Deniable Fully Homomorphic Encryption from LWE

Shweta Agrawal, Shafi Goldwasser, Saleet Mossel

Crypto, 2021 (To appear)

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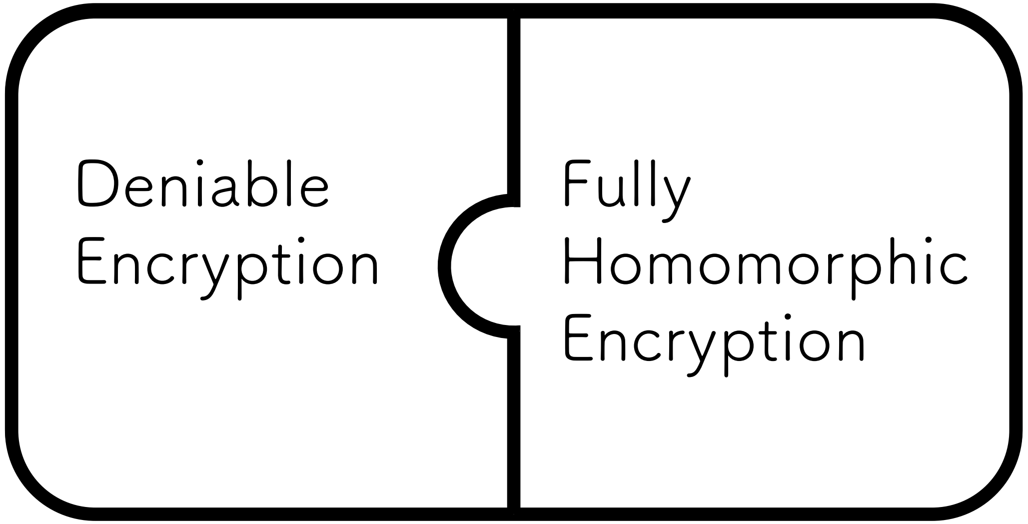
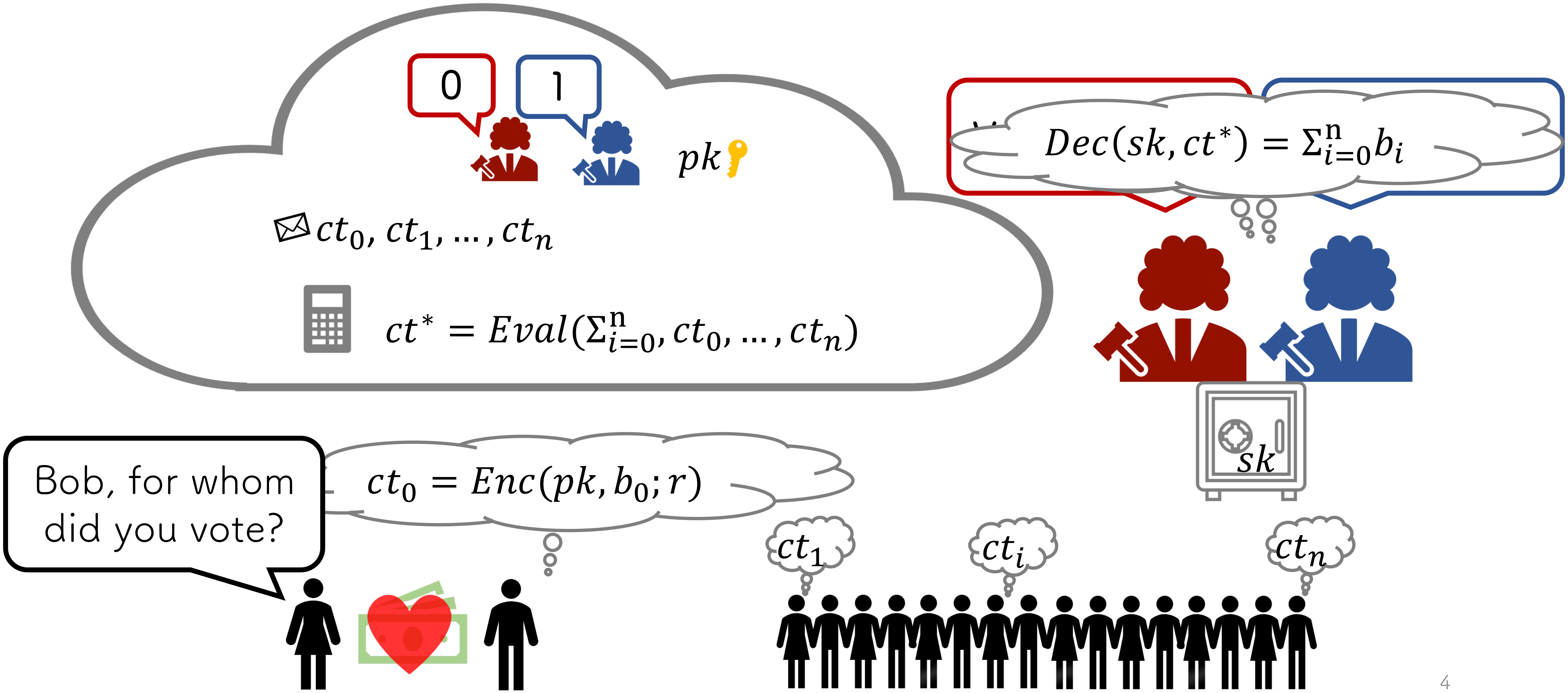
Most slides by Saleet Mossel



