Strong truthfulness in Peer Prediction with Overlapping Tasks

David C. Parkes

Computer Science
John A. Paulson School of Engineering
Harvard University

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Joint work with Victor Shnayder and Rafael Frongillo

Information Elicitation Without Verification

Illustrative examples:

- Participatory sensing
- Emotional response to content
- Consumer surveys
- Algorithm feedback
- Peer grading in MOOCs

Effort is costly. Need to reward informative responses, but without any ground truth; avoid unintended equilibria, collusion.

How to do this?

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- Minimal mechanisms : ask agents for information ('signal')
- Non-minimal: ask agents for signal, along with belief about signal of another agent.

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Simple Output agreement

- Two agents, joint *signal distribution* $P(X_1, X_2)$
- Signals $i, j \in \{1, \ldots, m\}$
- Take *reports* $\{r_1, r_2\}$, provide payment to each agent:

$$\text{report } r_1 \begin{array}{c} 1 \\ 2 \end{array} \overset{\text{report } r_2}{\overset{\begin{array}{cc} 1 & 2 \end{array}}{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}}$$

(Strict) proper: truthful reporting is a Bayes-Nash equilibrium.

Need $P(X_2 = 1 | X_1 = 1) > P(X_2 = 2 | X_1 = 1)$*; and there are uninformative equilibria with greater payment.*

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The Peer Prediction Method

Miller, Resnick and Zeckhauser, 2005:

- Receive report r_1, and form belief $b_1 = P(X_2 | X_1 = r_1)$
- Use proper scoring rule $t_1(b_1, r_2)$

Strict proper.

Problems: *(1) designer needs model; (2) uninformative equilibria with greater payment.*

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Faltings et al. (2012); Witkowski and Parkes (2012):

- Assume knowledge of *marginal probability*, $P(X)$

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Mechanism Desiderata

For peer prediction to be used in practice:

- Minimal mechanism
- (Strictly) Proper
- Low knowledge requirements on designer
- Truthful reports maximize expected payments:
  - *Strong-truthfulness* (in case of a tie ⇒ permutation)
  - *Informed-truthfulness* (in case of a tie ⇒ informed strategy)
- Heterogeneous agents (i.e., qualities, tastes)

Can assume multiple (independent) tasks.

What do we know?

- Jurca and Faltings, 2009.
  - $n \geq 4$ agents, knock-out pure, uninformative equil. *Ignore mixed equilibria, binary-signal only, require model.*
- Dasgupta and Ghosh, 2013.
  - Multiple tasks. Strict-proper, strong-truthful. *Binary-signal only.*
- Radonovic and Faltings, 2015.
  - Multiple tasks. Strict-proper, strong-truthful amongst symmetric strategies. *Multi-signal, but results only hold asymptotically, and need homogeneous agents.*
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Experimental evidence

- Gao, Mao, Chen and Adams, 2014.
  - This matters! mTurk experiment (see either *collusion*, or *confusion*.)

Our Contributions: Robust, Multi-Signal Methods

- *OA-mechanism*. Strict-proper and strong-truthful for multi-signal, *categorical domains* (generalizes DG'13).

With some domain knowledge:

- *01-mechanism*. Informed-truthful and proper for general, multi-signal domains (allow heterogeneity).
- *ABCD-mechanism*. Strong-truthful (symmetric) for general, multi-signal domains w/ het. *Not proper, but all equil strictly worse than truth in large system.*

Empirical analysis:

- ∼100 questions across ∼30 exercises in 17 MOOCs. Around 325,000 peer-evaluation responses.

