Corruption-Robust Contextual Search

Chara Podimata

(UC Berkeley → MIT)

Based on joint works with

Contextual Search Realizable Version

For rounds $t = 1, \ldots, T$:

[Cohen, Lobel, Paes Leme, **EC16/MS19**], [Lobel, Paes Leme, Vladu, **EC17/OR18**], [Paes Leme, Schneider, **FOCS18**], [Liu, Paes Leme, Schneider, **SODA21**]

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$Regret$ = total loss incurred – total loss for benchmark querying policy

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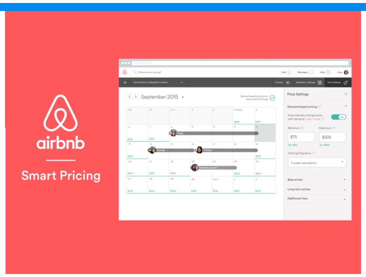

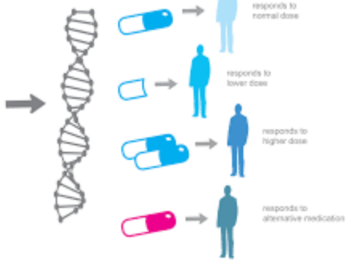

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Known Results

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Contextual Search with Adversarial Noise

Exists hidden $\theta^\star \in \mathbb{R}^d$ same across rounds.

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$$y_t^\star = \langle u_t, \theta^\star \rangle$$

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$y_t^\star = \langle u_t, \theta^\star \rangle + z_t$
$z_t \in [-1,1]$: adaptively and adversarially chosen

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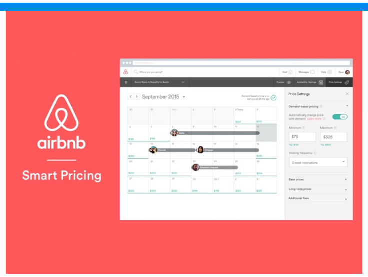

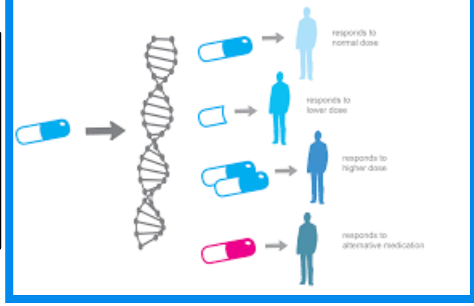

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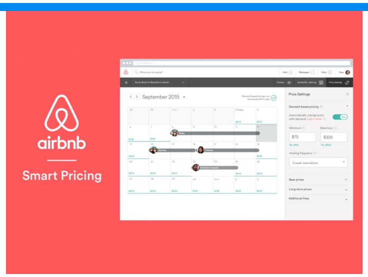

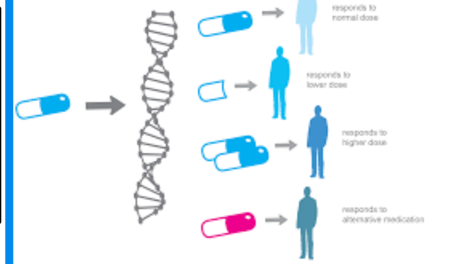

Desiderata for our Algorithms

✓ Graceful degradation of regret with C

✓ No knowledge of C assumed (*agnostic*)

Main Results

[Krishnamurthy, Lykouris, **P.**, Schapire, **STOC21/OR22**]

ε –ball loss: $Regret = O(C_0\, d^3 \log^3 1/\varepsilon)$

symmetric, pricing loss: $Regret = O(C_0\, d^3 \log^3 T)$

[Paes Leme, **P.**, Schneider, **COLT22**]

ε –ball loss: $Regret = O(C_0 + d \log 1/\varepsilon)$

symmetric loss: polytime, $Regret = O(C_1 + d \log T), where\ C_1 = \sum_t |z_t|$

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[Krishnamurthy, Lykouris, **P.**, Schapire, **STOC21/OR22**]

for $z_t \in \{0,1\}$: $C_0 = \#\{z_t = 1\}$

- $Regret = O(C_0\, d^3 \log^3 T)$ for pricing, symmetric loss
- $Regret = O(C_0\, d^3 \log^3 1/\varepsilon)$ for ε –ball loss

Runtime
$poly\,(d, \log T)^{poly(\log T)}$

Traditional Approach for Contextual Search Algorithms

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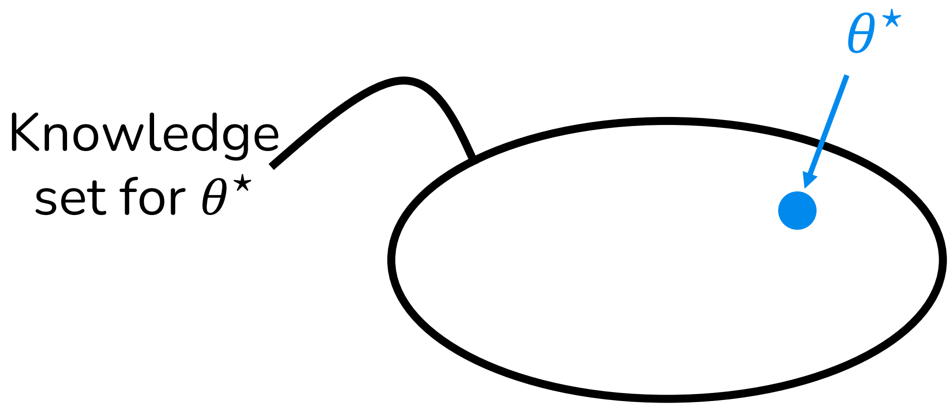

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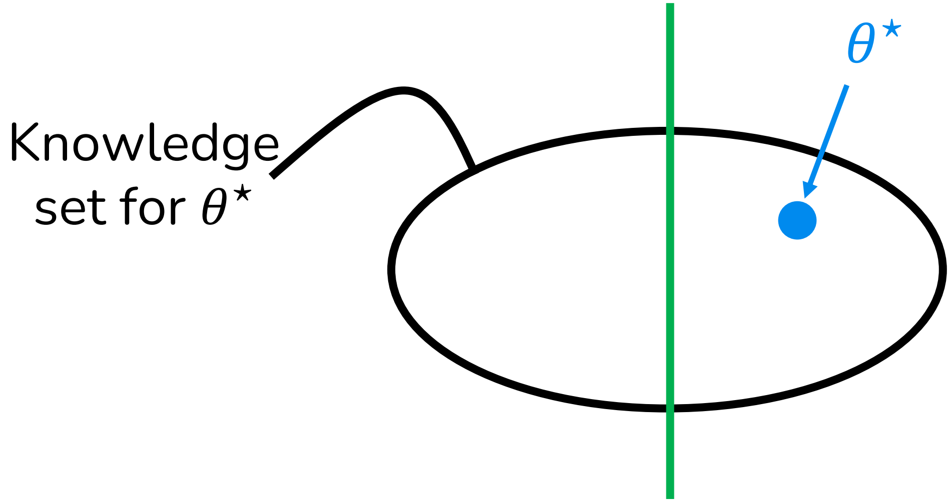

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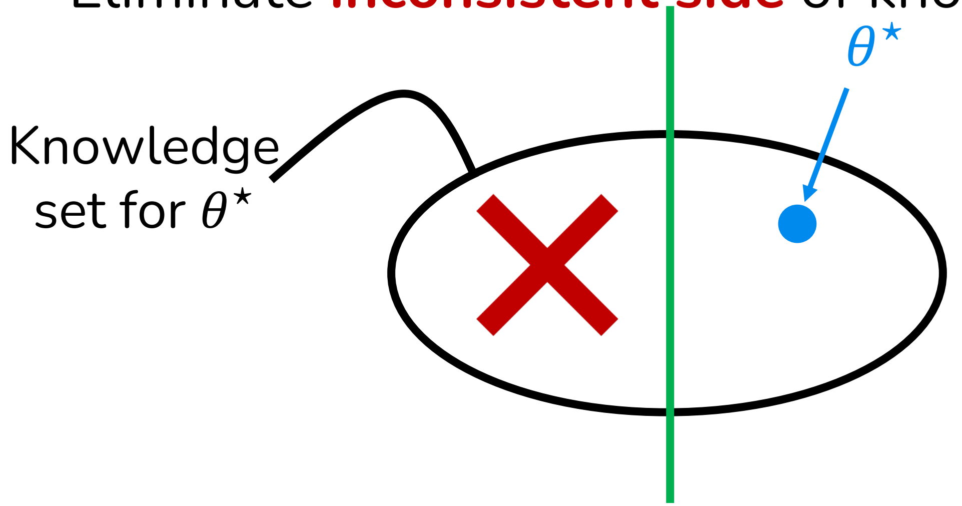

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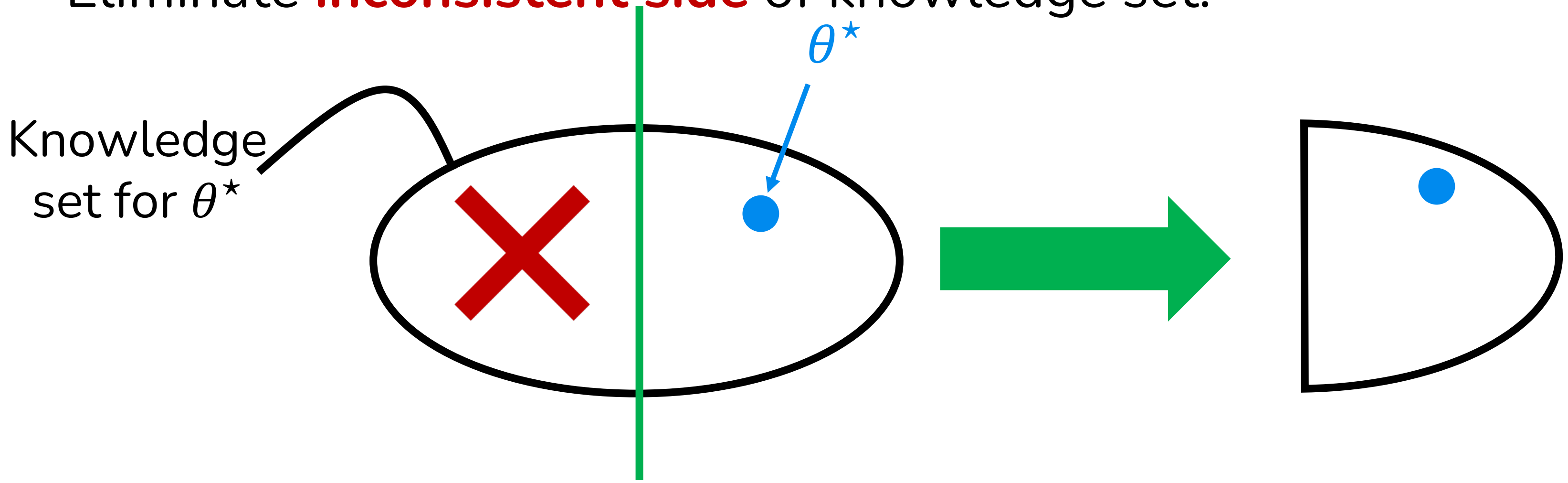

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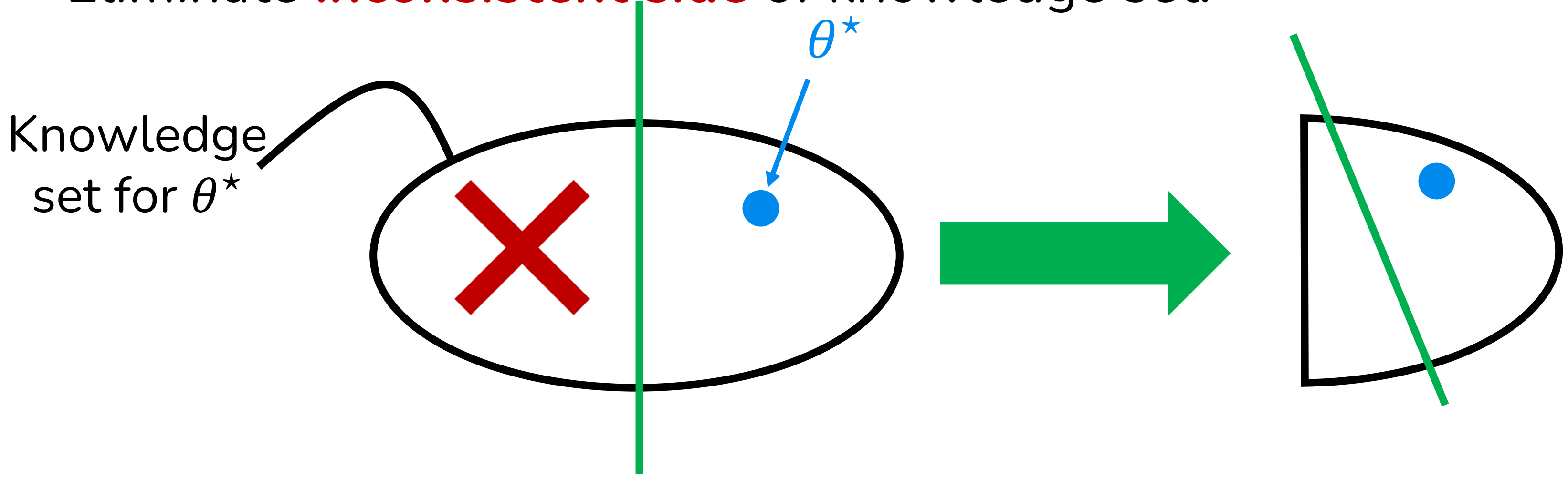