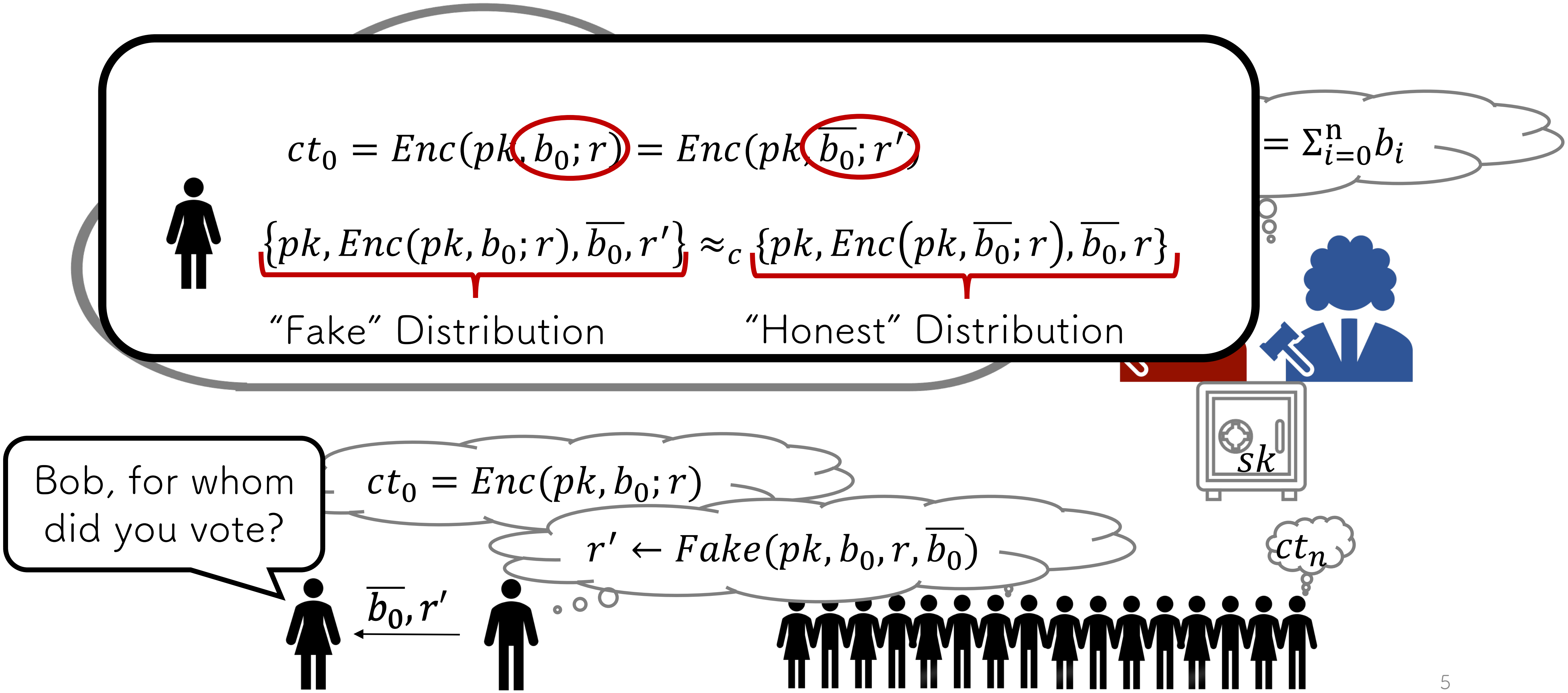
Deniable FHE

The notion of Deniable FHE

Deniable FHE



Deniable FHE



Elections require Deniability & FHE

- Benefit of Deniable Encryption in Elections:
  - Honest Participation

- Benefit of Fully Homomorphic Encryption in Elections:
  - Homomorphically compute the voting result

Any data-driven algorithm

Store Encrypted Data

Natural upgrade for all DE apps!

