Solving SVP in 2^n Time Using Discrete Gaussian Sampling

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Noah Stephens-Davidowitz

Before We Start

I'm going to talk about an exact algorithm. To break crypto, you only need to approximate SVP to within some polynomial factor.

(The fastest algorithm to provably break crypto runs in $2^{0.4n}$ time [Sch87, GN08, LWXZ11].)

Before We Start

This algorithm is easy to understand. If you aren't following, that is my fault. So, please interrupt me frequently.

Lattices

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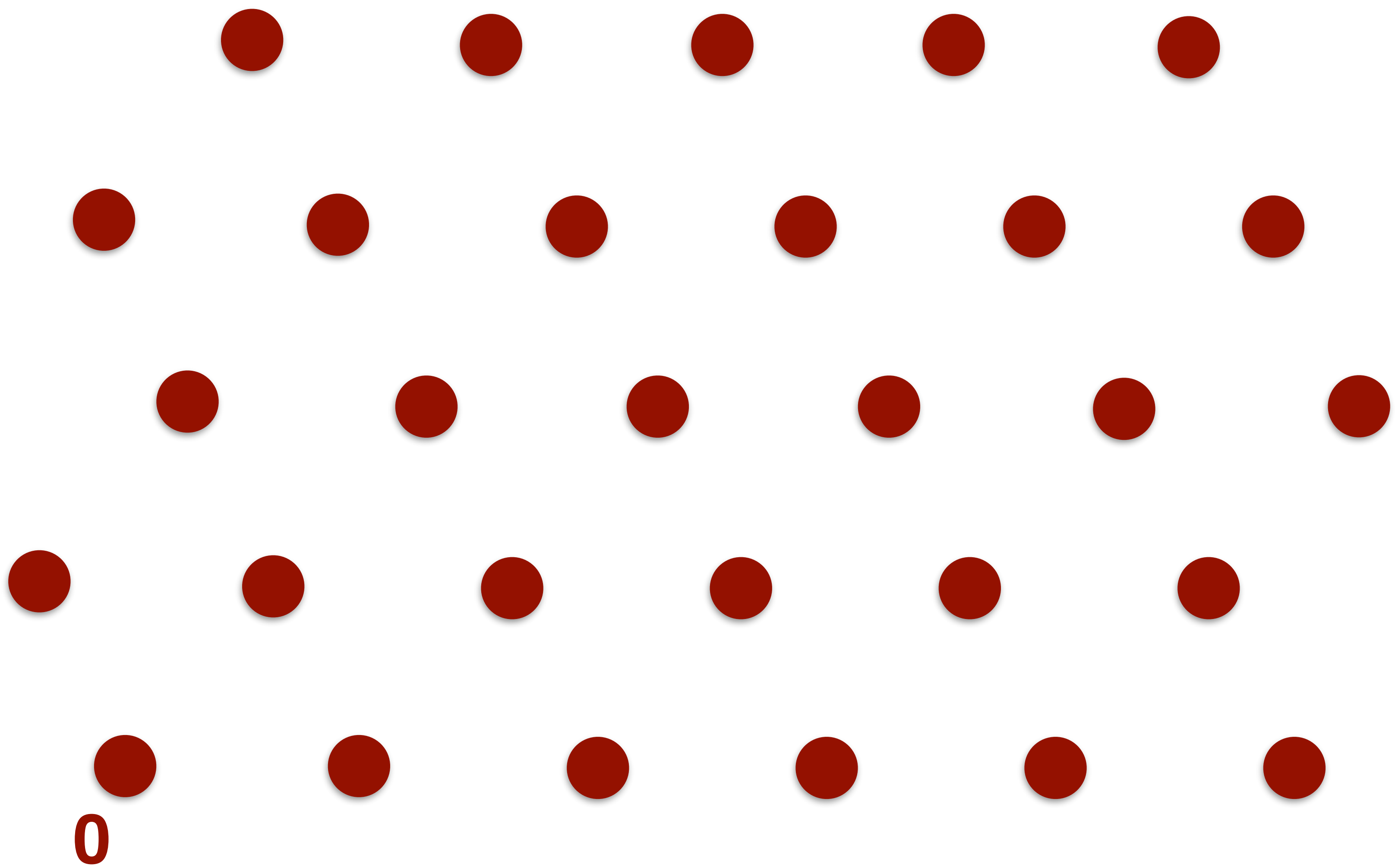

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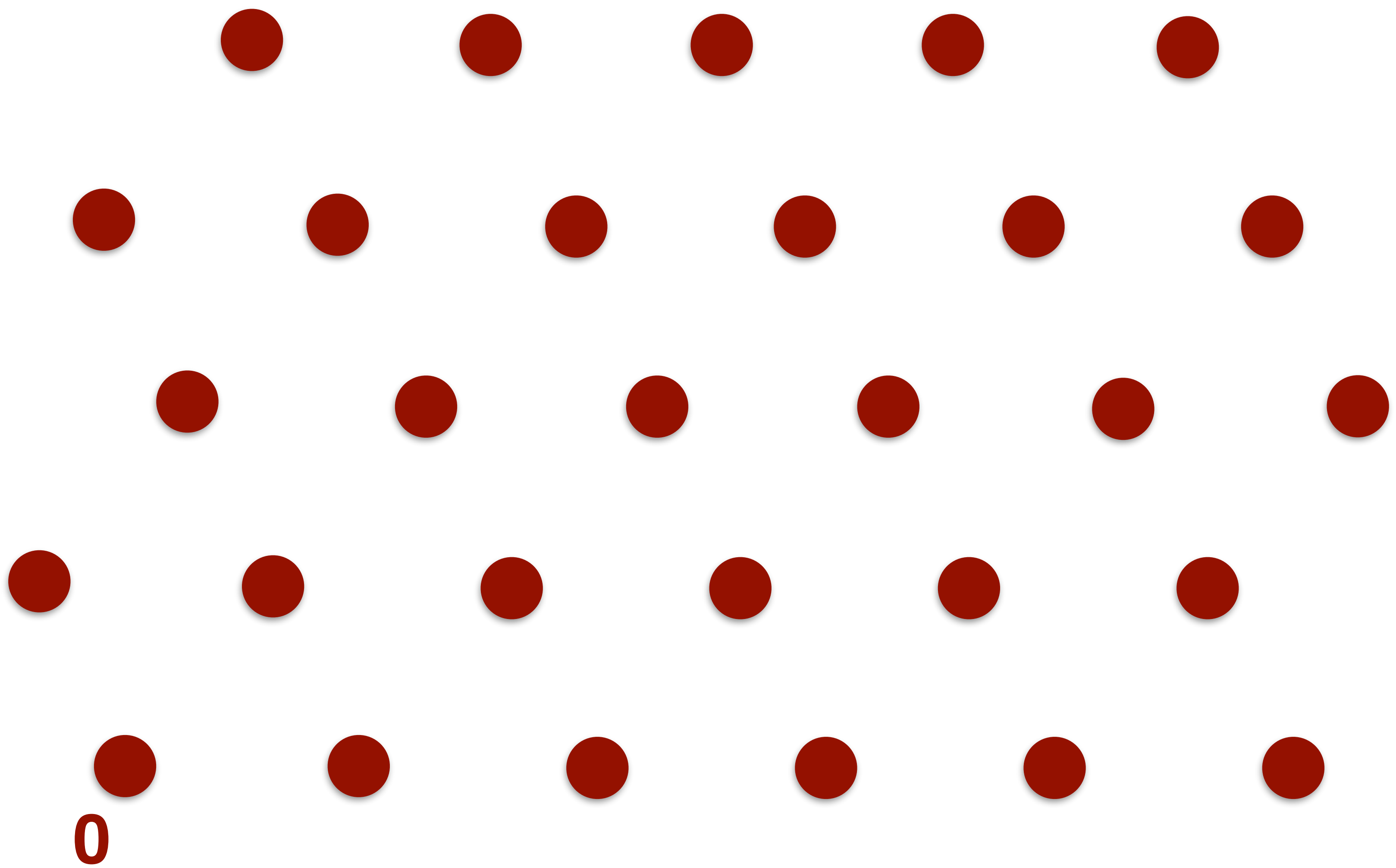
- Specified by a basis $\mathbf{b}_1, \ldots, \mathbf{b}_n$, linearly independent vectors



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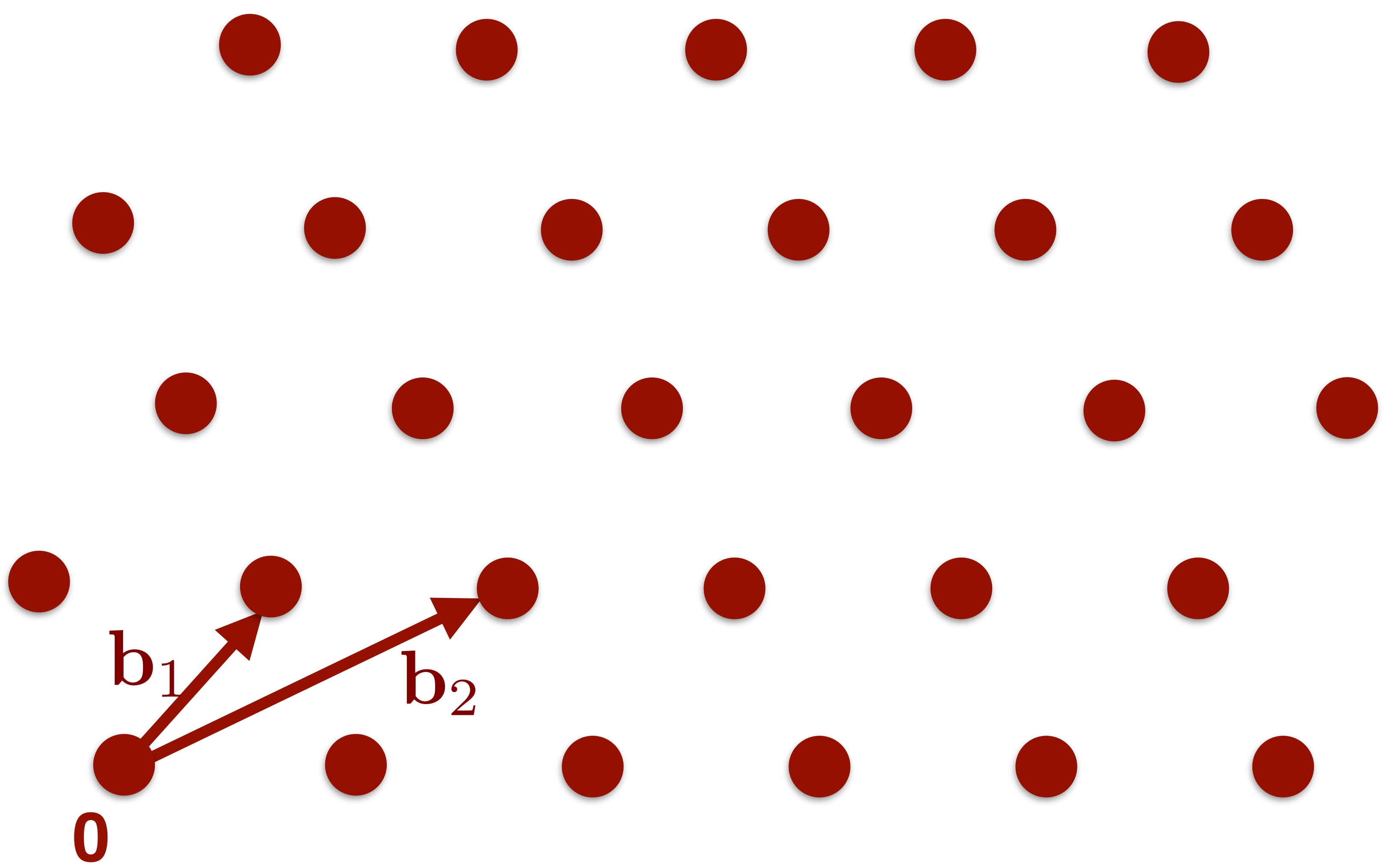

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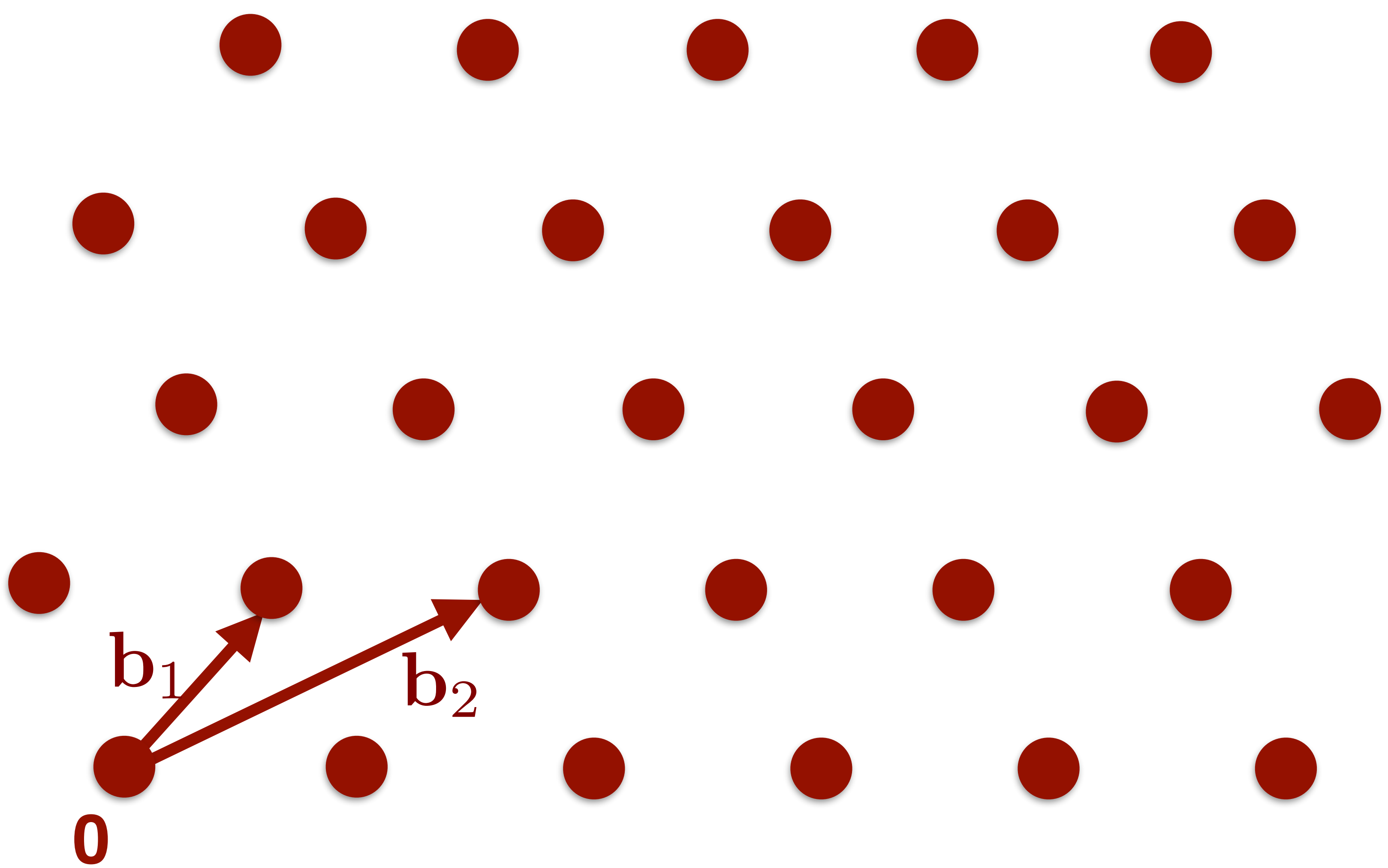
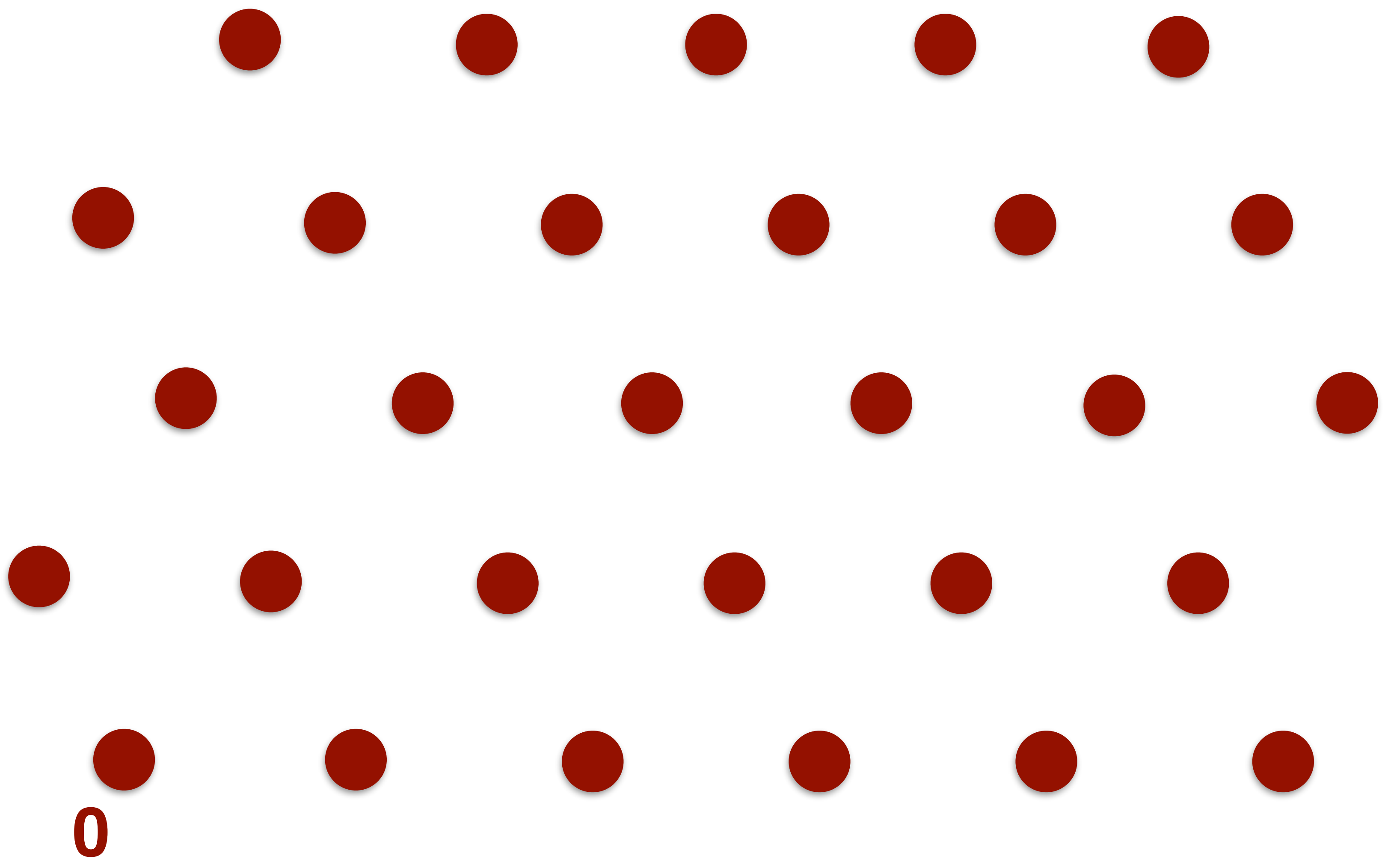
- Specified by a basis $\mathbf{b}_1, \ldots, \mathbf{b}_n$, linearly independent vectors
- $\mathcal{L} = \{a_1\mathbf{b}_1 + \cdots + a_n\mathbf{b}_n \mid a_i \in \mathbb{Z}\}$



The Shortest Vector Problem



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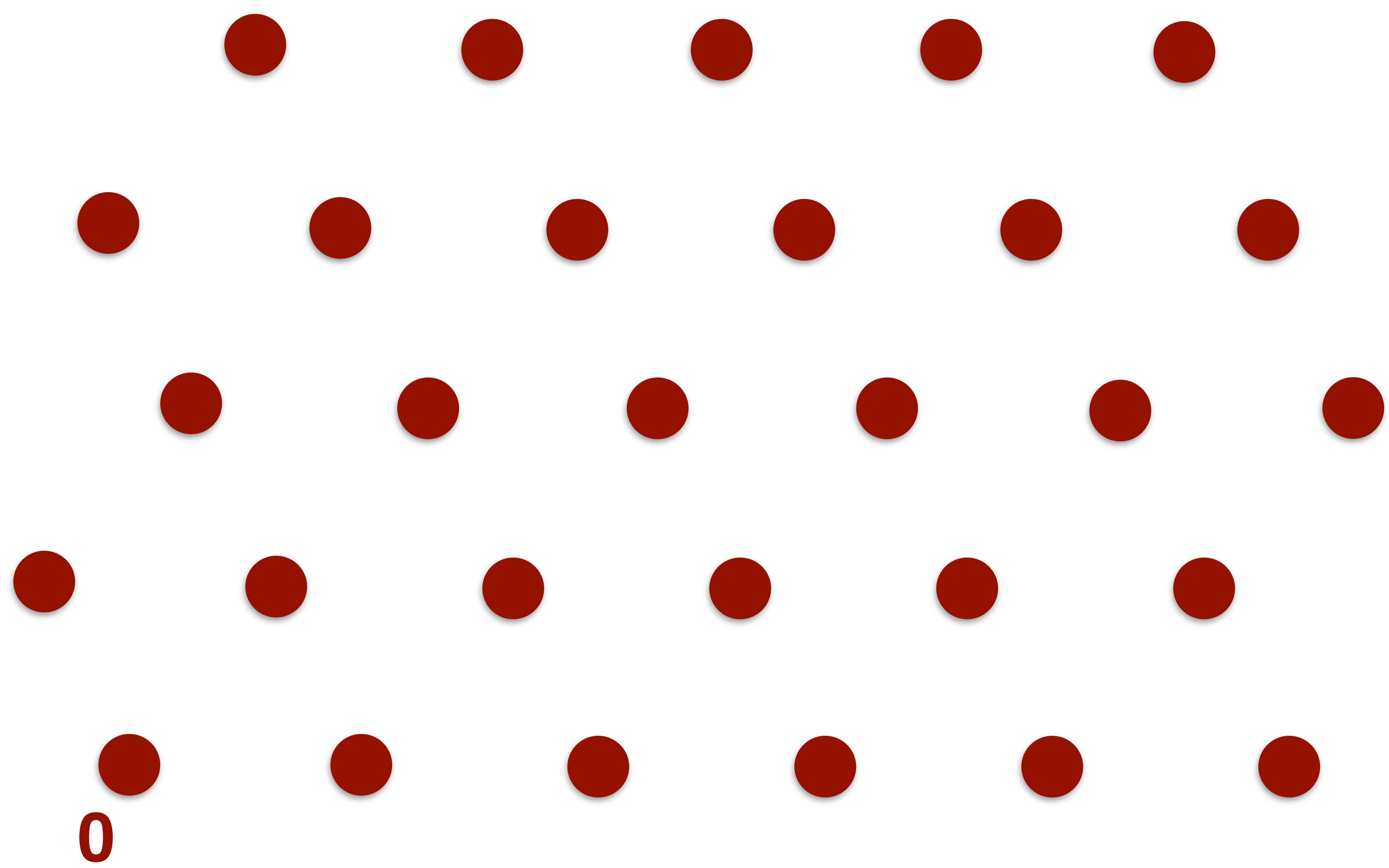
- $\text{SVP}(\mathcal{L}) =$ shortest non-zero $\mathbf{y} \in \mathcal{L}$



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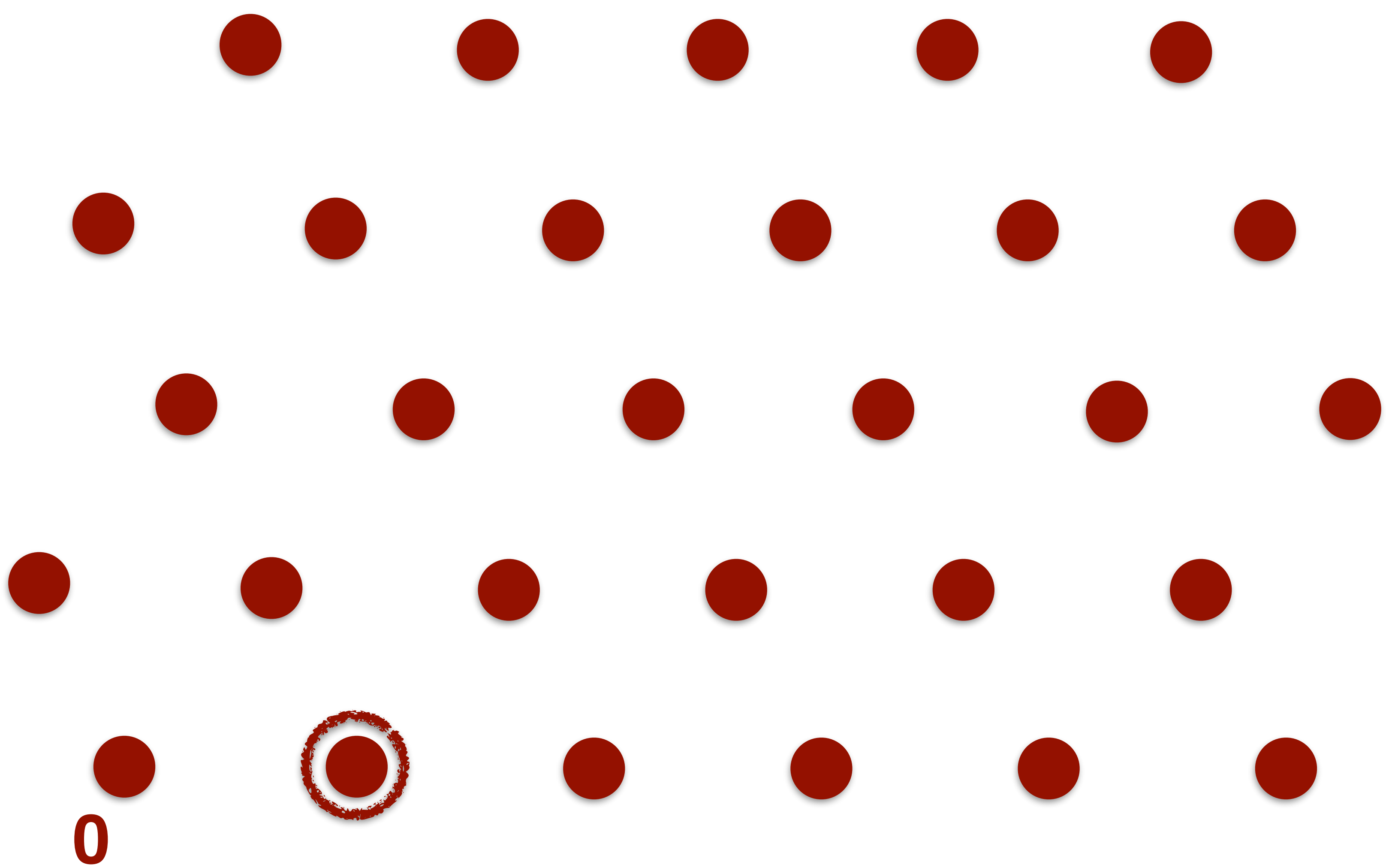