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Basic Set-up

- Agents 1, 2
- Tasks k (≥ 3); Signals $i, j \in \{1, \ldots, m\}$ (require effort)
- Joint distribution $P(X_1 = i, X_2 = j)$ (possibly asymmetric)
- Overlapping tasks: *shared* K_s, *agent 1* K_1, *agent 2* K_2.
- *Multi-task peer prediction*: for each $k \in K_s$, payment $\{r_1^k, r_2^k, r_1^{K_1}, r_2^{K_2}\} \mapsto \mathbb{R}$
- *Strategies*: $F_{ir} = P(r_1 = r | X_1 = i)$ $\quad G_{jr} = P(r_2 = r | X_2 = j)$
  - *Informed strategy*: $F_{ir} \neq F_{jr}$, some $i \neq j$, some r
  - *Truthful strategy*: F^*

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Solution Concepts

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- *Bayes-Nash equil.*
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Family of Mechanisms

Parameterized by score $S : \{1, \ldots, m\} \times \{1, \ldots, m\} \mapsto \mathbb{R}$

- Assign agents to tasks, get reports
- For shared $k \in K_s$, pay both 1 and 2

$$S(r_1^k, r_2^k) - S(r_1^\ell, r_2^m),$$

for $\ell \in K_1$ and $m \in K_2$ (can also take empirical average)

Idea: Reward 'excess agreement' not 'default agreement.'

- Zero payment if say '1' all the time, or random report.
- For S as the identify (output-agreement) matrix, this is multi-signal generalization of DG'13.

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Analysis: Expected Payment

For identity score-matrix:

$$E(F,G) = \sum_{ij} P(i,j) \sum_r F_{ir}G_{jr} - \sum_{ij} P(i)P(j) \sum_r F_{ir}G_{jr}$$
$$= \sum_{ij} \Delta_{ij} \sum_r F_{ir}G_{jr}.$$

Delta matrix:

- $\Delta_{ij} = P(i,j) - P(i)P(j)$; if $\Delta_{ij} > 0$ then $P(j|i) > P(j)$

Example: P: $\begin{pmatrix} 0.4 & 0.15 \\ 0.15 & 0.3 \end{pmatrix}$ $\Delta \approx \begin{pmatrix} 0.1 & -0.1 \\ -0.1 & 0.1 \end{pmatrix}$ or, $\begin{pmatrix} + & - \\ - & + \end{pmatrix}$

For general S:

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Deterministic Strategies

Lemma 1

Deterministic F, G maximize $E(F, G)$.

$$E(F, G) = \max_F \max_G h(F, G) = \max_F OBJ(F),$$

where $h(F, G)$ is linear in either argument. Fixing F, opt G is deterministic. $OBJ(F)$ is convex, and opt F is deterministic.

Can *focus on deterministic strategies*:

- sufficient to prove strong-truthful, or and informed-truthful.
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The game is to find 'which scores to pick' for each (i,j) pair.

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The OA-Mechanism

S is the identity matrix.

- Categorical domain:

$$\text{sig}(\Delta) : \begin{pmatrix} + & - & - \\ - & + & - \\ - & - & + \end{pmatrix}$$

- Image labeling {swim, fly, walk}, vs. grading {76, 78, 79, . . .}

Theorem 1

The OA-mechanism is strict-proper and strongly-truthful if the world is categorical.

Obtain DG'13 as a corollary. Theorem is tight.

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$$E(F^*, G^*) = \sum_i \Delta_{ii} = \sum_{ij:\Delta_{ij}>0} \Delta_{ij} \geq \sum_{ij} \Delta_{ij} \mathbb{1}(F(i) = G(j)) = E(F, G),$$

for all F, G.

Also need: tie in payment ⇒ permutation strategy.

- Case 1: Not permutation, and symmetric. Must be two i, j ($i \neq j$) that map to r. Assign $\Delta_{ij} < 0$ pair to score $S_{r,r} = 1$. Worse!
- Case 2: Asymmetric, e.g., agent 1 strategy $i \mapsto r$, agent 2 strategy $i \mapsto r'$. Assign $\Delta_{ii} > 0$ pair to score $S_{r,r'} = 0$. Worse!

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for all F, G.

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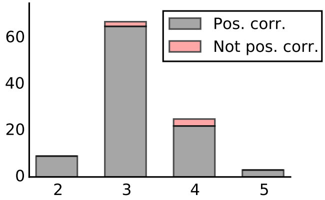
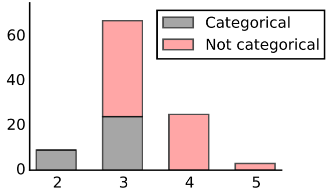
Are we done? Let's look at some data.