Deniable Encryption

- Introduced by **Canetti, Dwork, Naor and Ostrovsky 1997**
  - construction from trapdoor permutations, unique SVP
  - size of ct is the inverse of the detection probability

- Weak Deniable Encryption
  - can also lie about the encryption algorithm (Enc, Denc)
  - construction with compact ct and negligible deniability

- Lower bound (Efficiency vs. Deniability)
  - It **seems inherent** that the length of ct grows with the inverse of the detection probability in "separable" constructions.

- A significant step forward [SW14]
  - construction from iO and OWF
  - compact ct and negligible deniability

What does this mean given recent iO results?

Deniable Encryption

CDNO

- Based on TDP
- CT size inverse of detection prob

1997

SW

- Based on iO
- CT size indpt of detection prob

2014

In full model, nothing else known!

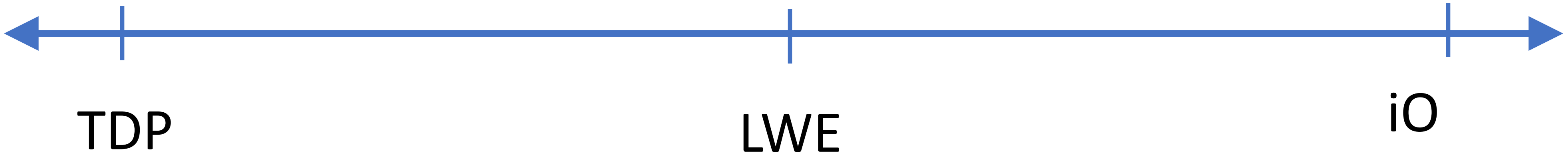
Our Results

- Notion of Deniable FHE (full and weak)
- Constructions based on Learning With Errors
- Compact ct : size does not depend on detection probability!
  - Our construction is separable (so not inherent)
  - Total encryption time grows with the inverse of the detection probability!
- Support large message space
  - All prior work encode large messages bit by bit
- Offline-Online Encryption
  - Online time independent of the detection probability