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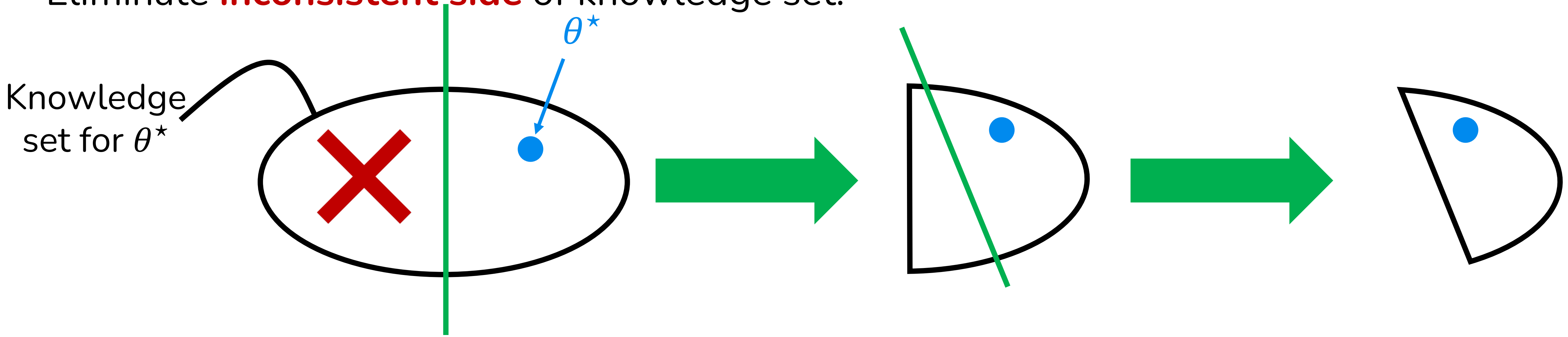

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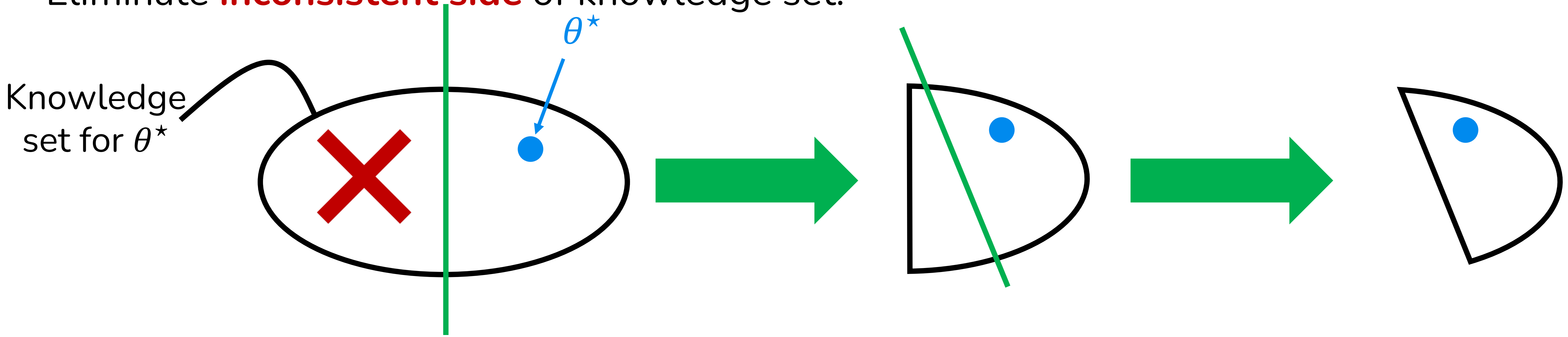

Aggressively introducing cuts → **fast**, **logarithmic** bounds

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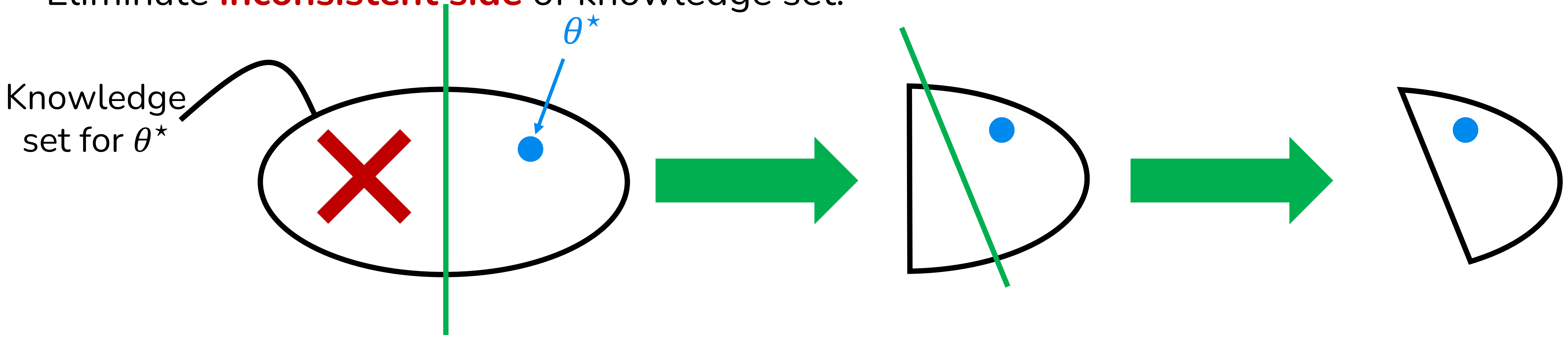

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$Regret \leq O(d \cdot \log T)$

[Lobel, Paes Leme, Vladu, **EC17/OR18**]

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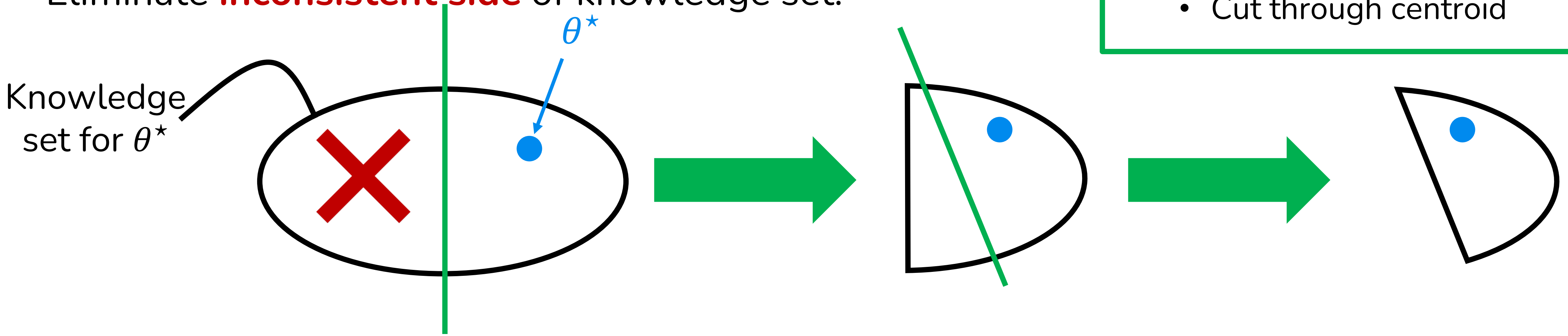
Important properties of cut

1. Never eliminate $\theta^\star$
   - Retain all parameters consistent with feedback
2. Volumetric progress
   - Cut through centroid



Aggressively introducing cuts → **fast**, **logarithmic** bounds

$Regret \leq O(d \cdot \log T)$

[Lobel, Paes Leme, Vladu, **EC17/OR18**]

Robustify-ing Knowledge-Set-Based Techniques

Idea Overview

1. Create a robust version of the knowledge-set-based algorithm that is robust to a **known** amount of corruption $\bar{c} \approx \log T$.
2. **Unknown** C: Run a variant of the Multi-Layering Race technique from [Lykouris, Mirrokni, Paes Leme, **STOC18**].

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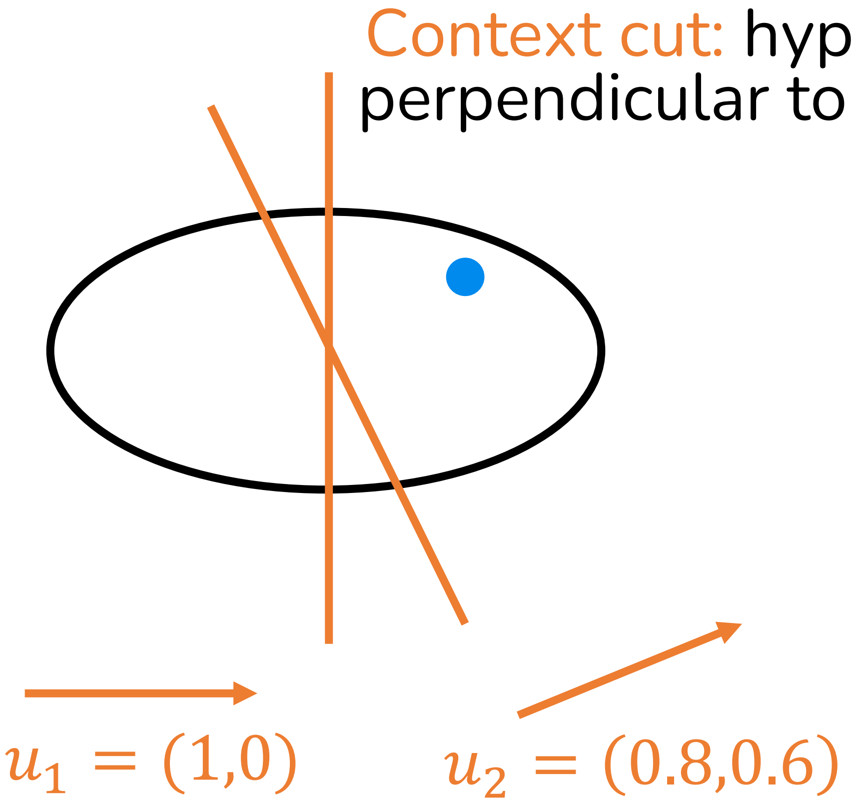

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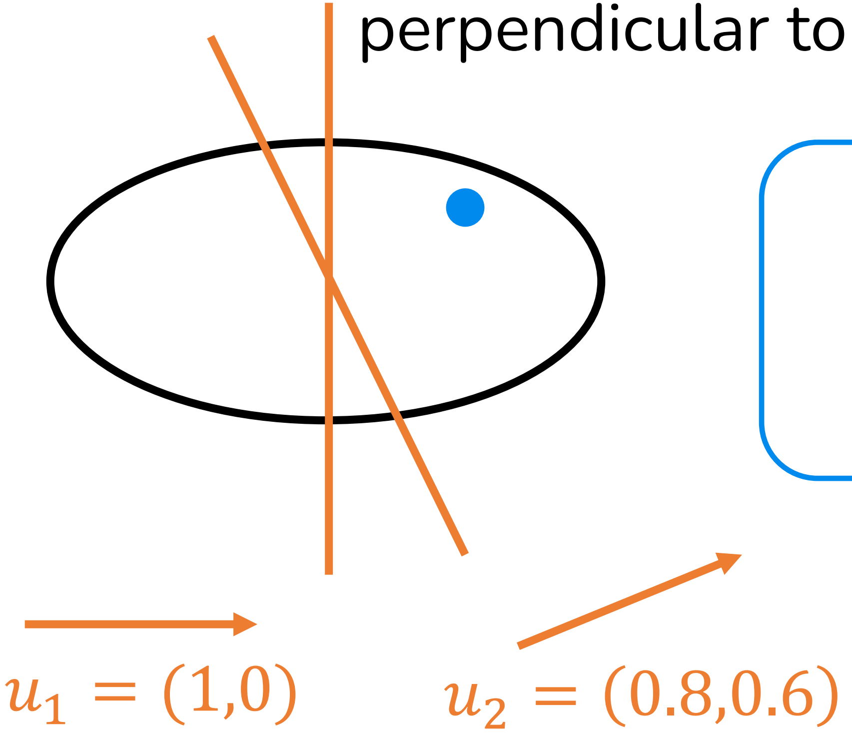

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Keep "penalty" for each parameter & make cut once a context cut fully retains protected region on one side.

- Protected region: all parameters with "penalty" $\leq \bar{c}$.