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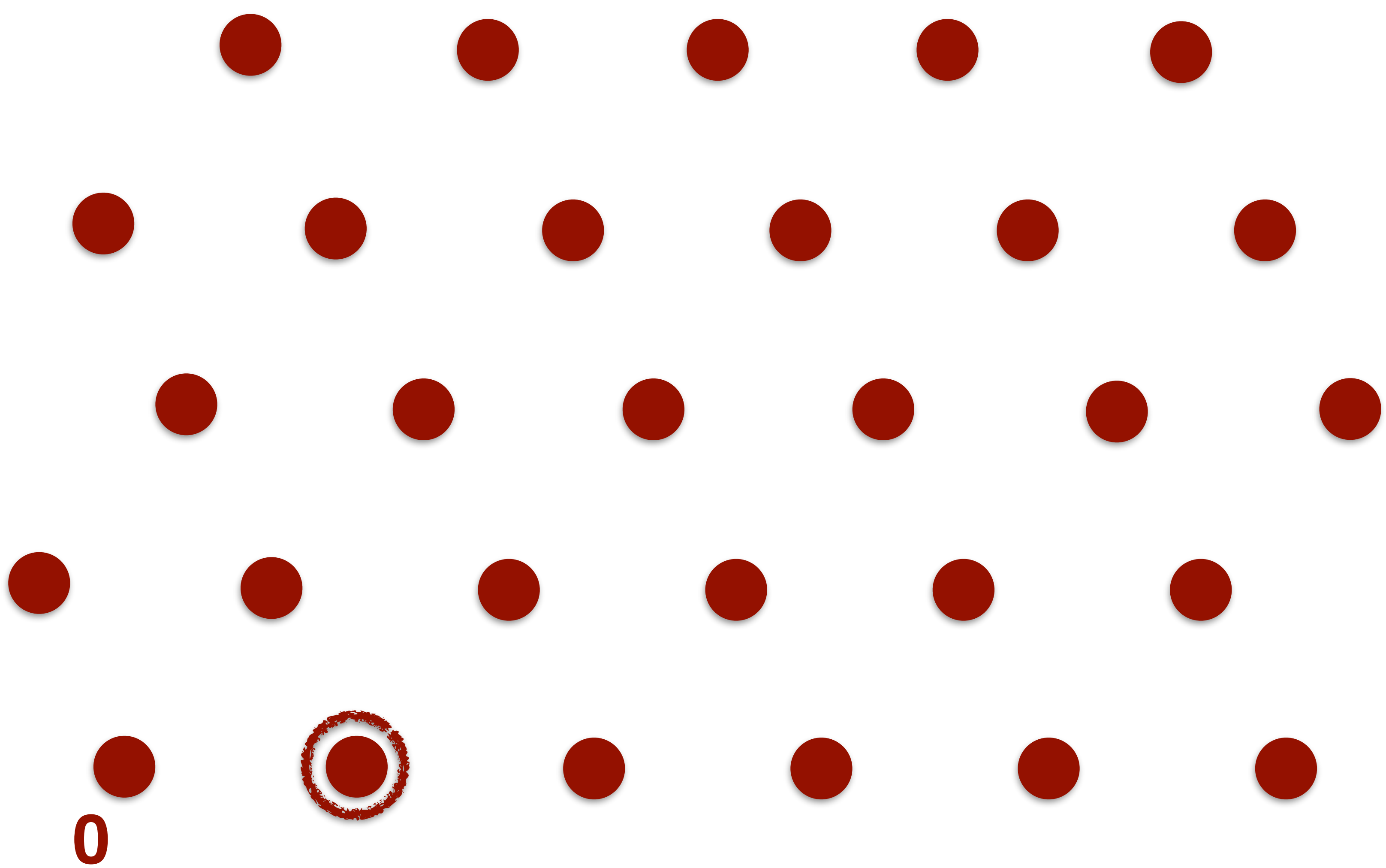

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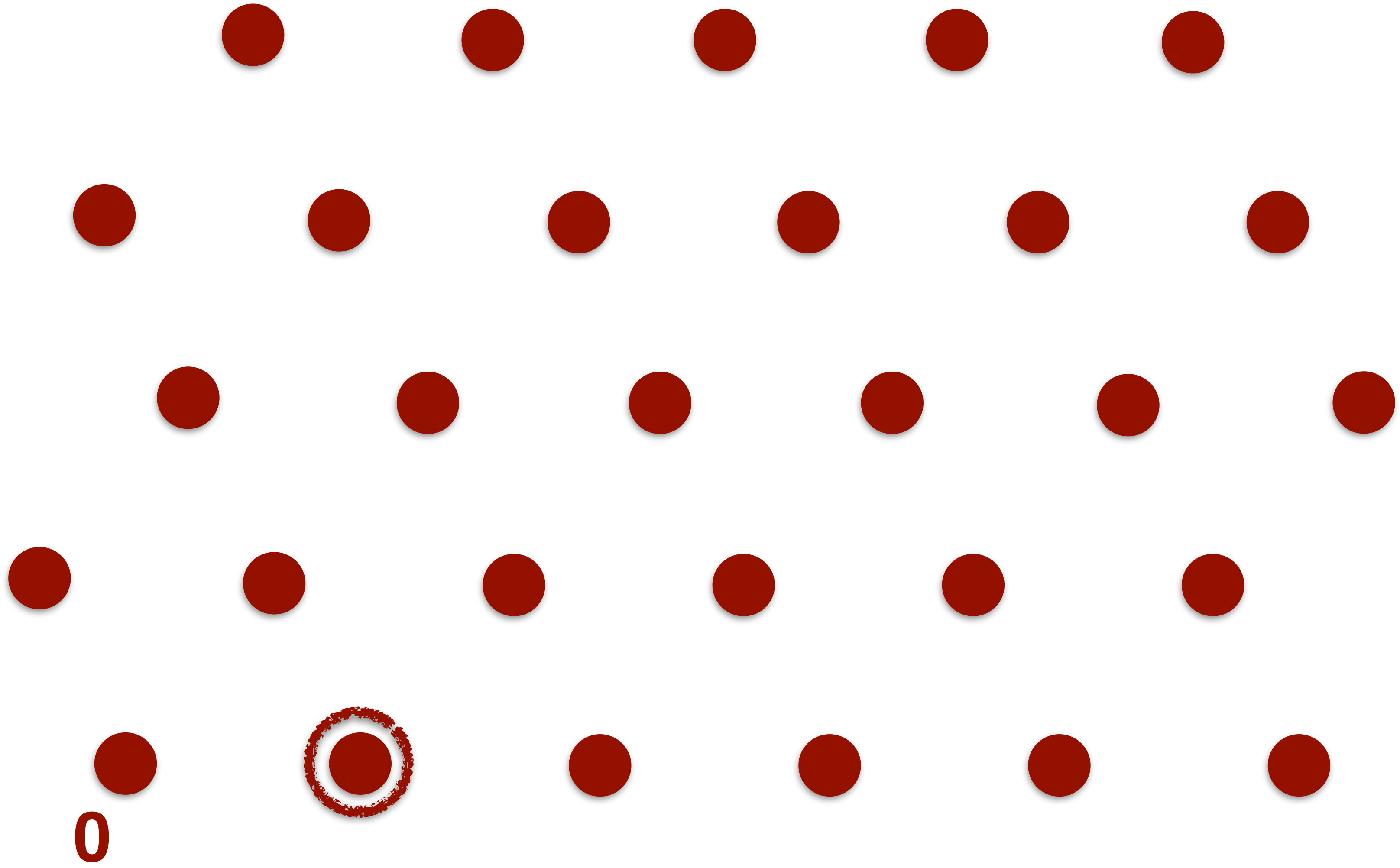

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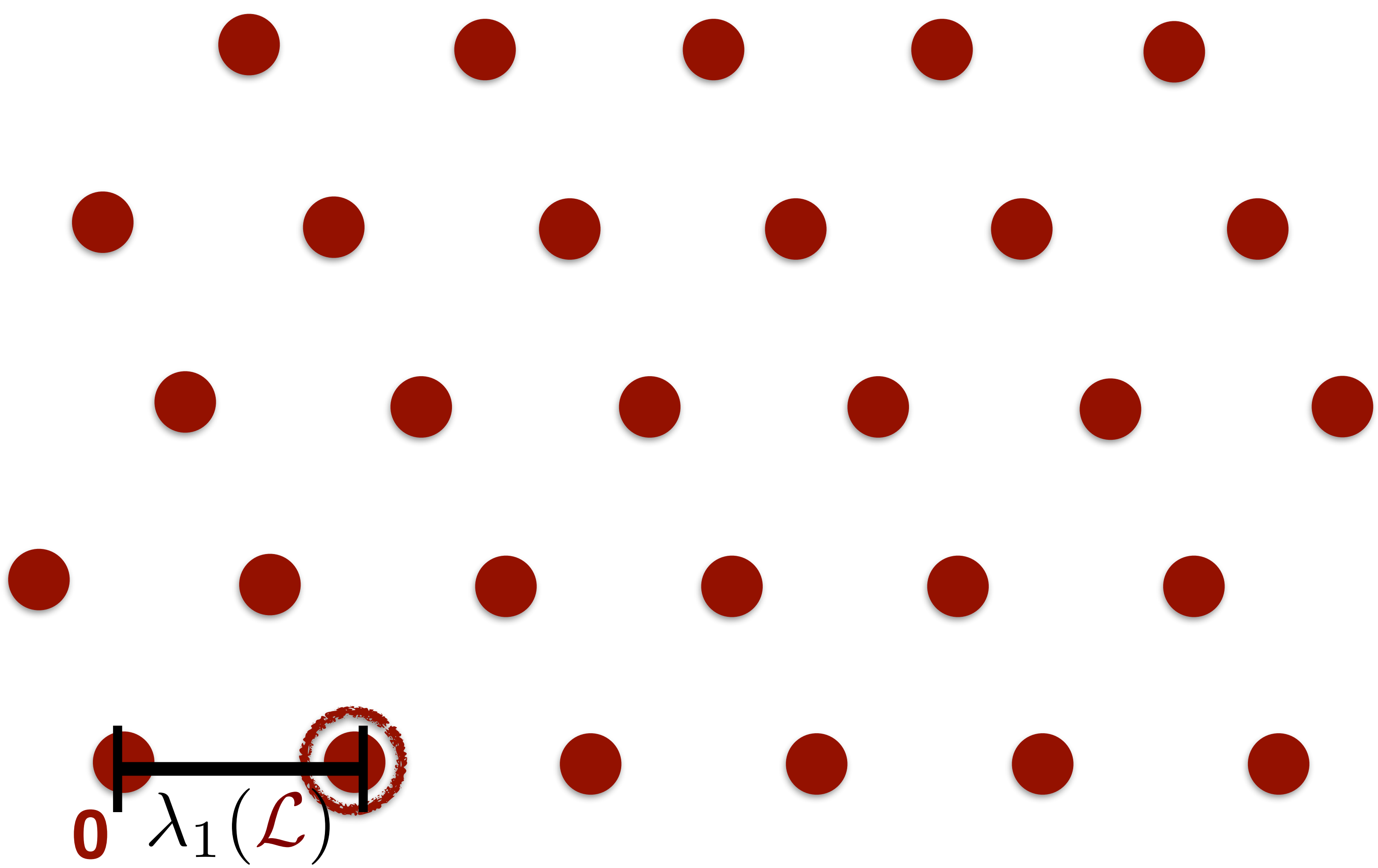

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This work (Discrete Gaussian sampling)	$2^{n+o(n)}$	$2^{n+o(n)}$

Our Algorithm

Gaussian Distribution

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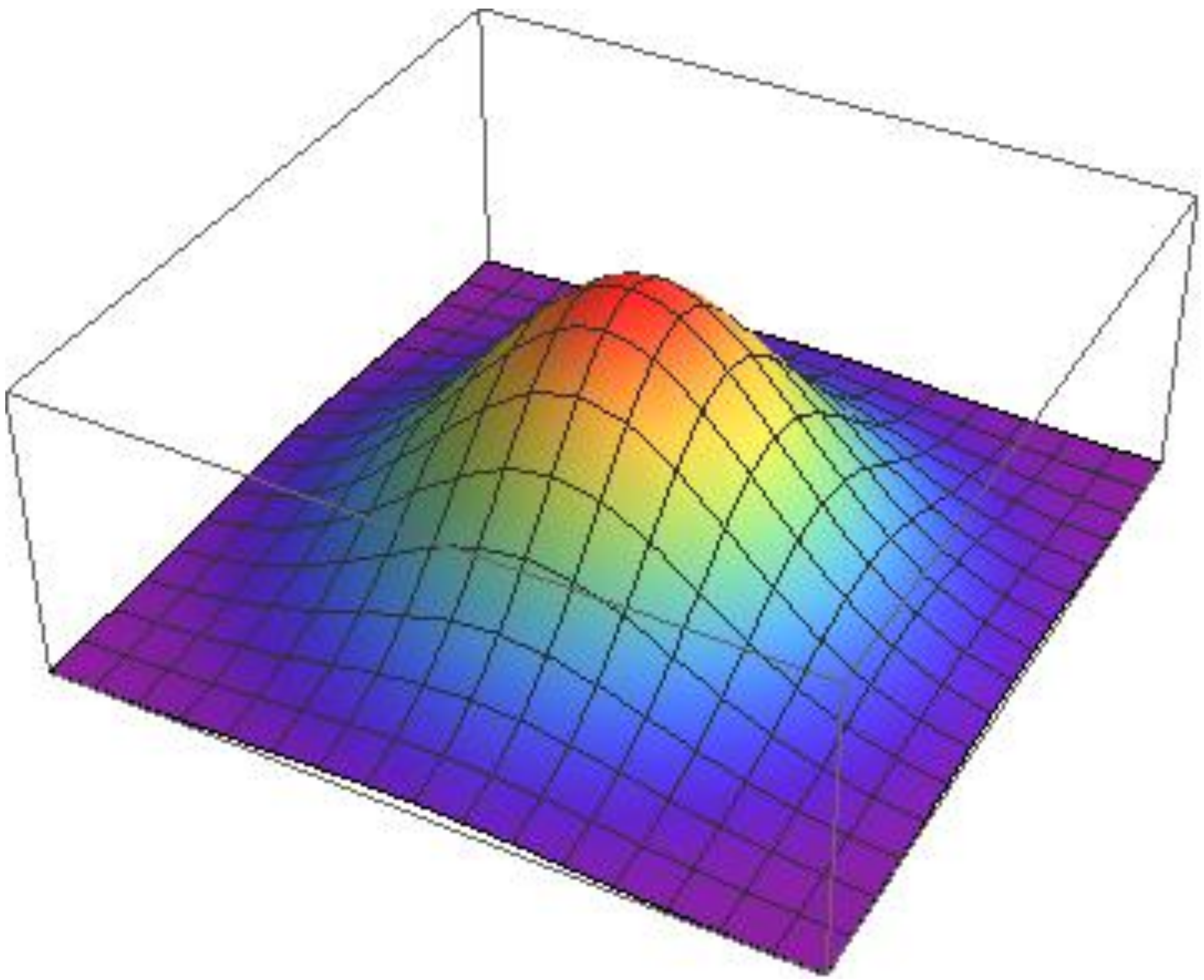

$s = 20$

Gaussian Distribution

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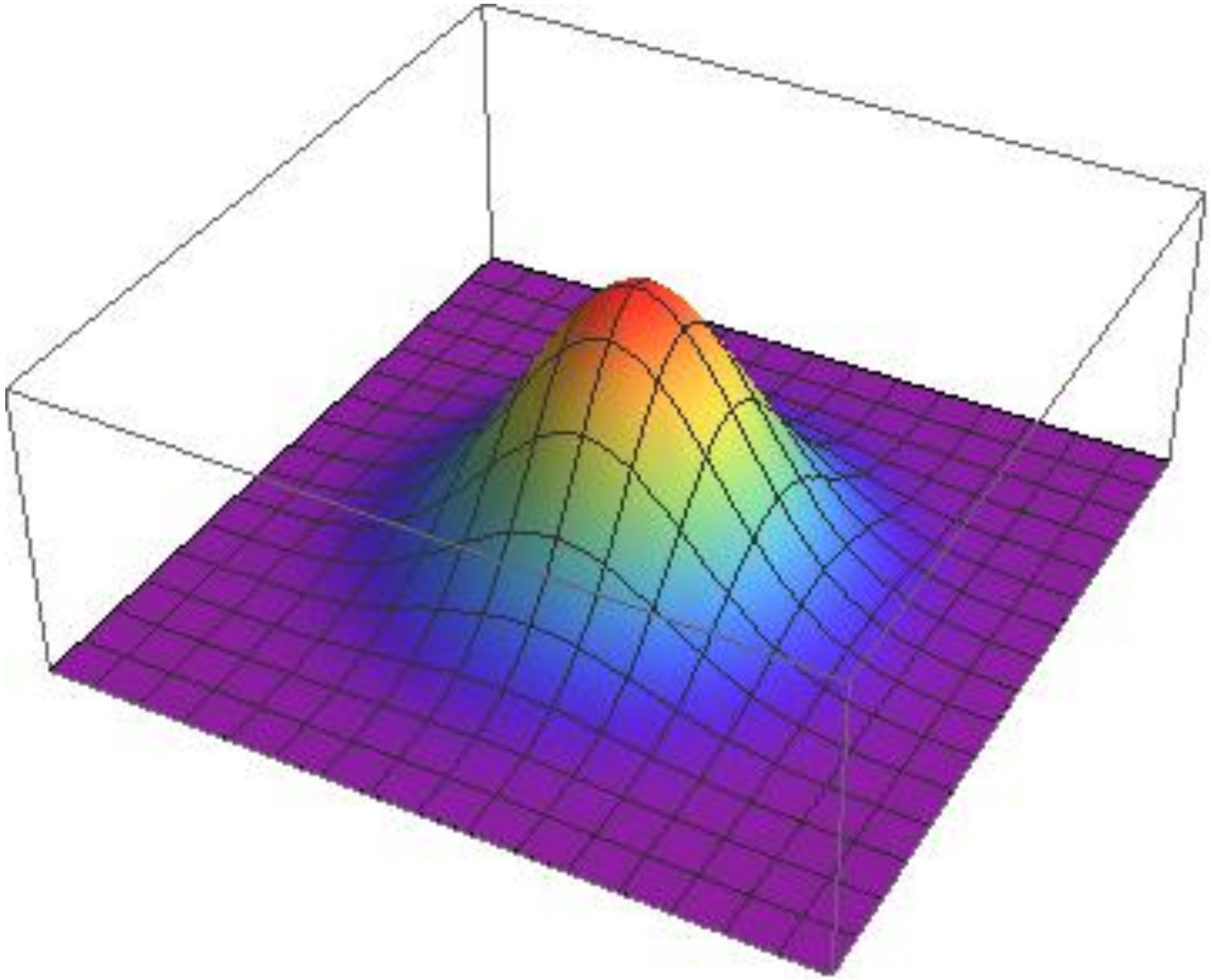

$s = 10$

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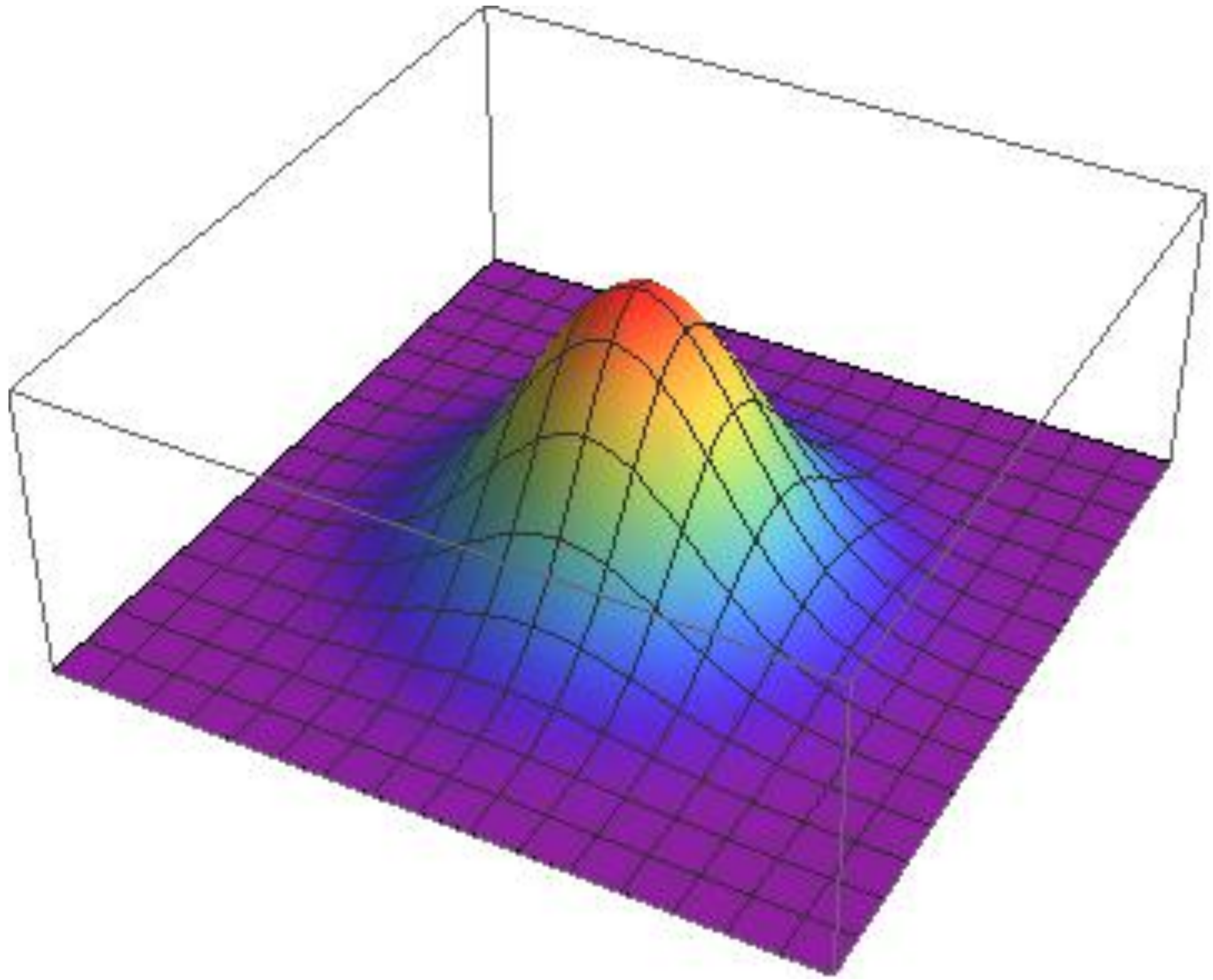

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Discrete Gaussian Distribution

$$D_{\mathcal{L},s} := \Pr[\mathbf{y}] \propto e^{-\|\mathbf{y}\|^2/s^2}$$

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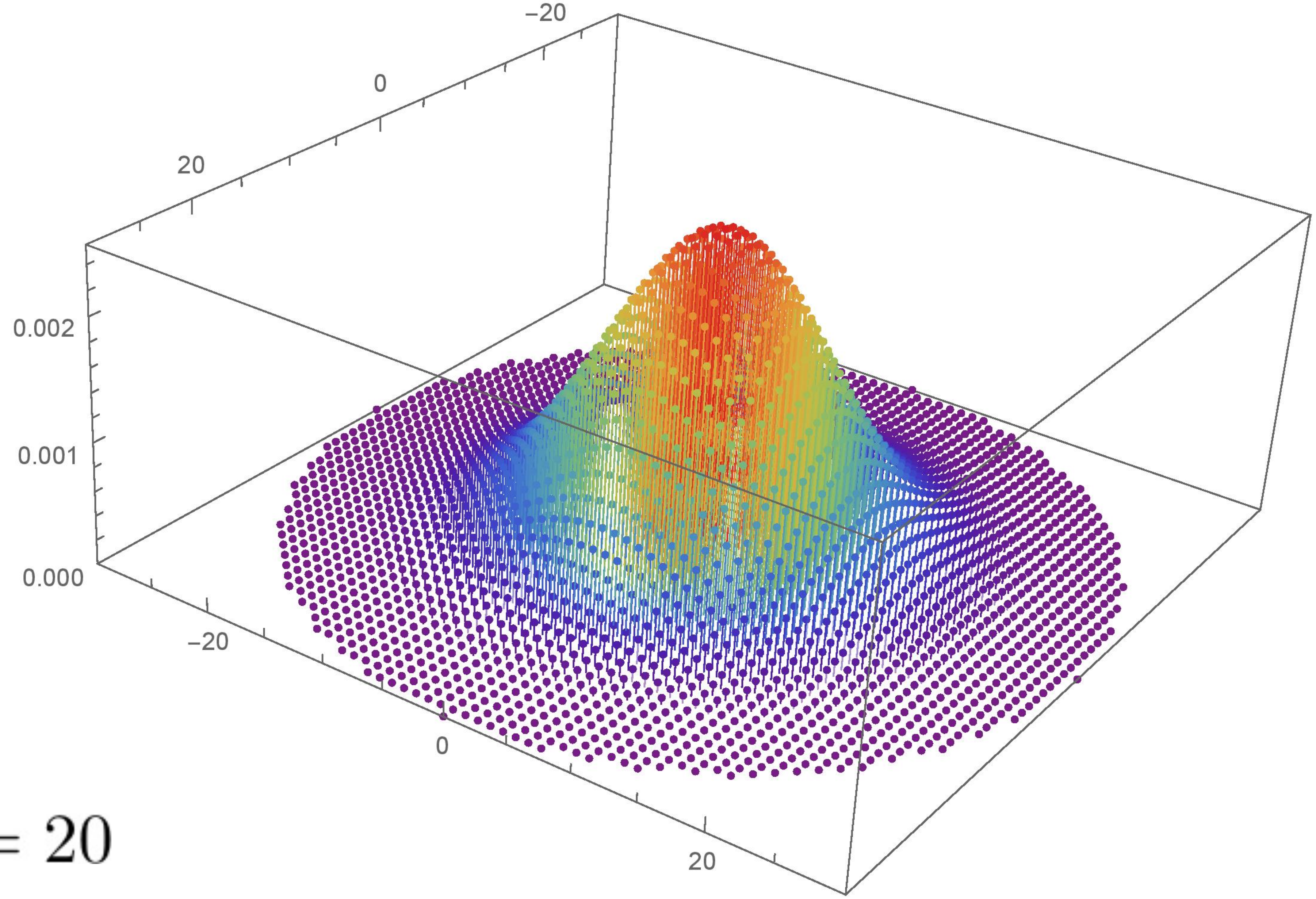