Our Results: Deniable Encryption

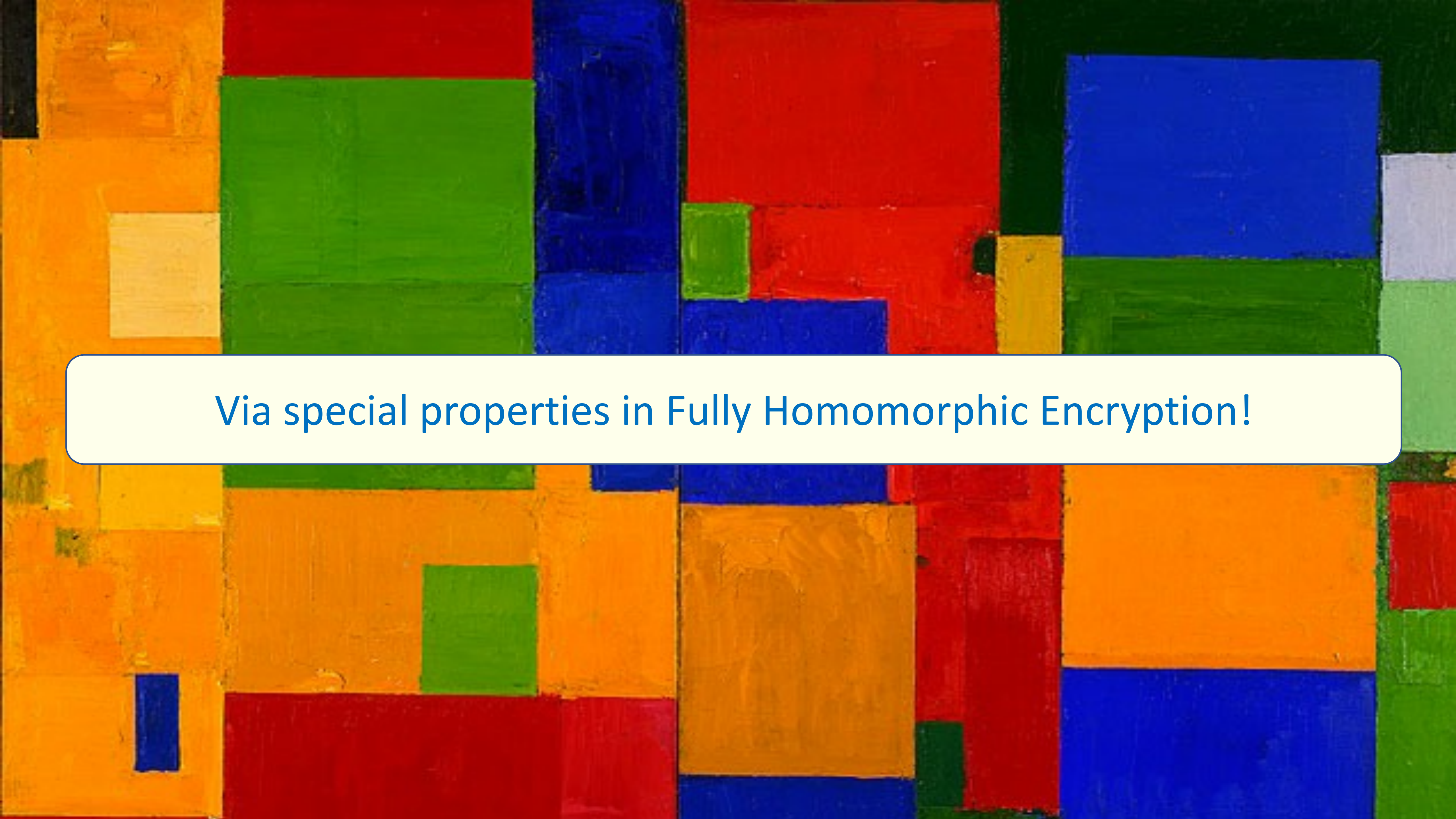
CDNO, 1997
CT size inverse of detection prob

SW, 2014
CT size indpt of detection prob



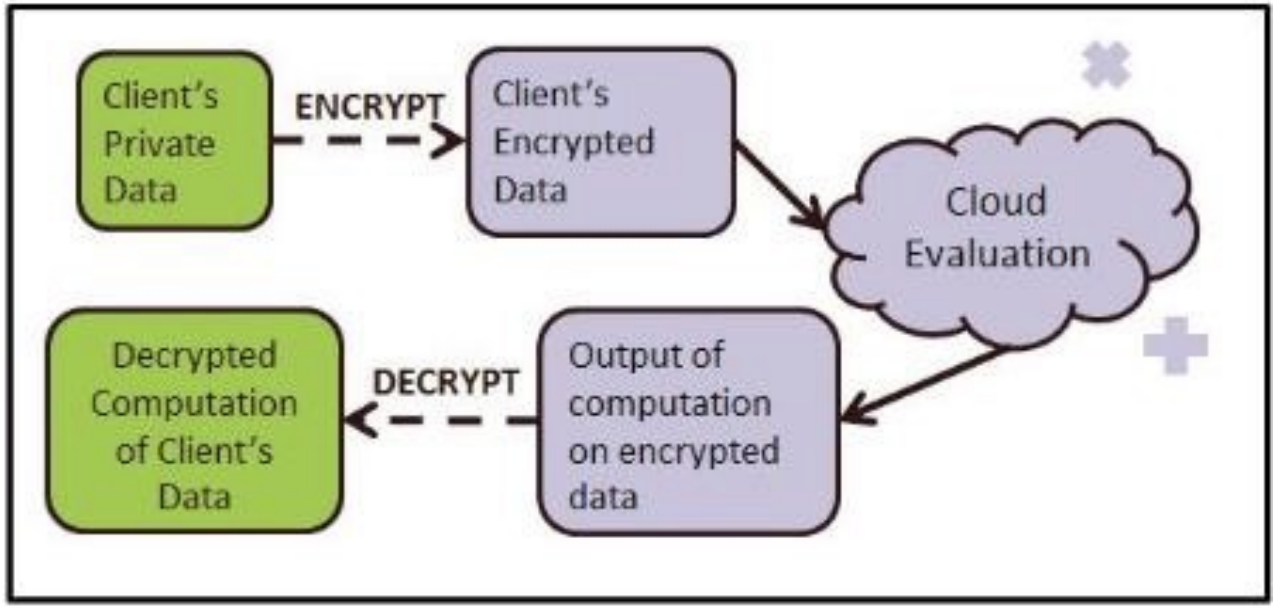
This Work

- CT size independent of detection prob
- (Offline) encryption time inverse of detection prob


Via special properties in Fully Homomorphic Encryption!