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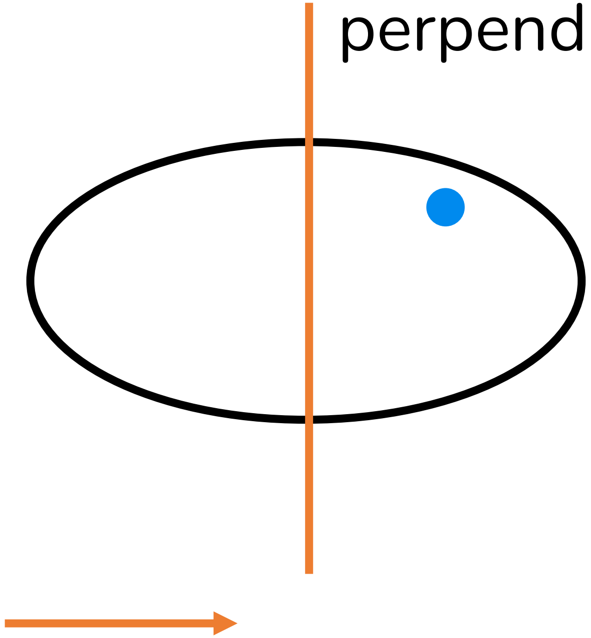

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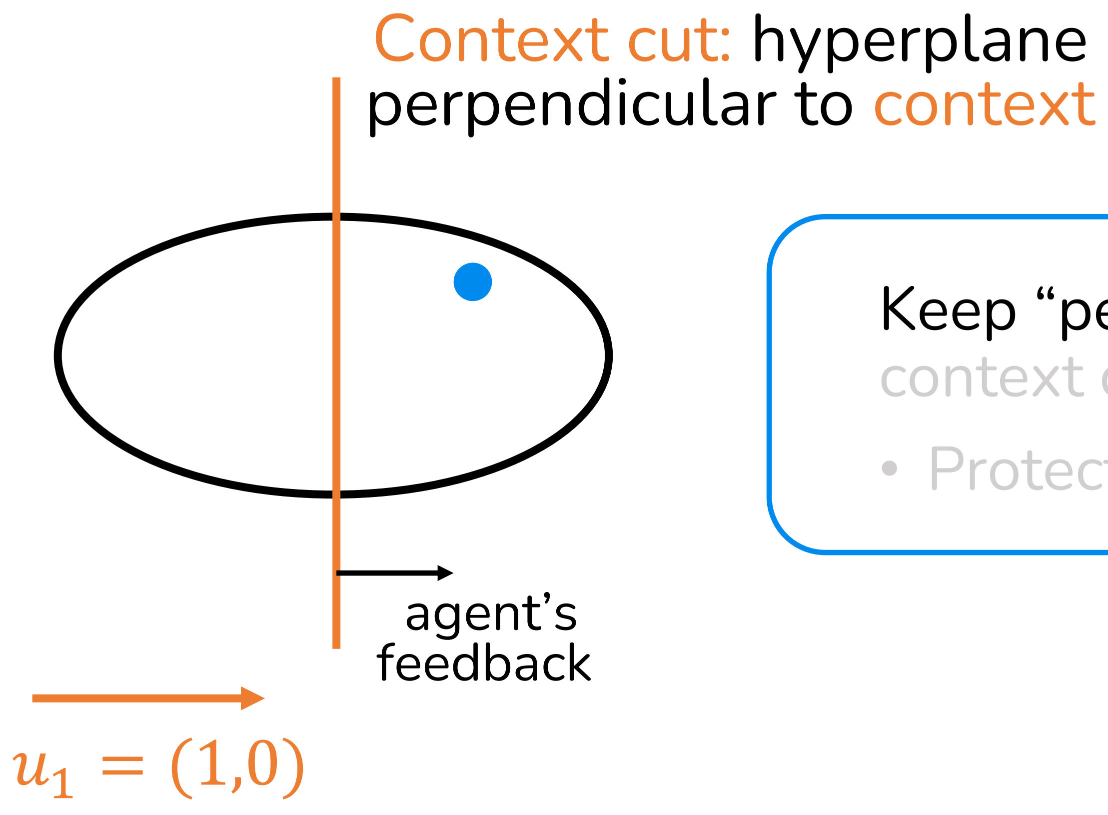

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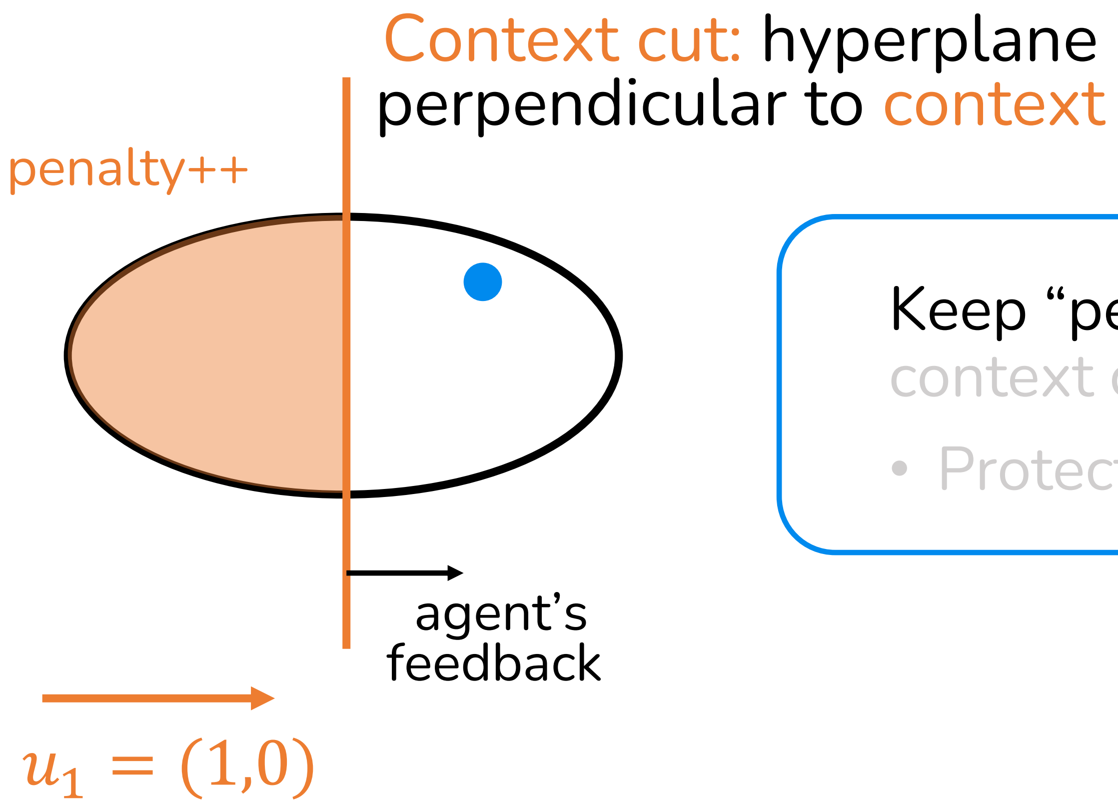

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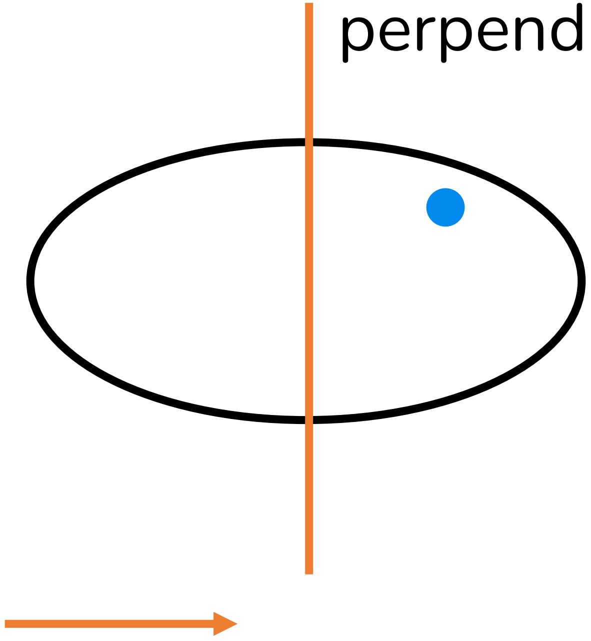

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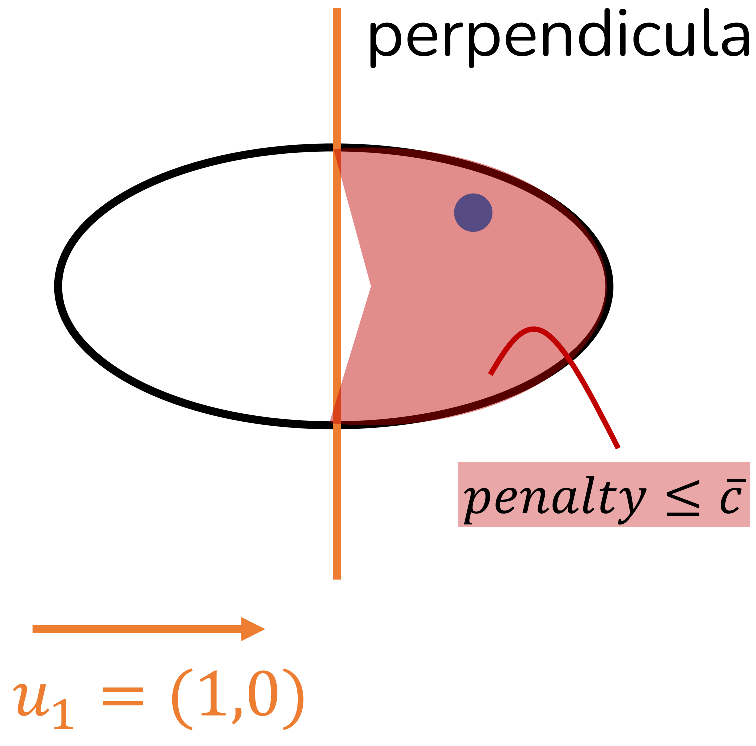

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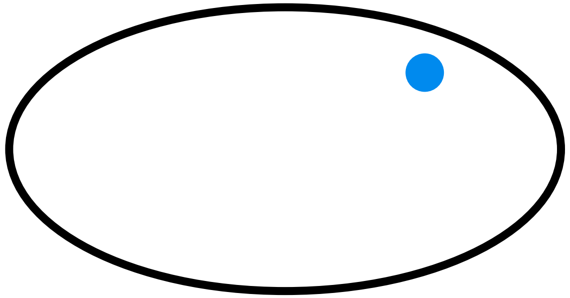

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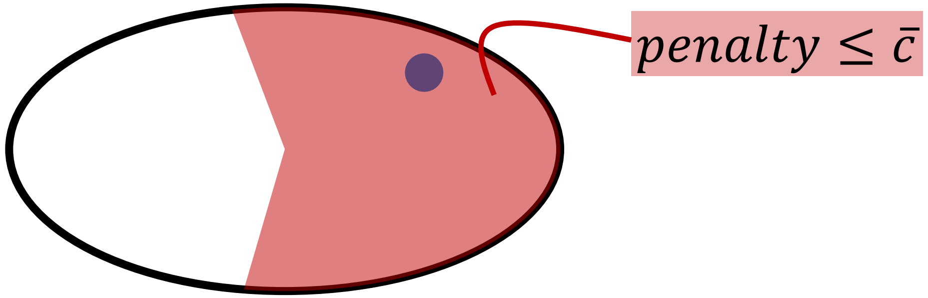

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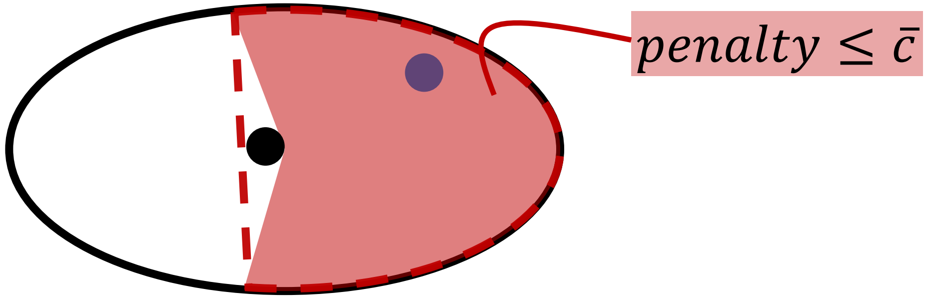

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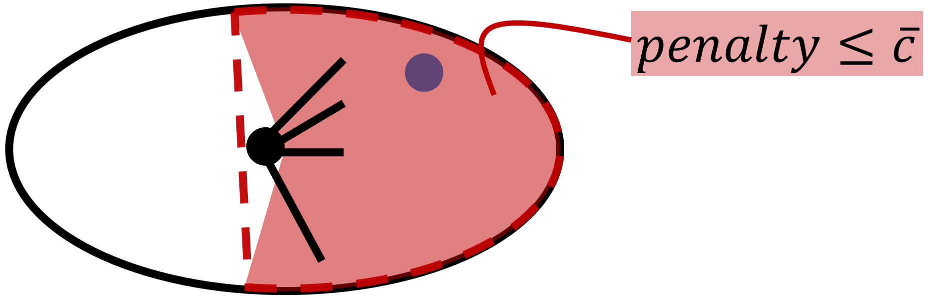

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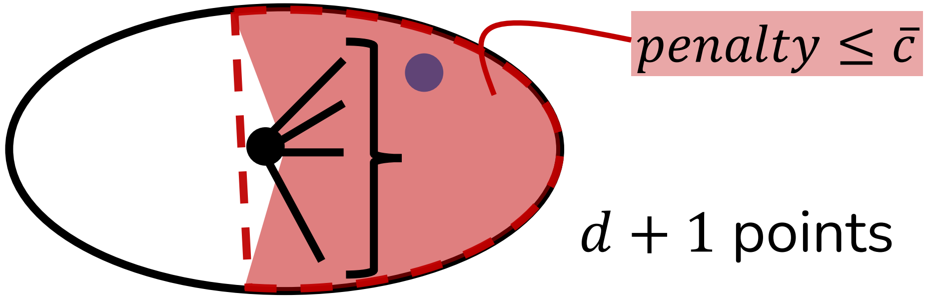

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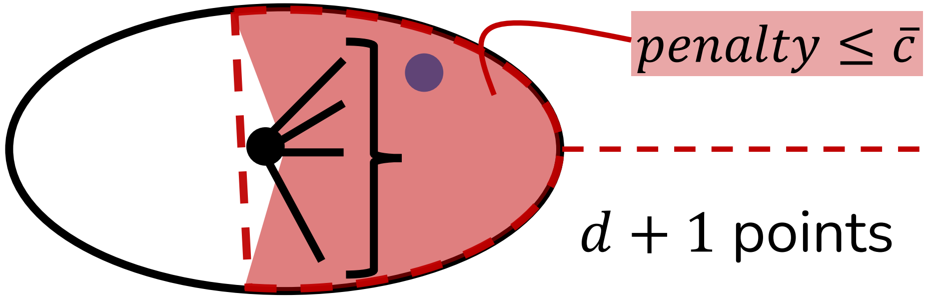

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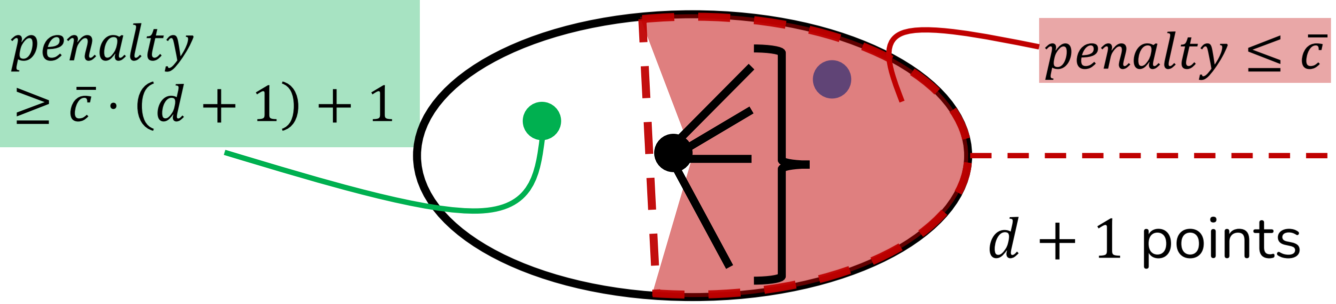

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Idea 4

Use Perceptron to find a valid cut.

