Logic and Quantum Information
Lecture I: Ringing the Bell

Samson Abramsky

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It is no longer safe to assume this!

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- Beyond algorithms and complexity it offers new challenges and opportunities across the range of Computer Science: in programming languages and methods, logic and semantics.
- There is a fascinating two-way interplay developing between Computer Science and Physics, extending to the foundations of both, as well as to more practical matters. Quantum technology — "hacking matter" — will be a huge feature of 21st Century science and engineering, and a lot of it will be to do with information.

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- Beyond algorithms and complexity it offers new challenges and opportunities across the range of Computer Science: in programming languages and methods, logic and semantics.
- There is a fascinating two-way interplay developing between Computer Science and Physics, extending to the foundations of both, as well as to more practical matters. Quantum technology — "hacking matter" — will be a huge feature of 21st Century science and engineering, and a lot of it will be to do with information.
- This is an exciting emerging area, attracting students with backgrounds in CS, Physics, Mathematics, Logic, Philosophy, ...

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- They also have profound consequences for our understanding of the very nature of physical reality.
- We shall describe recent work in which tools from Computer Science are used to shed new light on these phenomena.
- There are also striking and unexpected connections with a number of topics in **classical** computer science, including relational databases and constraint satisfaction.

First Loophole-free Bell test, 2015

NATURE | LETTER

日本語要約

Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres

B. Hensen, H. Bernien, A. E. Dréau, A. Reiserer, N. Kalb, M. S. Blok, J. Ruitenberg, R. F. L. Vermeulen, R. N. Schouten, C. Abellán, W. Amaya, V. Pruneri, M. W. Mitchell, M. Markham, D. J. Twitchen, D. Elkouss, S. Wehner, T. H. Taminiau & R. Hanson

Nature **526**, 682–686 (29 October 2015) doi:10.1038/nature15759

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More than 50 years ago[1], John Bell proved that no theory of nature that obeys locality and realism[2] can reproduce all the predictions of quantum theory: in any local-realist theory, the correlations between outcomes of measurements on distant particles satisfy an inequality that can be violated if the particles are entangled. Numerous Bell inequality tests have been reported[3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]; however, all experiments reported so far required additional assumptions to obtain a contradiction with local realism, resulting in 'loopholes'[13, 14, 15, 16]. Here we report a Bell experiment that is free of any such additional assumption and thus directly tests the principles underlying Bell's inequality. We use an event-ready scheme[17, 18, 19] that enables the generation of robust entanglement between distant electron spins (estimated state fidelity of 0.92 ± 0.03). Efficient spin read-out avoids the fair-sampling assumption (detection loophole[14, 15]), while the use of fast random-basis selection and spin read-out combined with a spatial separation of 1.3 kilometres ensure the required locality conditions[13]. We performed 245 trials that tested the CHSH–Bell inequality[20] $S \leq 2$ and found $S = 2.42 \pm 0.20$ (where S quantifies the correlation between measurement outcomes). A null-hypothesis test yields a probability of at most $P = 0.039$ that a local-realist model for space-like separated sites could produce data with a violation at least as large as we observe, even when allowing for memory[16, 21] in the devices. Our data hence imply statistically significant rejection of the local-realist null hypothesis. This conclusion may be further consolidated in future experiments; for instance, reaching a value of $P = 0.001$ would require approximately 700 trials for an observed $S = 2.4$. With improvements, our experiment could be used for testing less-conventional theories, and for implementing device-independent quantum-secure communication[22] and randomness certification[23, 24].