Fully Homomorphic Encryption

Can be built using LWE (BV11, BGV12, GSW13…)



Expressive Functionality: Supports arbitrary circuits

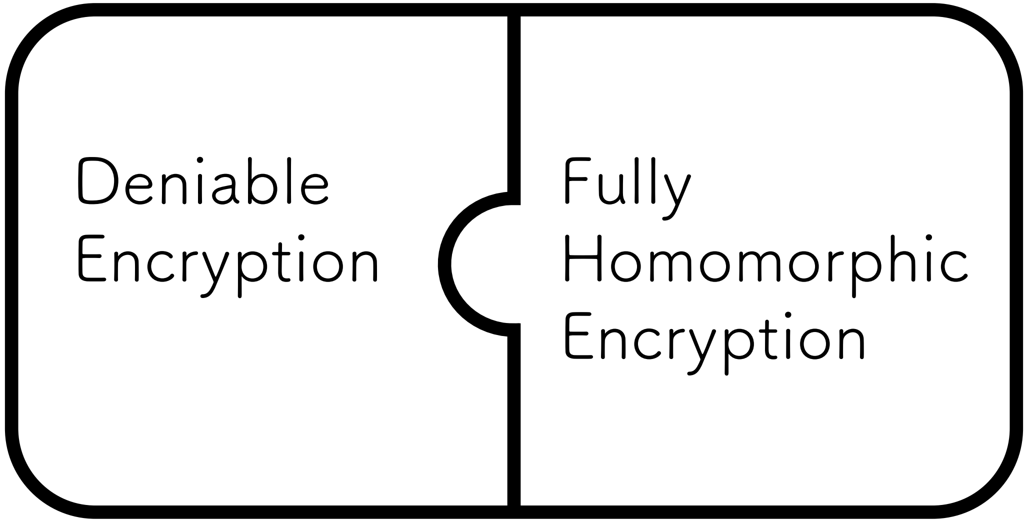
Compact ciphertext, independent of circuit size

Encryption and function evaluation commute!
Enc(f(x)) =* f(Enc(x))

* : roughly

Adding Deniability to the Mix

- A Deniable FHE scheme $(Gen, Enc, Eval, Dec, Fake)$
  - $(Gen, Enc, Eval, Dec)$ is an FHE scheme
  - $(Gen, Enc, Dec, Fake)$ is a Deniable Encryption scheme



Deniable FHE

A Deniable FHE scheme $(Gen, Enc, Eval, Dec, Fake)$ syntax

- $Gen \to (pk, sk)$
- $Enc(pk, m; r) = ct$
- $Dec(sk, ct) = b$
- $Eval(pk, f, ct_1, \dots, ct_k) = ct^*$
- $Fake(pk, b, r, \bar{b}) \to r'$

Deniable FHE

A Deniable FHE scheme $(Gen, Enc, Eval, Dec, Fake)$

1. Correctness
2. CPA-Security
3. Deniability
4. Compactness

Correctness versus Deniability

Correctness:

For every $\boldsymbol{f}$ and $\boldsymbol{m_1, \ldots, m_k}$:

$$\mathbf{Pr}\left[\boldsymbol{Dec}\left(\boldsymbol{sk}, \boldsymbol{Eval}(\boldsymbol{pk}, \boldsymbol{f}, \boldsymbol{ct_1}, \ldots, \boldsymbol{ct_k})\right) = \boldsymbol{f}(\boldsymbol{m_1}, \ldots, \boldsymbol{m_k})\right] = \mathbf{1} - \boldsymbol{negl}$$

where $\boldsymbol{ct_i} \leftarrow \boldsymbol{Enc}(\boldsymbol{pk}, \boldsymbol{m_i})$ and $(\boldsymbol{pk}, \boldsymbol{sk}) \leftarrow \boldsymbol{Gen}$

Cannot simultaneously satisfy <u>perfect</u> correctness and <u>deniability</u>

$\delta(\lambda)$- Deniability

We consider (inverse) polynomial deniability

For every bit $\boldsymbol{b}$, and PPT adversary $\boldsymbol{A}$

detection probability

$$\left|\mathbf{Pr}[A(pk, Enc(pk, b; r), b, r)] - \mathbf{Pr}\left[A\left(pk, Enc\left(pk, \bar{b}; r\right), b, r'\right)\right]\right| \leq \delta(\lambda)$$