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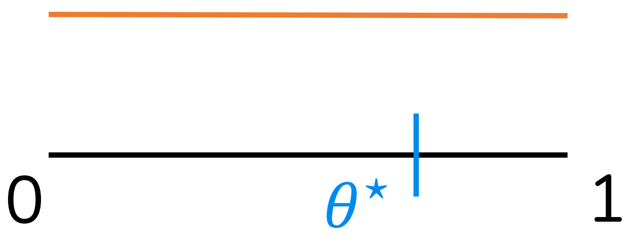

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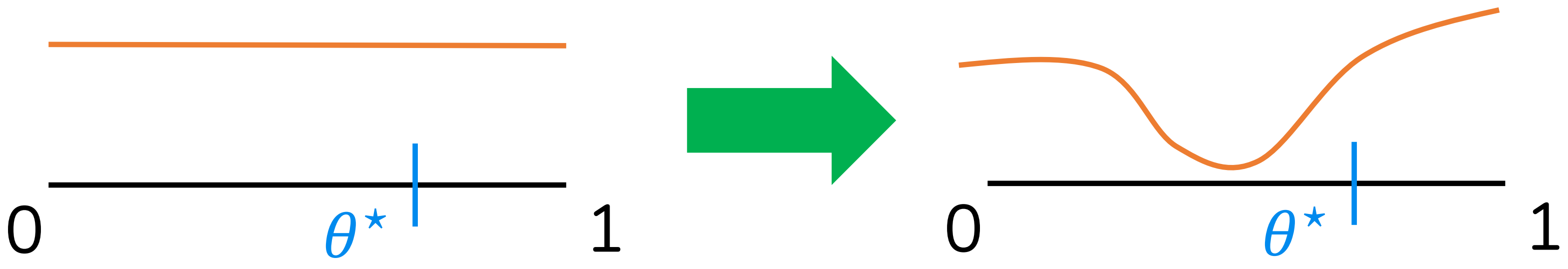

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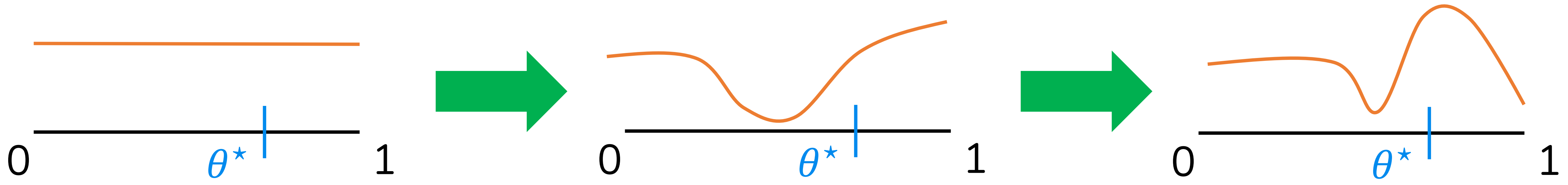

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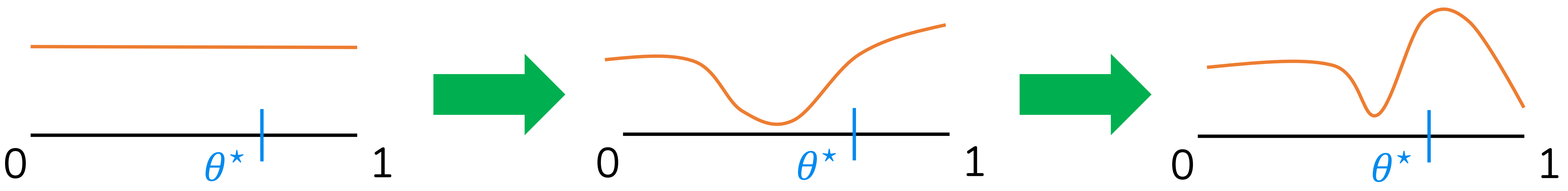
Seemingly more "forgiving" approach → faster bounds for corruption-robust

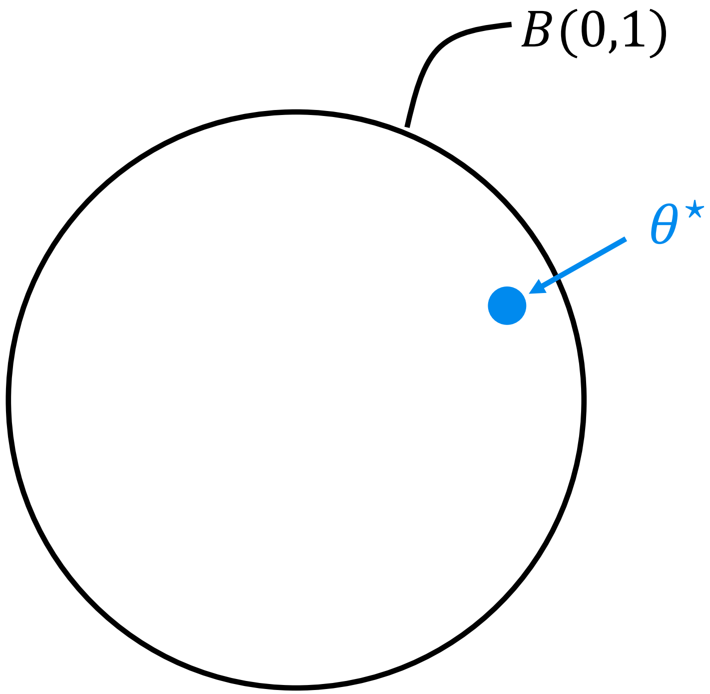


Algorithm for $\varepsilon-$ Ball Loss

Algorithm for ε – Ball Loss

$\boldsymbol{\varepsilon}$ – Window Median Algorithm

Initialize $f_1(x)$: uniform over $B(0,1)$.



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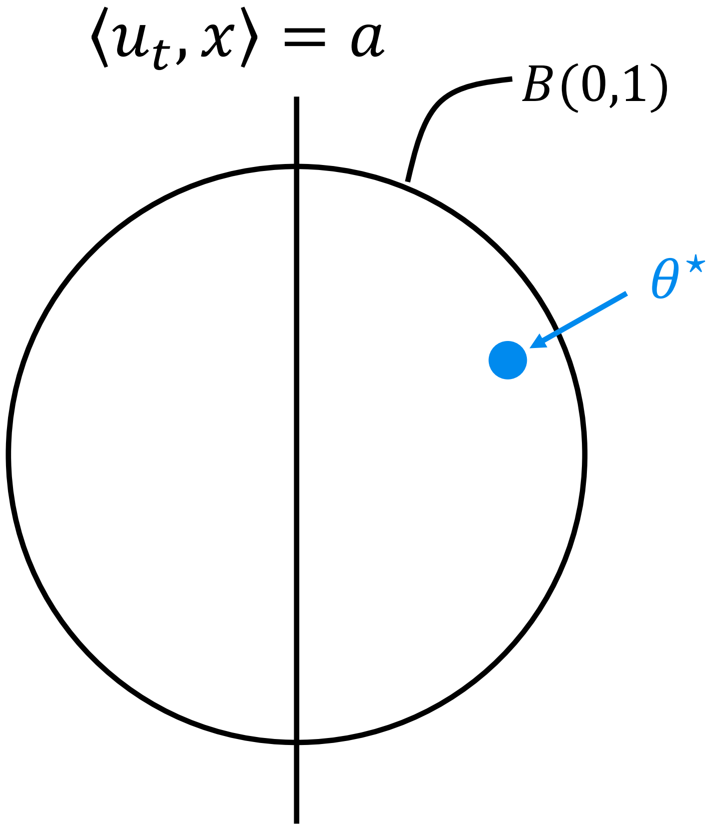

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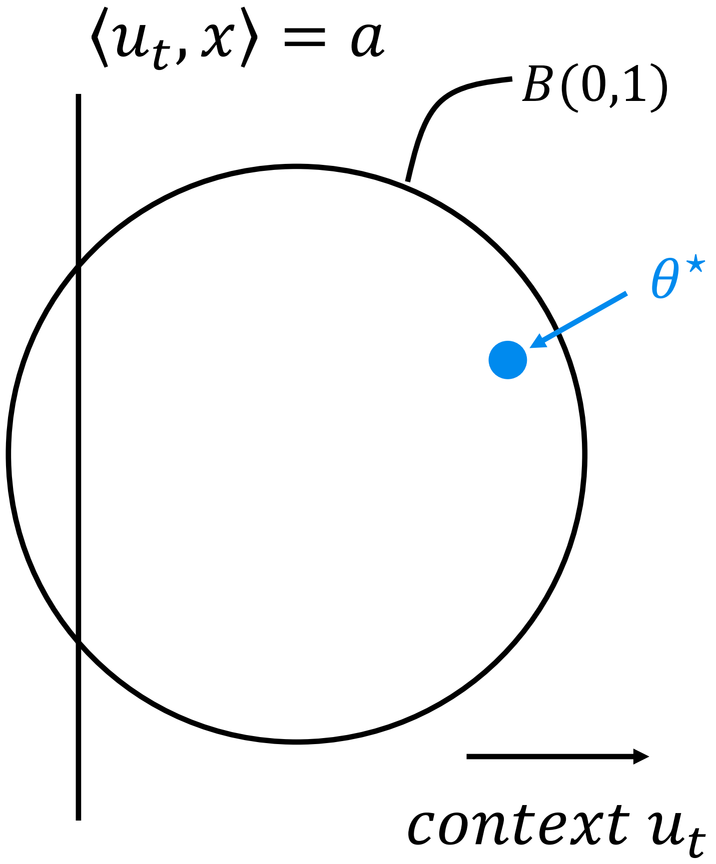

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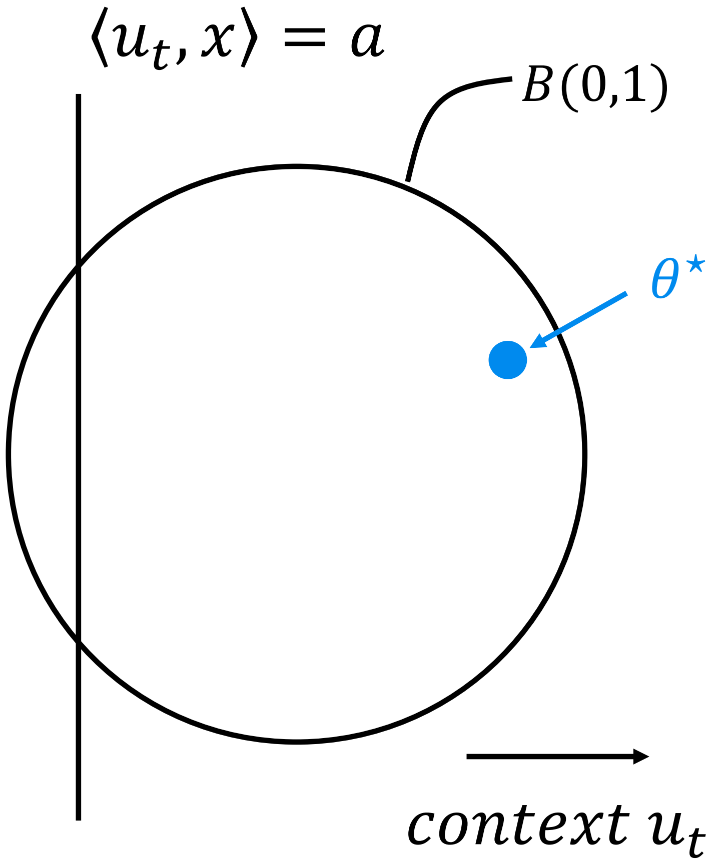