$s = 20$

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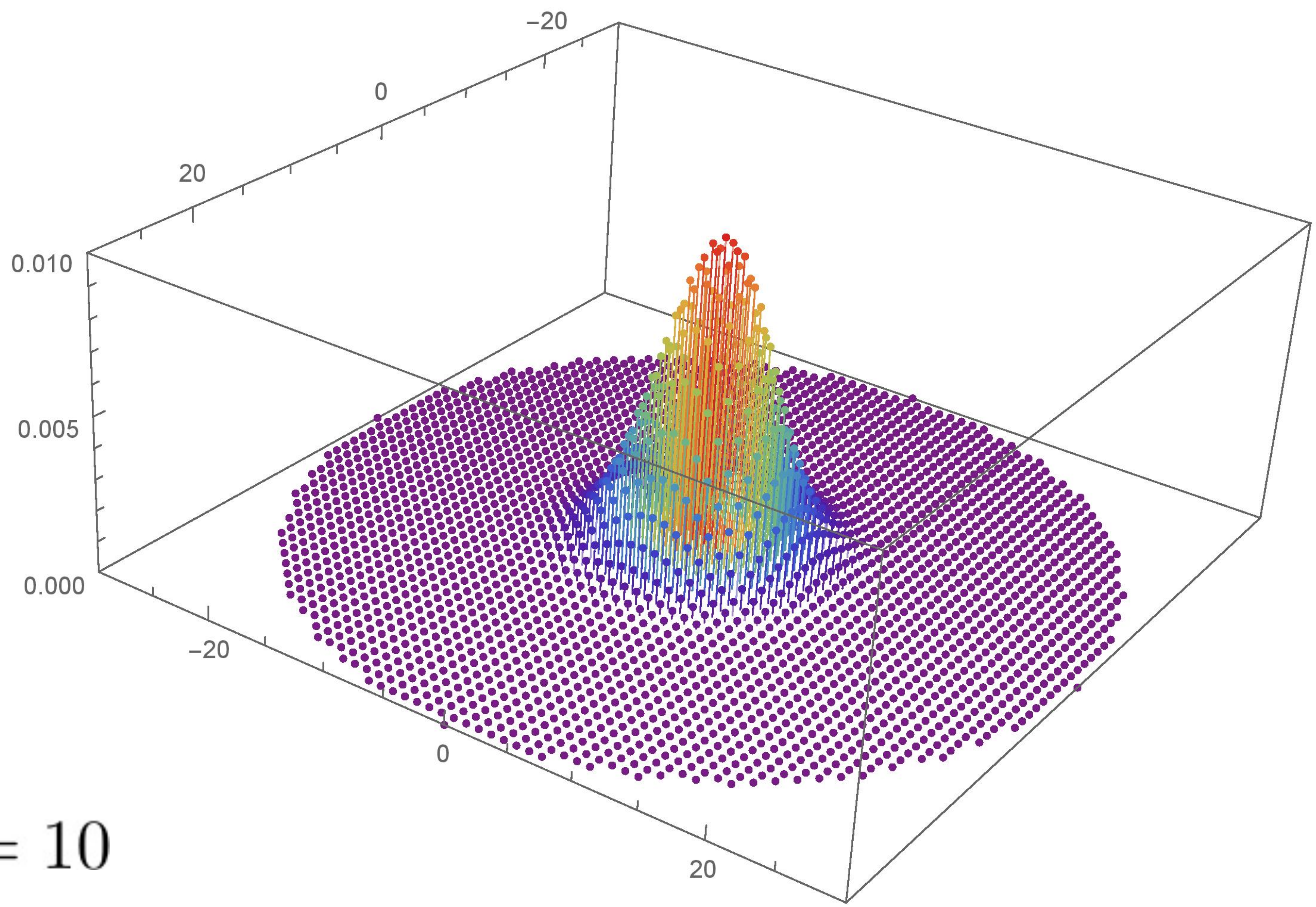

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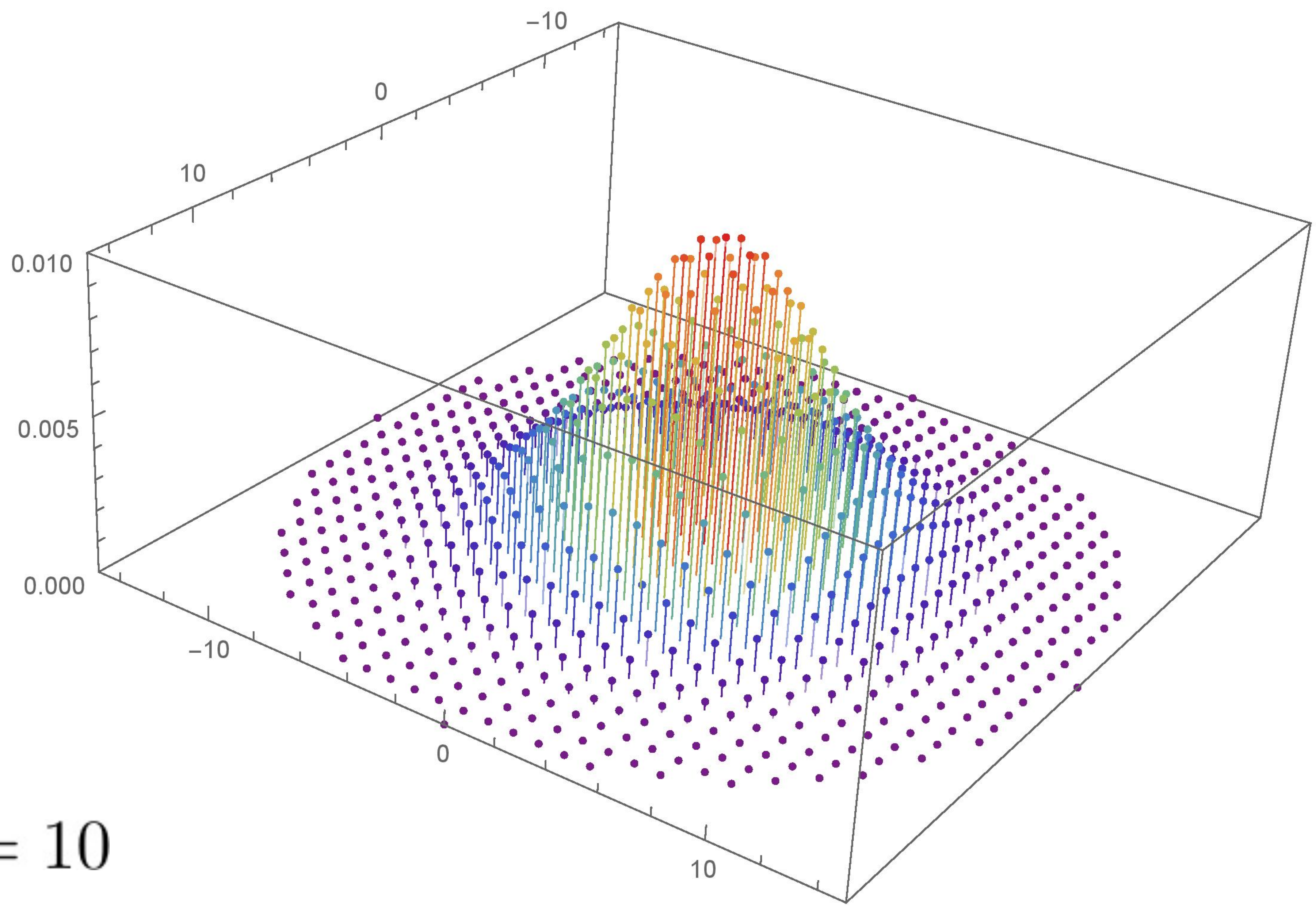

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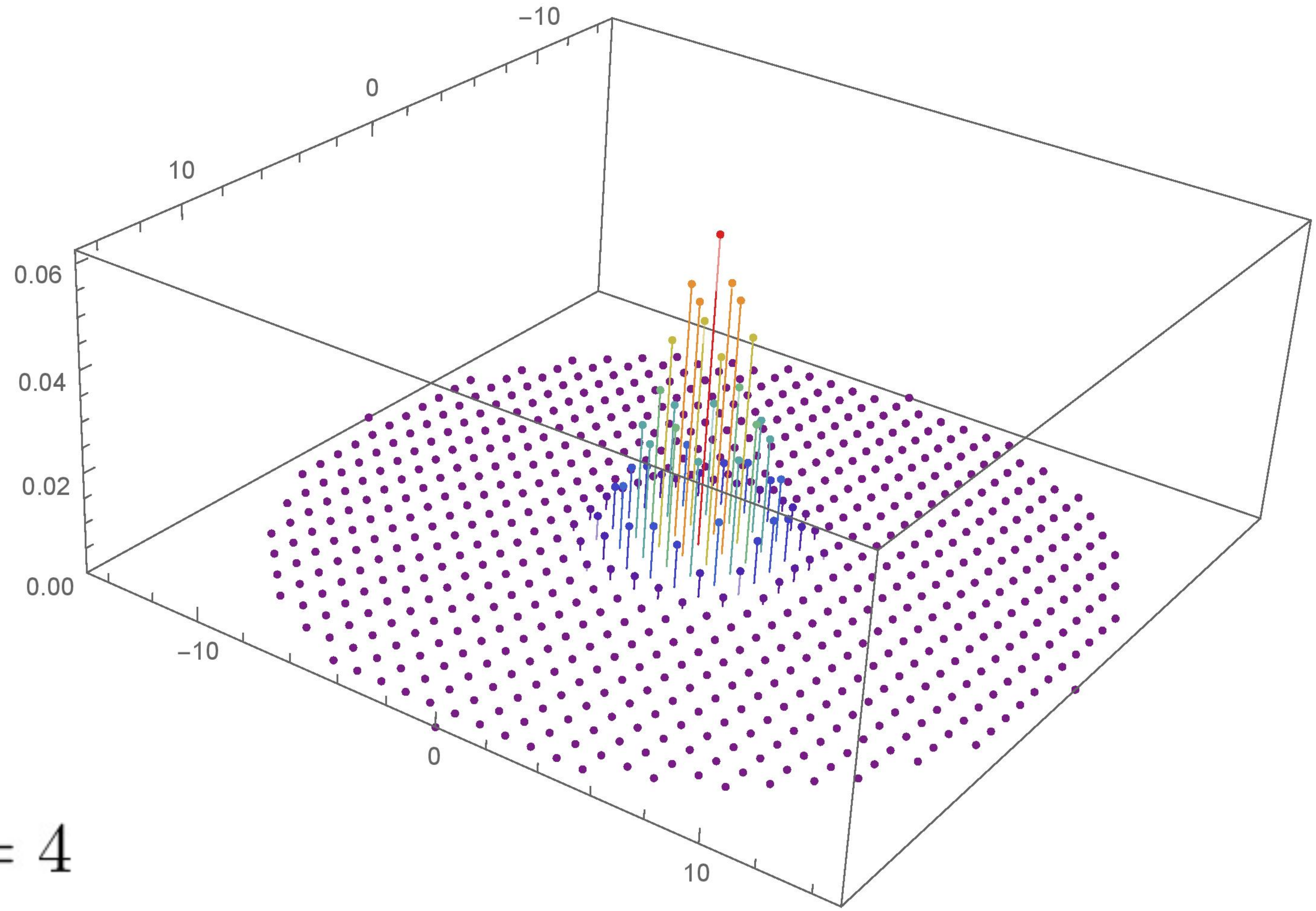

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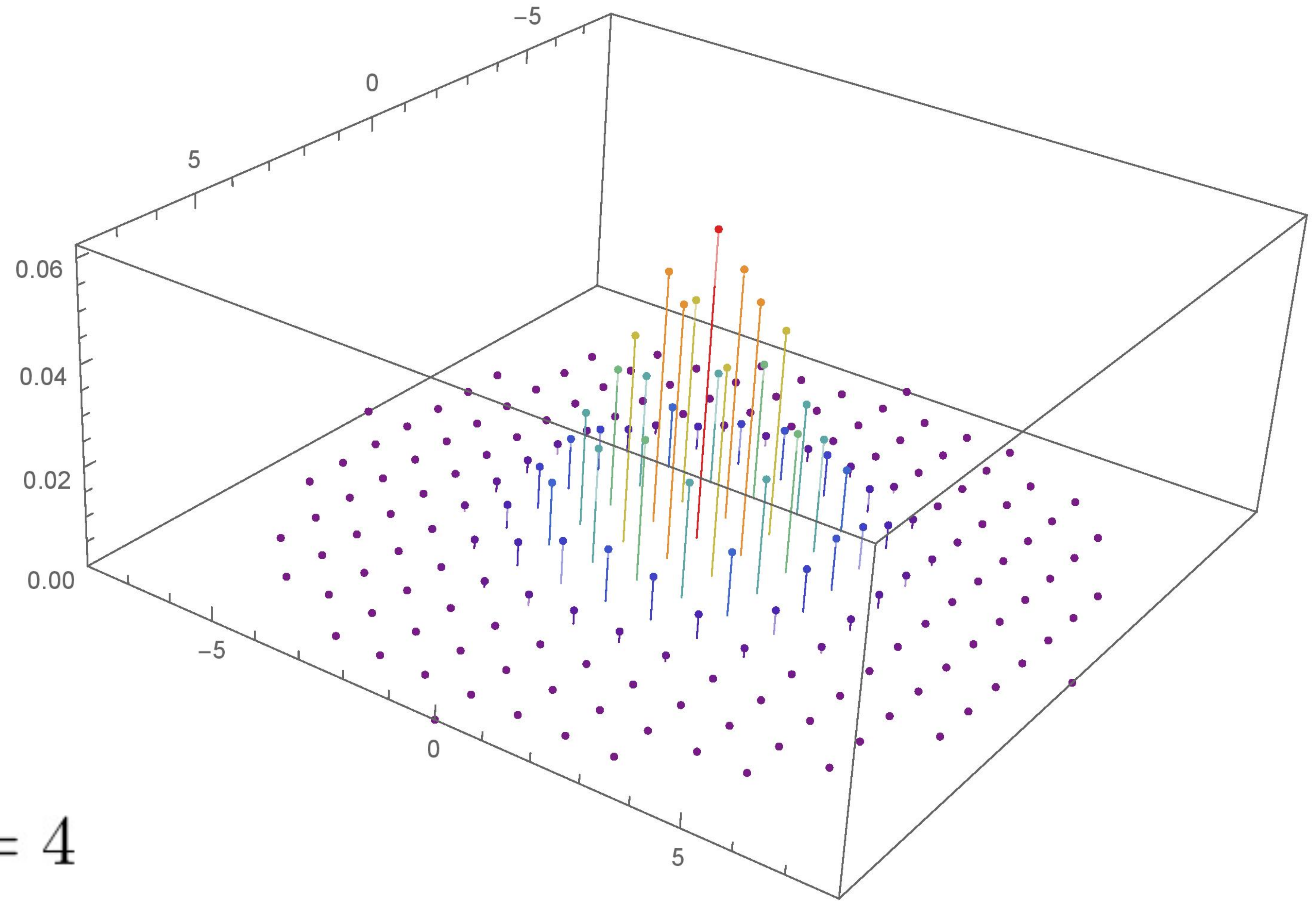

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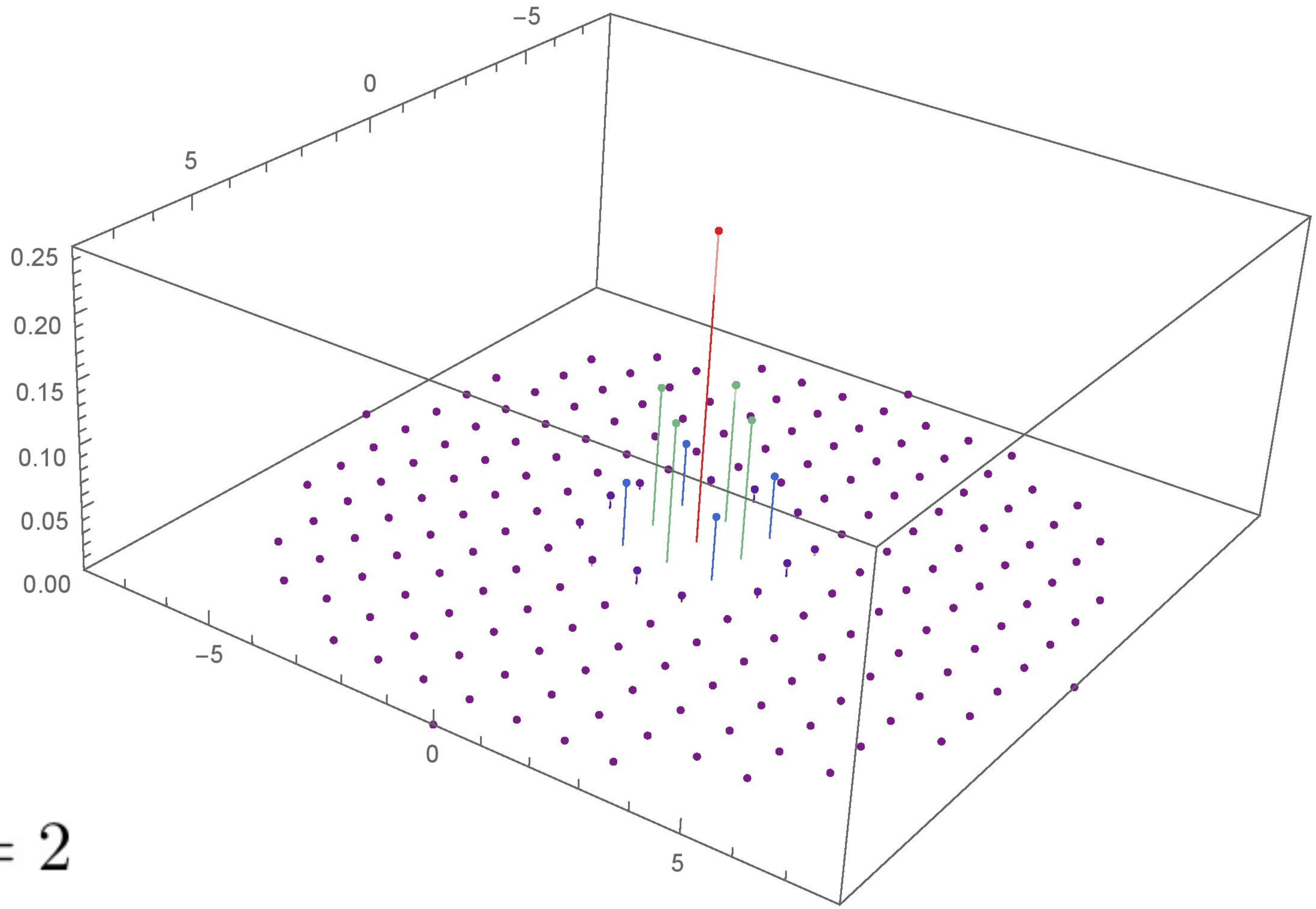

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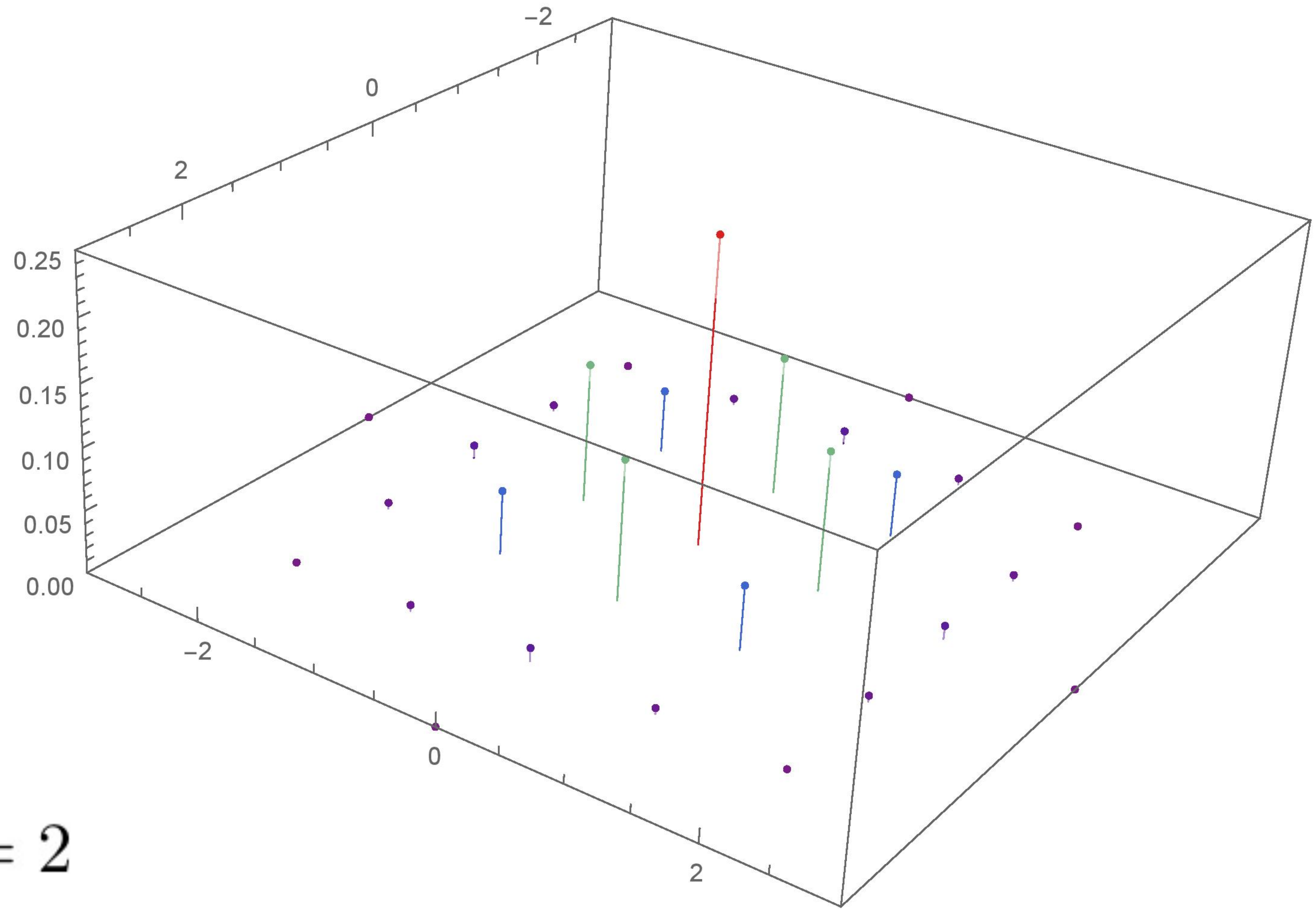

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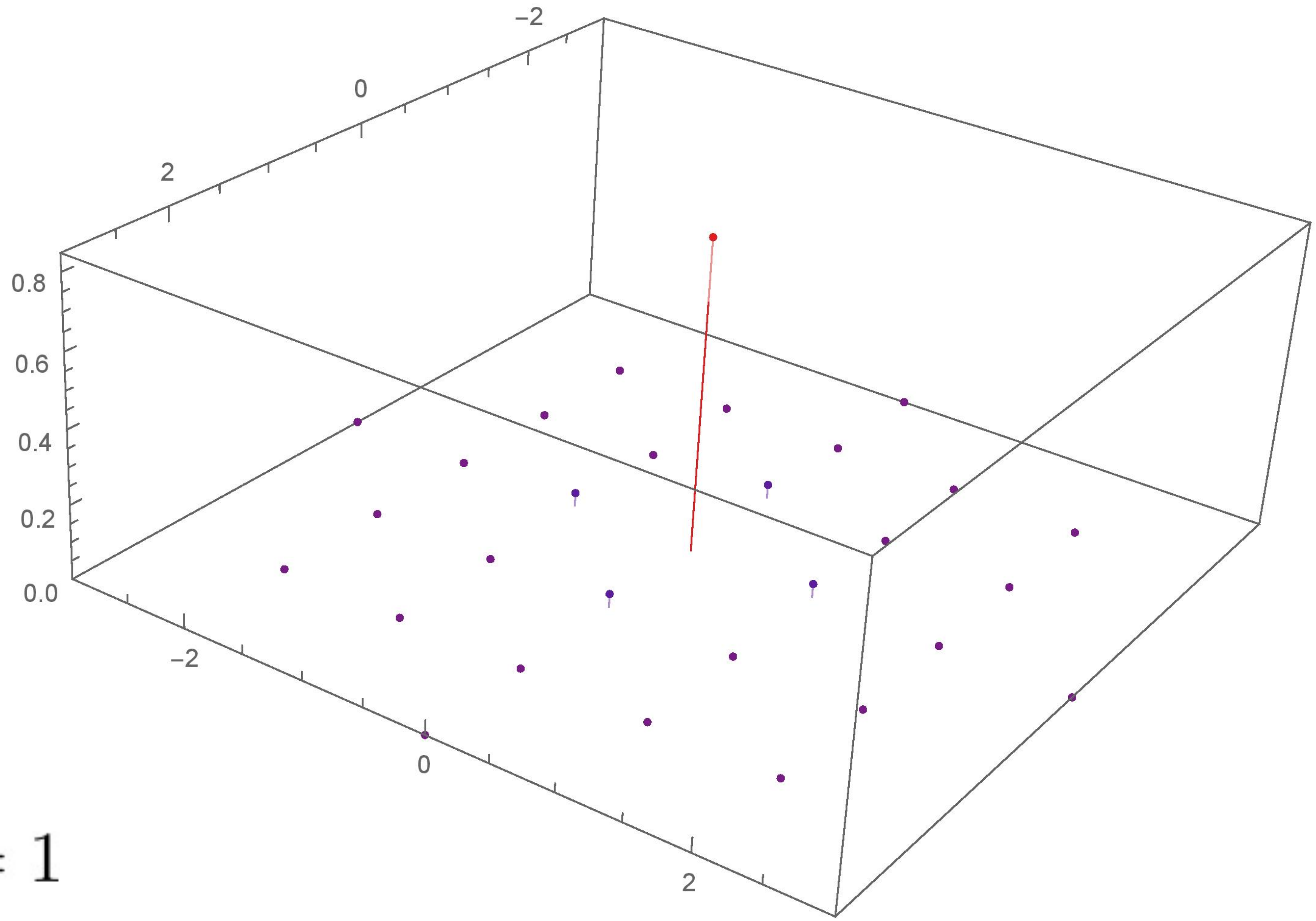

