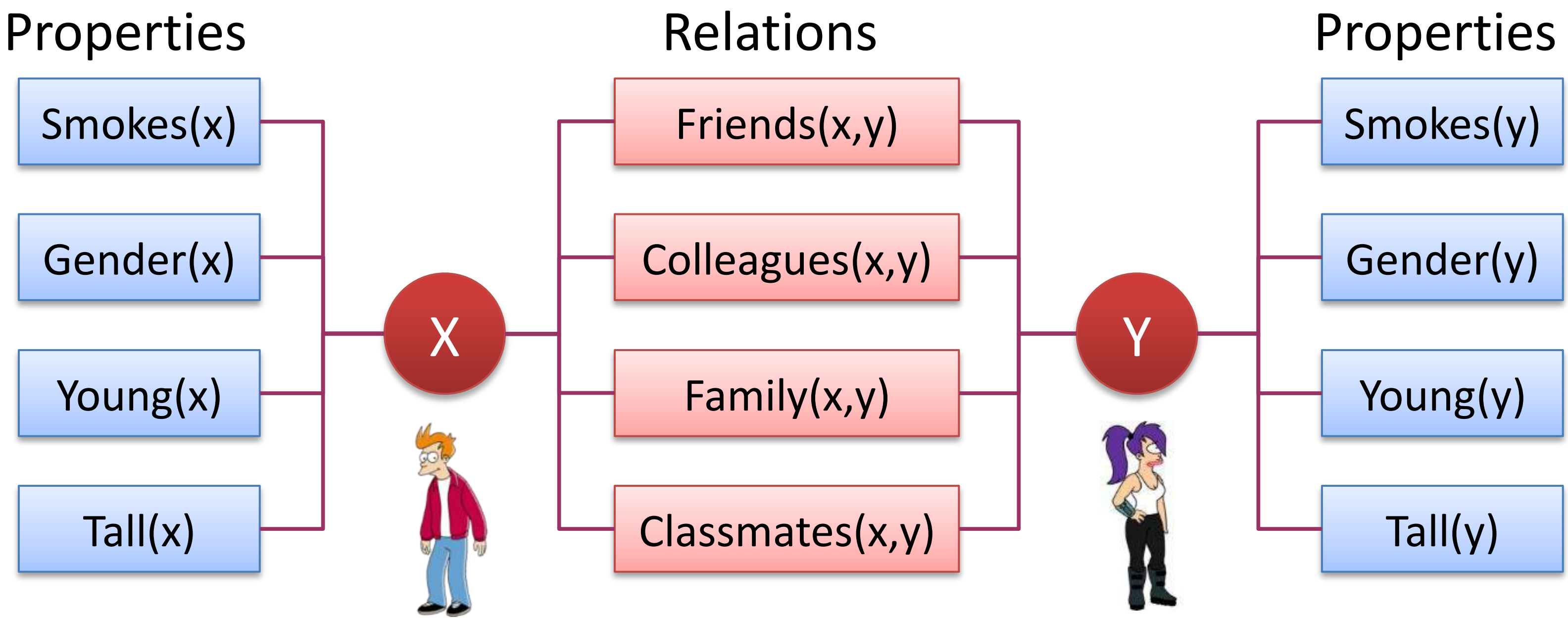
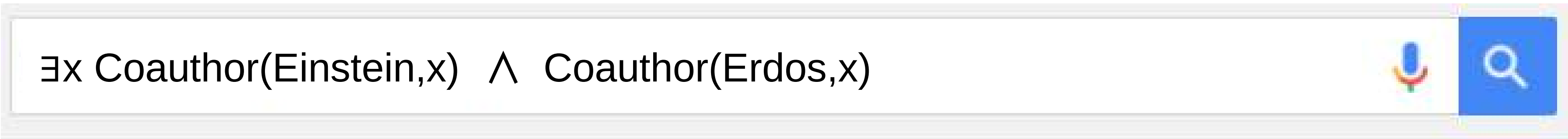
FO^2 is liftable!



"Smokers are more likely to be friends with other smokers."
"Colleagues of the same age are more likely to be friends."
"People are either family or friends, but never both."
"If X is family of Y, then Y is also family of X."
"If X is a parent of Y, then Y cannot be a parent of X."

What is this really good for?

What we'd like to do…



Ernst Straus

Kristian Kersting, …

Justin Bieber, …

Open World DB

- What if fact missing?

- Probability 0 for:

Coauthor

X	Y	P
Einstein	Straus	0.7
~~**Erdos**~~	~~**Straus**~~	~~**0.6**~~
Einstein	Pauli	0.9
Erdos	Renyi	0.7
Kersting	Natarajan	0.8
Luc	Paol	0.1
…	…	…

Q1 = ∃x Coauthor(Einstein,**x**) ∧ Coauthor(Erdos,**x**)

Open World DB

- What if fact missing?

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Q1 = ∃x Coauthor(Einstein,**x**) ∧ Coauthor(Erdos,**x**)

Q2 = ∃x Coauthor(Bieber,**x**) ∧ Coauthor(Erdos,**x**)

Open World DB

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- Probability 0 for:

Coauthor

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Q1 = ∃x Coauthor(Einstein,**x**) ∧ Coauthor(Erdos,**x**)

Q2 = ∃x Coauthor(Bieber,**x**) ∧ Coauthor(Erdos,**x**)

Q3 = Coauthor(Einstein,**Straus**) ∧ Coauthor(Erdos,**Straus**)

Coauthor

X	Y	P
Einstein	Straus	0.7
~~**Erdos**~~	~~**Straus**~~	~~**0.6**~~
Einstein	Pauli	0.9
Erdos	Renyi	0.7
Kersting	Natarajan	0.8
Luc	Paol	0.1
…	…	…

Open World DB

- What if fact missing?
- Probability 0 for:

Q1 = ∃x Coauthor(Einstein,**x**) ∧ Coauthor(Erdos,**x**)

Q2 = ∃x Coauthor(Bieber,**x**) ∧ Coauthor(Erdos,**x**)