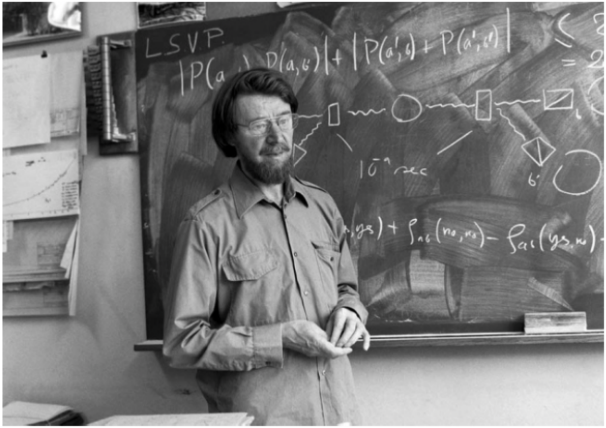
NATURE | NEWS

Quantum 'spookiness' passes toughest test yet

Experiment plugs loopholes in previous demonstrations of 'action at a distance', against Einstein's objections — and could make data encryption safer.

Zeeya Merali

27 August 2015



CERN

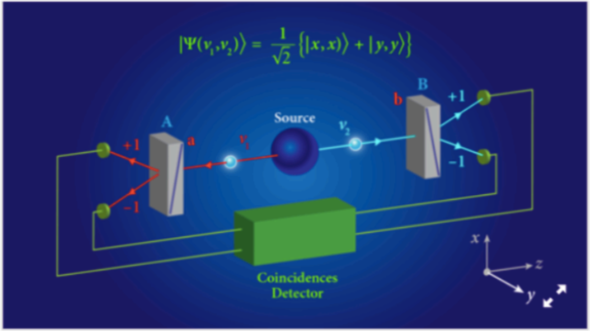
APS physics Physics

Viewpoint: Closing the Door on Einstein and Bohr's Quantum Debate

Alain Aspect, Laboratoire Charles Fabry, Institut d'Optique Graduate School, CNRS, Université Paris-Saclay, Palaiseau, France

December 16, 2015 • *Physics* 8, 123

By closing two loopholes at once, three experimental tests of Bell's inequalities remove the last doubts that we should renounce local realism. They also open the door to new quantum information technologies.



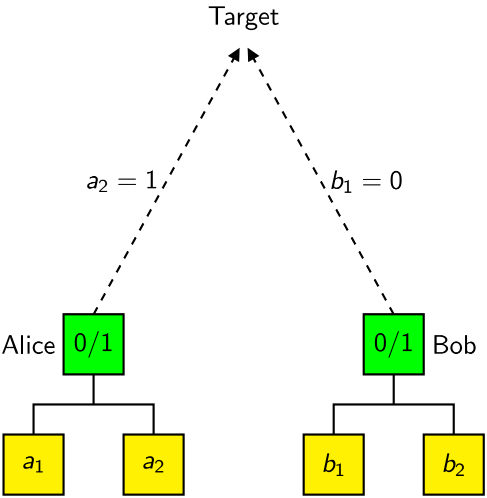
APS/Alan Stonebraker

Figure 1: An apparatus for performing a Bell test. A source emits a pair of entangled photons ν_1 and ν_2. Their polarizations are analyzed by polarizers A and B (grey blocks), which are aligned, respectively, along directions *a* and *b*. (*a* and *b* can be along *x*, ... **Show more**

Timeline

1932	von Neumann's Mathematical Foundations of Quantum Mechanics
1935	EPR Paradox, the Einstein-Bohr debate
1964	Bell's Theorem
1982	First experimental test of EPR and Bell inequalities (Aspect, Grangier, Roger, Dalibard)
1984	Bennett-Brassard quantum key distribution protocol
1985	Deutch Quantum Computing paper
1993	Quantum teleportation (Bennett, Brassard, Crépeau, Jozsa, Peres, Wooters)
1994	Shor's algorithm
2015	First loophole-free Bell tests (Delft, NIST, Vienna)

Alice and Bob look at bits



A Probabilistic Model Of An Experiment

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Example: The Bell Model

A	B	$(0,0)$	$(1,0)$	$(0,1)$	$(1,1)$
a_1	b_1	$1/2$	0	0	$1/2$
a_1	b_2	$3/8$	$1/8$	$1/8$	$3/8$
a_2	b_1	$3/8$	$1/8$	$1/8$	$3/8$
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The entry in row 2 column 3 says:

If Alice looks at a_1 *and Bob looks at* b_2, *then* $1/8$*th of the time, Alice sees a* 0 *and Bob sees a* 1.