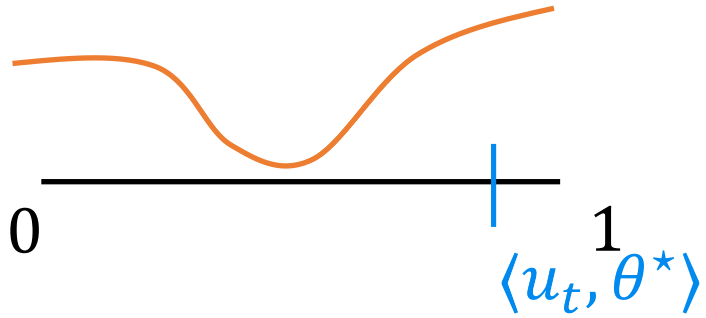

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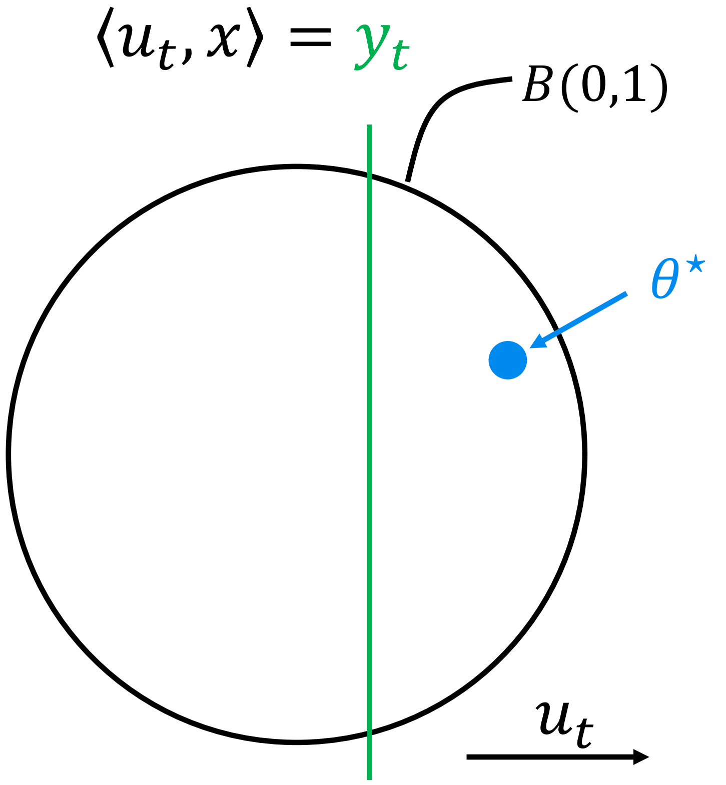

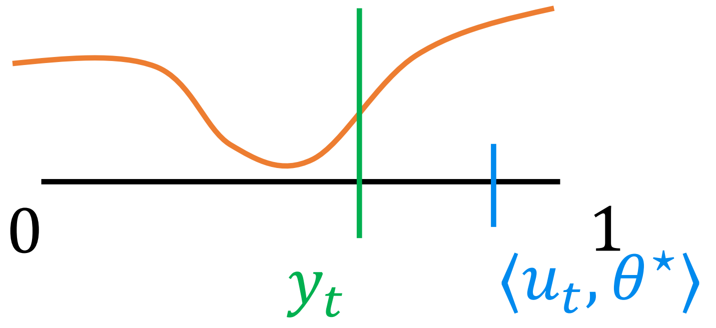

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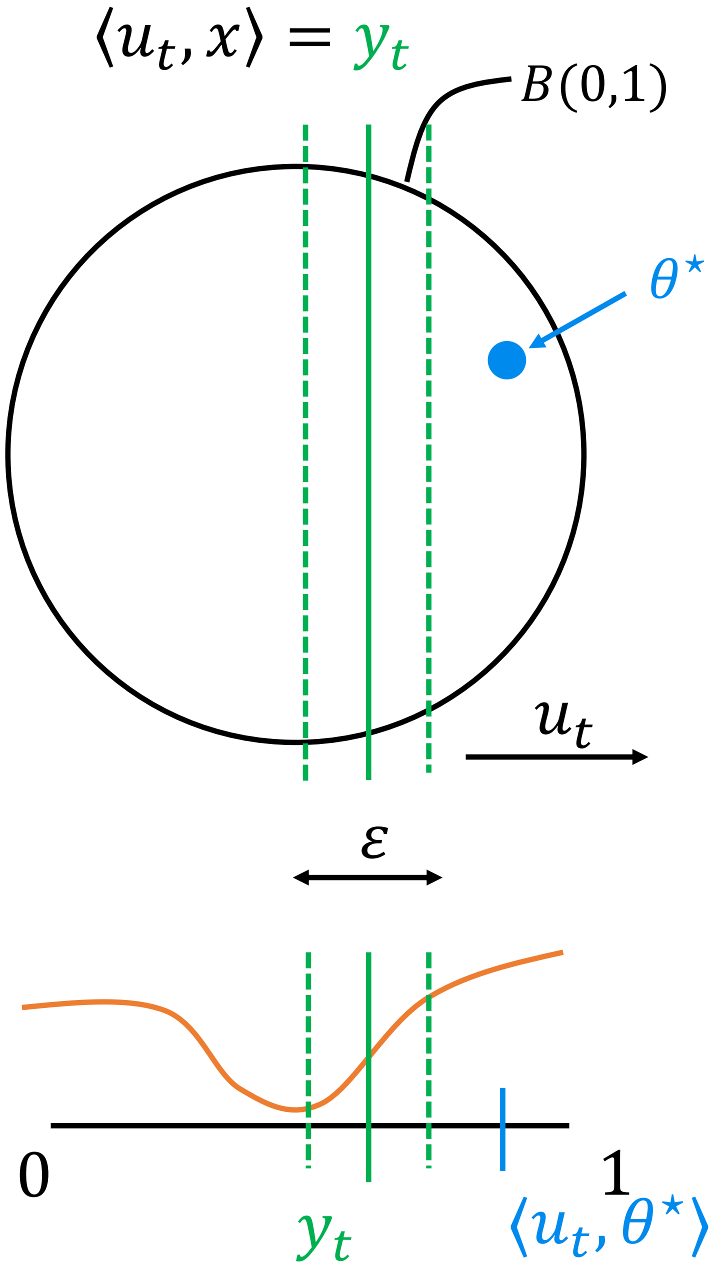

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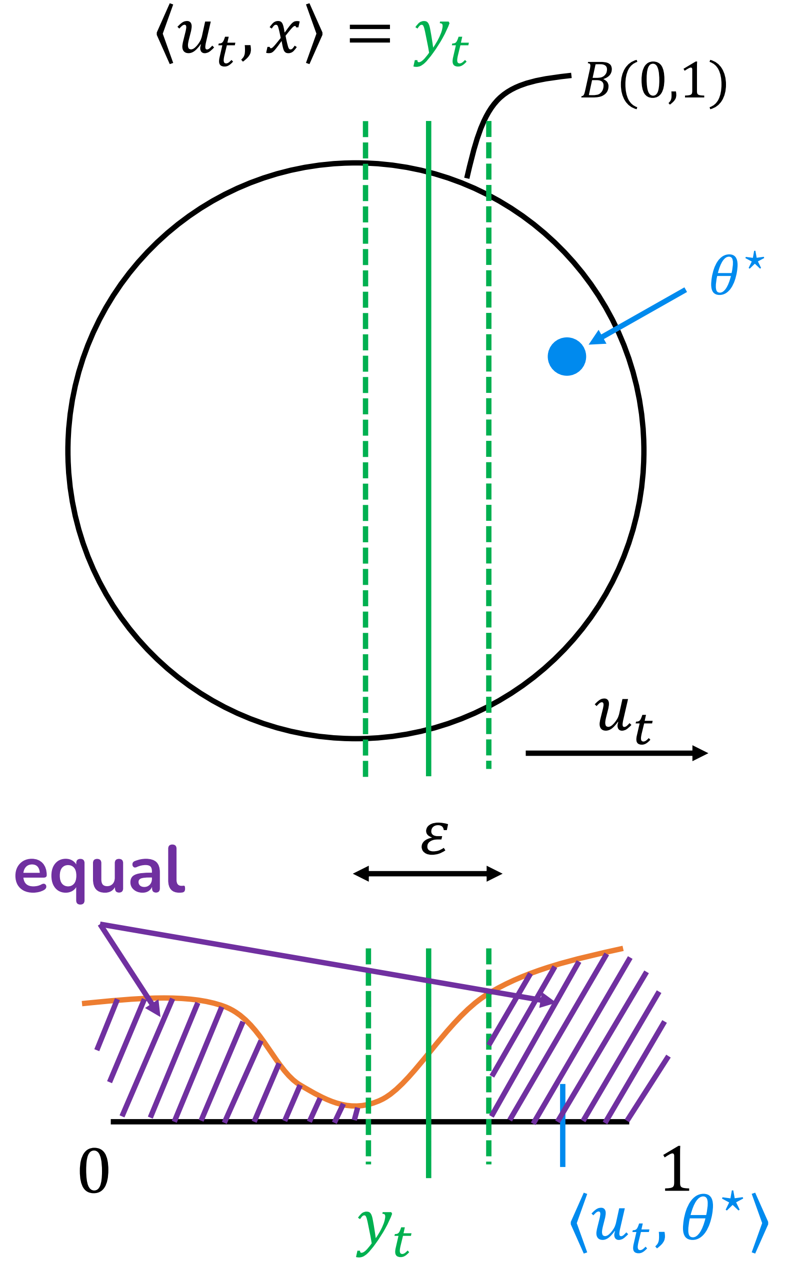

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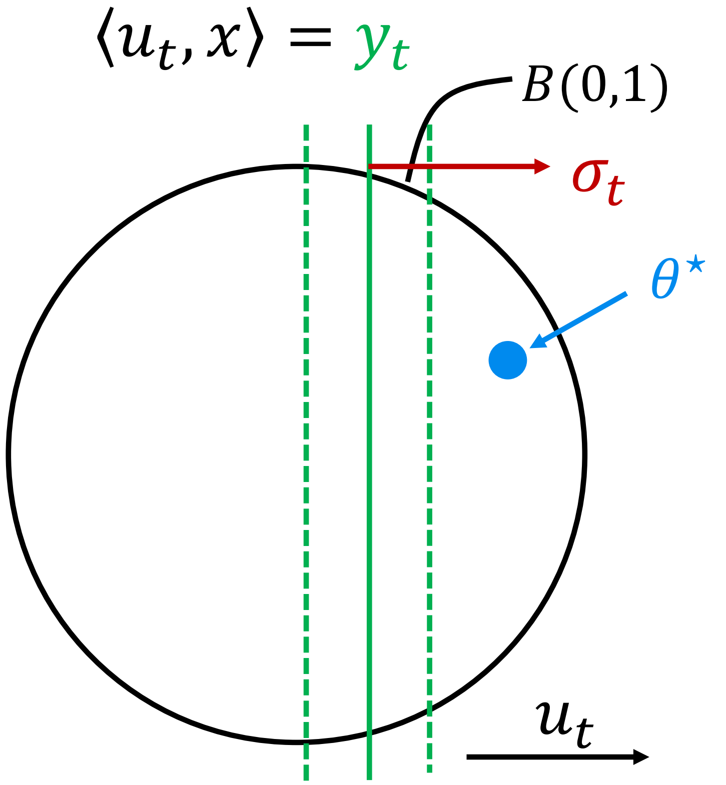

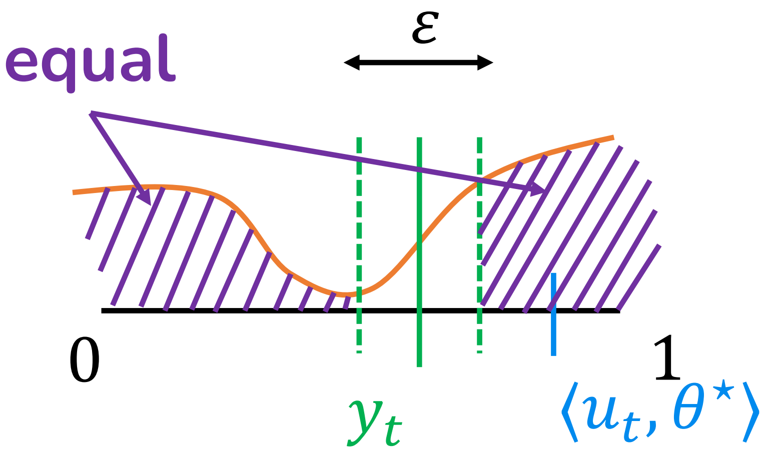

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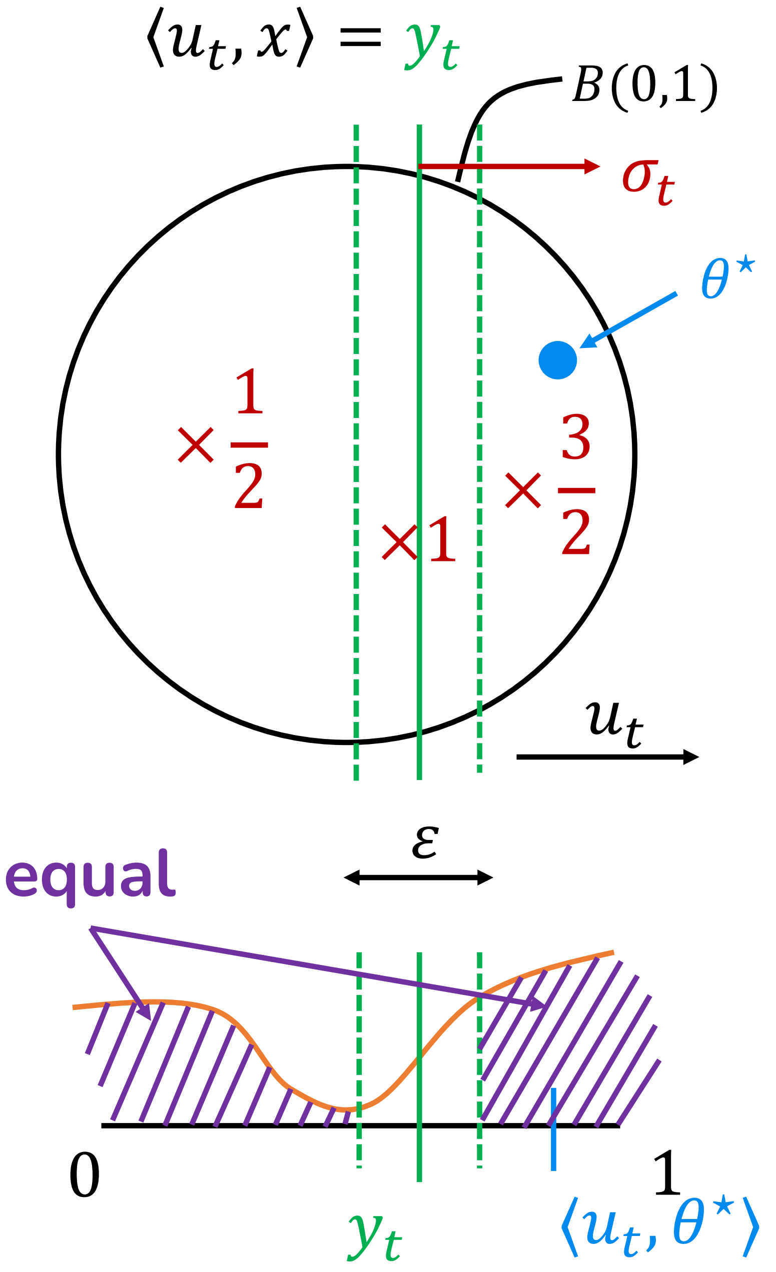