Q3 = Coauthor(Einstein,**Straus**) ∧ Coauthor(Erdos,**Straus**)

Q4 = Coauthor(Einstein,**Bieber**) ∧ Coauthor(Erdos,**Bieber**)

Open World DB

- What if fact missing?
- Probability 0 for:

Coauthor

X	Y	P
Einstein	Straus	0.7
~~**Erdos**~~	~~**Straus**~~	~~**0.6**~~
Einstein	Pauli	0.9
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Q2 = ∃x Coauthor(Bieber,**x**) ∧ Coauthor(Erdos,**x**)

Q3 = Coauthor(Einstein,**Straus**) ∧ Coauthor(Erdos,**Straus**)

Q4 = Coauthor(Einstein,**Bieber**) ∧ Coauthor(Erdos,**Bieber**)

Q5 = Coauthor(Einstein,**Bieber**) ∧ ¬Coauthor(**Einstein**,**Bieber**)

Intuition

X	Y	P
Einstein	Straus	0.7
~~**Erdos**~~	~~**Straus**~~	~~**0.6**~~
Einstein	Pauli	0.9
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…	…	…

Q1 = ∃x Coauthor(Einstein,**x**) ∧ Coauthor(Erdos,**x**)

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Q4 = Coauthor(Einstein,**Bieber**) ∧ Coauthor(Erdos,**Bieber**)

[Ceylan, Darwiche, Van den Broeck; KR'16]

Intuition

X	Y	P
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Q1 = ∃x Coauthor(Einstein,**x**) ∧ Coauthor(Erdos,**x**)

Q3 = Coauthor(Einstein,**Straus**) ∧ Coauthor(Erdos,**Straus**)

Q4 = Coauthor(Einstein,**Bieber**) ∧ Coauthor(Erdos,**Bieber**)

We know for sure that P(Q1) ≥ P(Q3), P(Q1) ≥ P(Q4)

[Ceylan, Darwiche, Van den Broeck; KR'16]

Intuition

X	Y	P
Einstein	Straus	0.7
~~**Erdos**~~	~~**Straus**~~	~~**0.6**~~
Einstein	Pauli	0.9
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Q1 = ∃x Coauthor(Einstein,**x**) ∧ Coauthor(Erdos,**x**)

Q3 = Coauthor(Einstein,**Straus**) ∧ Coauthor(Erdos,**Straus**)

Q4 = Coauthor(Einstein,**Bieber**) ∧ Coauthor(Erdos,**Bieber**)

Q5 = Coauthor(Einstein,**Bieber**) ∧ ¬Coauthor(**Einstein**,**Bieber**)

We know for sure that P(Q1) ≥ P(Q3), P(Q1) ≥ P(Q4)

and P(Q3) ≥ P(Q5), P(Q4) ≥ P(Q5)

[Ceylan, Darwiche, Van den Broeck; KR'16]

Intuition

X	Y	P
Einstein	Straus	0.7
~~**Erdos**~~	~~**Straus**~~	~~**0.6**~~
Einstein	Pauli	0.9
Erdos	Renyi	0.7
Kersting	Natarajan	0.8
Luc	Paol	0.1
…	…	…

Q1 = ∃x Coauthor(Einstein,**x**) ∧ Coauthor(Erdos,**x**)

Q3 = Coauthor(Einstein,**Straus**) ∧ Coauthor(Erdos,**Straus**)

Q4 = Coauthor(Einstein,**Bieber**) ∧ Coauthor(Erdos,**Bieber**)

Q5 = Coauthor(Einstein,**Bieber**) ∧ ¬Coauthor(**Einstein**,**Bieber**)

We know for sure that P(Q1) ≥ P(Q3), P(Q1) ≥ P(Q4)

and P(Q3) ≥ P(Q5), P(Q4) ≥ P(Q5) because P(Q5) = 0.

[Ceylan, Darwiche, Van den Broeck; KR'16]

Intuition

X	Y	P
Einstein	Straus	0.7
~~**Erdos**~~	~~**Straus**~~	~~**0.6**~~
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…	…	…

Q1 = ∃x Coauthor(Einstein,**x**) ∧ Coauthor(Erdos,**x**)

Q2 = ∃x Coauthor(Bieber,**x**) ∧ Coauthor(Erdos,**x**)

Q3 = Coauthor(Einstein,**Straus**) ∧ Coauthor(Erdos,**Straus**)

Q4 = Coauthor(Einstein,**Bieber**) ∧ Coauthor(Erdos,**Bieber**)

Q5 = Coauthor(Einstein,**Bieber**) ∧ ¬Coauthor(**Einstein**,**Bieber**)

We know for sure that P(Q1) ≥ P(Q3), P(Q1) ≥ P(Q4)

and P(Q3) ≥ P(Q5), P(Q4) ≥ P(Q5) because P(Q5) = 0.

We have strong evidence that P(Q1) ≥ P(Q2).

[Ceylan, Darwiche, Van den Broeck; KR'16]

Bayesian Learning Loop

Bayesian view on learning:

1. Prior belief:

   P(Coauthor(Straus,Pauli)) = 0.01

2. Observe page

   P(Coauthor(Straus,Pauli| ) = 0.2

3. Observe page

   P(Coauthor(Straus,Pauli)|  , ) = 0.3

Principled and sound reasoning!