$s = 1$

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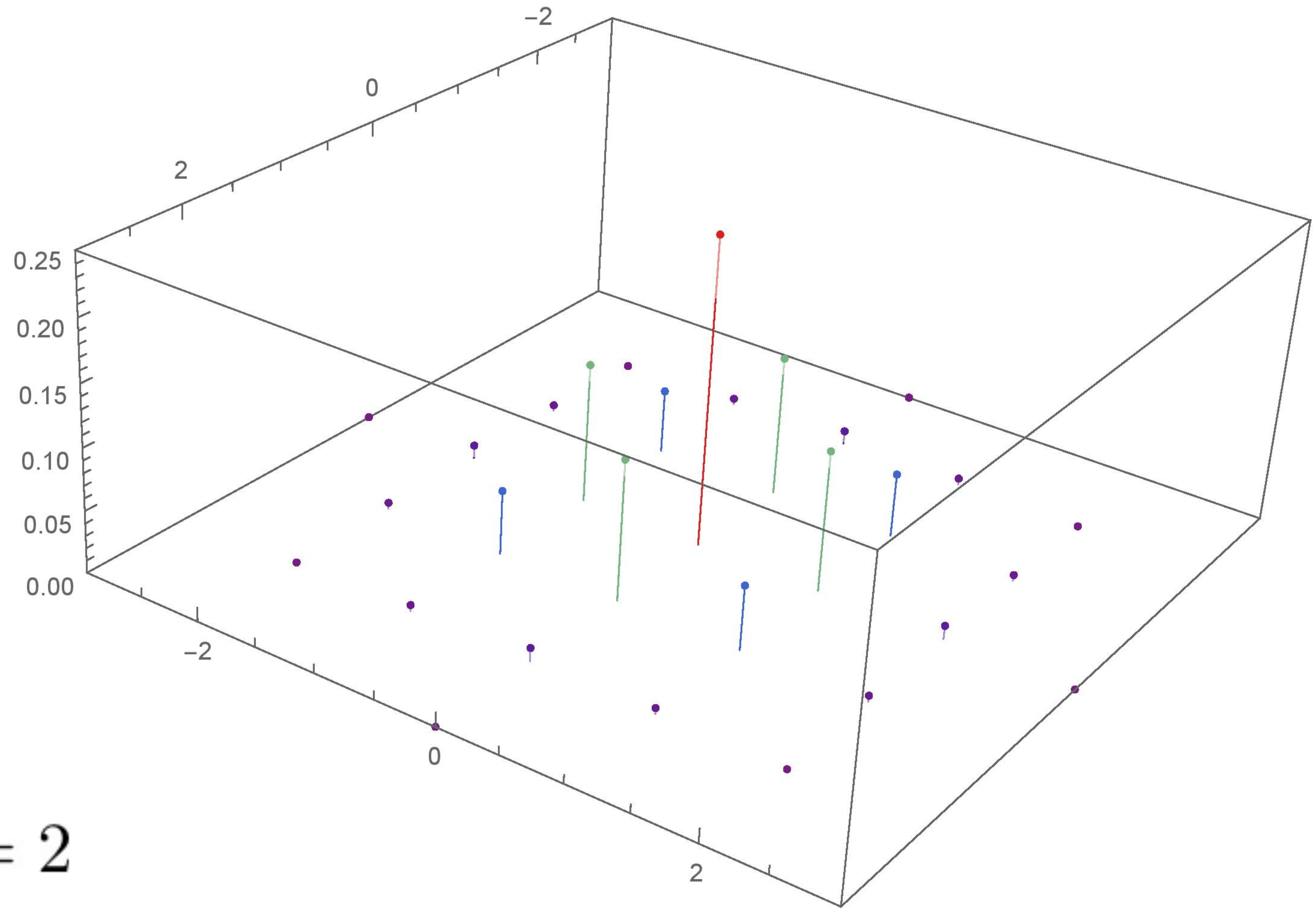

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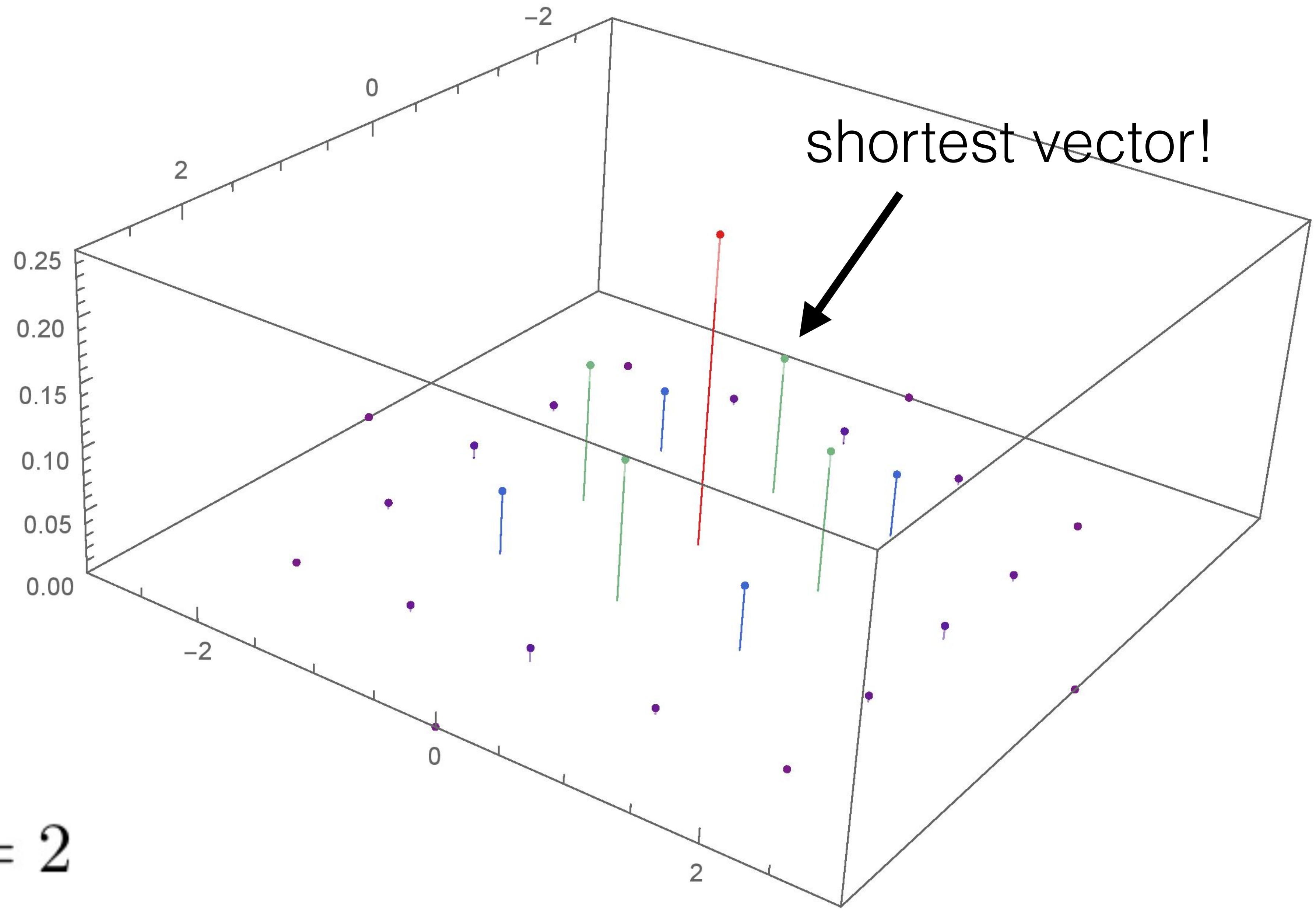

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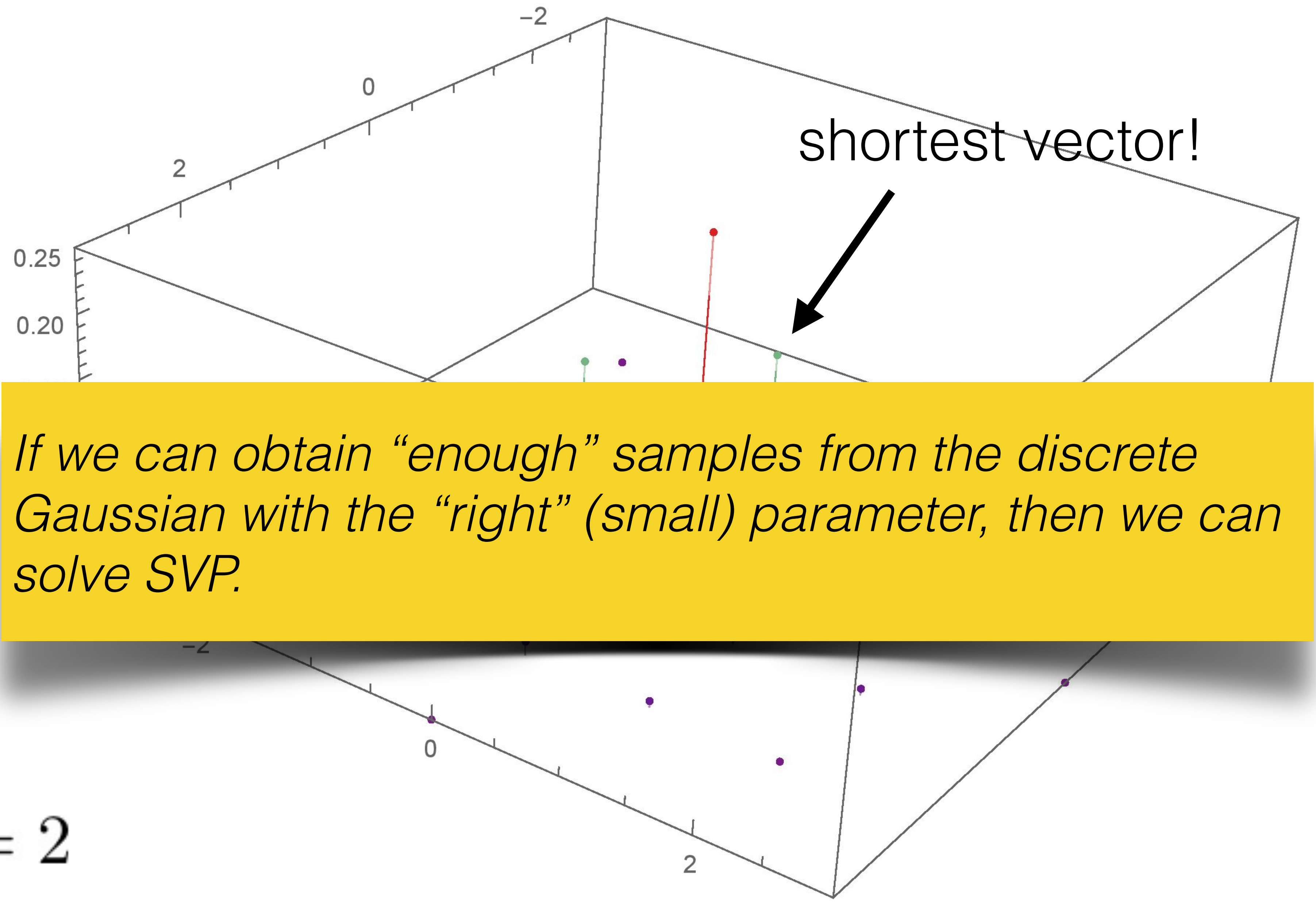

If we can obtain "enough" samples from the discrete Gaussian with the "right" (small) parameter, then we can solve SVP.

$s = 2$

Discrete Gaussian Distribution

We need at most $\approx 1.38^n$ vectors with $s \approx \lambda_1(\mathcal{L})/\sqrt{n}$ [KL78].

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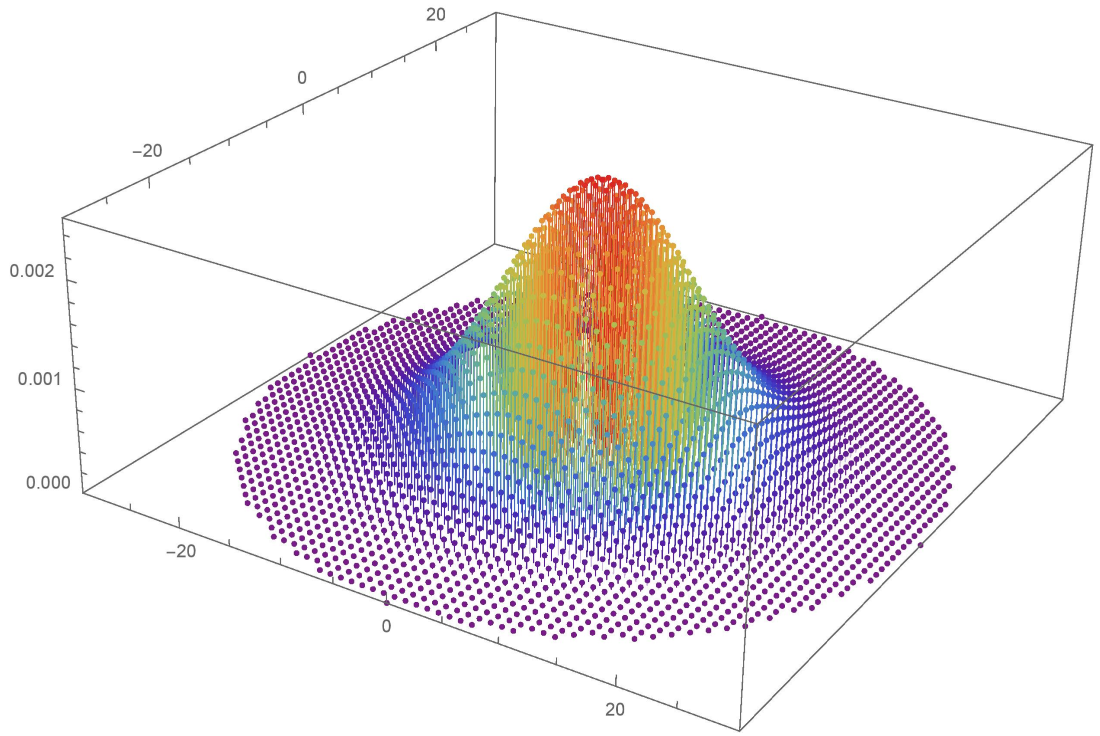
[GPV08] show how to sample in this regime in polynomial time.

(Previously could not do much better, even in exponential time.)

Discrete Gaussian Distribution

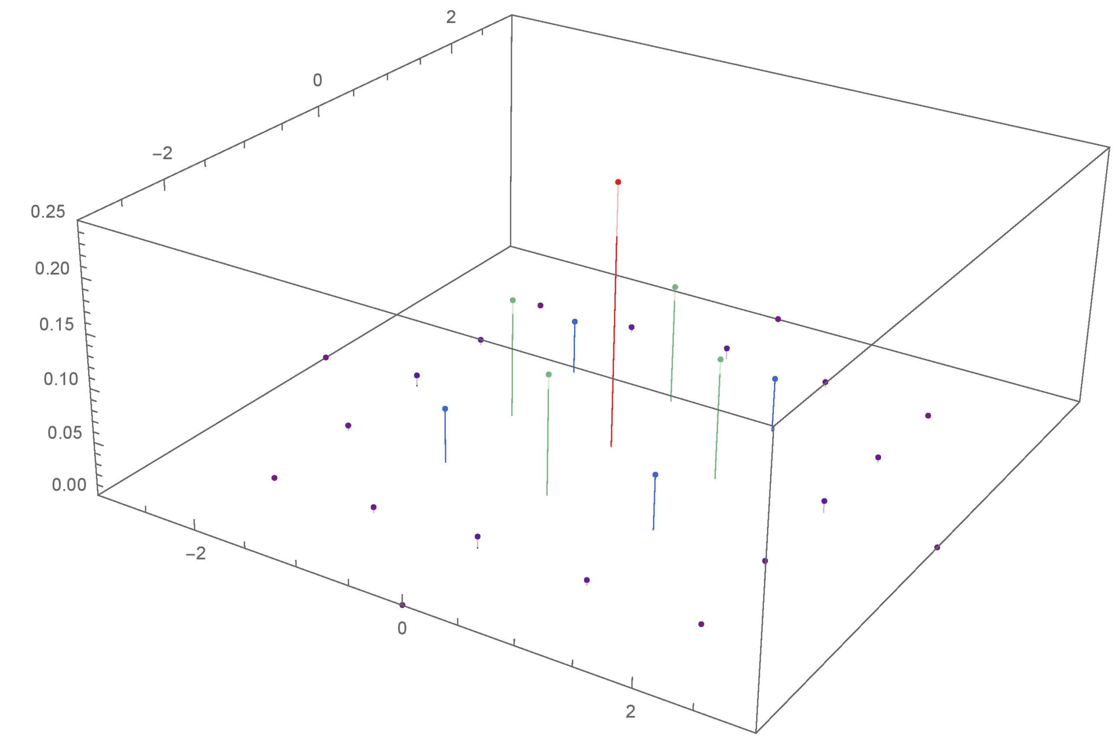
Easy

[GPV08]



$$s \gg \lambda_1(\mathcal{L})$$

Hard



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Discrete Gaussian Distribution

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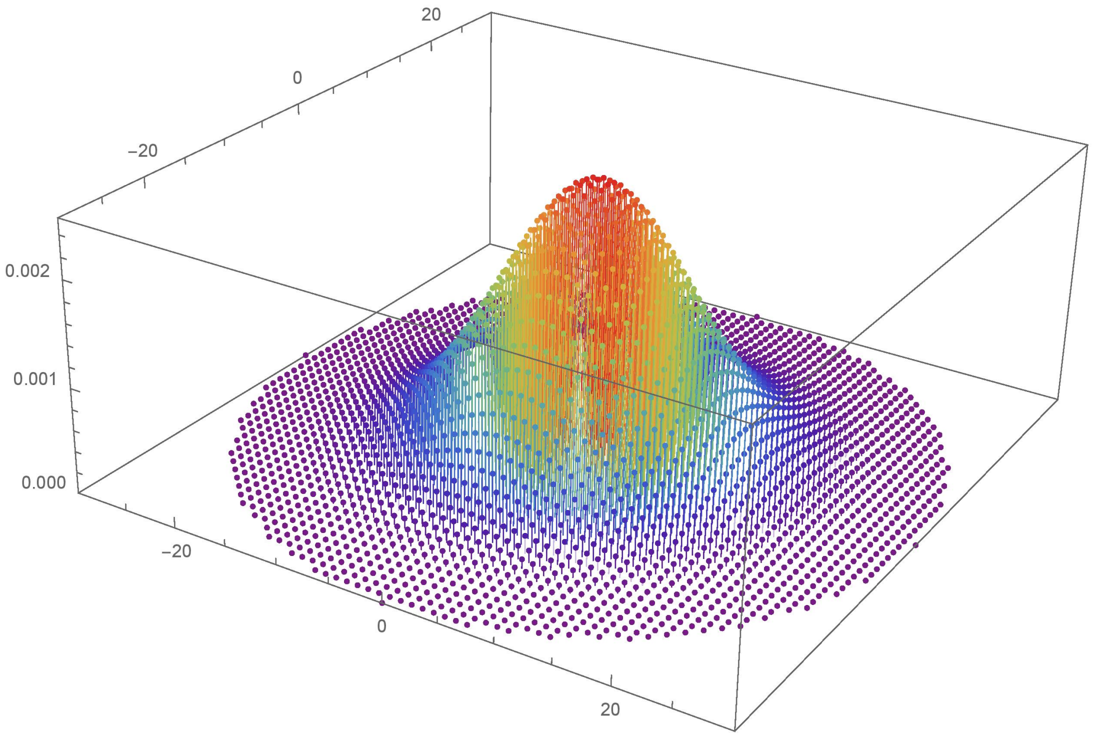

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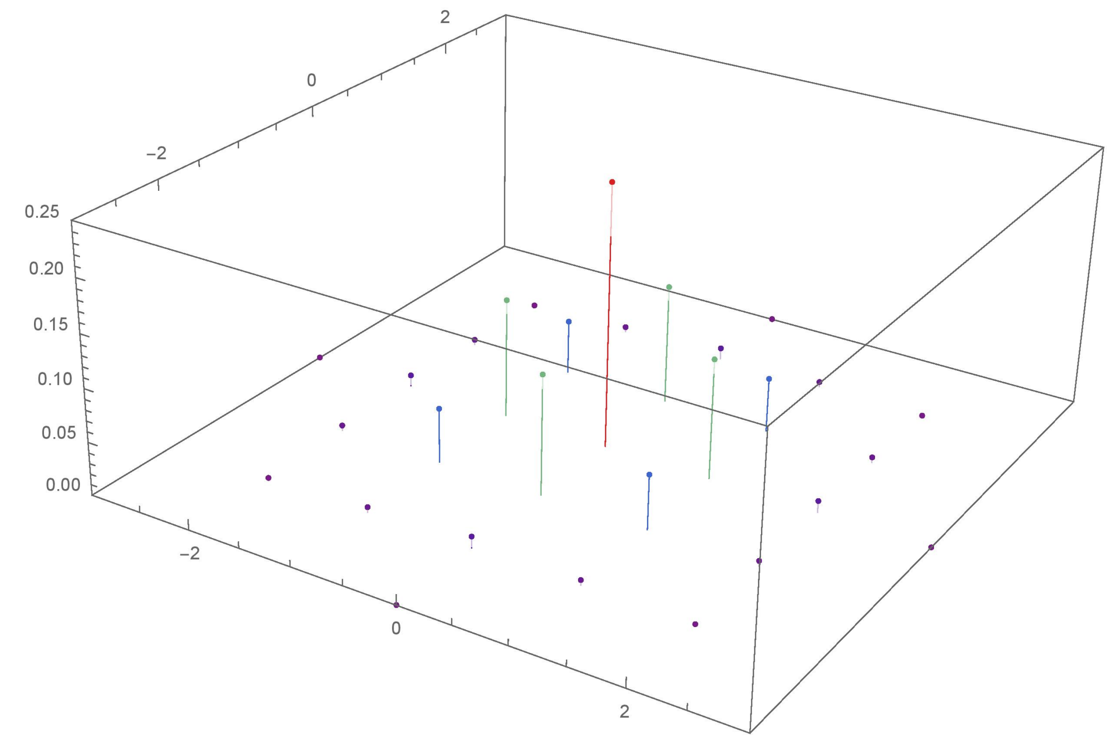

$$s \approx \lambda_1(\mathcal{L})/\sqrt{n}$$

Can we use samples from the LHS to get samples from the RHS?

Discrete Gaussian Distribution

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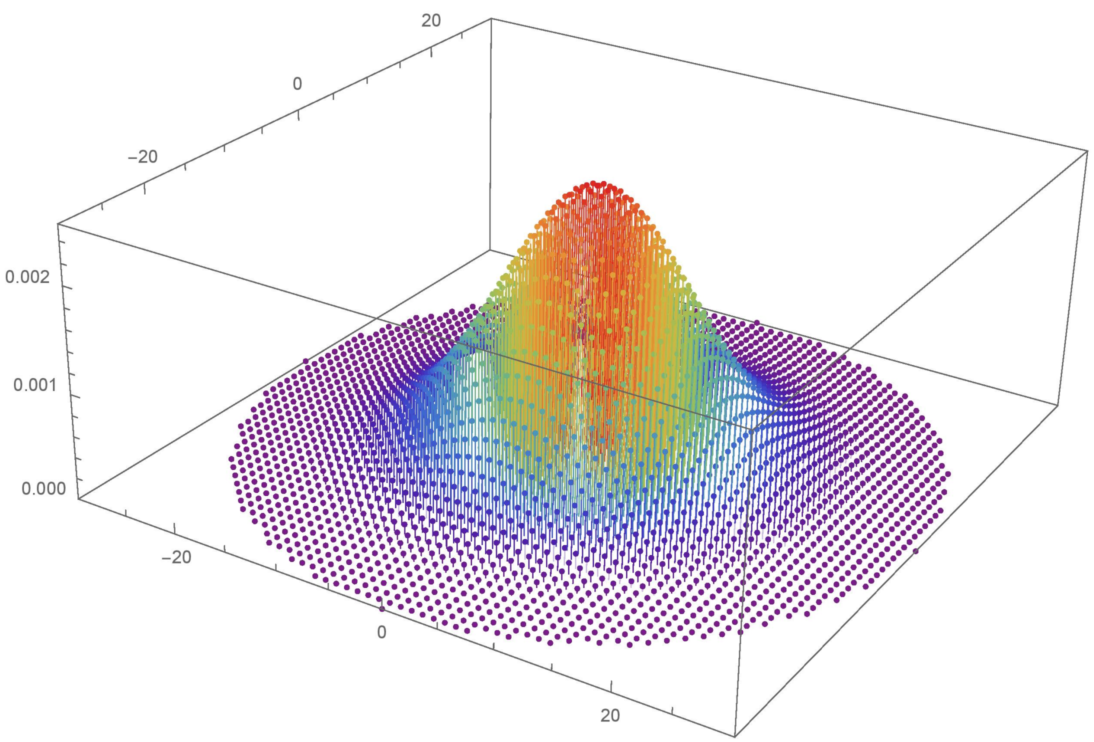
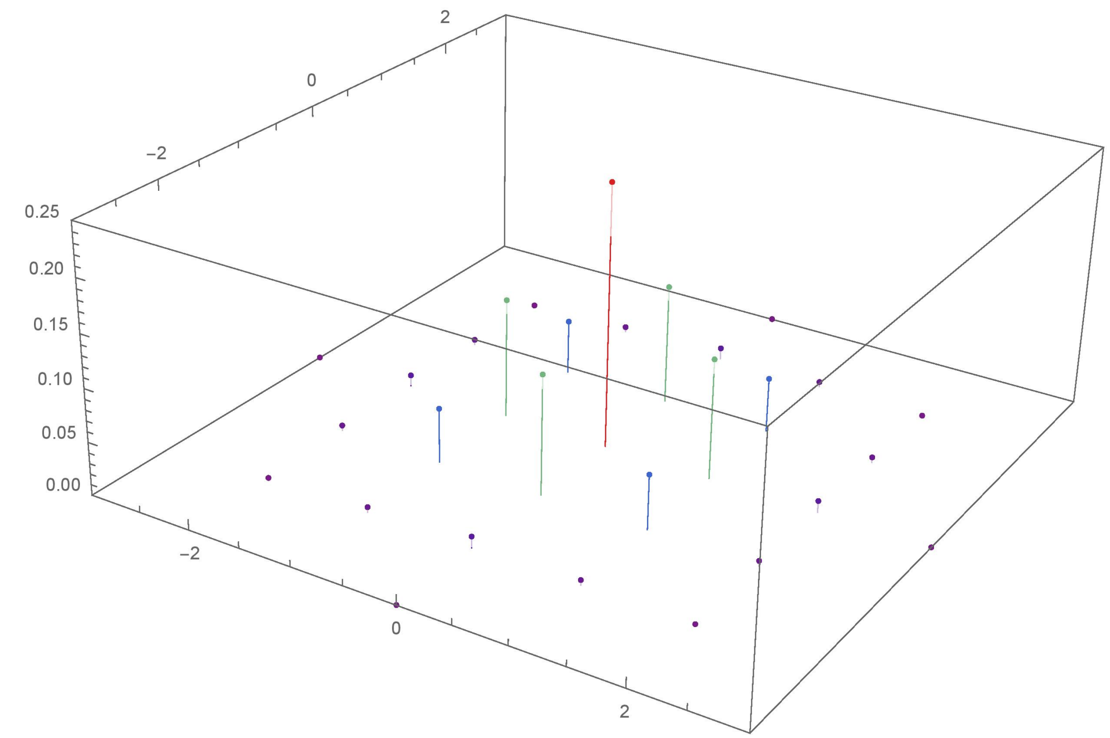