"Honest" Distribution

"Fake" Distribution

where $(pk, sk) \leftarrow Gen$, $r \leftarrow \{0,1\}^{\ell'}$, and $r' \leftarrow Fake(pk, \bar{b}, r, b)$

Evaluation & Deniability Compactness

a) For every f and $m_1, \ldots, m_k$:

$$|Eval(pk, f, ct_1, \ldots, ct_k)| \leq poly$$

where $ct_i \leftarrow Enc(pk, m_i)$ and $(pk, sk) \leftarrow Gen$

Independent of k and the complexity of f

b) For every m:

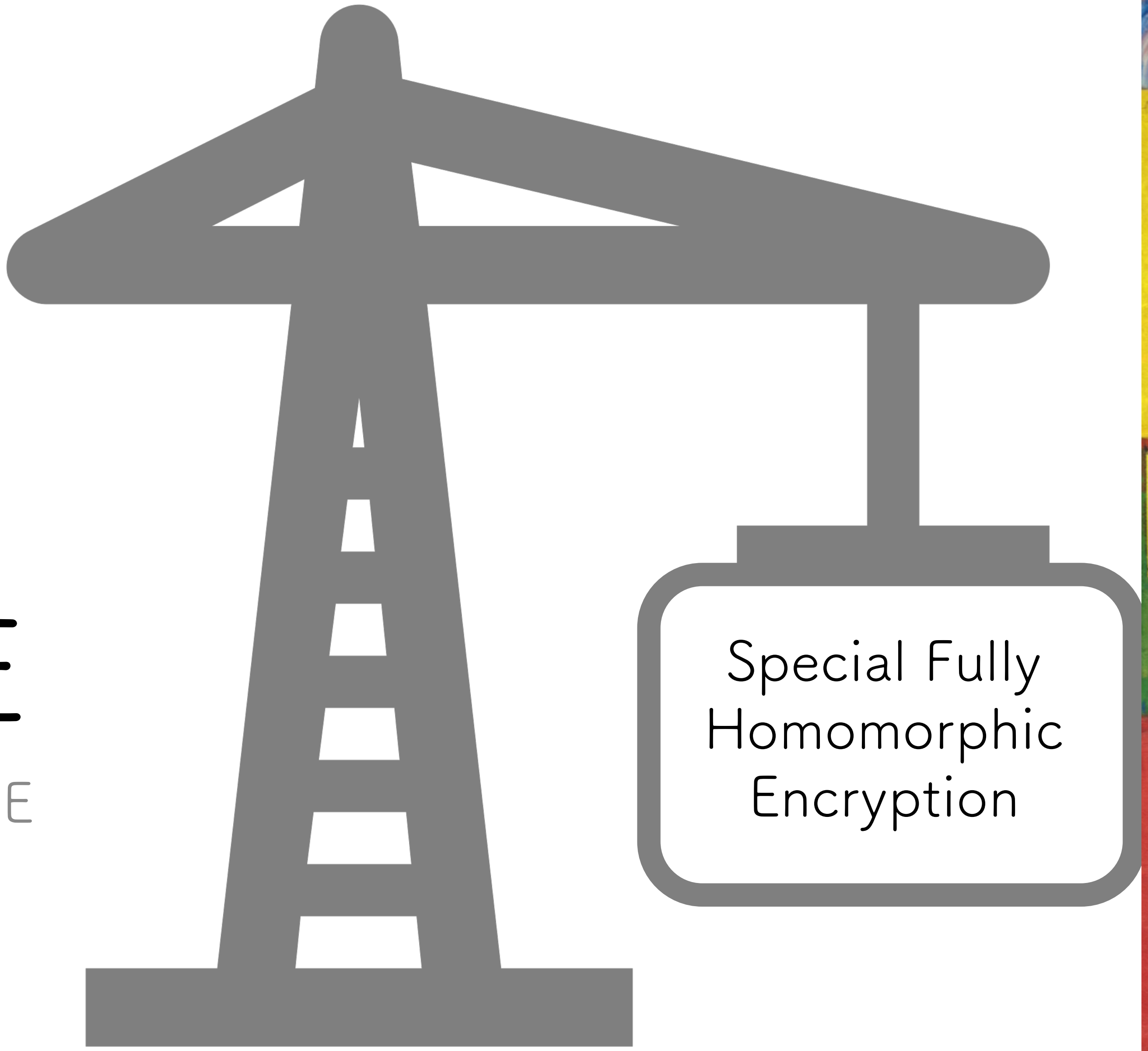
$$|Enc(pk, m)| \leq poly$$

where $(pk, sk) \leftarrow Gen$, regardless of the encryption running time

Independent of the detection probability

Deniable FHE

Our Construction of Deniable FHE



FHE from LWE: A Very Brief Recap

- All* known FHE schemes add noise in CT for security.
- Homomorphic evaluation of CTs (eval(f, $ct_1 \cdots ct_n$)) cause noise to grow
- Kills correctness after noise grows too much
- Limits number of homomorphic operations

How to keep going: Gentry's bootstrapping [Gen09]!

The Magic of Bootstrapping

- Assume that an FHE is powerful enough to support evaluation of its own decryption circuit Dec.

- By correctness of decryption, $\mathrm{Dec}(ct_x, sk) = x$

$$\mathrm{Dec}\left(\boxed{x}\ , sk \right) = x$$