Problem: Broken Learning Loop

Bayesian view on learning:

1. Prior belief:

   $P(\texttt{Coauthor(Straus,Pauli)}) = 0$

2. Observe page

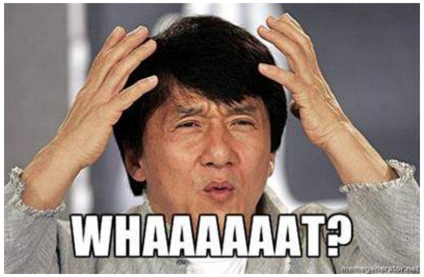
   $P(\texttt{Coauthor(Straus,Pauli} \mid$  $) = 0.2$

3. Observe page

   $P(\texttt{Coauthor(Straus,Pauli)} \mid$  ,  $) = 0.3$

[Ceylan, Darwiche, Van den Broeck; KR'16]

Problem: Broken Learning Loop



Bayesian view on learning:

1. Prior belief:

   $P(\mathtt{Coauthor(Straus,Pauli)}) = 0$

2. Observe page

   $P(\mathtt{Coauthor(Straus,Pauli} \mid$  $) = 0.2$

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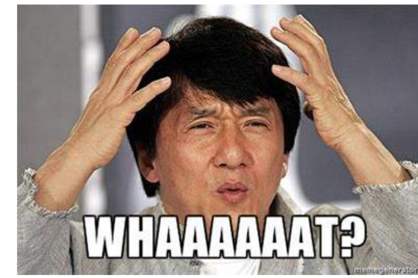
   $P(\mathtt{Coauthor(Straus,Pauli)} \mid$  ,  $) = 0.3$

[Ceylan, Darwiche, Van den Broeck; KR'16]

Problem: Broken Learning Loop



Bayesian view on learning:

1. Prior belief:

   P(Coauthor(Straus,Pauli)) = 0

2. Observe page

   P(Coauthor(Straus,Pauli| ) = 0.2

3. Observe page

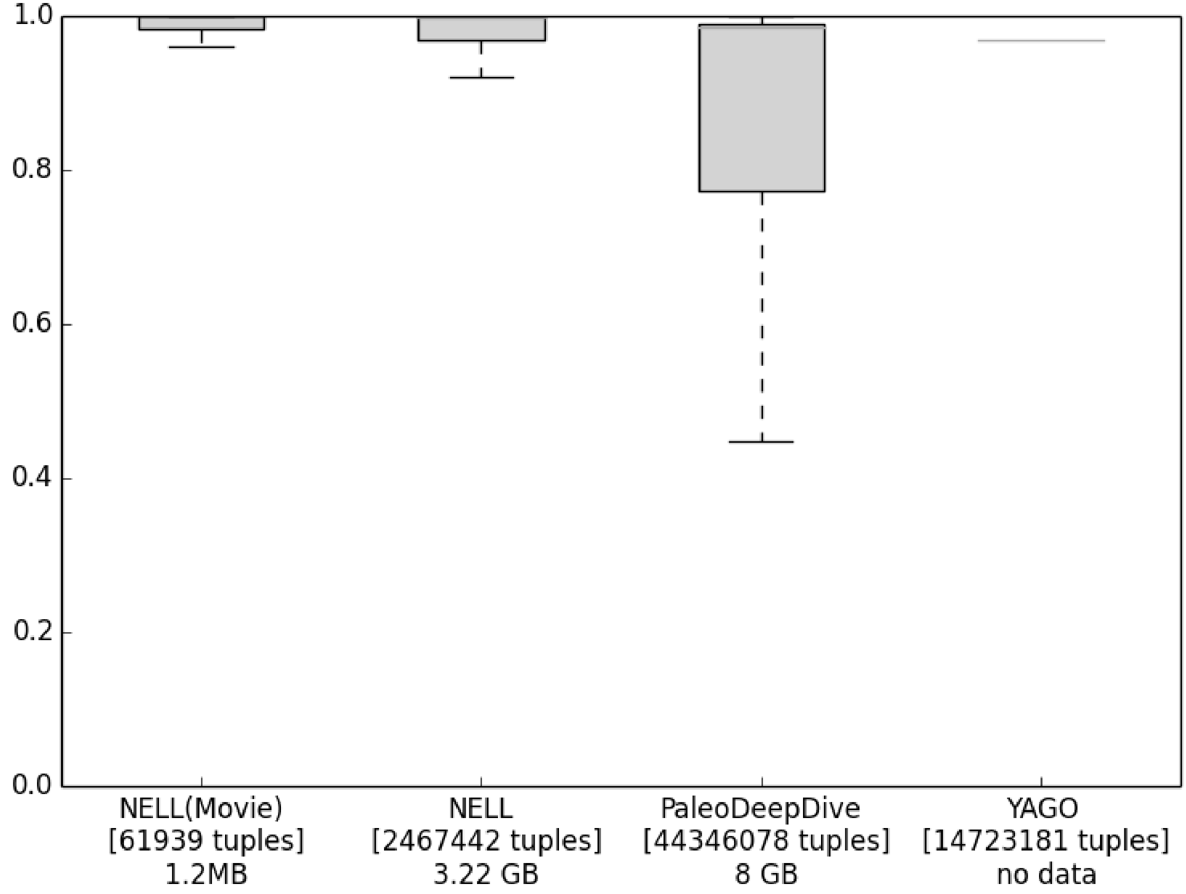
   P(Coauthor(Straus,Pauli)|  , ) = 0.3

This is mathematical nonsense!

[Ceylan, Darwiche, Van den Broeck; KR’16]

Problem: Curse of Superlinearity

- Reality is worse!
- Tuples are **intentionally** missing!



Problem: Curse of Superlinearity

Sibling

x	y	P
…	…	…

Facebook scale

[Ceylan, Darwiche, Van den Broeck; KR'16]

Problem: Curse of Superlinearity

Sibling

x	y	P
…	…	…

Facebook scale
⇒ 200 Exabytes of data"

[Ceylan, Darwiche, Van den Broeck; KR'16]

Problem: Curse of Superlinearity

Sibling

x	y	P
…	…	…

Facebook scale
⇒ 200 Exabytes of data"

All Google storage is a couple exabytes…

Randall Munroe. Google's datacenters on punch cards, 2015.