$s \gg \lambda_1(\mathcal{L})$

Our goal

Hard



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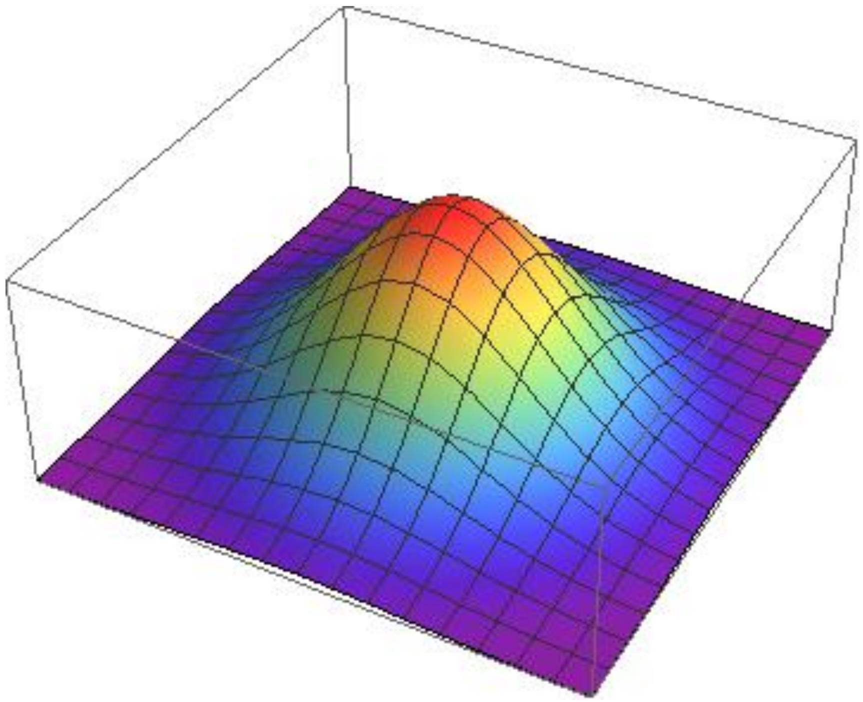

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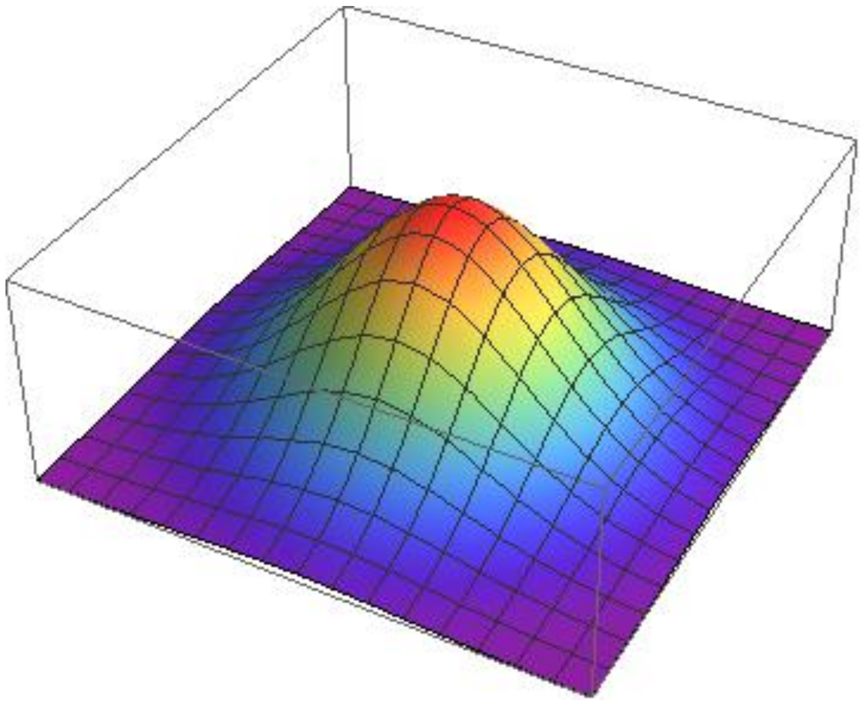

2

Converting Gaussian Vectors

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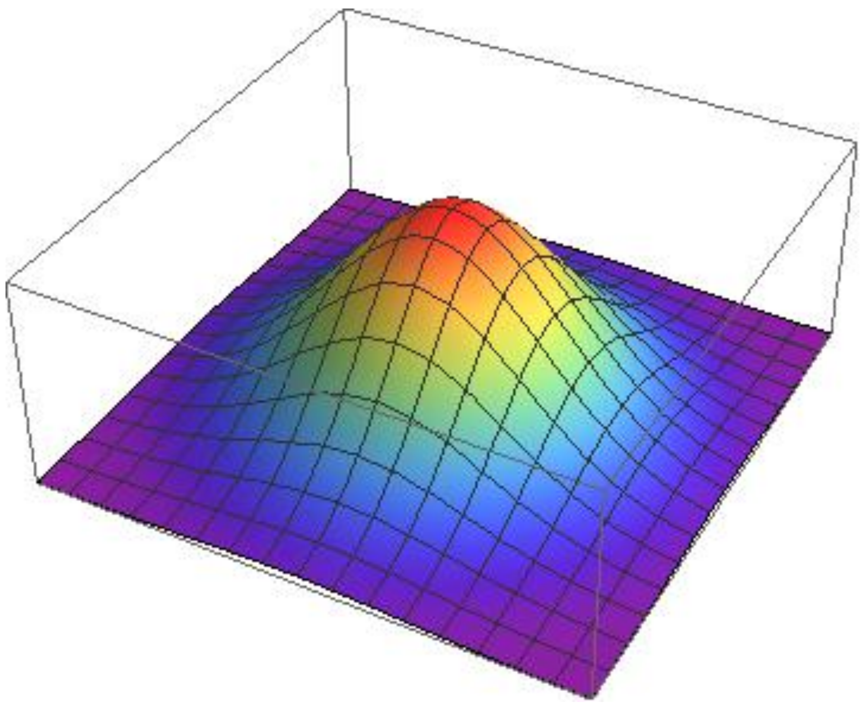
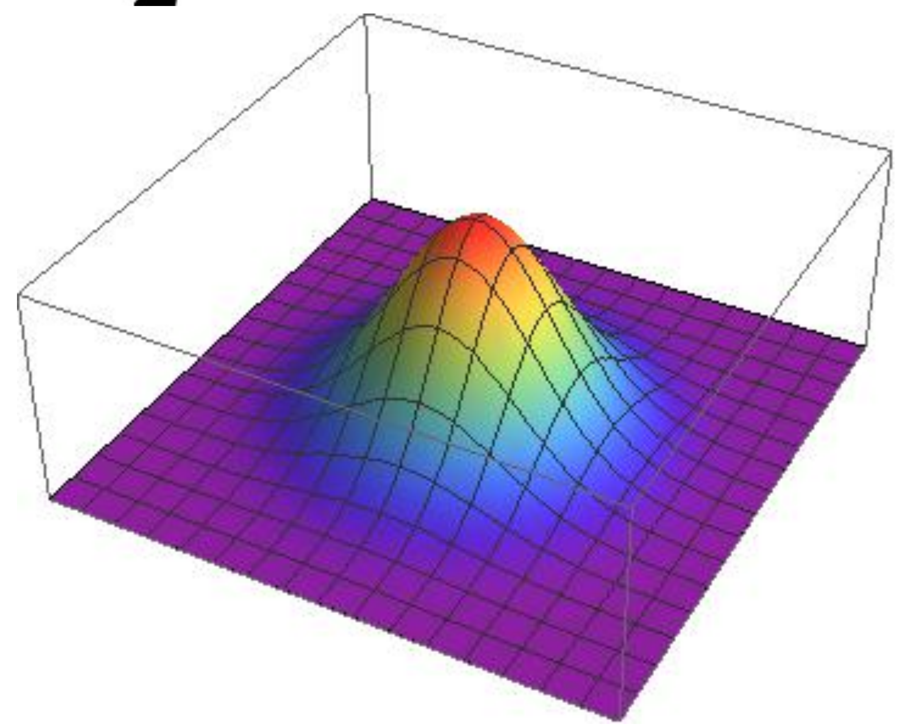


2

=

$\frac{\mathbf{x}}{2} \sim \mathsf{Gauss}(s/2)$



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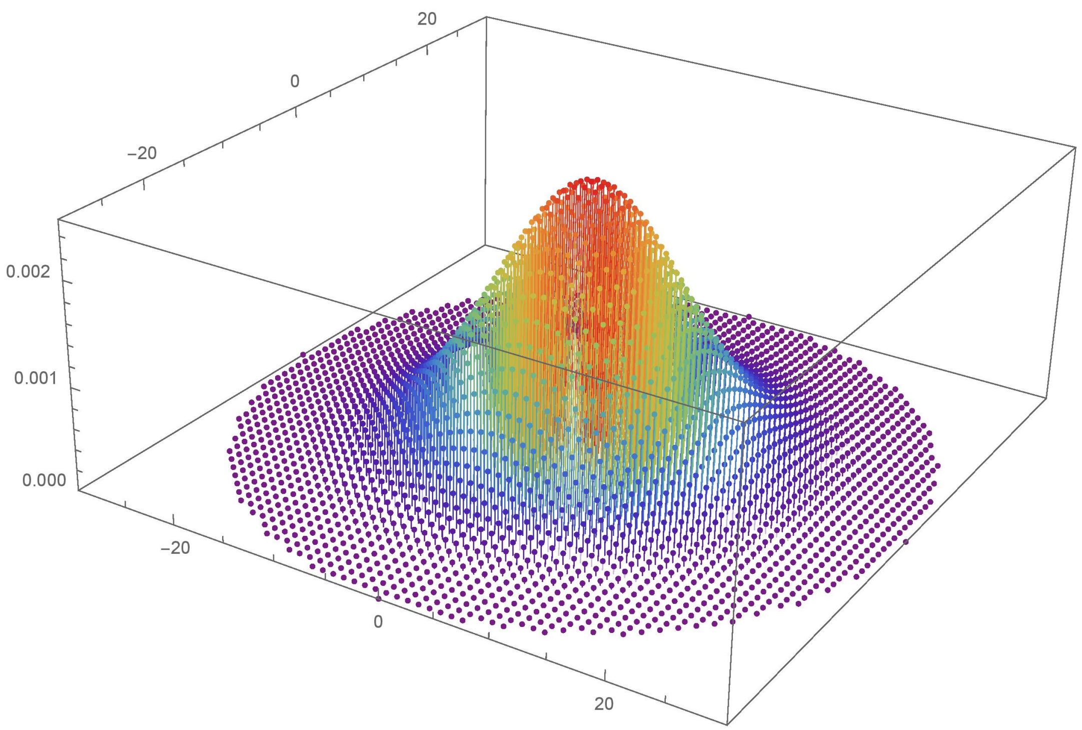

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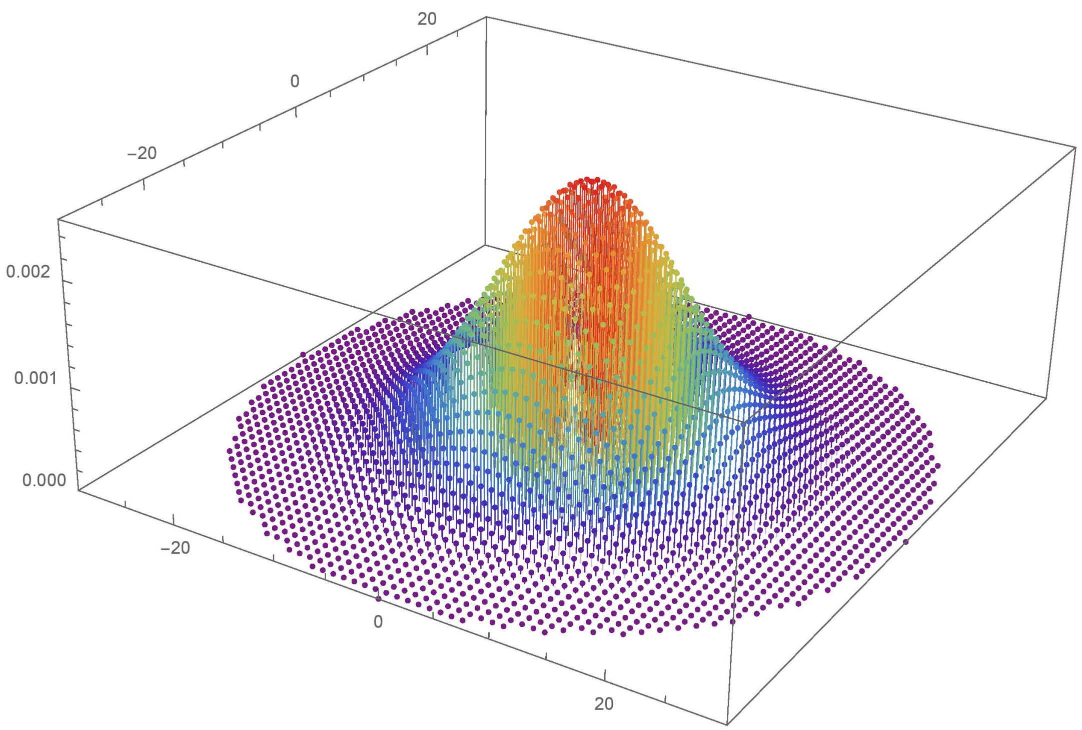

$$\frac{}{2}$$

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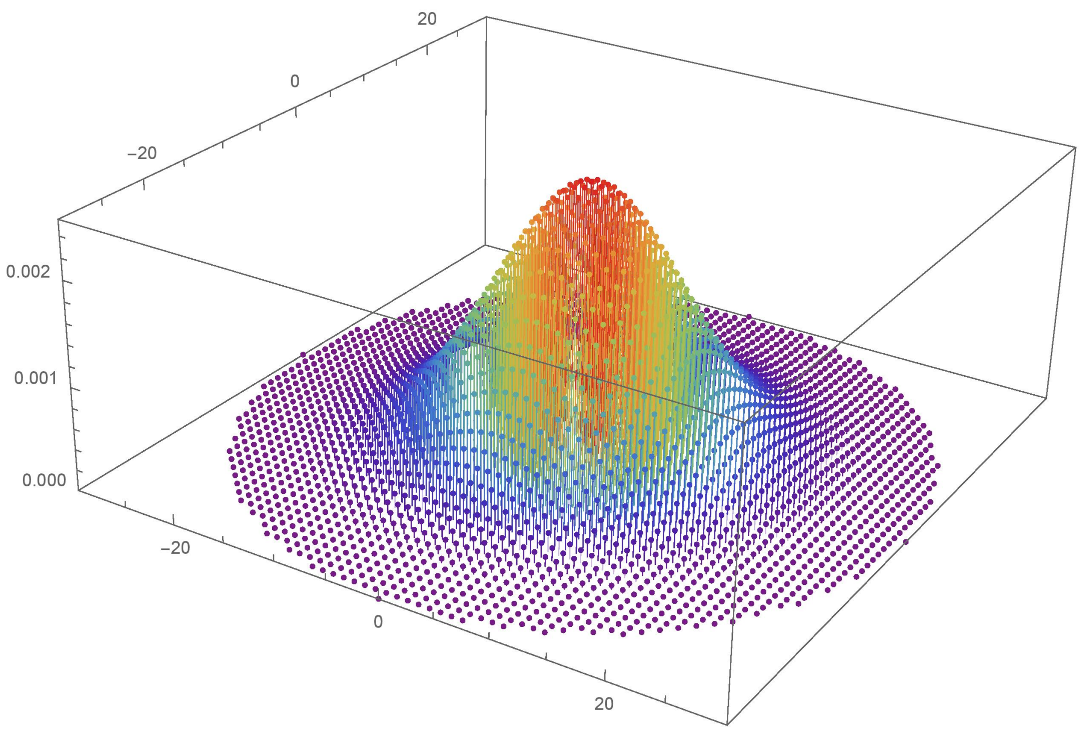
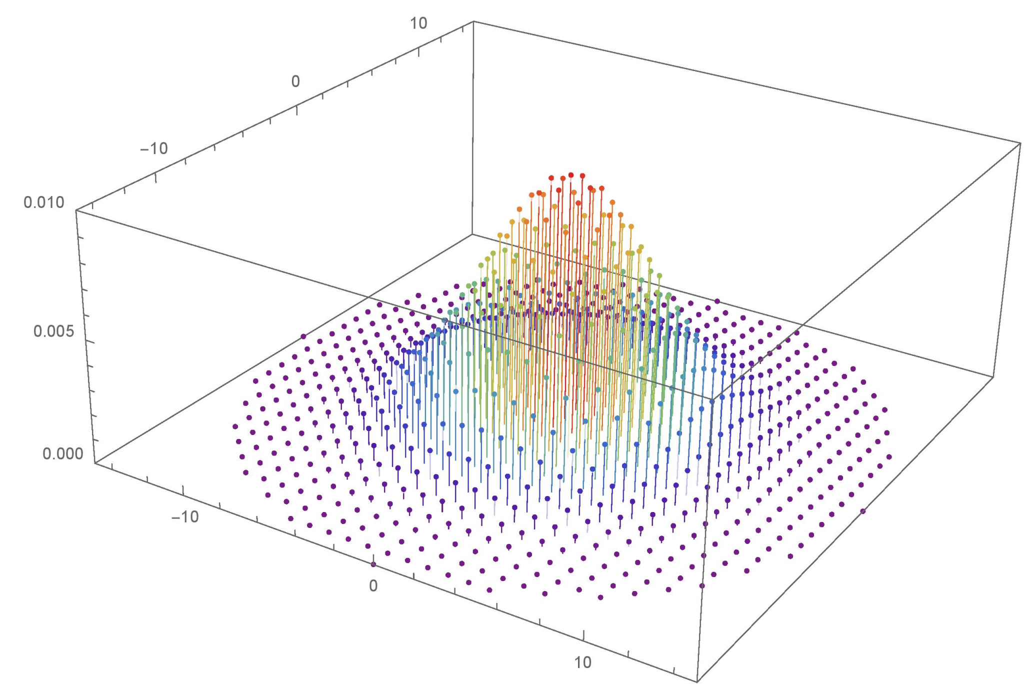


$$\frac{\quad}{2} \stackrel{?}{=}$$



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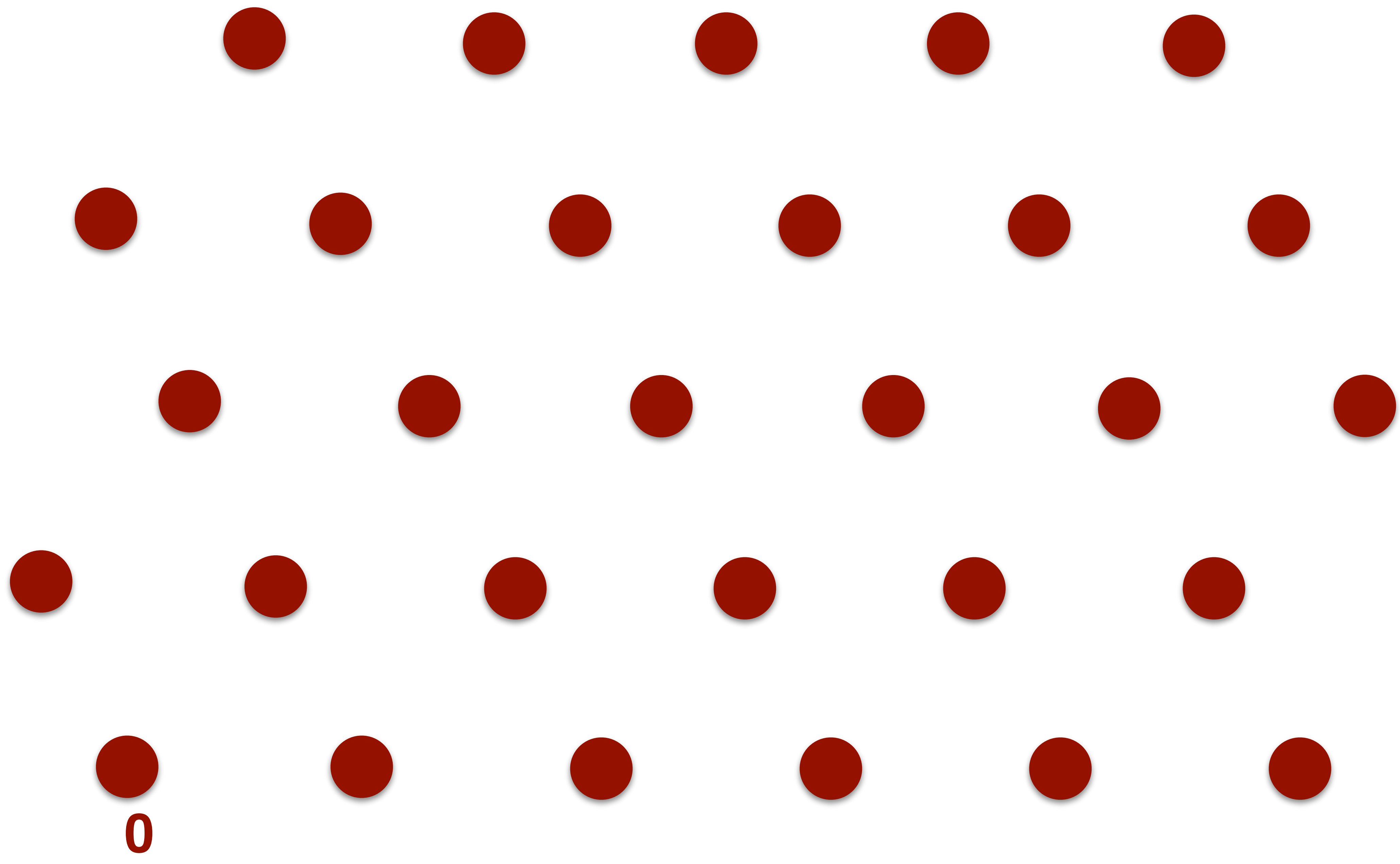

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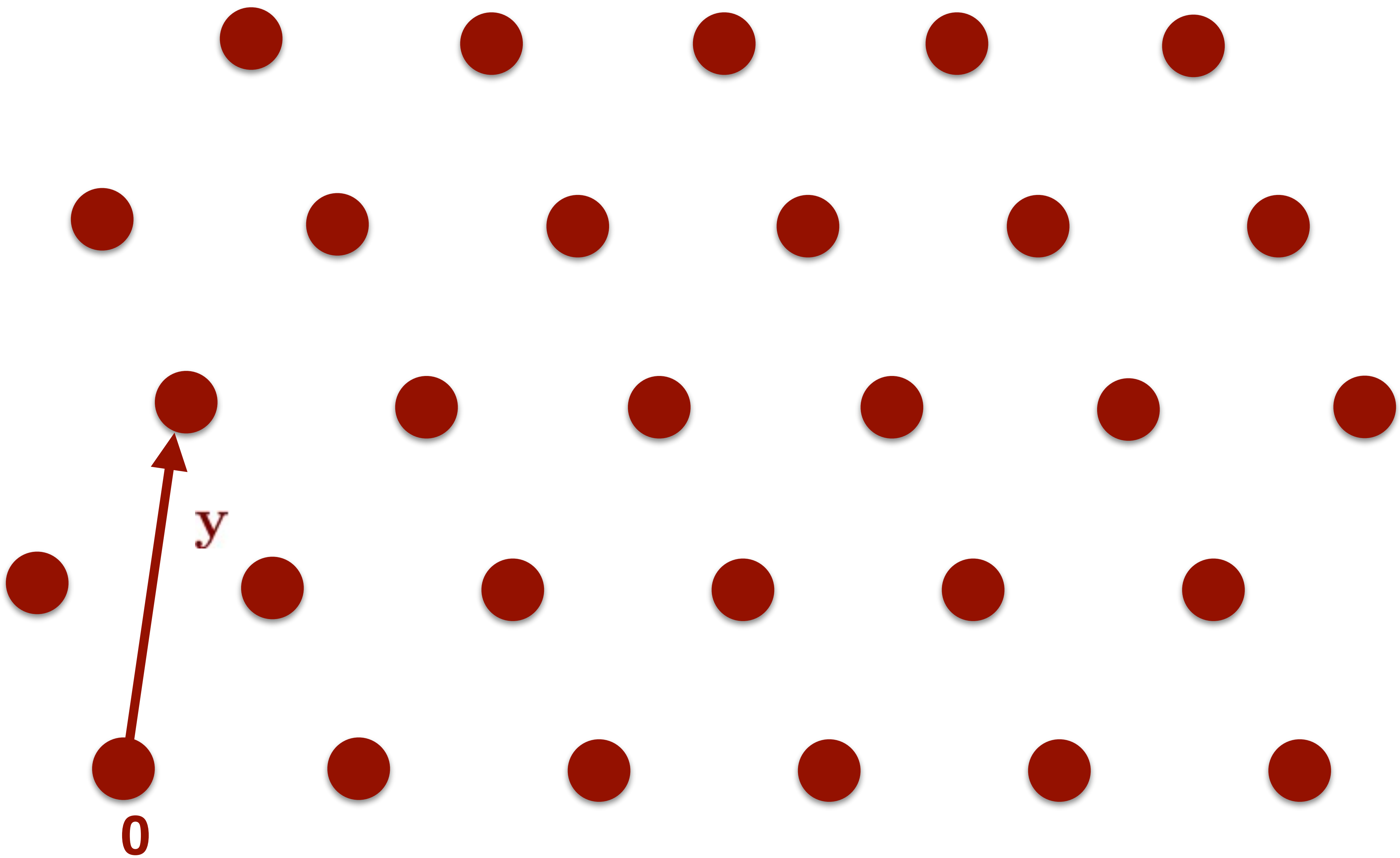

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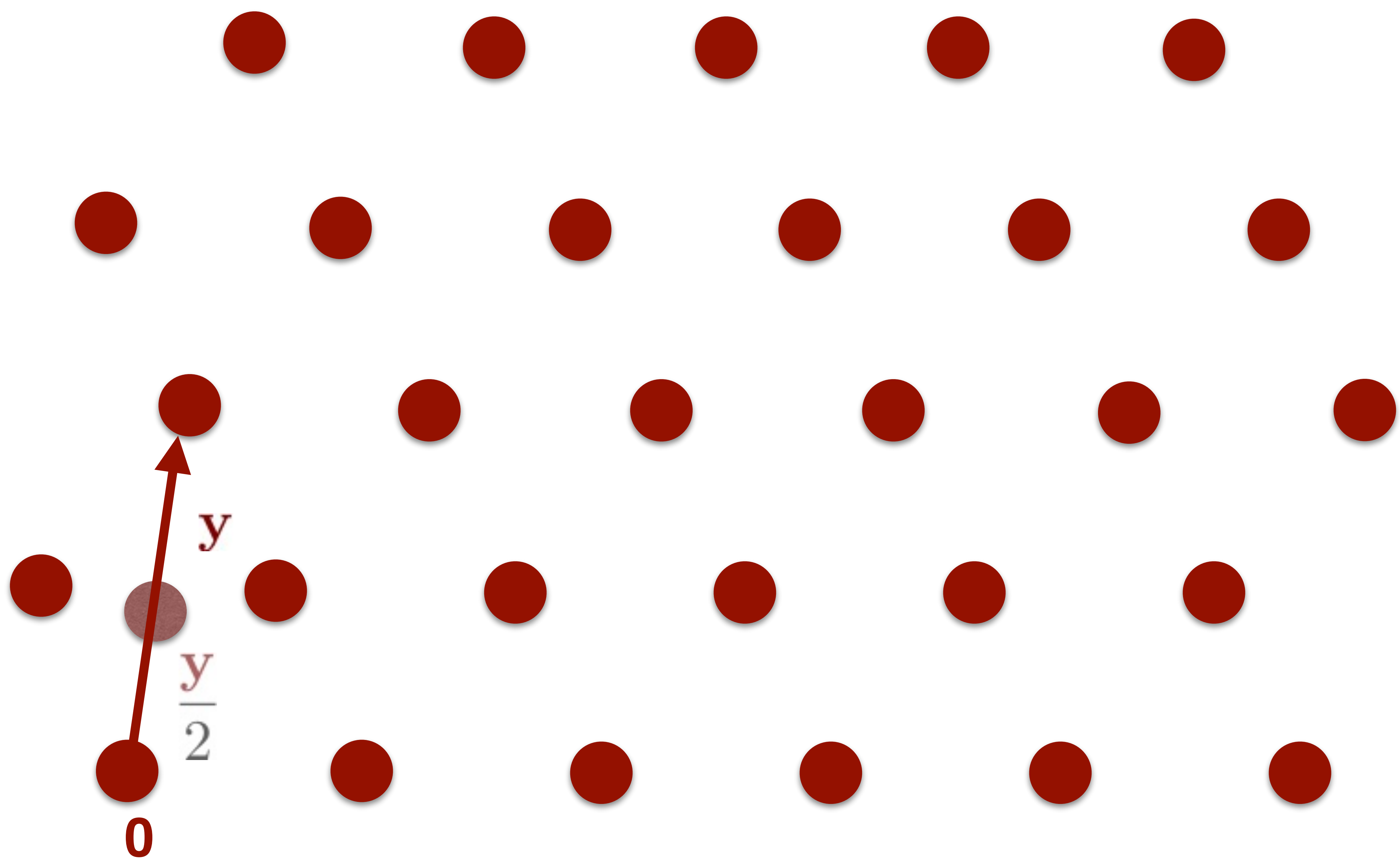