- Peer-evaluation responses to 100 questions across 30 exercises in 17 MOOCs
- Vast majority of questions have $m \in \{2, 3, 4\}$.
  - Example rubric element: "Not much of a style at all", "Communicative style", and "Strong, flowing writing style".





The 01-mechanism

$$S_{ij} = \begin{cases} 1 & \text{, if } \Delta_{ij} > 0 \\ 0 & \text{o.w.} \end{cases} \quad \Delta = \begin{pmatrix} + & + & - \\ + & + & - \\ - & - & + \end{pmatrix} \quad S = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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For general domains, the 01-mechanism is informed truthful (and proper).

$$E(F^*, G^*) = \sum_{ij} \Delta_{ij} S_{ij} = \sum_{ij:\Delta_{ij}>0} \Delta_{ij} \geq E(F, G), \quad \forall F, G$$

Also need to show uninformed $\Rightarrow$ strictly less payment.
Fix G, consider uninformed F (e.g., $F(i) = `1'$, for all i). Have $E(F, G) = \sum_{ij} \Delta_{ij} S_{1,G(j)} < E(F^*, G^*)$.

Note: indifference between '1 → 1, 2 → 2' and '1 → 1, 2 → 1'.

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Parameterized $0 \leq a < b < c < d$

Scores: $S_{ii} = \begin{cases} b & \text{, if } \Delta_{ii} \leq 0 \\ c & \text{o.w.} \end{cases}$ $\quad S_{ij} = \begin{cases} a & \text{, if } \Delta_{ij} \leq 0 \\ d & \text{o.w.} \end{cases}$

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For general domains, the ABCD-mechanism is strong-truthful amongst symmetric strategies.

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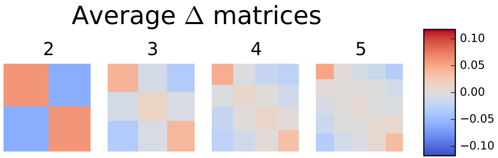
- 17 courses, 104 questions, ∼325,000 reports.



Positive correlation.

For models of size 4 and 5, see failure of categorical (e.g., score 2 is +ve correlated with score 3.) *Ordinal domain*.

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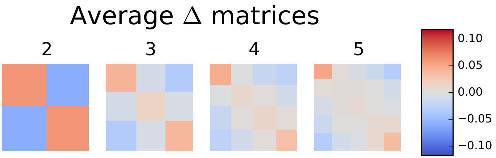

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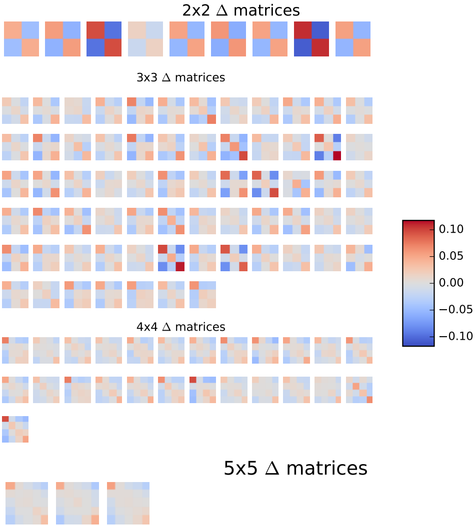

Empirical observations

34 of 104 worlds are categorical:

- All mechanisms are strong-truthful and strict-proper.

In the other 70 worlds:

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- 01-mechanism is informed-truthful and proper. It is also *strict-proper* in 19/70 worlds.
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Discussion

- Is there a proper and strong-truthful (symmetric) mechanism for general domains? Perhaps leveraging two S matrices?
- Can heterogeneity be handled (e.g., “pushing” reports towards categorical)?
- Prior-free design: can we use observed data to design and then apply a score matrix?
- Population learning: does strong- or informed-truthful promote convergence to truthful equilibrium?
- Richer models of effort.
- Experiments and applications.

Thank you