[Ceylan, Darwiche, Van den Broeck; KR'16]

Open-World Prob. Databases

Intuition: tuples can be added with P < λ

Q2 = Coauthor(Einstein,**Straus**) ∧ Coauthor(Erdos,**Straus**)

P(Q2) ≥ 0

Coauthor

X	Y	P
Einstein	Straus	0.7
Einstein	Pauli	0.9
Erdos	Renyi	0.7
Kersting	Natarajan	0.8
Luc	Paol	0.1
…	…	…

Open-World Prob. Databases

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…	…	…

Coauthor

X	Y	P
Einstein	Straus	0.7
Einstein	Pauli	0.9
Erdos	Renyi	0.7
Kersting	Natarajan	0.8
Luc	Paol	0.1
…	…	…
Erdos	**Straus**	**λ**

Open-World Prob. Databases

Intuition: tuples can be added with $P < \lambda$

Q2 = Coauthor(Einstein,**Straus**) ∧ Coauthor(Erdos,**Straus**)

$0.7 * \lambda \geq P(Q2) \geq 0$

Coauthor

X	Y	P
Einstein	Straus	0.7
Einstein	Pauli	0.9
Erdos	Renyi	0.7
Kersting	Natarajan	0.8
Luc	Paol	0.1
…	…	…

Coauthor

X	Y	P
Einstein	Straus	0.7
Einstein	Pauli	0.9
Erdos	Renyi	0.7
Kersting	Natarajan	0.8
Luc	Paol	0.1
…	…	…
Erdos	**Straus**	**λ**

UCQ

- Lower bound = closed-world probability
- Upper bound = probability after adding **all** tuples with probability λ

UCQ

- Lower bound = closed-world probability
- Upper bound = probability after adding **all** tuples with probability λ

- Polynomial time ☺

UCQ

- Lower bound = closed-world probability
- Upper bound = probability after adding **all** tuples with probability λ

- Polynomial time ☺

  Quadratic blow-up ☹
- 200 exabytes … again ☹

Closed-World Lifted Query Eval

$Q = \exists x \, \exists y \, \text{Scientist}(x) \wedge \text{Coauthor}(x,y)$

$P(Q) = 1 - \Pi_{A \in \text{Domain}} (1 - P(\text{Scientist}(A) \wedge \exists y \, \text{Coauthor}(A,y)))$

Closed-World Lifted Query Eval

$Q = \exists x\, \exists y\, \text{Scientist}(x) \wedge \text{Coauthor}(x,y)$

Decomposable ∀-Rule

$$P(Q) = 1 - \Pi_{A \in \text{Domain}} (1 - P(\text{Scientist}(A) \wedge \exists y\, \text{Coauthor}(A,y)))$$

Closed-World Lifted Query Eval

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Decomposable ∀-Rule

$$P(Q) = 1 - \Pi_{A \in Domain} (1 - P(Scientist(A) \wedge \exists y \, Coauthor(A,y))$$

$$= 1 - (1 - P(Scientist(A) \wedge \exists y \, Coauthor(A,y))$$
$$\times (1 - P(Scientist(B) \wedge \exists y \, Coauthor(B,y))$$
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$$\ldots$$

Closed-World Lifted Query Eval

$Q = \exists x\, \exists y\, \text{Scientist}(x) \wedge \text{Coauthor}(x,y)$

Decomposable ∀-Rule

$$P(Q) = 1 - \Pi_{A \in \text{Domain}} (1 - P(\text{Scientist}(A) \wedge \exists y\, \text{Coauthor}(A,y))$$

$$\begin{aligned} = 1 - (&1 - P(\text{Scientist}(A) \wedge \exists y\, \text{Coauthor}(A,y)) \\ \times (&1 - P(\text{Scientist}(B) \wedge \exists y\, \text{Coauthor}(B,y)) \\ \times (&1 - P(\text{Scientist}(C) \wedge \exists y\, \text{Coauthor}(C,y)) \\ \times (&1 - P(\text{Scientist}(D) \wedge \exists y\, \text{Coauthor}(D,y)) \\ \times (&1 - P(\text{Scientist}(E) \wedge \exists y\, \text{Coauthor}(E,y)) \\ \times (&1 - P(\text{Scientist}(F) \wedge \exists y\, \text{Coauthor}(F,y)) \\ &\ldots \end{aligned}$$

Complexity PTIME

Closed-World Lifted Query Eval

$Q = \exists x\, \exists y\, \text{Scientist}(x) \wedge \text{Coauthor}(x,y)$

$$P(Q) = 1 - \Pi_{A \in \text{Domain}} (1 - P(\text{Scientist}(A) \wedge \exists y\, \text{Coauthor}(A,y))$$

$= 1 - (1 - P(\text{Scientist}(A) \wedge \exists y\, \text{Coauthor}(A,y))$
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…

Closed-World Lifted Query Eval

$Q = \exists x \; \exists y \; Scientist(x) \wedge Coauthor(x,y)$

$$P(Q) = 1 - \Pi_{A \in Domain} (1 - P(Scientist(A) \wedge \exists y \; Coauthor(A,y))$$

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$$\times (1 - P(Scientist(F) \wedge \exists y \; Coauthor(F,y))$$
$$\ldots$$

⇨ No supporting facts in database!

Closed-World Lifted Query Eval

$Q = \exists x \, \exists y \, Scientist(x) \wedge Coauthor(x,y)$

$$P(Q) = 1 - \Pi_{A \in Domain} (1 - P(Scientist(A) \wedge \exists y \, Coauthor(A,y))$$

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…

⇨ No supporting facts in database!

⇨ Probability 0 in closed world

Closed-World Lifted Query Eval

$Q = \exists x \, \exists y \, Scientist(x) \wedge Coauthor(x,y)$

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⇒ Ignore these queries!

Closed-World Lifted Query Eval

$Q = \exists x\ \exists y\ \text{Scientist}(x) \land \text{Coauthor}(x,y)$

$P(Q) = 1 - \prod_{A \in \text{Domain}} (1 - P(\text{Scientist}(A) \land \exists y\ \text{Coauthor}(A,y))$

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$\times (1 - P(\text{Scientist}(B) \land \exists y\ \text{Coauthor}(B,y))$
$\times (1 - P(\text{Scientist}(C) \land \exists y\ \text{Coauthor}(C,y))$
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…

⇒ No supporting facts in database!

⇒ Probability 0 in closed world

⇒ Ignore these queries!

Complexity linear time!