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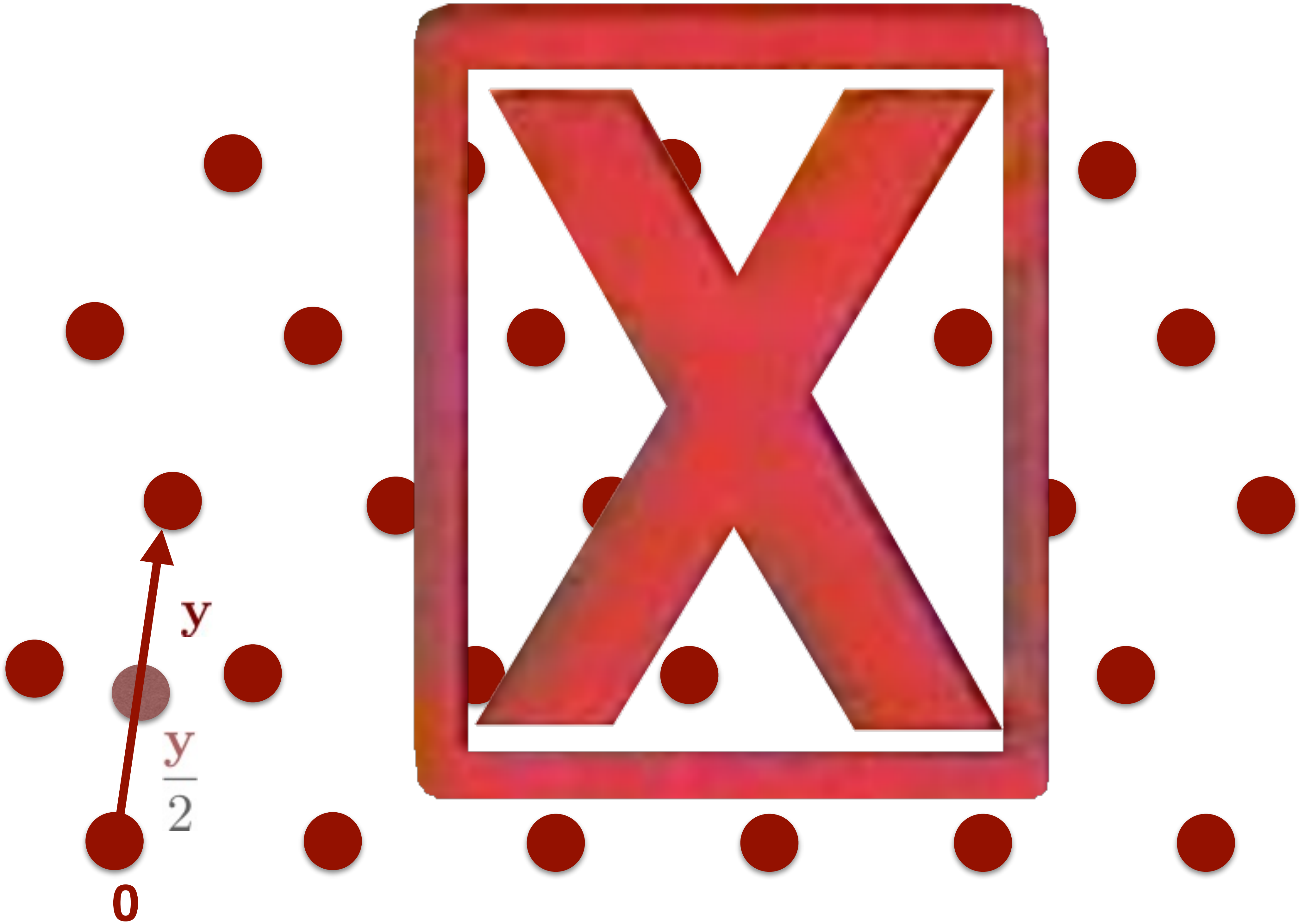

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Progress!

Unfortunately, this requires us to throw out a lot of vectors.

We only keep one from every $\approx 2^n$ vectors each time we do this, leading to a very slow algorithm!

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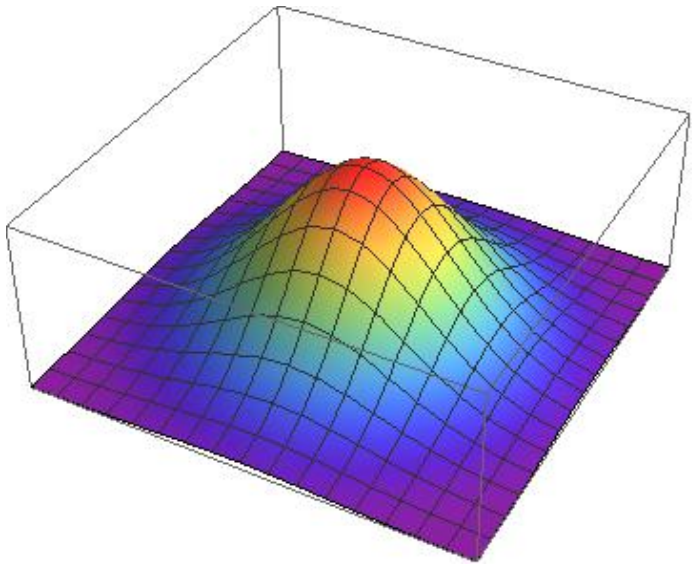

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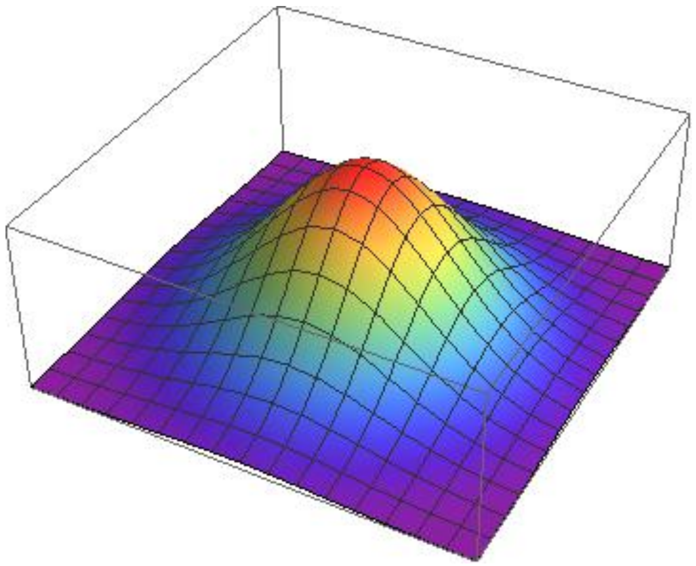
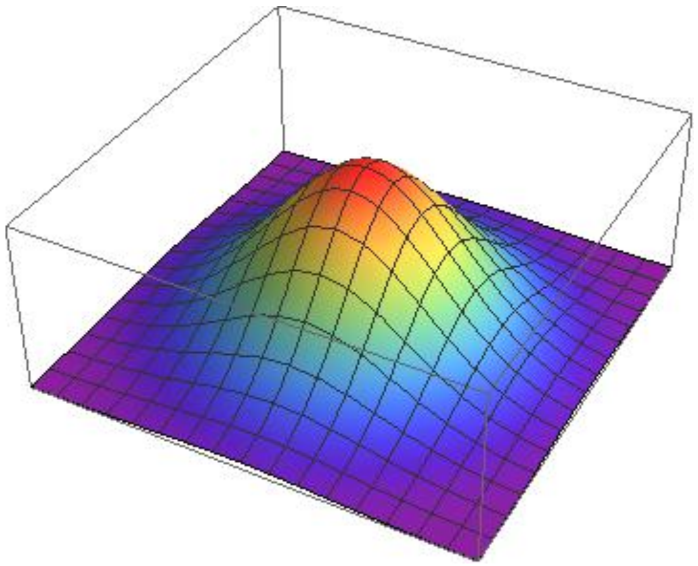
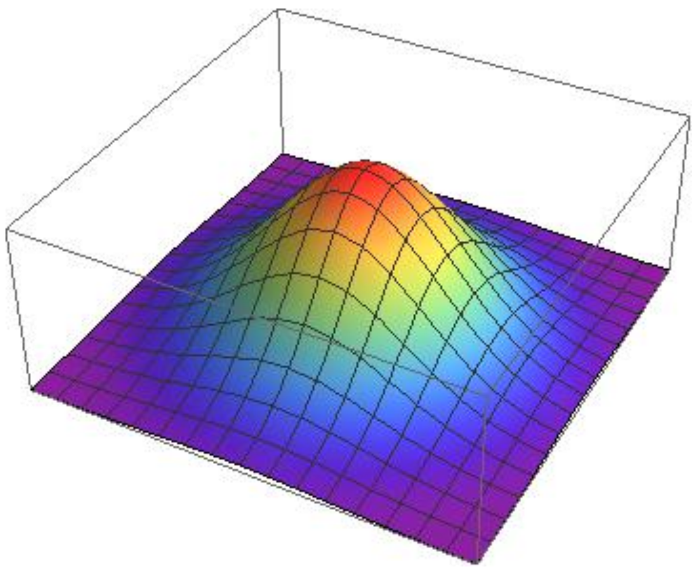




Converting Gaussian Vectors

$\mathbf{x}_1 \sim \text{Gauss}(s)$ $\mathbf{x}_2 \sim \text{Gauss}(s)$



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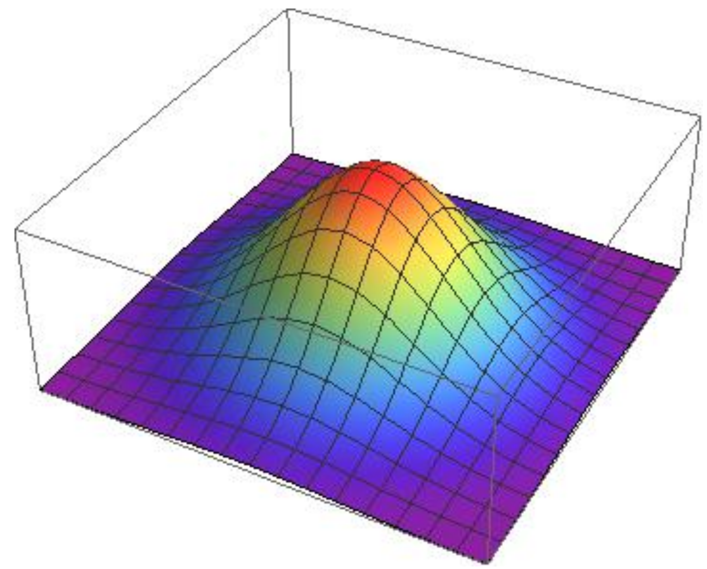
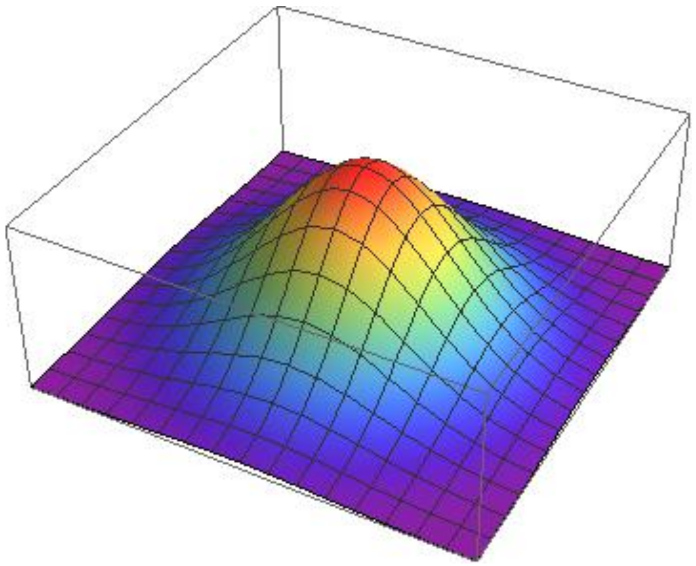


2

Converting Gaussian Vectors

$\mathbf{x}_1 \sim \mathsf{Gauss}(s)$ $\mathbf{x}_2 \sim \mathsf{Gauss}(s)$ $\frac{\mathbf{x}_1 + \mathbf{x}_2}{2} \sim \mathsf{Gauss}(s/\sqrt{2})$



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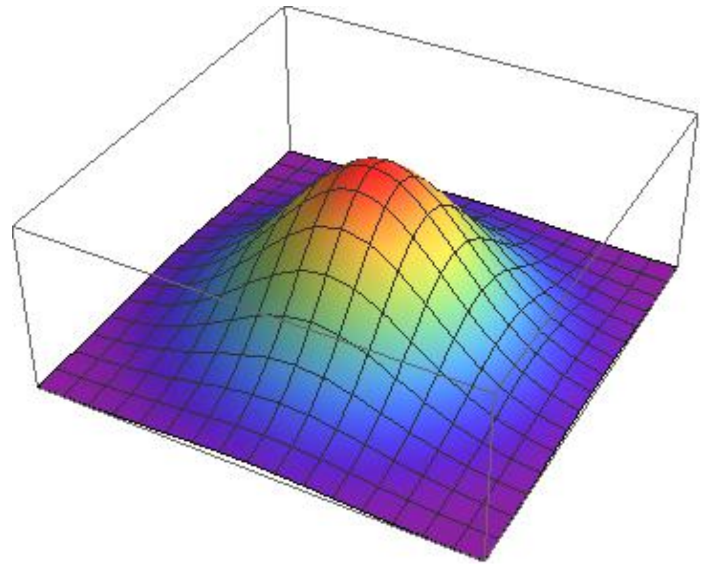

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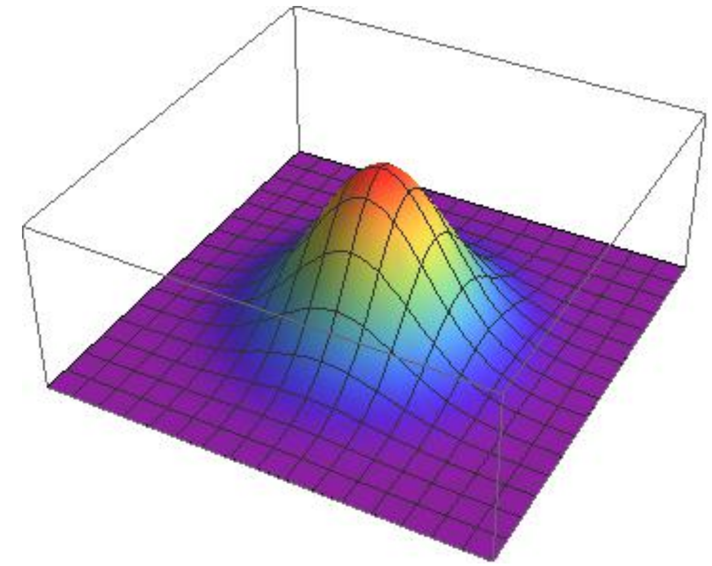

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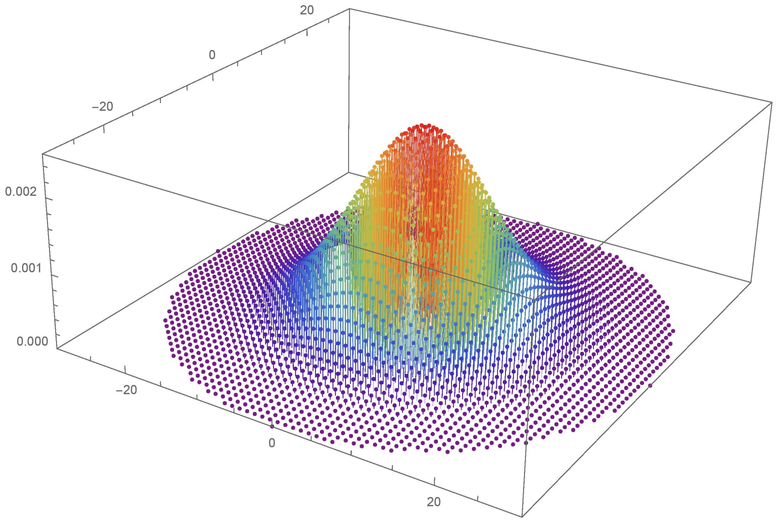
2

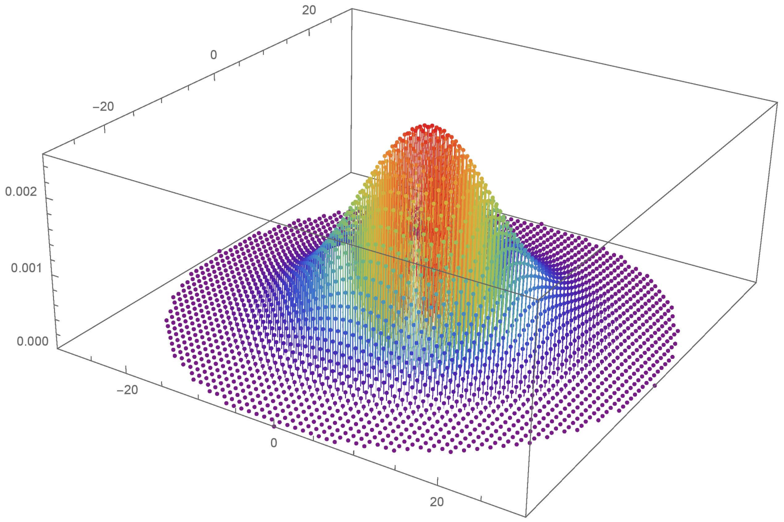
Converting Gaussian Vectors

Converting Gaussian Vectors

$\mathbf{y}_1 \sim D_{\mathcal{L},s}$

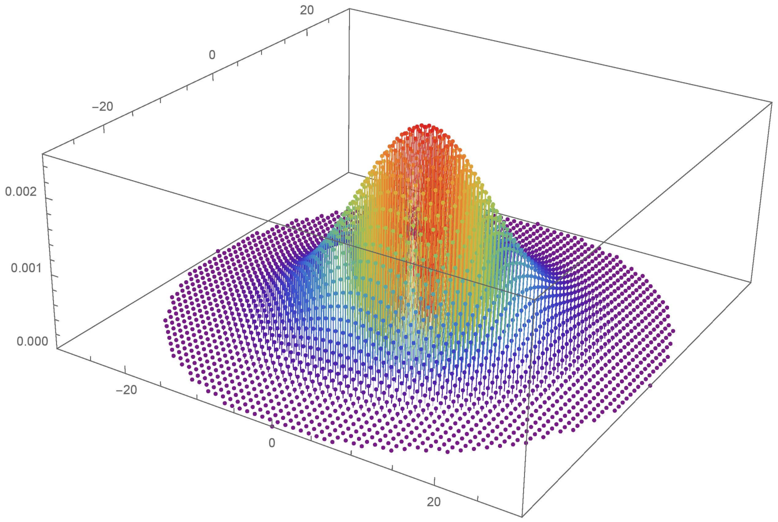


$\mathbf{y}_2 \sim D_{\mathcal{L},s}$



Converting Gaussian Vectors

$\mathbf{y}_1 \sim D_{\mathcal{L},s}$ $\mathbf{y}_2 \sim D_{\mathcal{L},s}$



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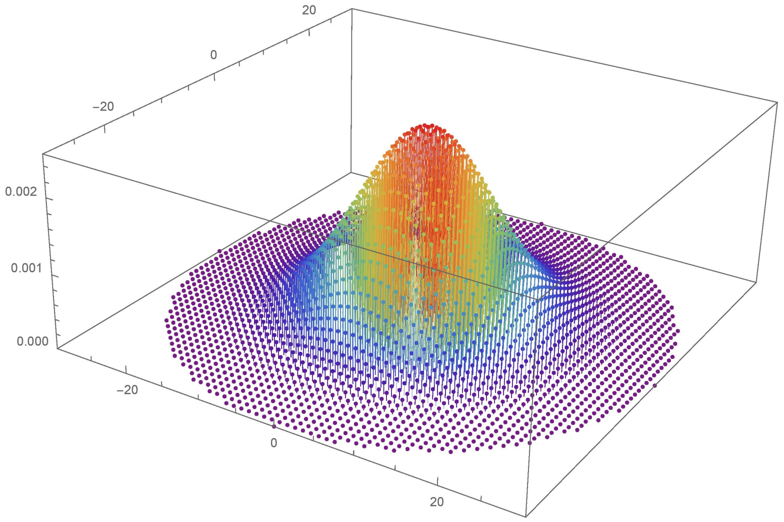

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2

Converting Gaussian Vectors

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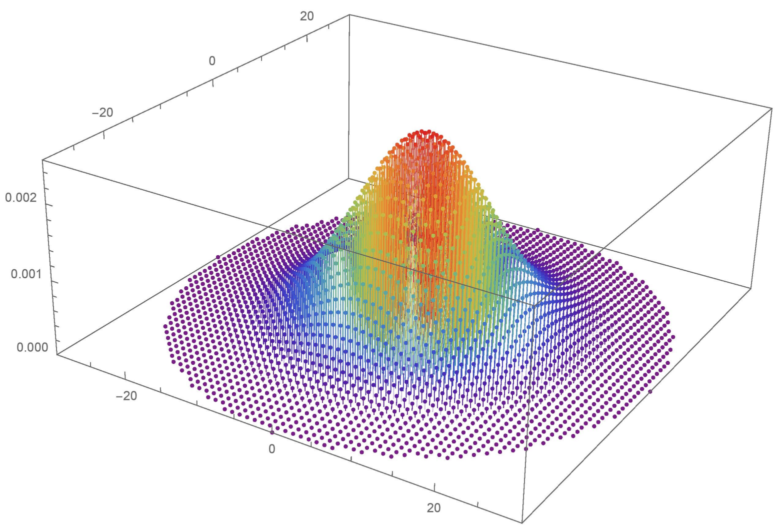

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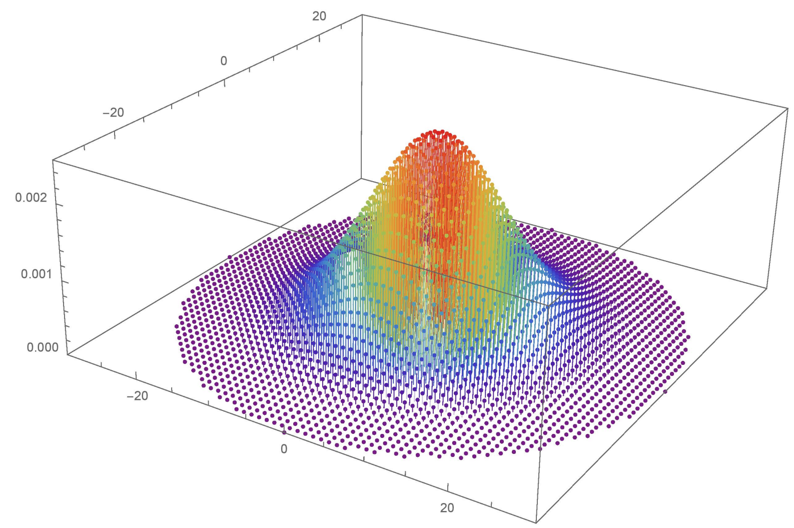
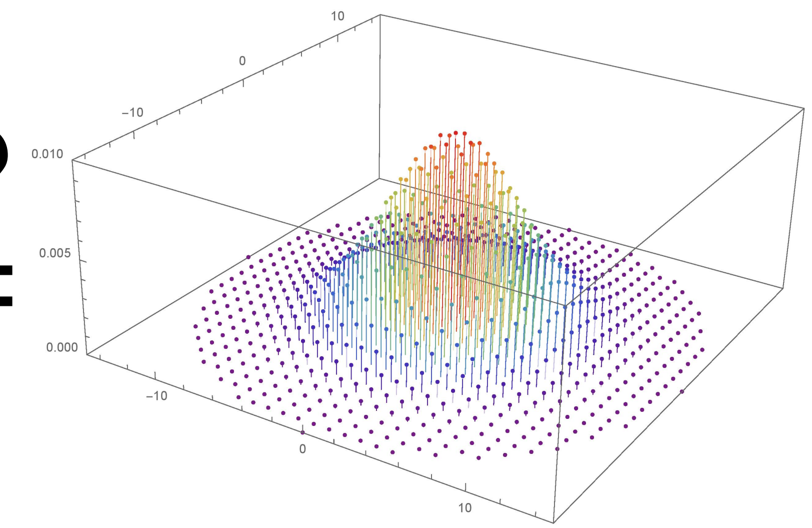
$$\mathbf{y}_2 \sim D_{\mathcal{L},s}$$