- Define circuit $\mathrm{Dec}_{ct}(sk) = \mathrm{Dec}(sk, ct)$
- By correctness of homomorphic evaluation, $\mathrm{Eval}(F, ct_x) = ct(F(x))$

$$\mathrm{Eval}\left(\mathrm{Dec}_{ct}, \boxed{sk} \right) = \boxed{\mathrm{Dec}_{ct}(sk)} = \boxed{x}$$

The Magic of Bootstrapping

- Originally introduced to reduce noise in evaluated ciphertext

- Homomorphic evaluation of decryption
  - removes large old noise
  - adds small new noise (size small since decryption shallow)

This work: Oblivious Sampling of FHE ciphertexts!

The Magic of Bootstrapping

- Assume that decryption always outputs 0 or 1
  - even if input ct is not well formed

- Then, bootstrapping always outputs proper encryption of 0 or 1!

$$\text{Eval}\left(\text{Dec}_{ct}, \boxed{sk} \right) = \boxed{\text{Dec}_{ct}(sk)} = \boxed{x}$$

Even if input "ct" is a random element in ciphertext space!

The Magic of Bootstrapping

- Assume that decryption outputs 0 w.o.p for random input

- Then, bootstrapping outputs encryption of 0 w.o.p for random input

$$\text{Eval}\left(\text{Dec}_{\text{rand}}, \boxed{\text{sk}}\right) = \boxed{\text{Dec}_{\text{rand}}(\text{sk})} = \boxed{0}$$

Given enc(sk), run dec homomorphically on random to generate encryption of 0 w.o.p!