Open-World Lifted Query Eval

$$Q = \exists x \, \exists y \; Scientist(x) \wedge Coauthor(x,y)$$

$$P(Q) = 1 - \Pi_{A \in Domain} (1 - P(Scientist(A) \wedge \exists y \; Coauthor(A,y))$$

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$\quad \ldots$

⇨ No supporting facts in database!

⇨ Probability p in closed world

Open-World Lifted Query Eval

$Q = \exists x\, \exists y\, \text{Scientist}(x) \wedge \text{Coauthor}(x,y)$

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…

⇨ No supporting facts in database!

⇨ Probability p in closed world

Complexity PTIME!

Open-World Lifted Query Eval

$Q = \exists x\ \exists y\ \text{Scientist}(x) \wedge \text{Coauthor}(x,y)$

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⇒ No supporting facts in database!

⇒ Probability p in closed world

⇒ All together, probability $(1-p)^k$
Do symmetric lifted inference

Open-World Lifted Query Eval

Q = ∃x ∃y Scientist(x) ∧ Coauthor(x,y)

P(Q) = 1 - $\Pi_{A \in \text{Domain}}$ (1 - P(Scientist(A) ∧ ∃y Coauthor(A,y))

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x (1 - P(Scientist(F) ∧ ∃y Coauthor(F,y))
…

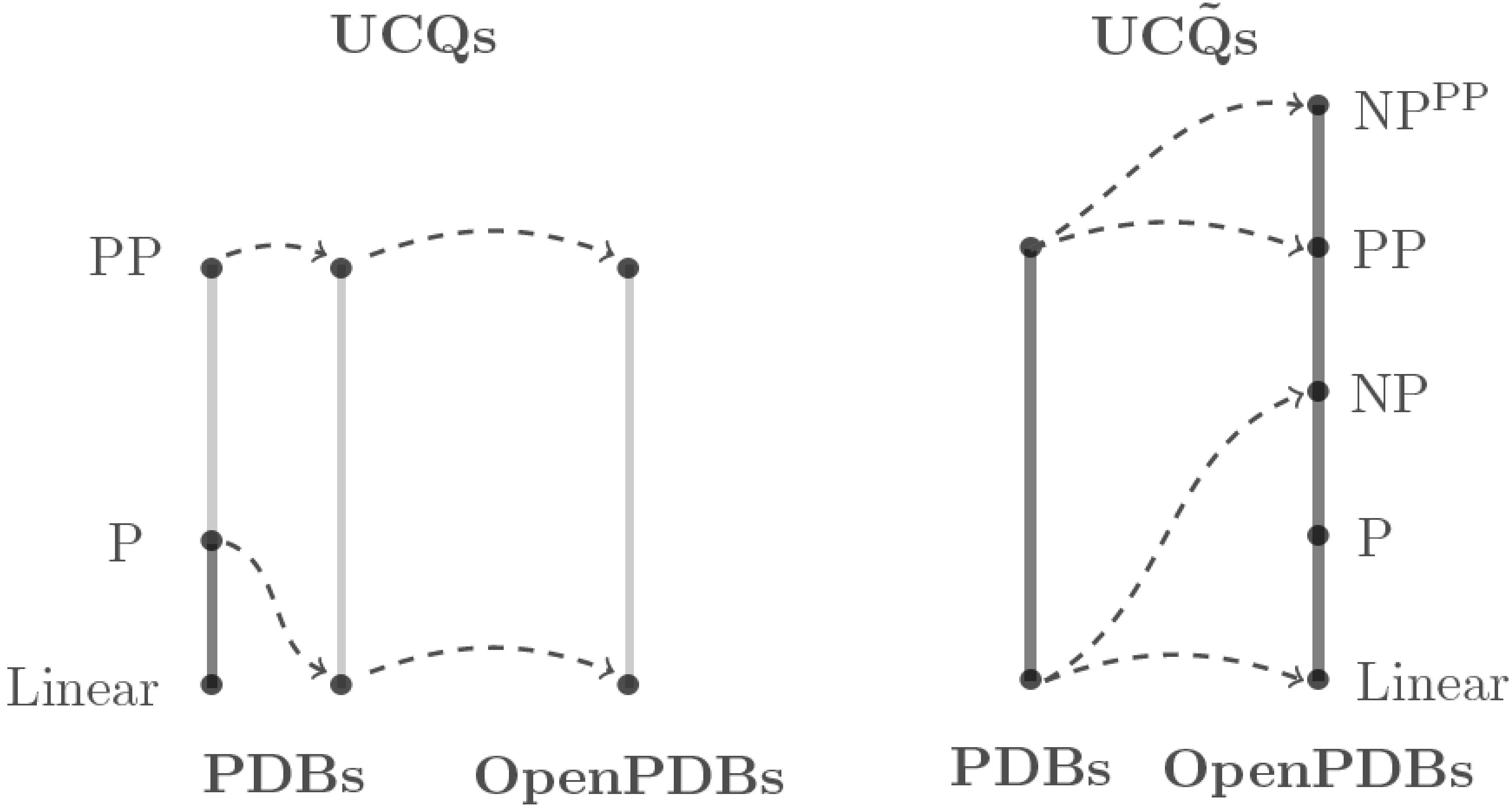
⇨ No supporting facts in database!

⇨ Probability p in closed world

⇨ All together, probability $(1-p)^k$
Do symmetric lifted inference

Complexity linear time!