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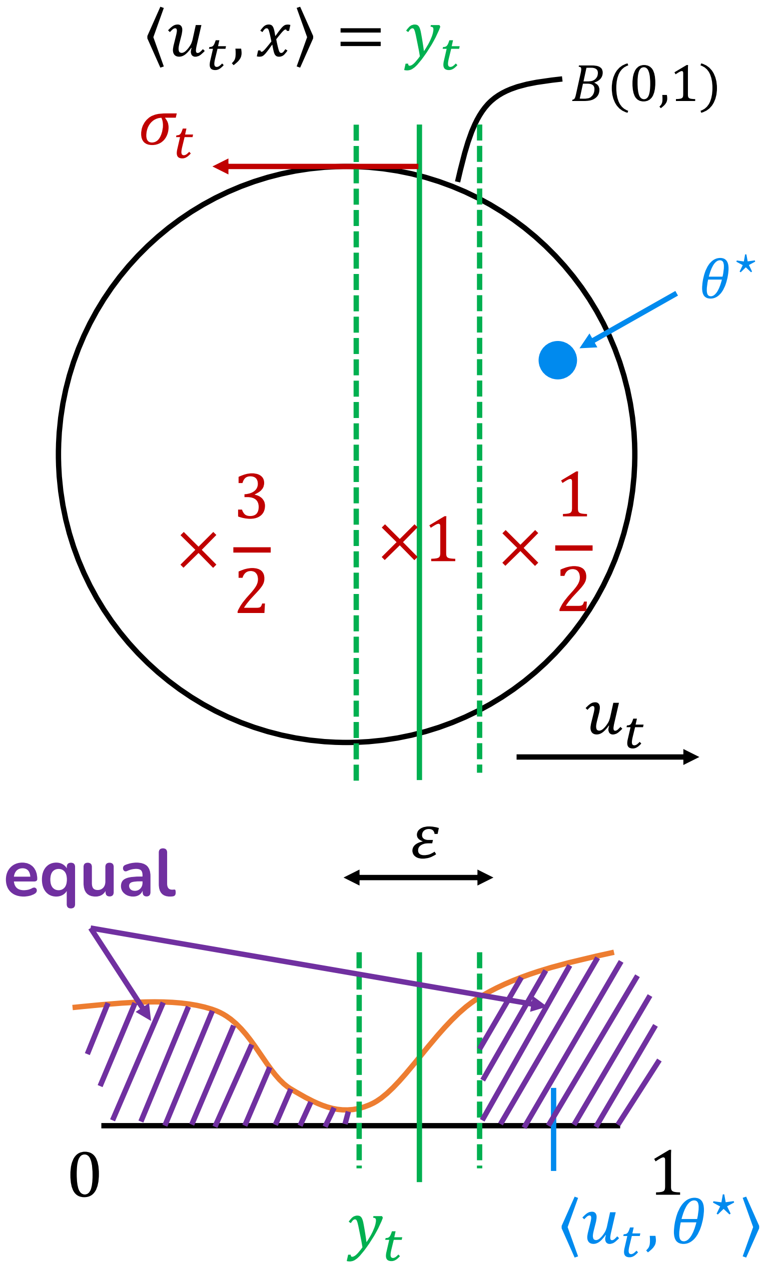

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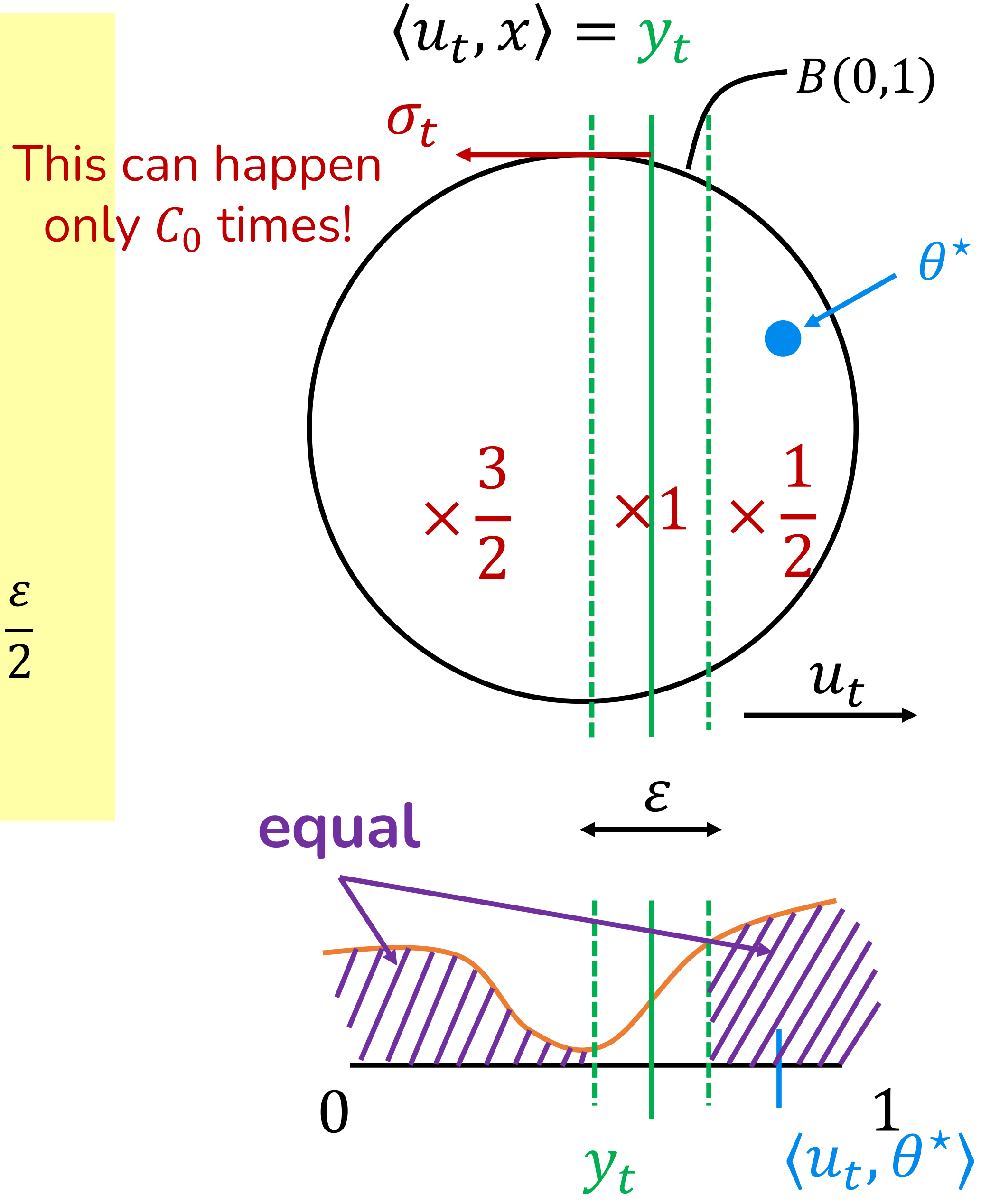

Algorithm for ε – Ball Loss

ε – Window Median Algorithm

Initialize $f_1(x)$: uniform over $B(0,1)$.

For rounds $t = 1, \dots, T$:

- Observe u_t and query $y_t = \varepsilon - window - median(f_t)$
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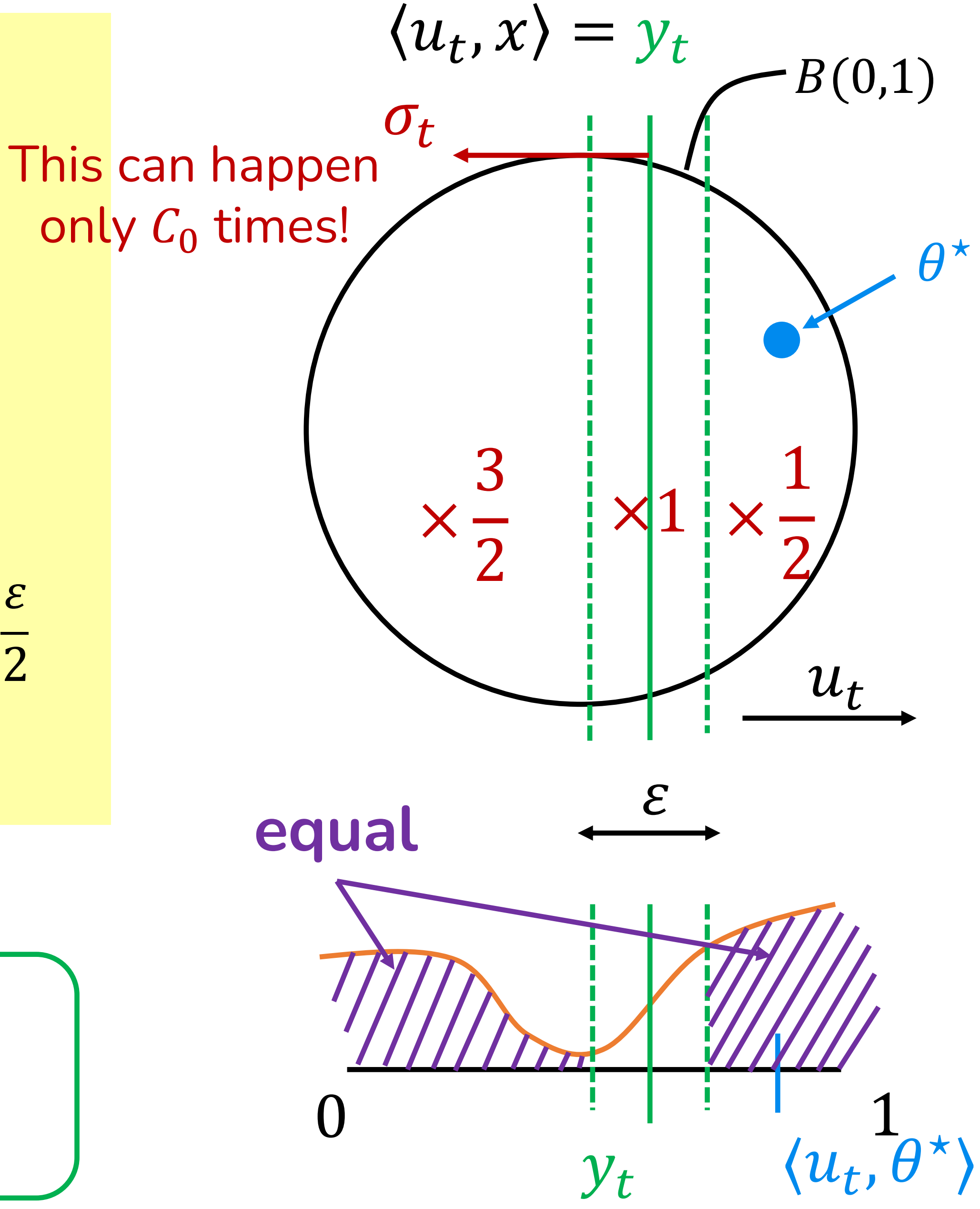