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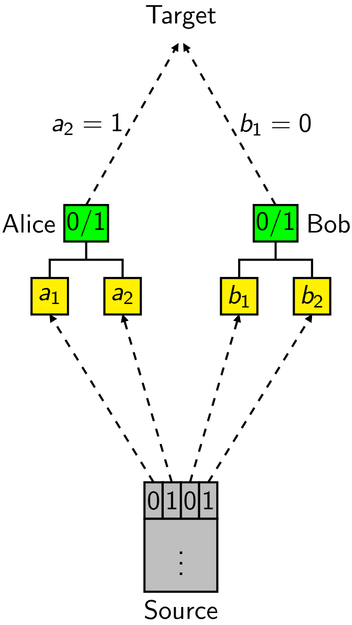
How can we explain this behaviour?

Classical Correlations: The Classical Source



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Using elementary probability theory, we can calculate:

$$p_N \leq \text{Prob}(\bigvee_{i=1}^{N-1} \neg\phi_i) \leq \sum_{i=1}^{N-1} \text{Prob}(\neg\phi_i) = \sum_{i=1}^{N-1} (1 - p_i) = (N-1) - \sum_{i=1}^{N-1} p_i.$$

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Hence we obtain the inequality

$$\sum_{i=1}^{N} p_i \leq N - 1.$$

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If we read 0 as true and 1 as false, the highlighted entries in each row of the table are represented by the following propositions:

$$\begin{array}{lclclclcl}
\varphi_1 & = & (a_1 \wedge b_1) & \vee & (\neg a_1 \wedge \neg b_1) & = & a_1 & \leftrightarrow & b_1 \\
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The violation of the logical Bell inequality is $1/4$.

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The support of the Hardy model:

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Hence the Hardy model achieves a violation of $p_1 = \text{Prob}(a \wedge b)$ for the logical Bell inequality.

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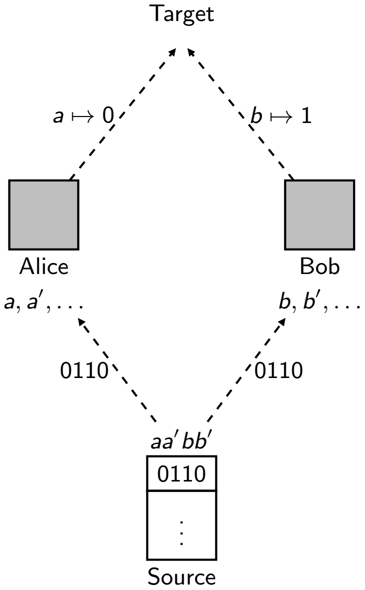
However, this view is **impossible to sustain** in the light of our **actual observations of (micro)-physical reality**.

Hidden Variables: The Mermin instruction set picture



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Alice and Bob's choices are now of **measurement setting** (e.g. which direction to measure spin) rather than "which register to load".

The Quantum Case: Spin Measurements

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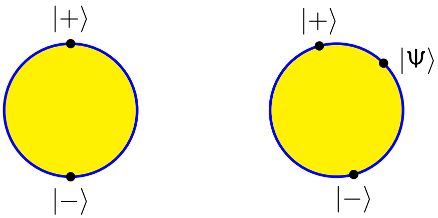
States of the system can be described by complex unit vectors in $\mathbb{C}^2$. These can be visualized as points on the unit 2-sphere:



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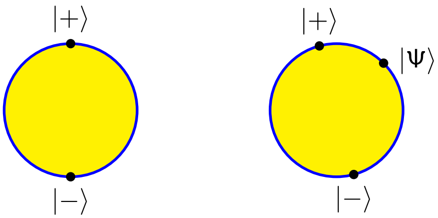

Spin can be measured in any direction; so there are a continuum of possible measurements. There are **two possible outcomes** for each such measurement; spin in the specified direction, or in the opposite direction. These two directions are represented by a pair of orthogonal vectors. They are represented on the sphere as a pair of **antipodal points**.