Complexity Results



$\text{Linear} \subseteq \text{P} \subseteq \text{NP} \subseteq \text{PP} \subseteq \text{P}^{\text{PP}} \subseteq \text{NP}^{\text{PP}} \subseteq \text{PSpace} \subseteq \text{ExpTime}$

[Ceylan'16]

Parameter Learning

- **Given:** A set of first-order logic **formulas**

  w FacultyPage(x) ∧ Linked(x,y) ⇒ CoursePage(y)

  A set of training **databases**

- **Learn:** The associated maximum-likelihood **weights**

$$\frac{\partial}{\partial w_j} \log \Pr_w(db) = n_j(db) - \mathbb{E}_w[n_j]$$

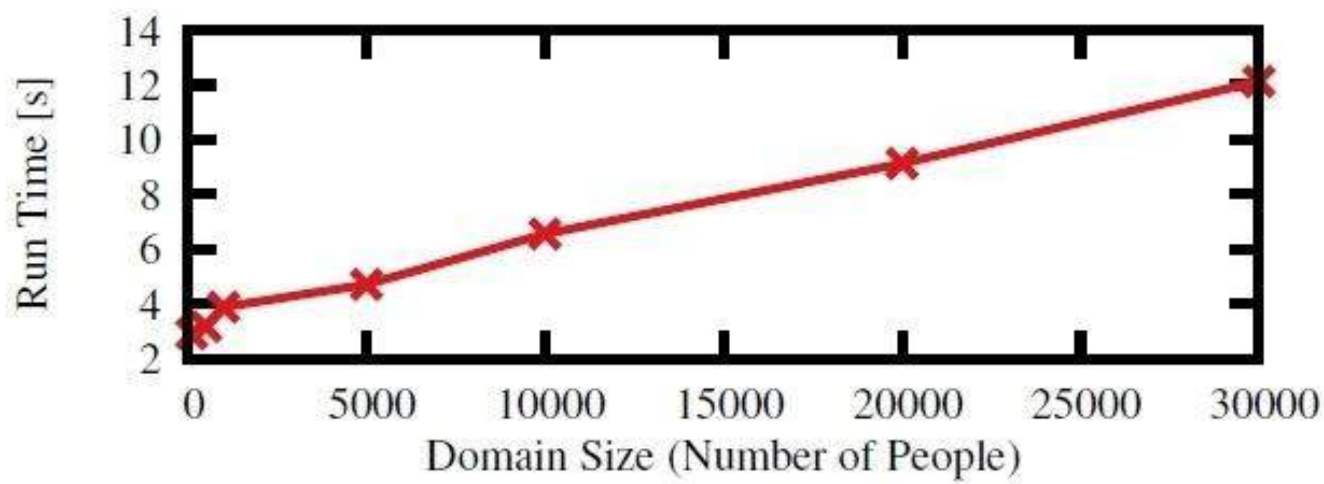
Count in databases
Efficient

Expected counts
Requires inference

$$\mathbb{E}_w[n_F] = \Pr(F\theta_1) + \cdots + \Pr(F\theta_m)$$

Lifted Parameter Learning

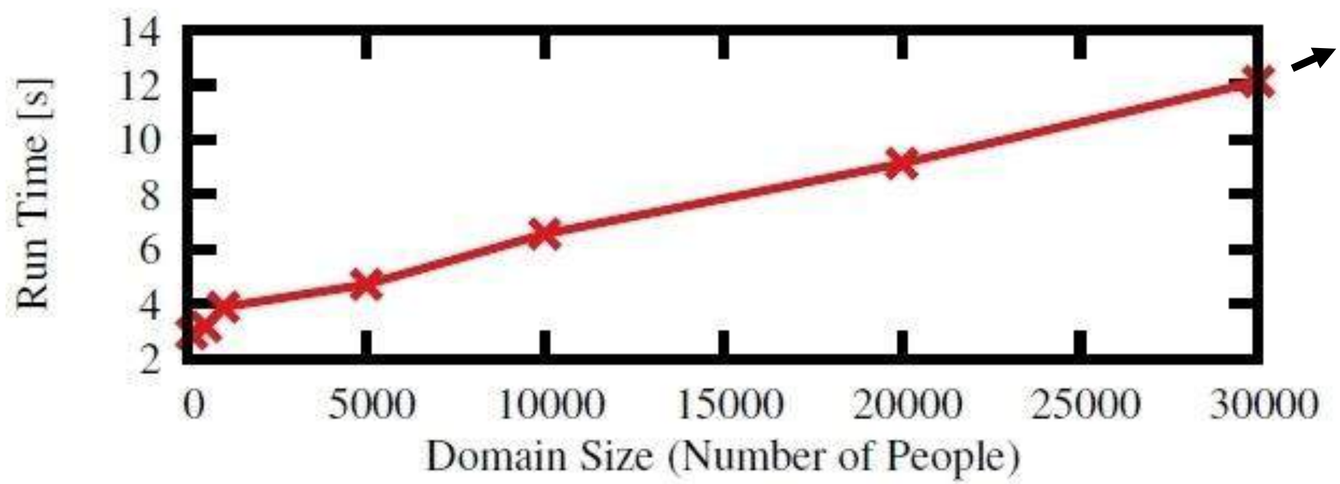
- **Given:** A set of first-order logic **formulas**
  A set of training **databases**
- **Learn:** Maximum-likelihood **weights**
- **Idea:** Lift the gradient computation



[Van Haaren et al.; MLJ'15]

Lifted Parameter Learning

- **Given:** A set of first-order logic **formulas**
  A set of training **databases**
- **Learn:** Maximum-likelihood **weights**
- **Idea:** Lift the gradient computation



900,030,000 random variables

[Van Haaren et al.; MLJ'15]

Lifted Structure Learning

- **Given:** A set of training **databases**
- **Learn:** A set of first-order logic **formulas**
  The associated maximum-likelihood **weights**
- **Idea:** Learn liftable models (regularize with symmetry)

	IMDb			*UWCSE*		
	Baseline	Lifted Weight Learning	Lifted Structure Learning	Baseline	Lifted Weight Learning	Lifted Structure Learning
Fold 1	-548	-378	**-306**	-1,860	-1,524	**-1,477**
Fold 2	-689	-390	**-309**	-594	-535	**-511**
Fold 3	-1,157	-851	**-733**	-1,462	-1,245	**-1,167**
Fold 4	-415	-285	**-224**	-2,820	-2,510	**-2,442**
Fold 5	-413	-267	**-216**	-2,763	-2,357	**-2,227**

[Van Haaren et al.; MLJ'15]

What is the broader picture?