Main Result

- ε – ball loss: $Regret = O(C_0 + d \log 1/\varepsilon)$

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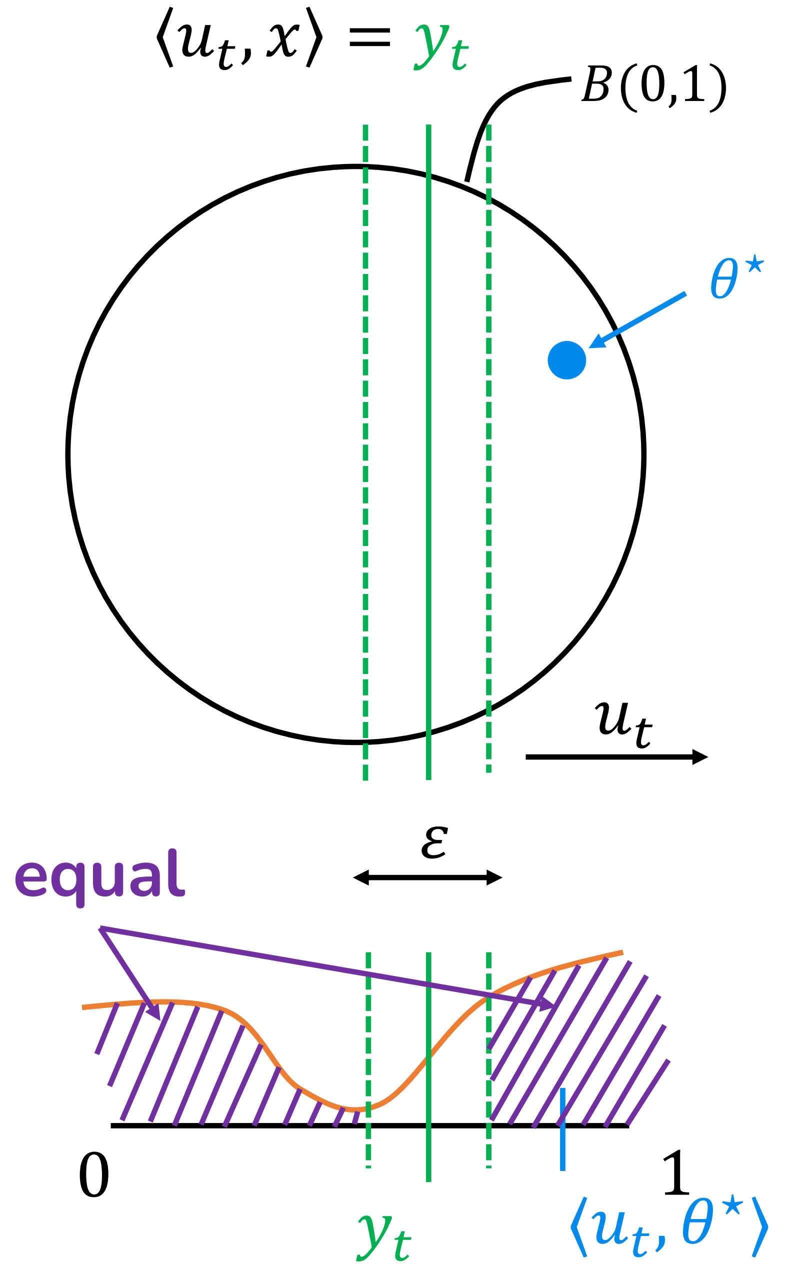
Proof Idea



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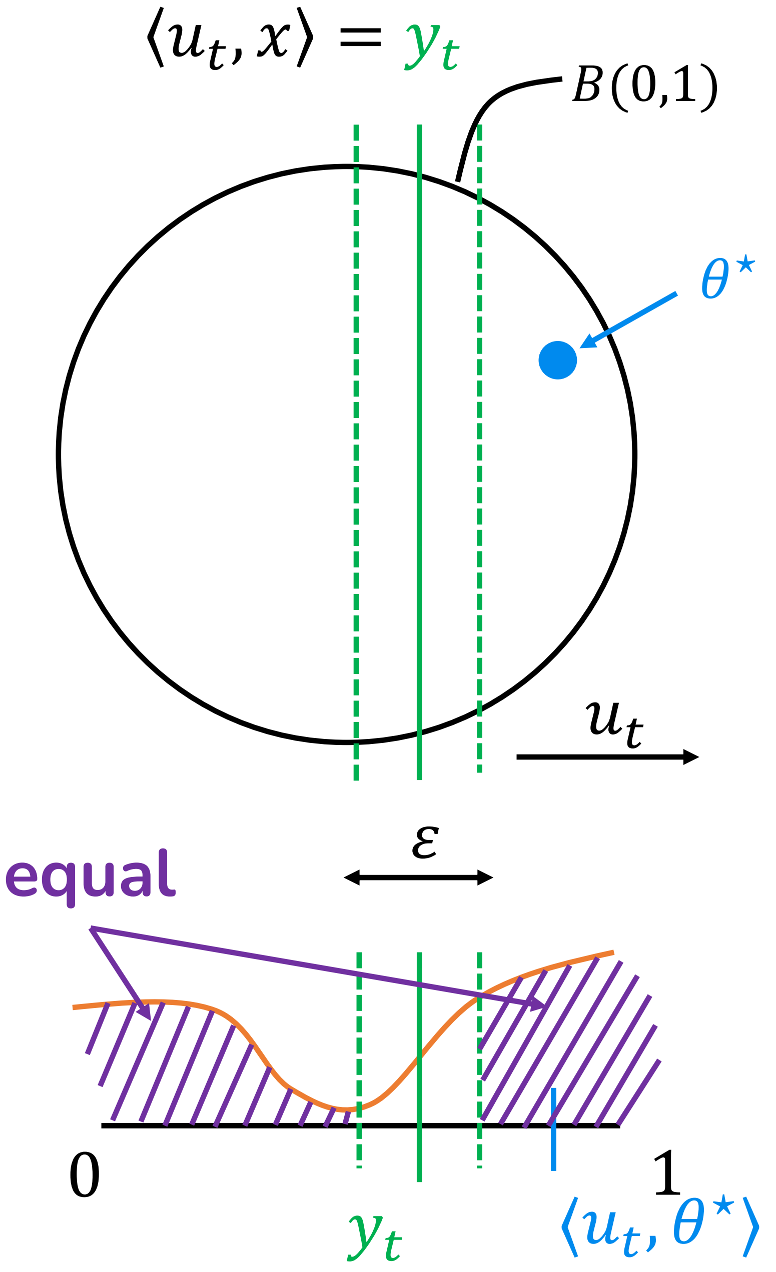
1. Given updates above, $f_t(\cdot)$ is **always a density**.



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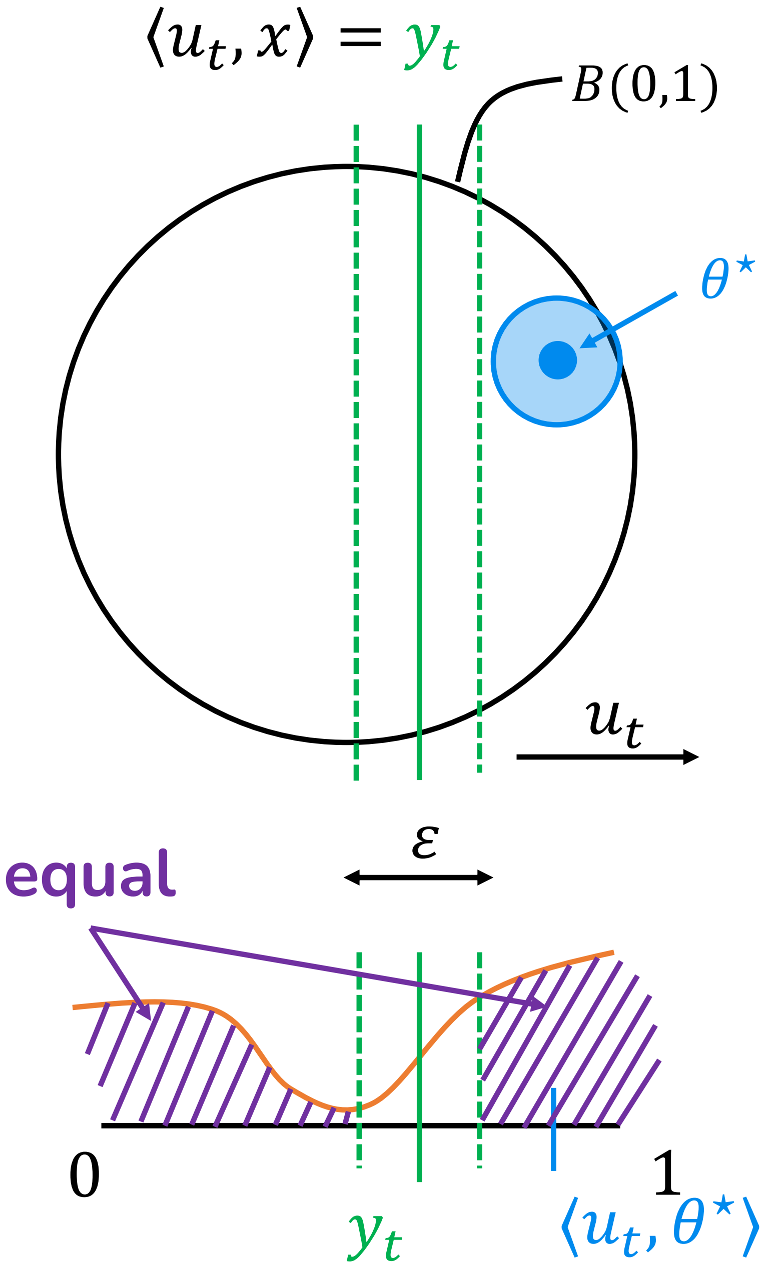
2. Potential $\Phi_t = \int_{B(\theta^\star, \varepsilon/2)} f_t(x)dx$:



Algorithm for ε – Ball Loss

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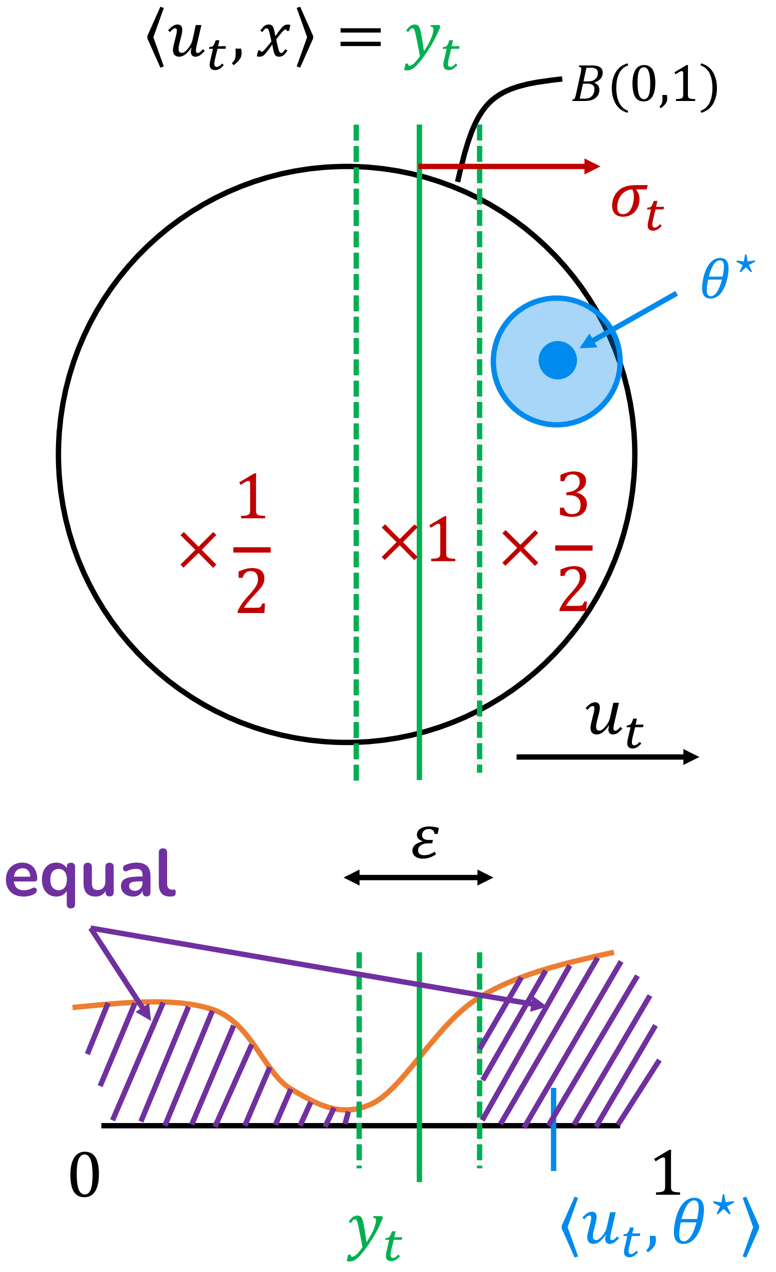