$\overset{?}{=}$

$$\frac{\mathbf{y}_1 + \mathbf{y}_2}{2} \sim D_{\mathcal{L},s/\sqrt{2}}$$



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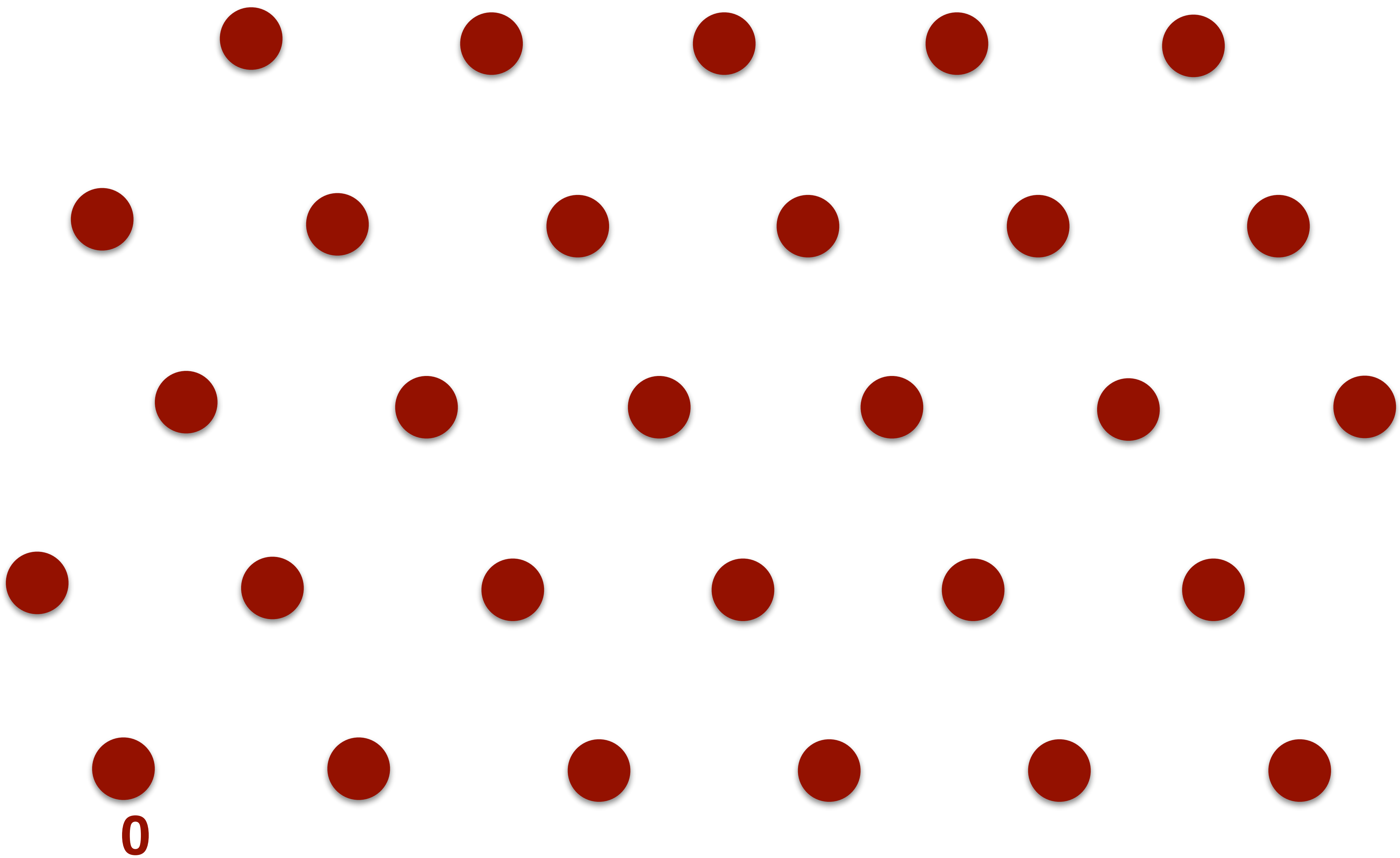

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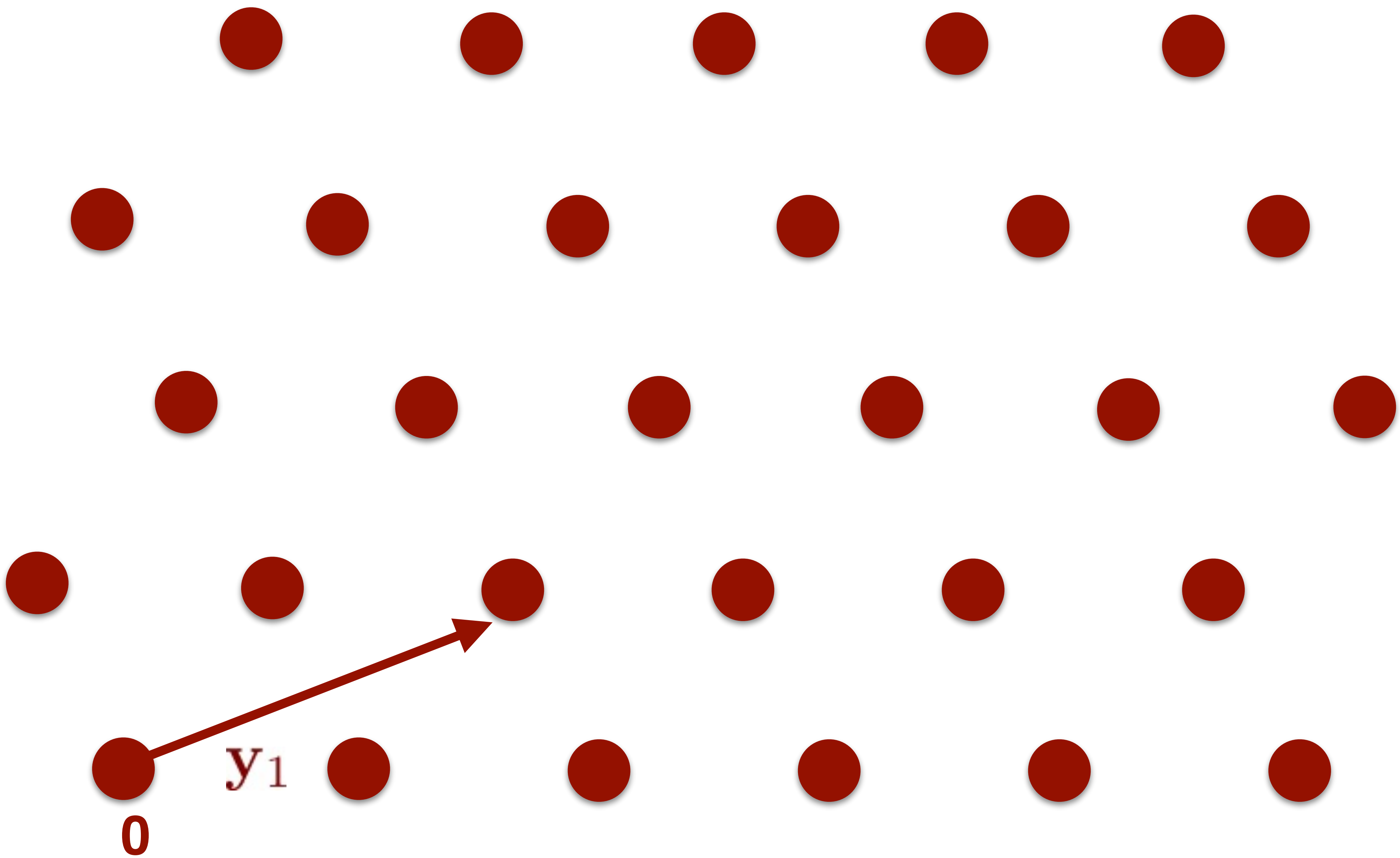

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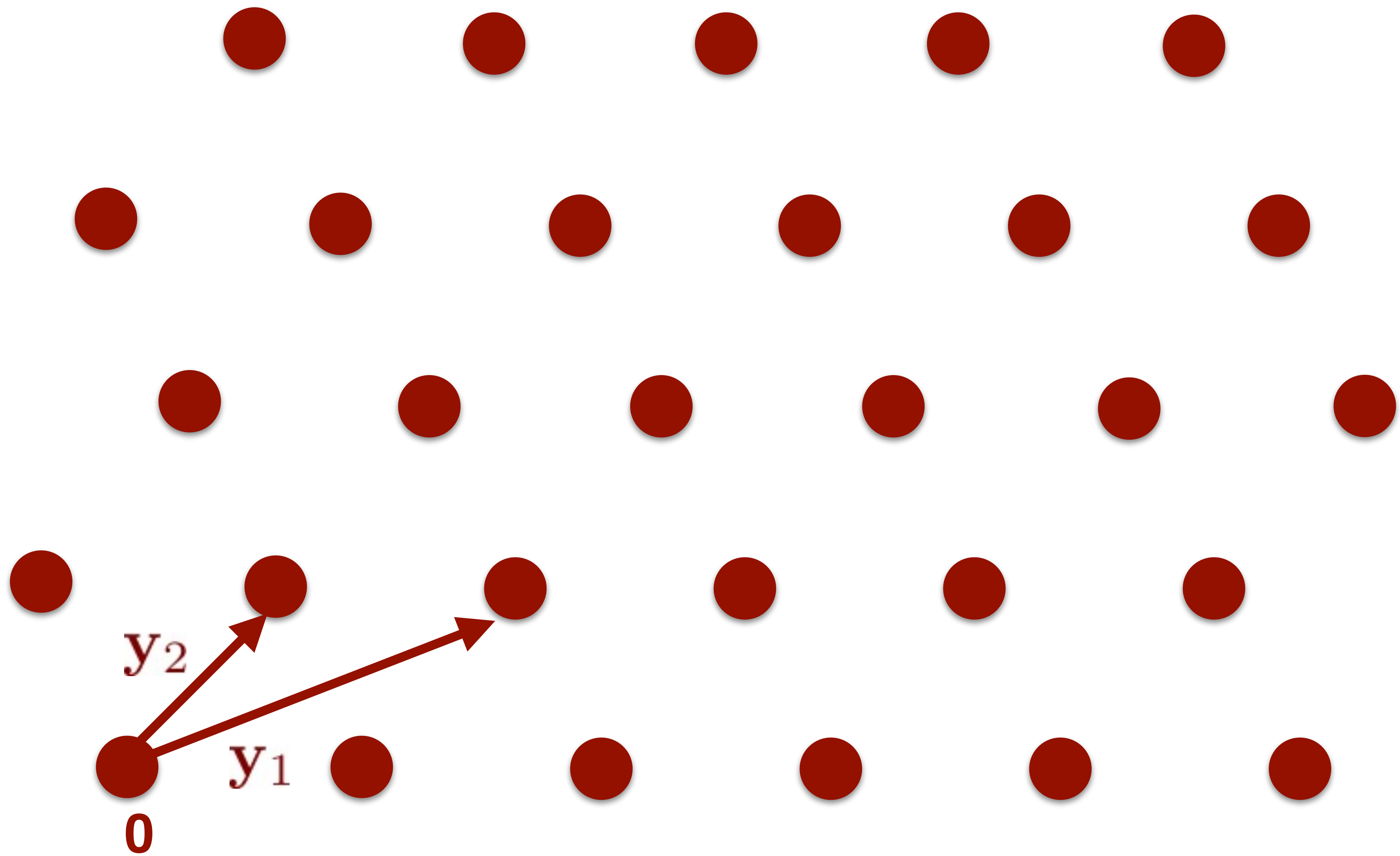

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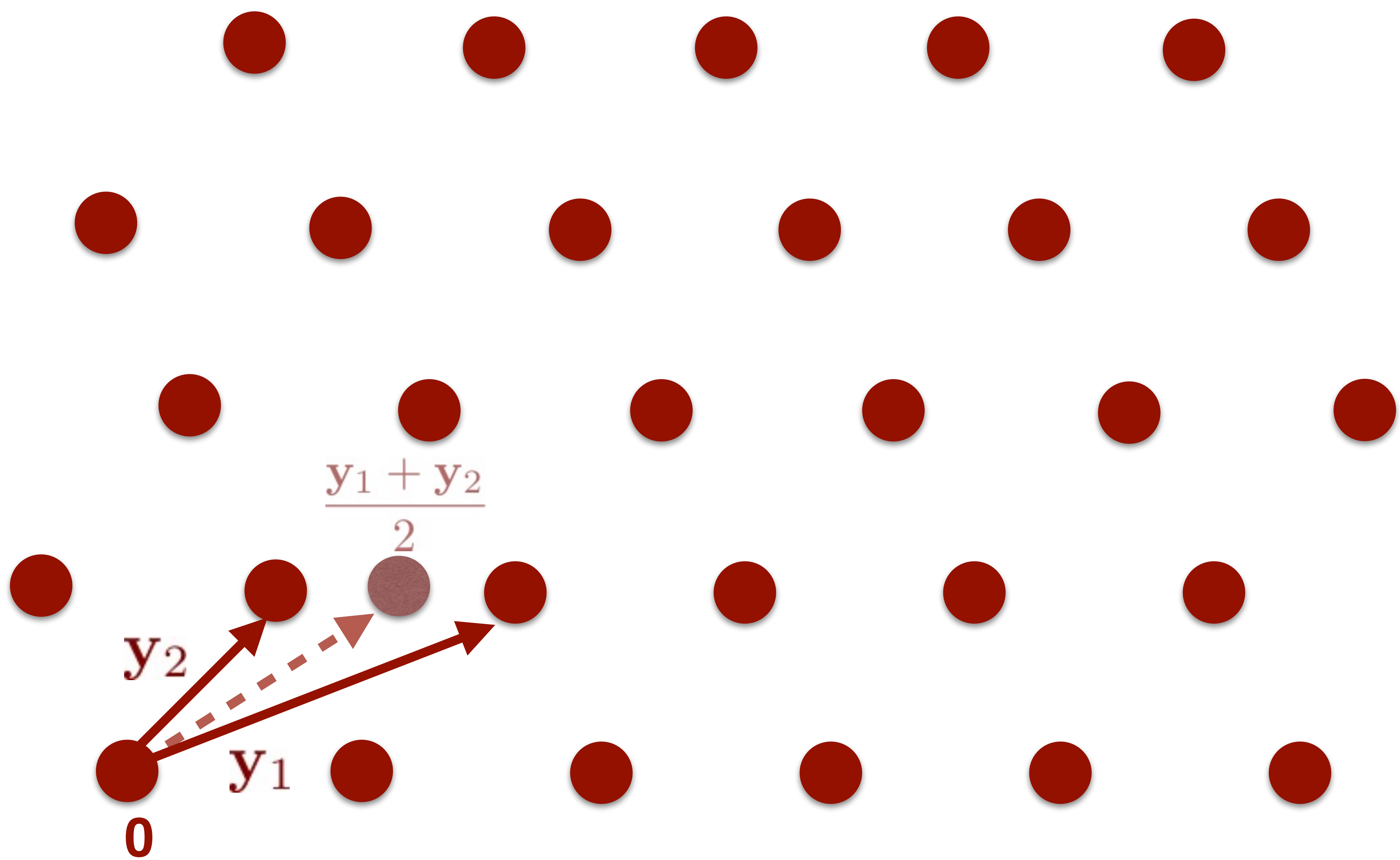

Converting Gaussian Vectors

Converting Gaussian Vectors

What about the average of two discrete Gaussian vectors *conditioned on* the result being in the lattice?

Converting Gaussian Vectors

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When do we have $\frac{\mathbf{y}_1 + \mathbf{y}_2}{2} \in \mathcal{L}$?

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$$\frac{\mathbf{y}_1 + \mathbf{y}_2}{2} = \frac{a_{1,1} + a_{2,1}}{2} \cdot \mathbf{b}_1 + \cdots + \frac{a_{1,n} + a_{2,n}}{2} \cdot \mathbf{b}_n$$

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When do we have $\frac{\mathbf{y}_1 + \mathbf{y}_2}{2} \in \mathcal{L}$?

$\mathbf{y}_1 = a_1$ … $a_{2,n}\mathbf{b}_n$

$\frac{\mathbf{y}_1 \ldots}{}$ … $\mathbf{b}_n$

We have $\frac{\mathbf{y}_1 + \mathbf{y}_2}{2} \in \mathcal{L}$ *if and only if* $\mathbf{y}_1, \mathbf{y}_2$ *are in the same* ***coset*** *of* $2\mathcal{L}$.

(Note that there are 2^n *cosets.)*

$$\frac{\mathbf{y}_1 + \mathbf{y}_2}{2} \in \mathcal{L} \iff a_{1,i} \equiv a_{2,i} \mod 2$$

$$\iff \mathbf{y}_1 \equiv \mathbf{y}_2 \mod 2\mathcal{L}$$

Converting Gaussian Vectors

What about the average of two discrete Gaussian vectors *conditioned on* the result being in the lattice?

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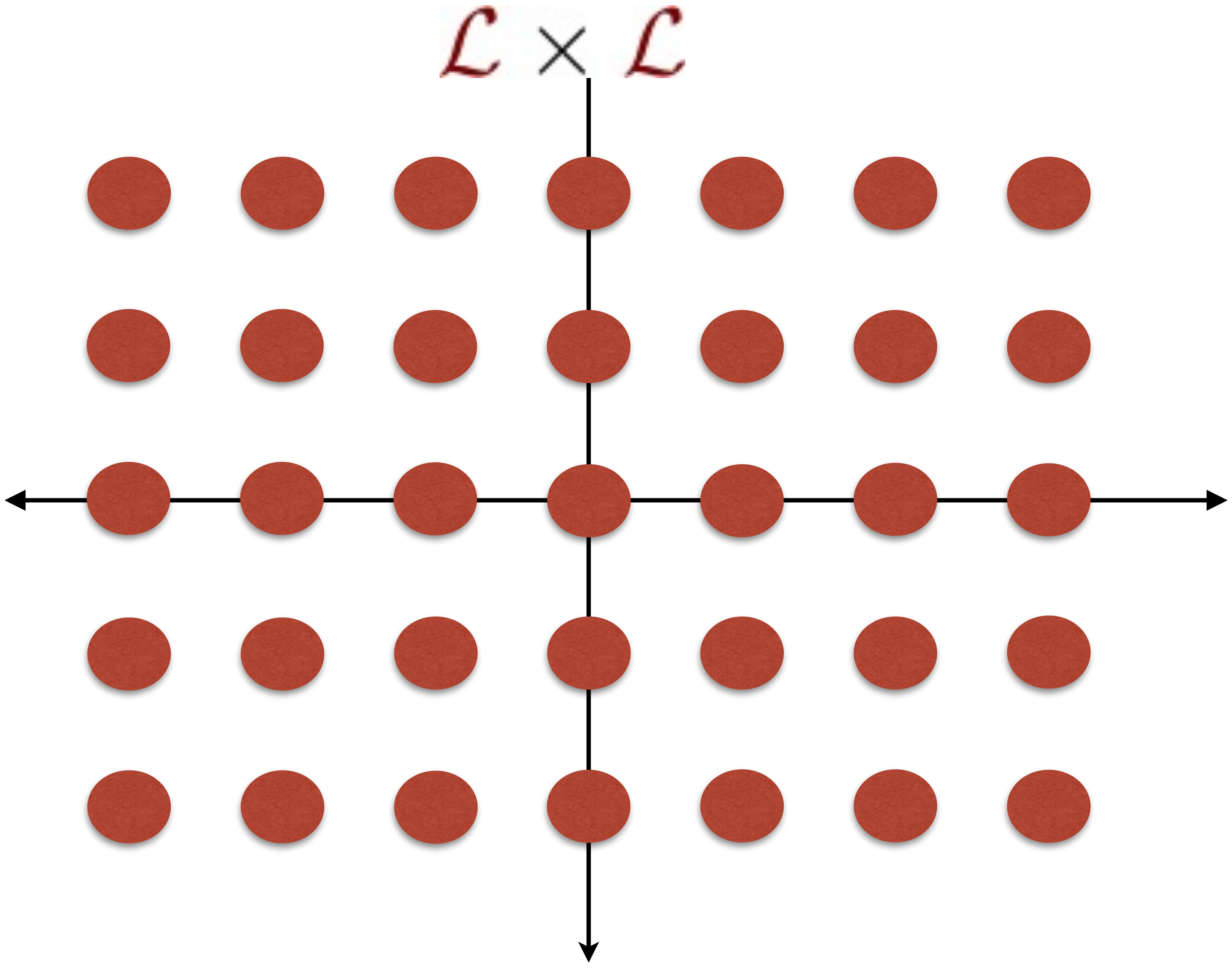

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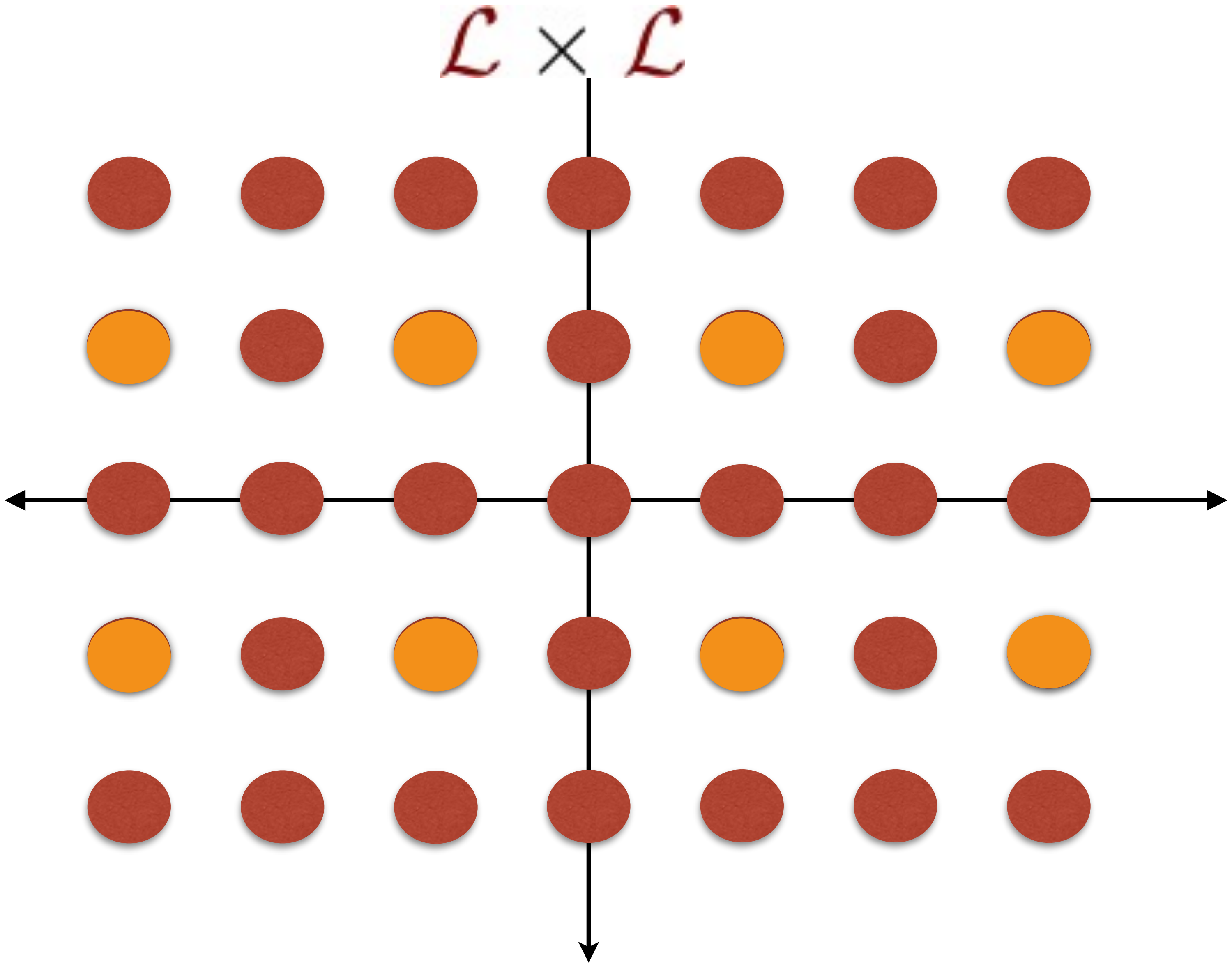

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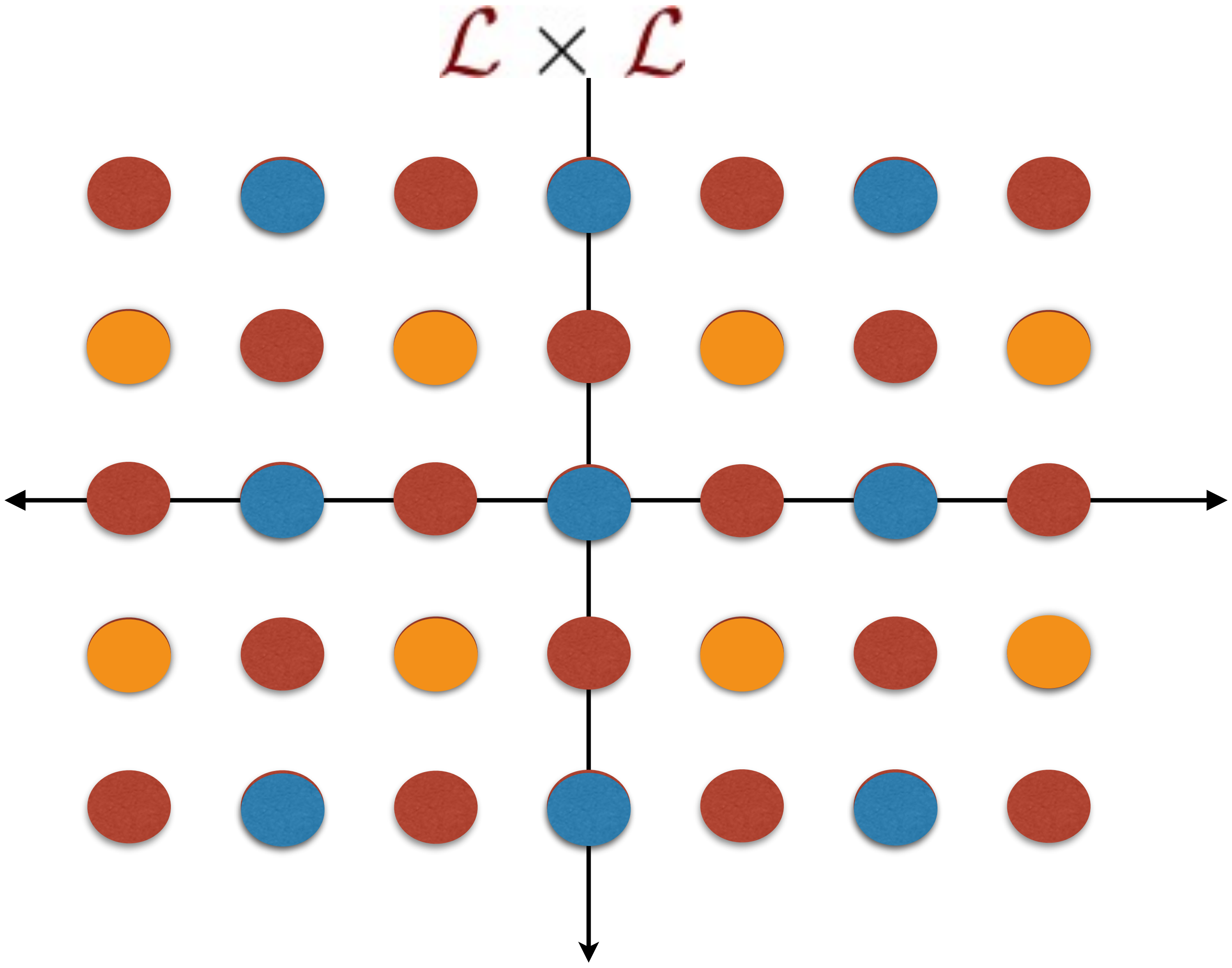