Uncertainty in AI

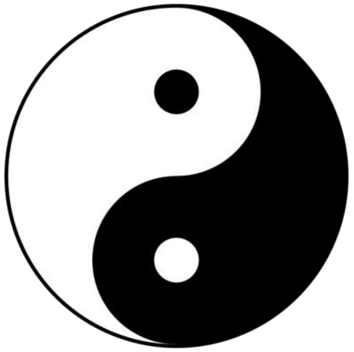
Probability Distribution

=

Qualitative

+

Quantitative



Generalized Model Counting

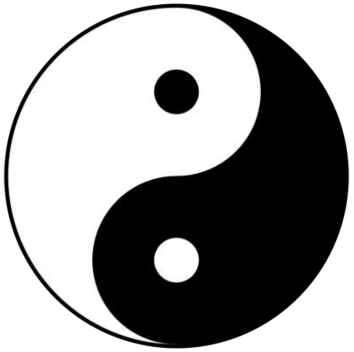
Probability Distribution

=

Logic

+

Weights



Generalized Model Counting

Probability Distribution	Logical Syntax
=	Model-theoretic Semantics
Logic	+
+	Weight function w(.)
Weights	

Weighted Model Integration

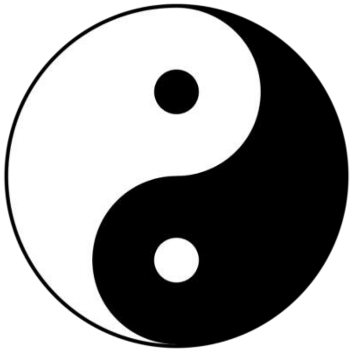
Probability Distribution

=

SMT(LRA)

+

Weights



[Belle et al. IJCAI'15, UAI'15]

Weighted Model Integration

Probability Distribution

=

SMT(LRA)

+

Weights

0 ≤ height ≤ 200
0 ≤ weight ≤ 200
0 ≤ age ≤ 100
age < 1 ⇒
height+weight ≤ 90

+

w(height))=height-10
w(¬height)=3*height^2
w(¬weight)=5
...

[Belle et al. IJCAI'15, UAI'15]

Probabilistic Programming

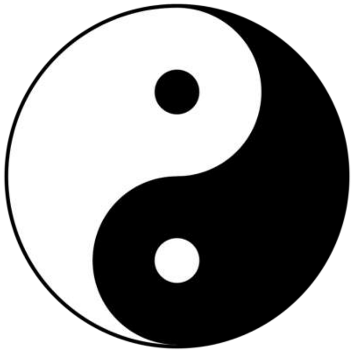
Probability Distribution

=

Logic Programs

+

Weights



[Fierens et al., TPLP'15]

Probabilistic Programming

Probability Distribution

=

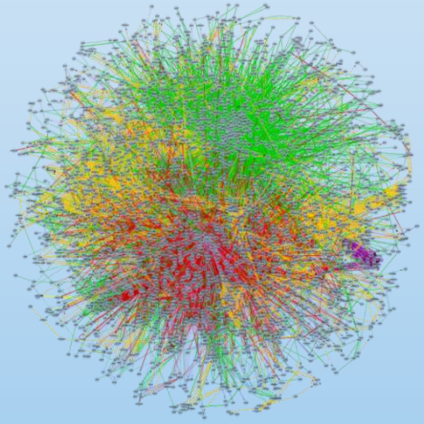
Logic Programs

+

Weights

```
path(X,Y) :-
        edge(X,Y).
path(X,Y) :-
        edge(X,Z), path(Z,Y).
```

+



[Fierens et al., TPLP'15]

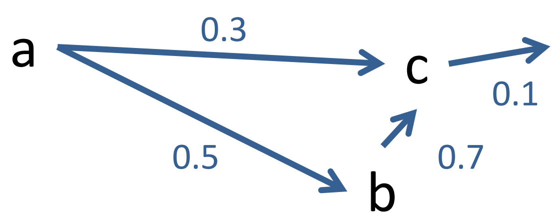
Probabilistic Functional Programming

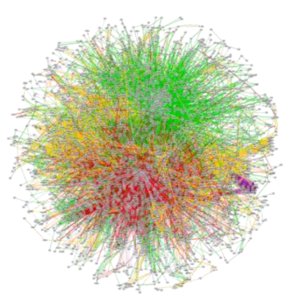
Discrete probabilistic reachability program:

Logic Program (ProbLog) = Functional Program (Scala-like)

```
path(X,Y) :- edge(X,Y).
path(X,Y) :- edge(X,Z),
             path(Z,Y).
edge(X,Y) :- …random vars…
```

```
def path(start,end,visited=List())={
    if(start == end)
        return true
    if(visited.contains(start))
        return false
    return start.neighbors.exists{
        path(_,end,(visited+start))
    }
}
nodeA.neighbors = …random vars…
nodeB.neighbors = …random vars…
```





First-Order Knowledge Compilation

Markov Logic

3.14	Smokes(x) ∧ Friends(x,y) ⇒ Smokes(y)

[Van den Broeck,PhD'13]

First-Order Knowledge Compilation

Markov Logic

3.14 Smokes(x) ∧ Friends(x,y) ⇒ Smokes(y)

Weight Function

w(Smokes)=1
w(¬Smokes)=1
w(Friends)=1
w(¬Friends)=1
w(F)=3.14
w(¬F)=1

FOL Sentence

∀x,y, F(x,y) ⇔ [Smokes(x) ∧ Friends(x,y) ⇒ Smokes(y)]

[Van den Broeck,PhD'13]

First-Order Knowledge Compilation

Markov Logic: 3.14 Smokes(x) ∧ Friends(x,y) ⇒ Smokes(y)

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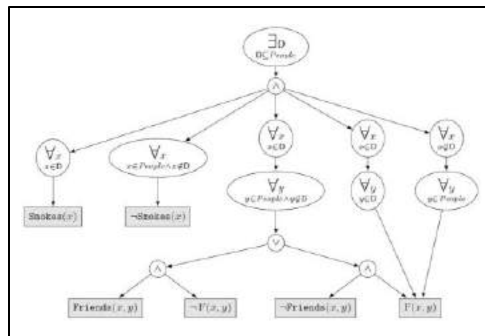
↓ Compile?