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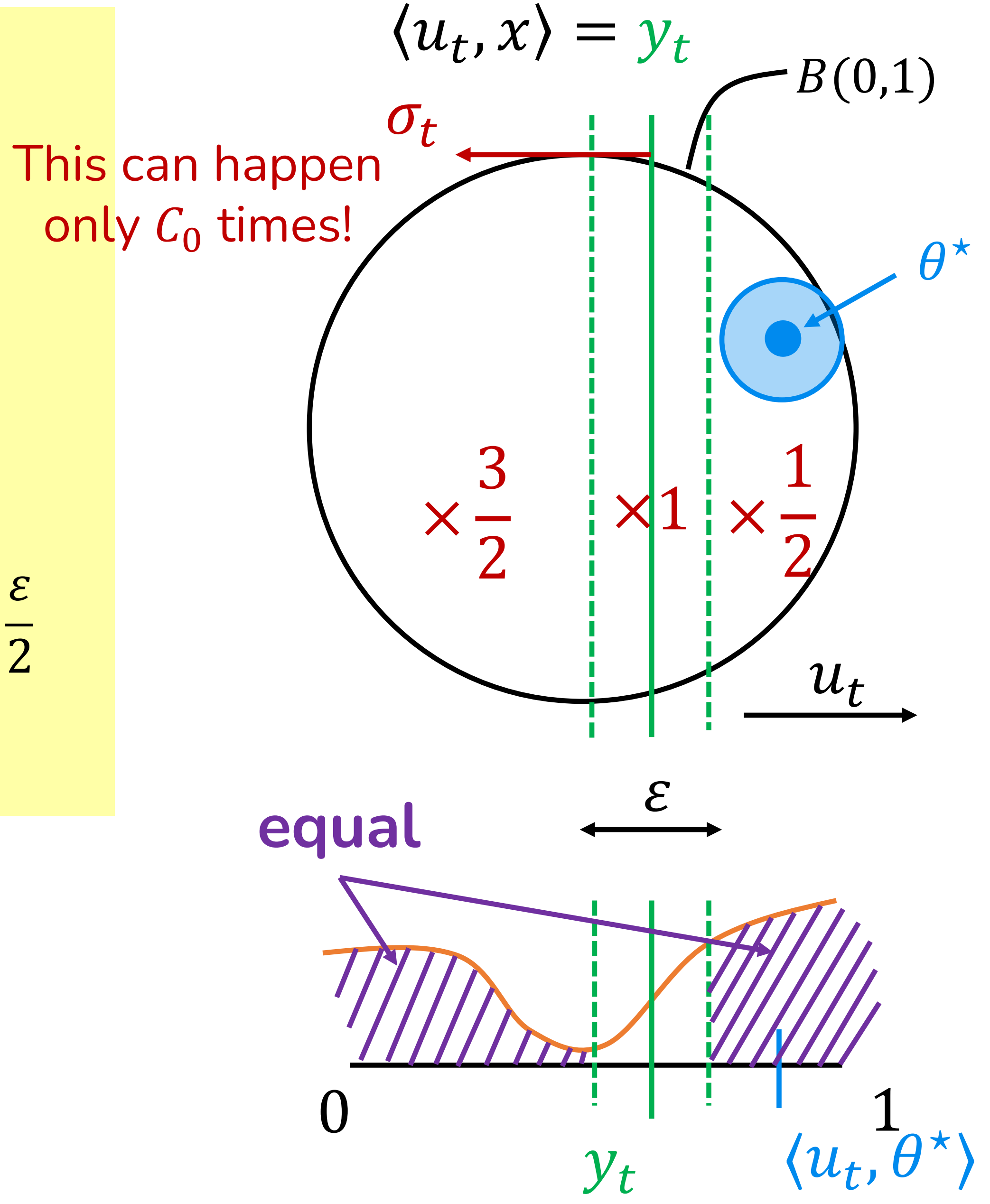

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- **decreases by $1/2$** in corrupted ones (**C_0 in total**)

Algorithm for ε – Ball Loss

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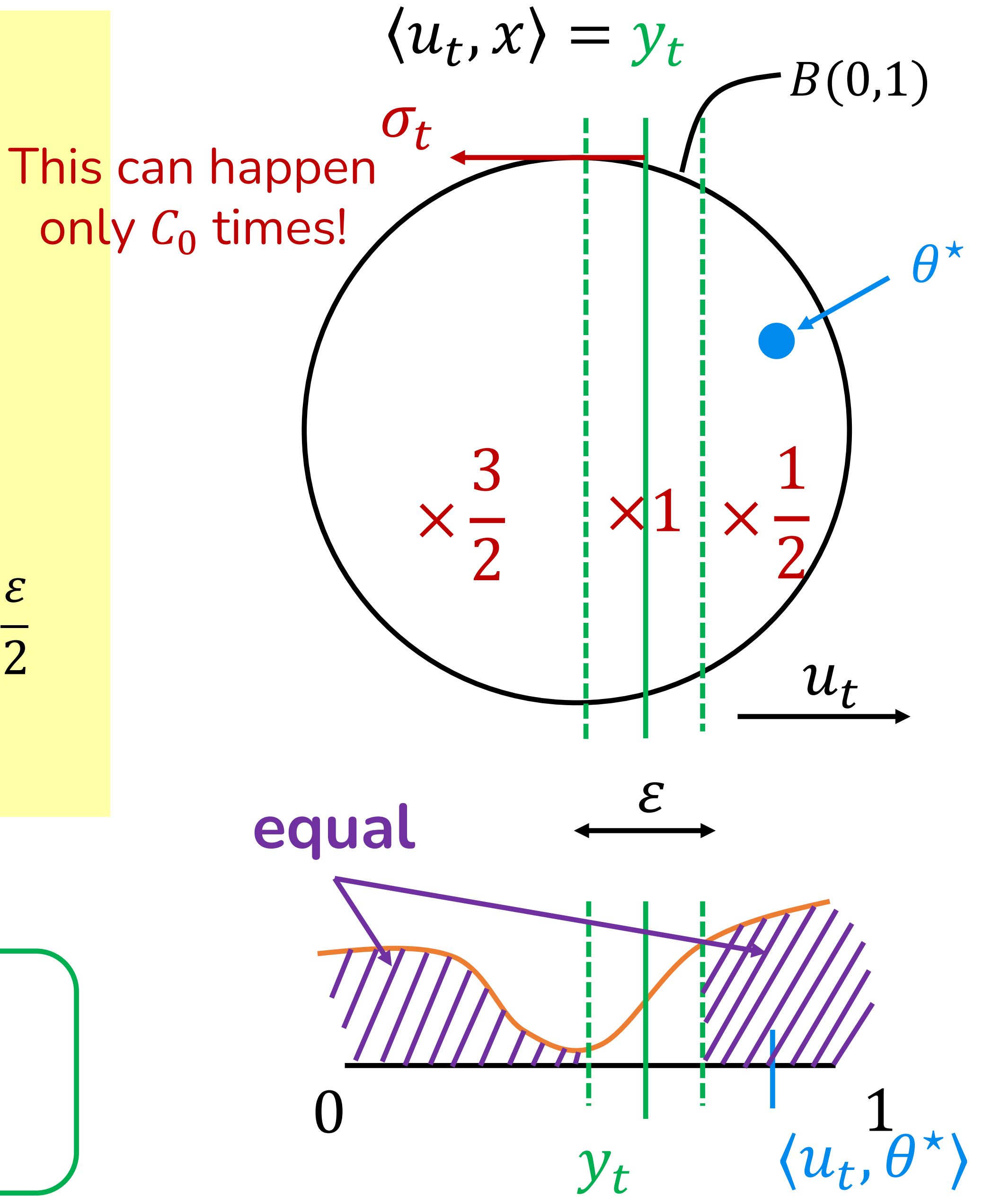

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Algorithm for ε – Ball Loss

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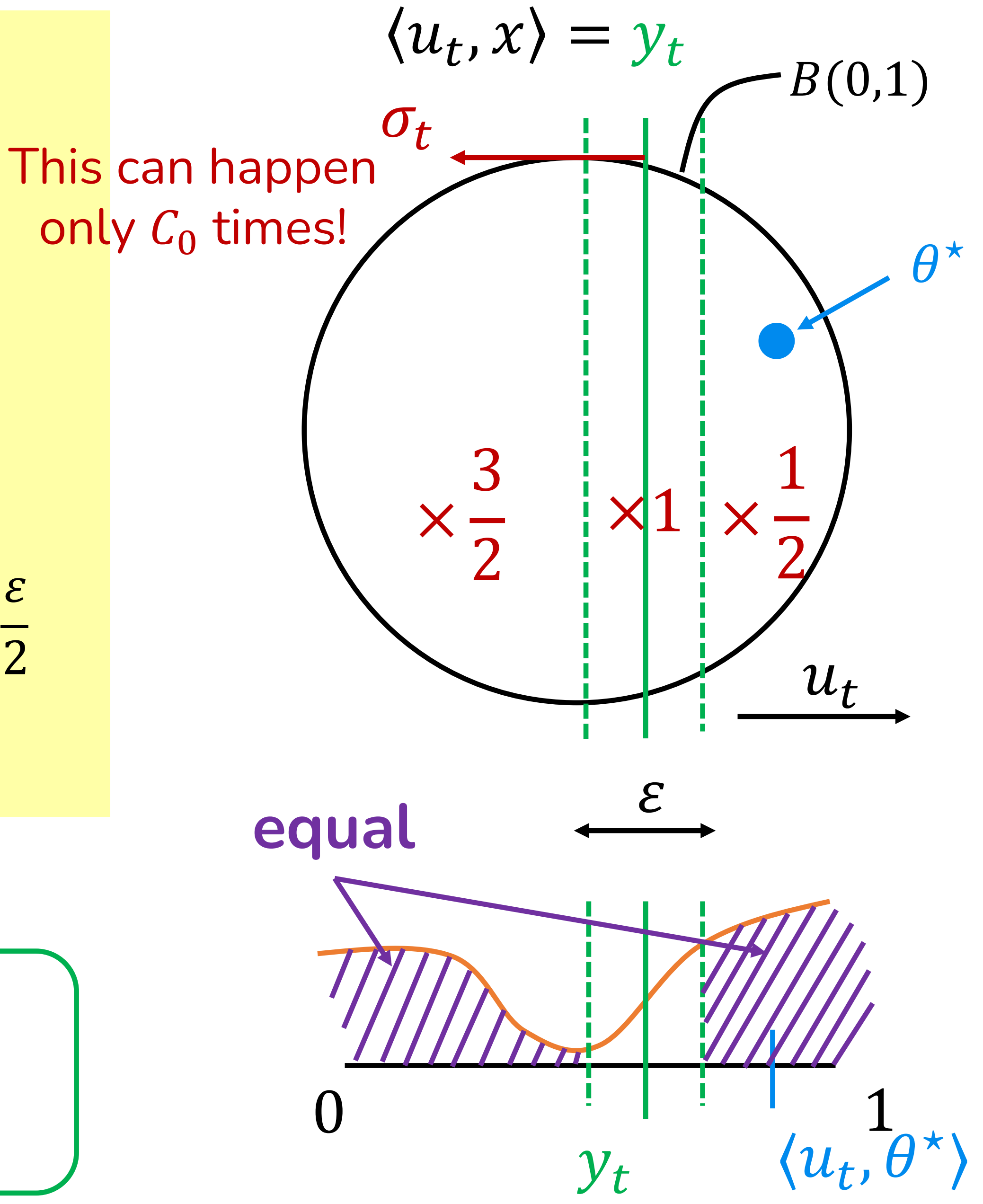

Main Result

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Algorithm for ε − Ball Loss

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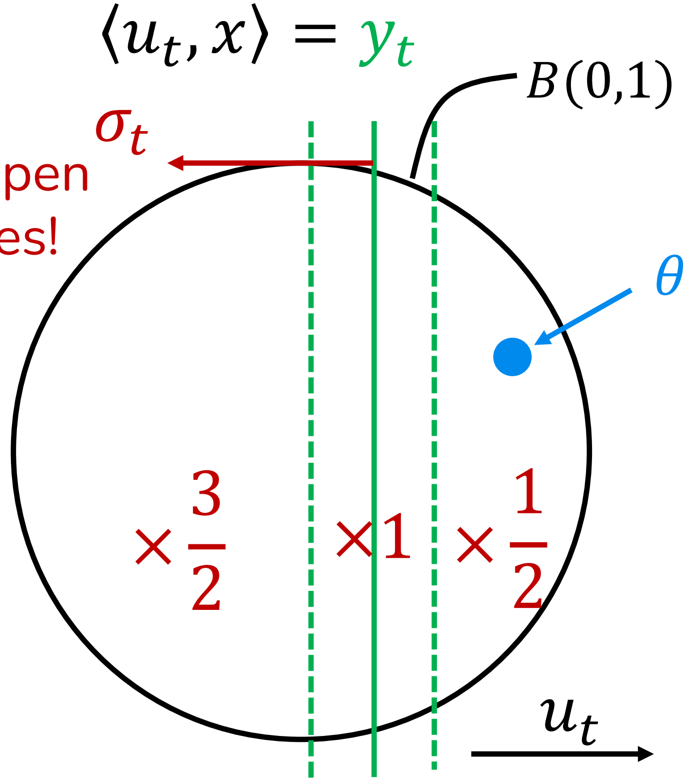
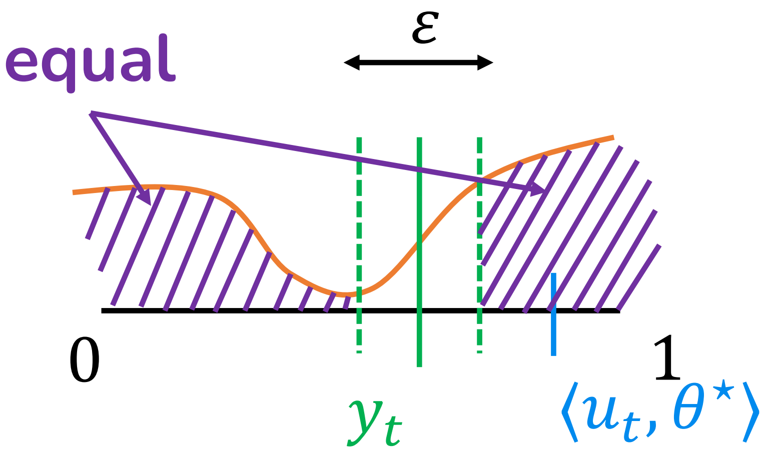
This can happen only C_0 times!





Runtime ≈ $O(T^d\, poly(d,T))$

Main Result

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Efficient Algorithm for Symmetric Loss

Main Result

Polytime algorithm with $Regret = O(C_1 + d \log T)$ for symmetric loss,

where $C_1 = \sum_t |z_t|$.

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Idea

- Maintain "structured" $f_t(\cdot)$, such that it always is a **log-concave density**.
- Query centroid of distribution: $y_t = cg_t = \int x \, f_t(x) dx$.
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efficient sampling
([Applegate & Kannan, STOC91])

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Main Result

Corruption-robust contextual search algorithms with rates:

- ε −ball loss: $Regret = O(C_0 + d \log 1/\varepsilon)$
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- pricing loss: $Regret = O(C_0\, d^3 \log^3 T)$

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Open Questions

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1. Variant of **distribution-based algorithms** for **pricing loss**.

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