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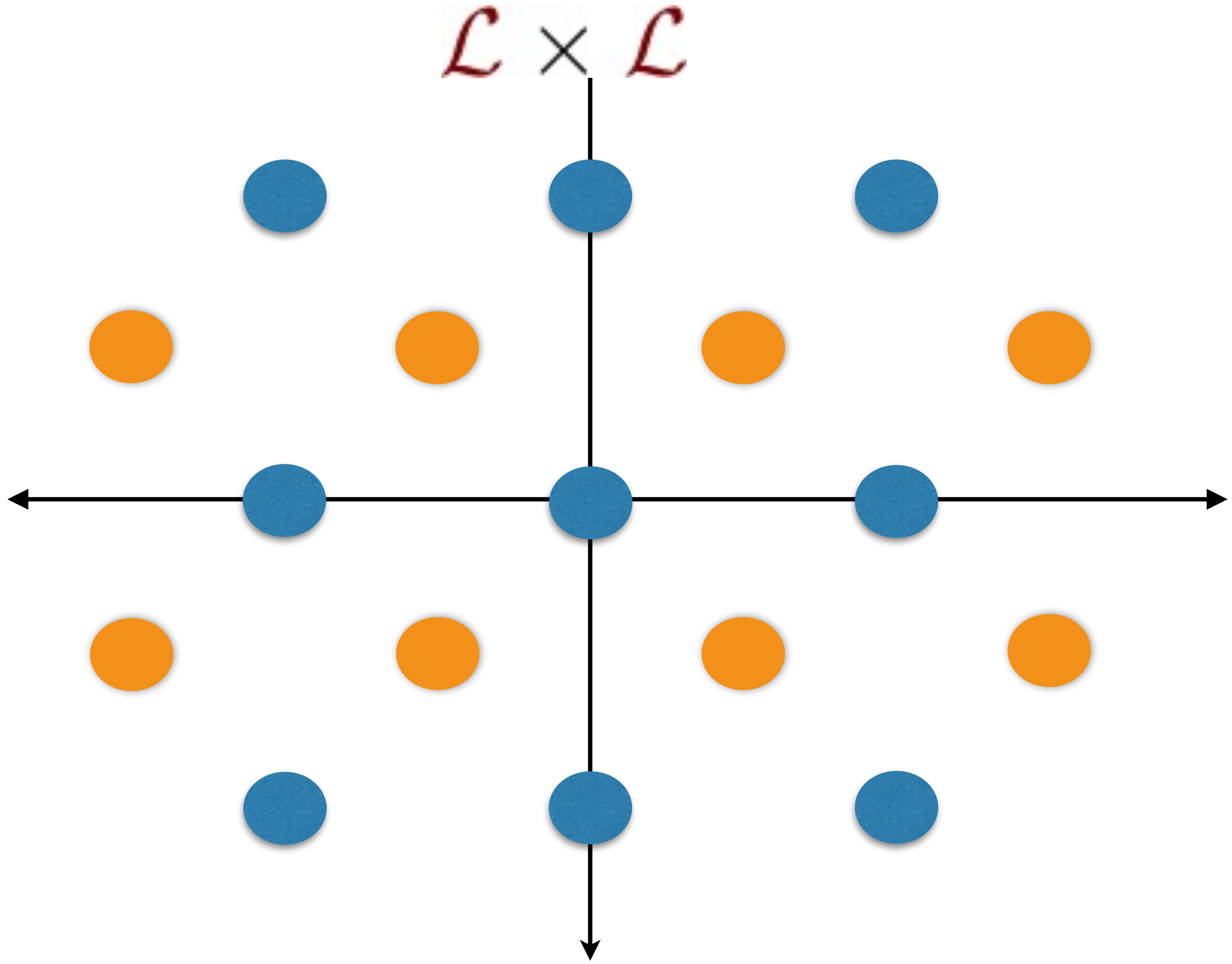

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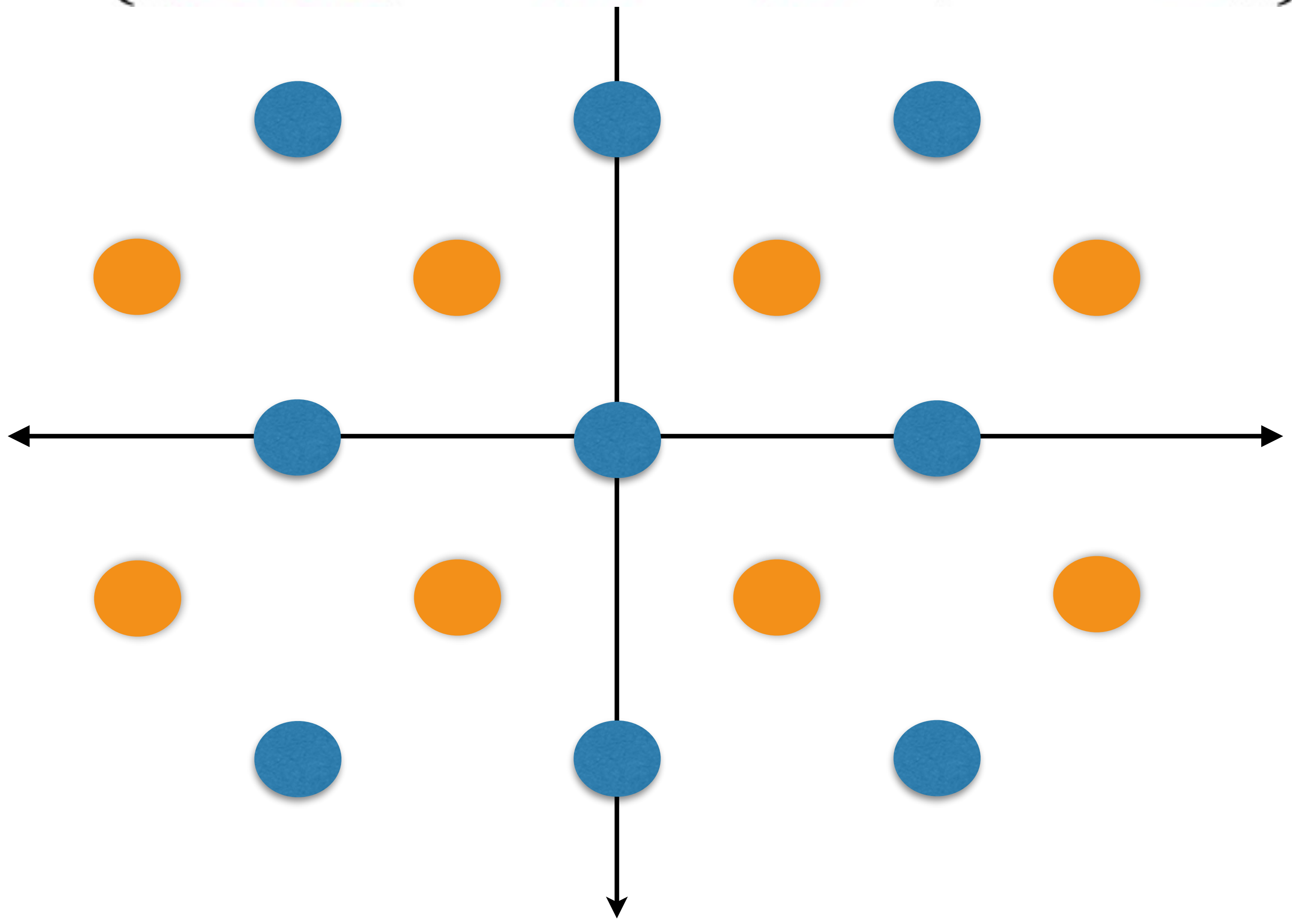
Converting Gaussian Vectors

$$\mathcal{L}^\dagger := \{(\mathbf{y_1}, \mathbf{y_2}) \ : \ \mathbf{y_1} \equiv \mathbf{y_2} \pmod{2\mathcal{L}}\}$$



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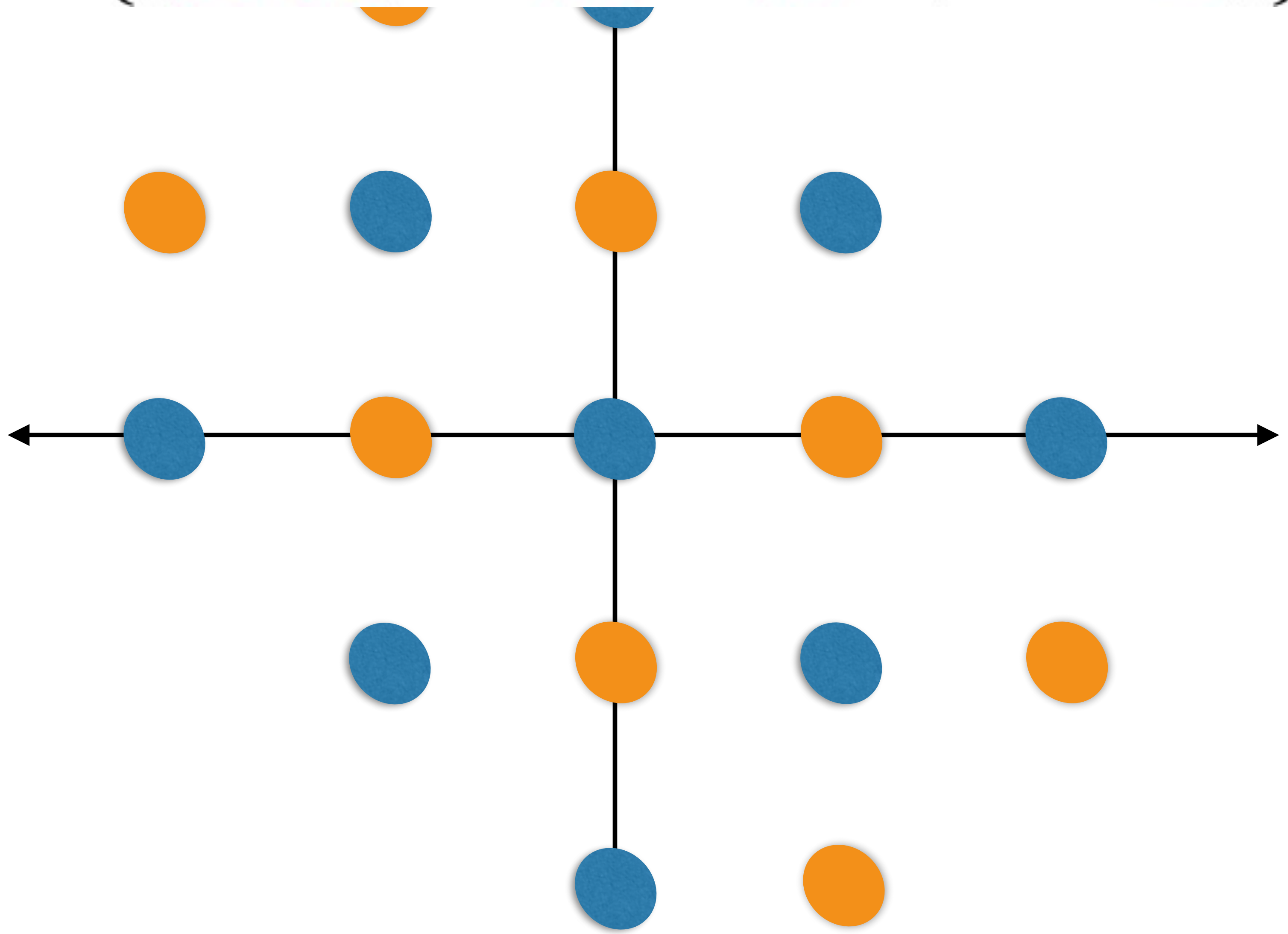

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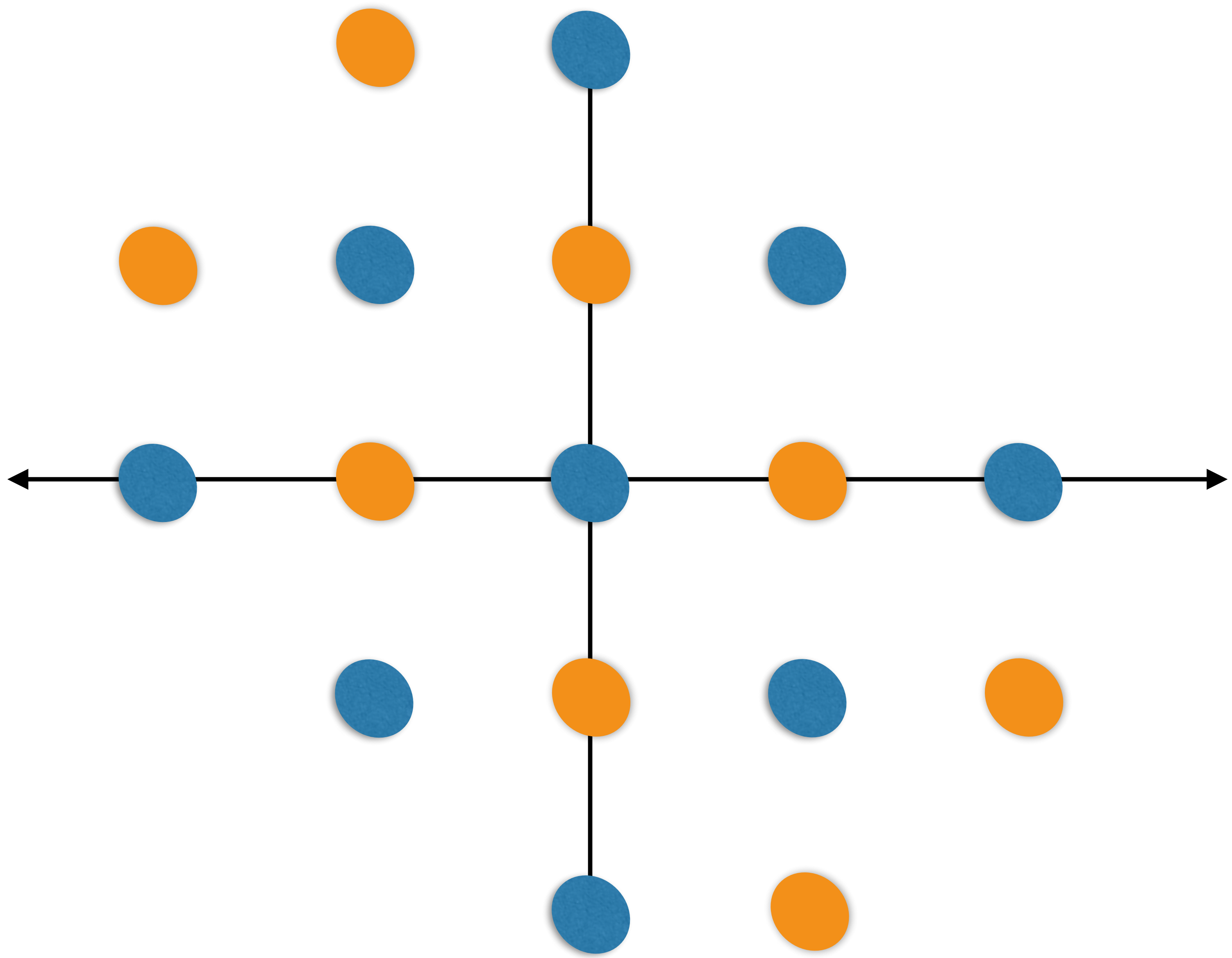
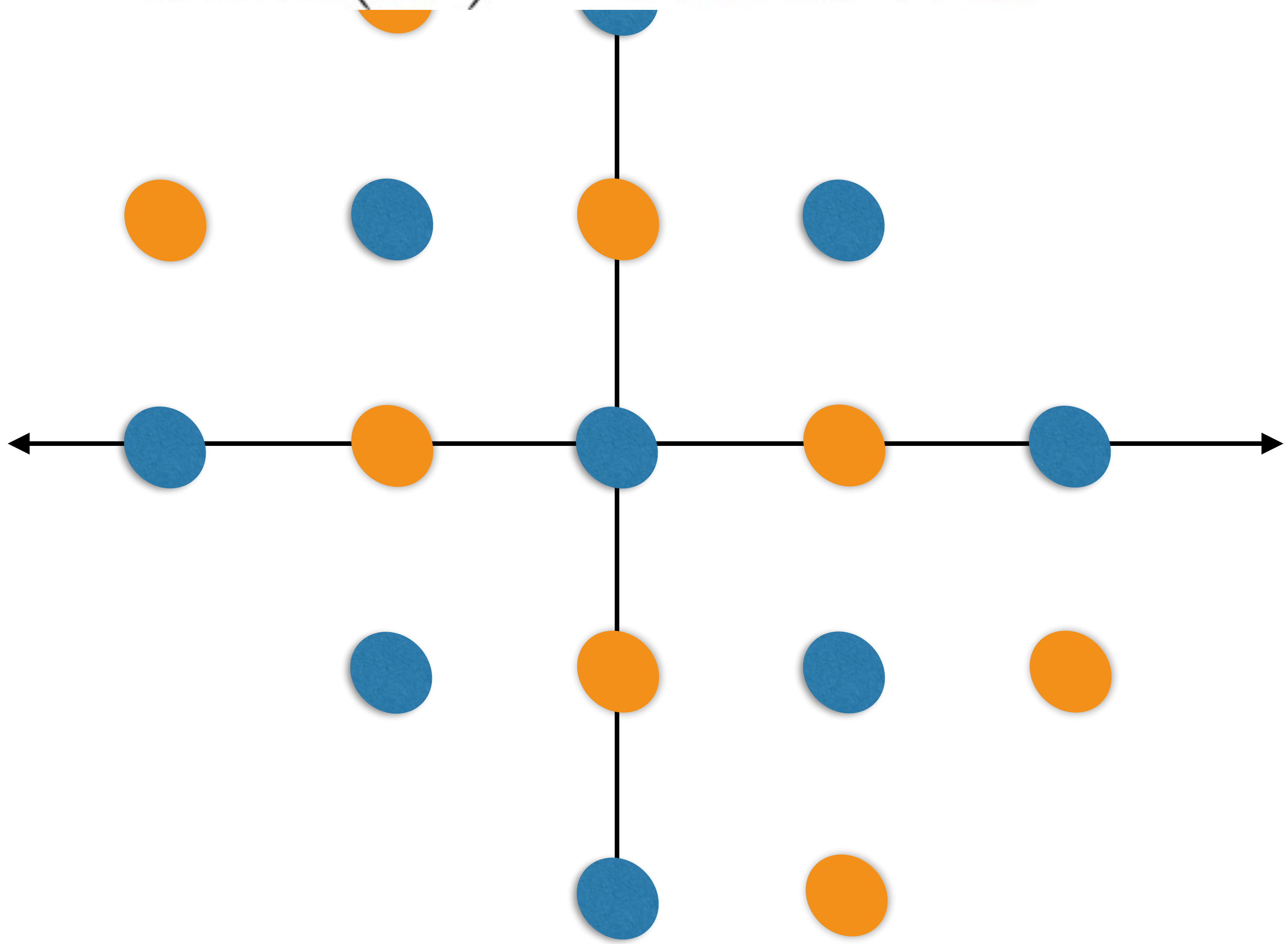


What about the average of two discrete Gaussian vectors *conditioned on* the result being in the lattice?

Converting Gaussian Vectors

$$\text{rotate}(\mathcal{L}^\dagger) = \sqrt{2}\mathcal{L} \times \sqrt{2}\mathcal{L}$$



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$$(\mathbf{y}_1, \mathbf{y}_2) \sim D_{\mathcal{L}^\dagger, s} \Rightarrow \mathsf{rotate}(\mathbf{y}_1, \mathbf{y}_2) \sim D_{\sqrt{2}\mathcal{L} \times \sqrt{2}\mathcal{L}, s}$$

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$$\mathsf{rotate}(\mathbf{y}_1, \mathbf{y}_2) := \left(\frac{\mathbf{y}_1 + \mathbf{y}_2}{\sqrt{2}}, \frac{\mathbf{y}_1 - \mathbf{y}_2}{\sqrt{2}}\right)$$

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Progress!

$$\Big(\frac{\mathbf{y}_1 + \mathbf{y}_2}{2}, \frac{\mathbf{y}_1 - \mathbf{y}_2}{2}\Big) = \frac{\mathsf{rotate}(\mathbf{y}_1, \mathbf{y}_2)}{\sqrt{2}} \sim D_{\mathcal{L} \times \mathcal{L}, s/\sqrt{2}}$$

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$$(\mathbf{y}_1, \mathbf{y}_2) \sim D_{\mathcal{L}^{\dagger},s} \Rightarrow \text{rotate}(\mathbf{y}_1, \mathbf{y}_2) \sim D_{\sqrt{2}\mathcal{L} \times \sqrt{2}\mathcal{L},s}$$

If we sample $\mathbf{y_1}, \mathbf{y_2} \sim D_{\mathcal{L},s}$ *,*
then their average will be distributed as $D_{\mathcal{L},s/\sqrt{2}}$*,*
if we condition on the result being in the lattice.

ogress!

$$\left(\frac{\mathbf{y}_1 + \mathbf{y}_2}{2}, \frac{\mathbf{y}_1 - \mathbf{y}_2}{2}\right) = \frac{\text{rotate}(\mathbf{y}_1, \mathbf{y}_2)}{\sqrt{2}} \sim D_{\mathcal{L} \times \mathcal{L},s/\sqrt{2}}$$

Sampling from the Conditional Distribution

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$$\Pr_{\mathbf{y}_1, \mathbf{y}_2 \sim D_{\mathcal{L}, s}} \left[\frac{\mathbf{y}_1 + \mathbf{y}_2}{2} = \mathbf{y} \;\middle|\; \frac{\mathbf{y}_1 + \mathbf{y}_2}{2} \in \mathcal{L} \right]$$

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$$\Pr_{\mathbf{y}_1,\mathbf{y}_2\sim D_{\mathcal{L},s}}\left[\frac{\mathbf{y}_1+\mathbf{y}_2}{2}=\mathbf{y}\;\middle|\;\frac{\mathbf{y}_1+\mathbf{y}_2}{2}\in\mathcal{L}\right]$$

$$\propto \sum_{\text{coset }\mathbf{c}} \Pr[D_{\mathcal{L},s}\in\mathbf{c}]^2\cdot \Pr_{\mathbf{y}_1,\mathbf{y}_2\sim D_{\mathcal{L},s}}\left[\frac{\mathbf{y}_1+\mathbf{y}_2}{2}=\mathbf{y}\;\middle|\;\mathbf{y}_1,\mathbf{y}_2\in\mathbf{c}\right]$$

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Input: $\approx 2^n$ samples from $D_{\mathcal{L},s}$

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1. Separate the vectors into "buckets" according to their coset.

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2. Pair a number of vectors from each coset proportional to $|\{\mathbf{y}\in\mathbf{c}\}|^2$

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$$\propto \sum_{\text{coset } \mathbf{c}} \Pr[D_{\mathcal{L}} \ldots]^2 \ldots \Pr\left[\frac{\mathbf{y}_1+\mathbf{y}_2}{\ldots} \ldots \mathbf{y} \;\middle|\; \mathbf{y}_1,\mathbf{y}_2 \in \mathbf{c}\right]$$

Output will be $D_{\mathcal{L},s/\sqrt{2}}$!!!

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$$\approx M \cdot \frac{\sum_{\mathbf{c}} \Pr[D_{\mathcal{L},s} \in \mathbf{c}]^2}{\max_{\mathbf{c}} \Pr[D_{\mathcal{L},s} \in \mathbf{c}]}$$

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This can be as bad as $\frac{M}{2^{n/2}}$ after a single step!

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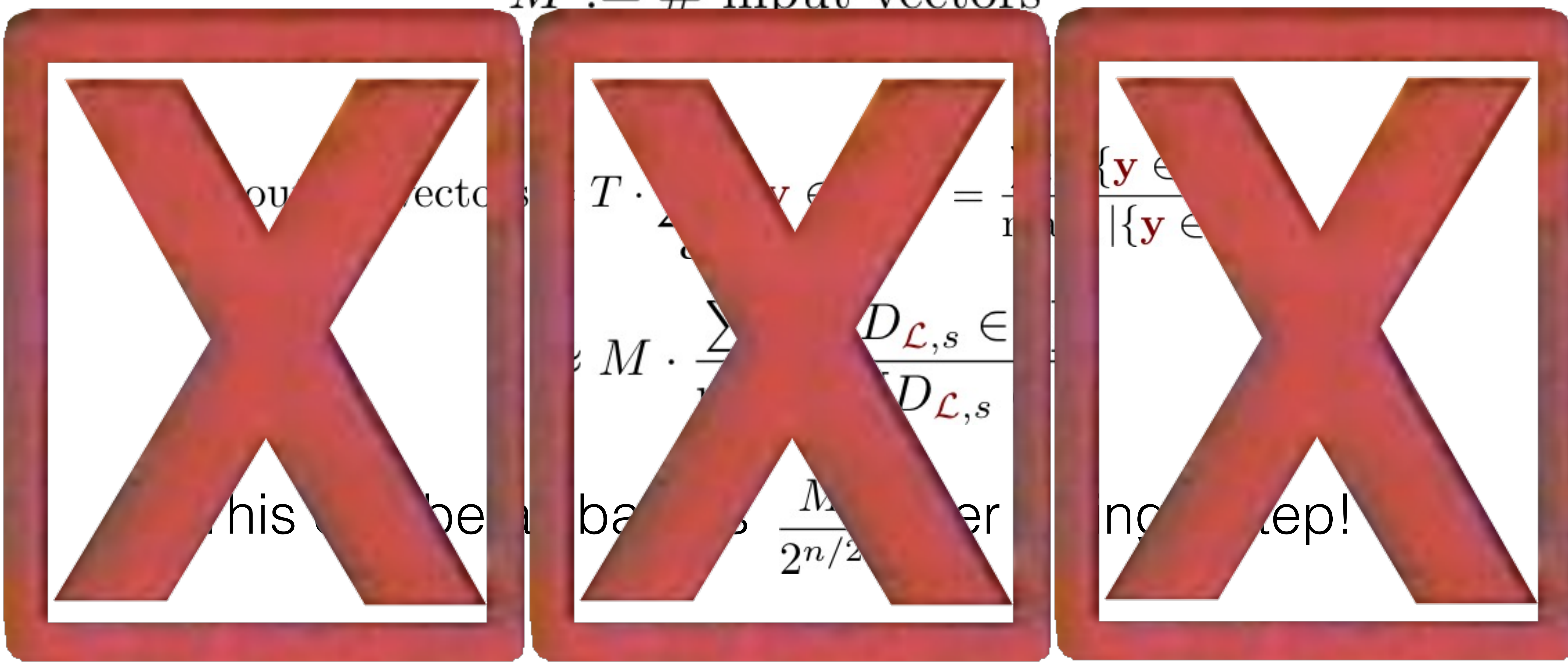

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4. We can then simply output a shortest non-zero vector from our samples.

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  - (Not equivalence because the reduction in the other direction requires 1.38^n samples.)

Open Questions/Future Work

- Other uses for discrete Gaussian sampling at arbitrary parameters?
- Faster discrete Gaussian sampling?
- Is centered discrete Gaussian sampling NP-hard? (Conjecture: No. Can we prove it?)
- Lower bounds for CVP/SVP assuming SETH (or something similar)?

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