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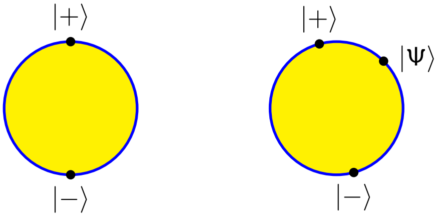

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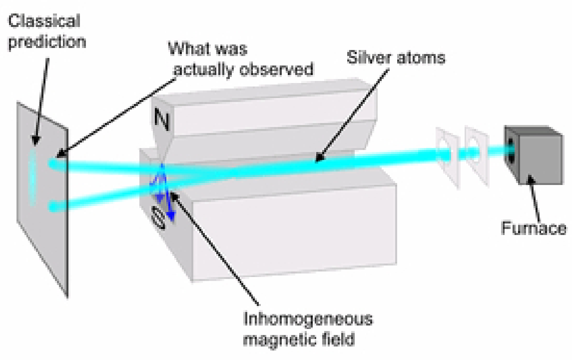
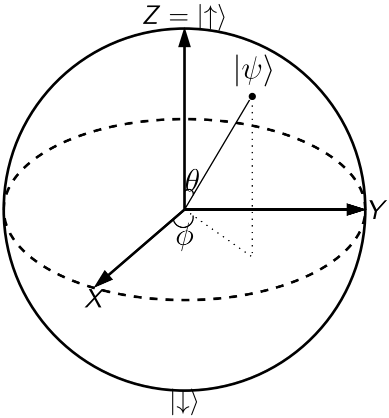
Note the appearance of **quantization** here: there are not a continuum of possible outcomes for each measurement, but only two!

The Stern-Gerlach Experiment



The Bloch sphere representation of qubits



Quantum Entanglement

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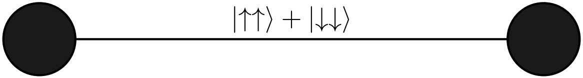
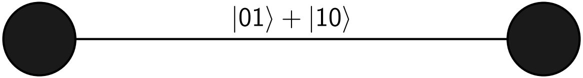


EPR state:



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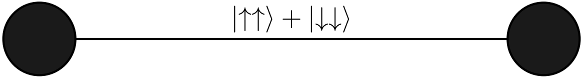

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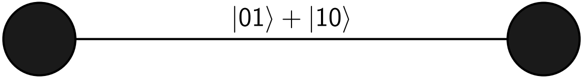

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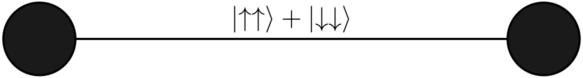

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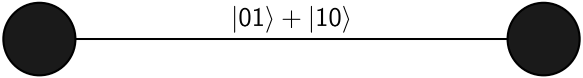

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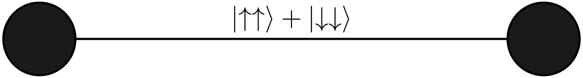

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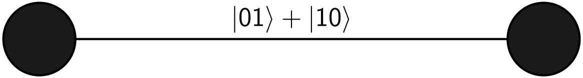

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Bell's theorem: QM is **essentially non-local**.

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a_1	b_2	$3/8$	$1/8$	$1/8$	$3/8$
a_2	b_1	$3/8$	$1/8$	$1/8$	$3/8$
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This is really how the world is!

The XOR Game

Alice and Bob play a cooperative game against Nature:

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A probability table defines a **strategy** for this game. The **success probability** for this strategy is:

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These are exactly the probabilities of events we used in our derivation of the logical Bell inequality.

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The Bell table exceeds this bound. Since it is quantum realizable, it shows that quantum resources yield a **quantum advantage** in an information-processing task.