First-Order d-DNNF Circuit



[Van den Broeck,PhD'13]

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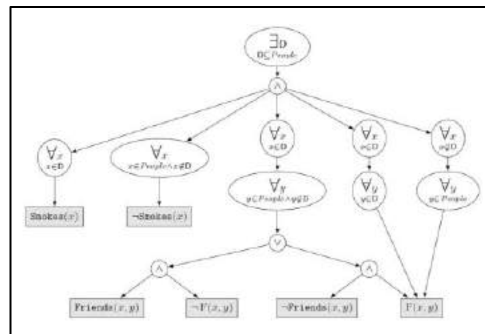

Domain

Alice
Bob
Charlie

[Van den Broeck,PhD’13]

First-Order Knowledge Compilation

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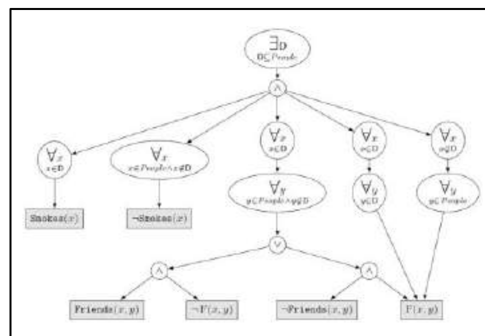

Domain

Alice
Bob
Charlie

Z = WFOMC = 1479.85

[Van den Broeck,PhD'13]

First-Order Knowledge Compilation

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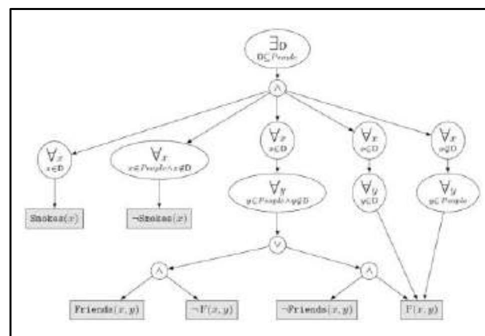

Domain

Alice
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Charlie

Z = WFOMC = 1479.85

Evaluation in time polynomial in domain size!

[Van den Broeck,PhD'13]

First-Order Knowledge Compilation

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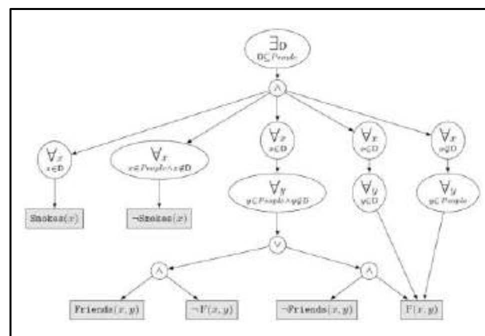
Compile?

First-Order d-DNNF Circuit



Domain

Alice
Bob
Charlie

Z = WFOMC = 1479.85

Evaluation in time polynomial in domain size!

= Lifted!

[Van den Broeck,PhD’13]

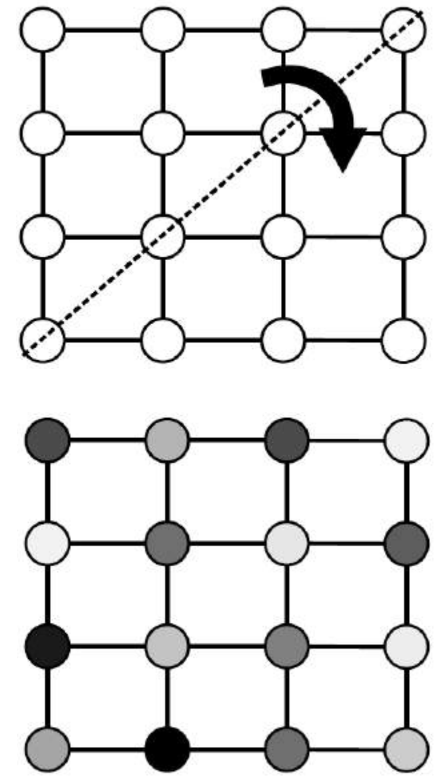
Approximate Symmetries in MCMC

- Exploit approximate symmetries:
  - Exact symmetry g

    $\Pr(\mathbf{x}) = \Pr(\mathbf{x}^{g})$
  - Approximate symmetry g

    $\Pr(\mathbf{x}) \approx \Pr(\mathbf{x}^{g})$

Pr [] ≈ Pr []



- Approximate lifted inference (MCMC)

[Van den Broeck, Darwiche; NIPS’13], [Van den Broeck, Niepert; AAAI’15]

Exchangeability

- FO2 characterized by a particular form of partial exchangeability
  - You can construct a statistic that defines probability
  - This is what our algorithms do
- Other forms of exchangeability open …
- Connection to computability?

Results for Symmetric Inference

Theorem
There exists Q in FO^3 s.t. FOMC(Q, n) is $\#P_1$ hard
There exists CQ Q s.t. WFOMC(Q, n) is $\#P_1$ hard

Theorem WFOMC(Q, n) is in PTIME
- For any Q in FO^2
- For any gamma-acyclic Q

- Corresponding decision problem = the spectrum problem
- Data complexity: { Spec(Q) | Q in FO} = NP_1 [Fagin'74]
- Combined complexity: NP-complete for FO^2, PSPACE-complete for FO
- 0-1 Laws connection

[Van den Broeck'14 ,Beame'15]

Conclusions

- Relational probabilistic reasoning is **frontier** and **integration** of AI, KR, ML, DB, TH, etc.
- We need
  - relational models and logic
  - probabilistic models and statistical learning
  - algorithms that scale
- Open-world data model
  - semantics make sense
  - FREE for UCQs
  - expensive otherwise

Long-Term Outlook

Probabilistic inference and learning exploit

~ 1988: conditional independence

~ 2000: contextual independence (local structure)

Long-Term Outlook

Probabilistic inference and learning exploit

~ 1988: conditional independence

~ 2000: contextual independence (local structure)

~ 201?: **symmetry & exchangeability & first-order**

Thanks!

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