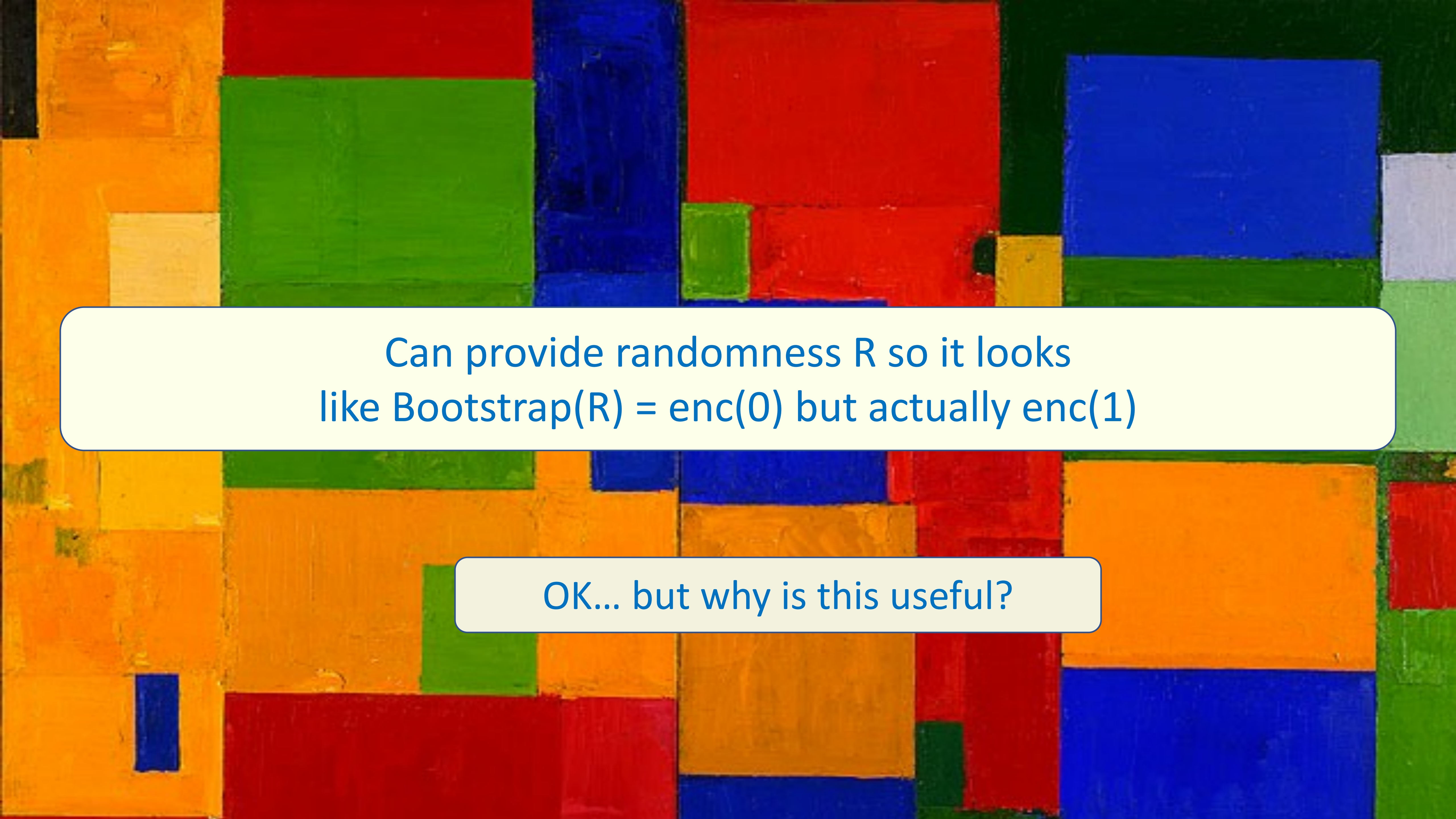
But, wait a minute…

- Given encryption of 1, decryption outputs 1 w.o.p
- Encryption of 1 is indistinguishable from random!

$$\mathrm{Eval}\left(\mathrm{Dec}_{ct1},\ \boxed{sk}\right) = \boxed{\mathrm{Dec}_{ct1}(sk)} = \boxed{1}$$

- Can pretend as if ct1 = enc(1) is a random string

Pretend bootstrapping outputs enc(0) but actually enc(1)!



Can provide randomness R so it looks like Bootstrap(R) = enc(0) but actually enc(1)

OK… but why is this useful?

Leveraging our trick (binary msg space)

- Let $B(x) = Eval(pk, Dec_x, ct_{sk})$ the bootstrapping procedure
  - recall $Dec_x(sk) = Dec(sk, x)$

- Denote homomorphic addition (mod 2) as

$$Eval(pk, +, ct_a, ct_b) = ct_a \oplus ct_b$$

$B(R_i)$
$= Enc(x_i)$

$$B(R_1) \oplus \cdots \oplus B(R_n) = \text{Enc}\,(\text{Parity}\,(x_1, \ldots, x_n))$$

Construction

Gen:

1. $(pk, sk) \leftarrow Gen$
2. $ct_{sk} \leftarrow Enc(pk, sk)$
3. Output $pk = (pk, ct_{sk}), sk = sk$

Construction

$rand = (x_1, \ldots, x_n, \{R_i\}_{x_i=0}, \{r_i\}_{x_i=1})$

$Enc(pk, b)$:

1. Sample $x_1, \ldots, x_n \leftarrow \{0,1\}$ s.t. $\sum_i x_i = b \ (mod\ 2)$
2. For $x_i = 0$, sample $R_i \leftarrow \mathcal{R}^{\ell}$
3. For $x_i = 1$, sample $r_i \leftarrow \{0,1\}^{\ell'}$ and set $R_i = Enc(pk, 1; r_i)$
4. Compute $ct = B(R_1) \oplus \cdots \oplus B(R_n)$
5. Output ct

$B(\mathcal{R}^{\ell})$ is a valid encryption of 0 w.h.p

Construction

$rand = (x_1, \ldots, x_n, \{R_i\}_{x_i=0}, \{r_i\}_{x_i=1})$

$Fake(pk, b, rand, \bar{b})$:

1. If $b = \bar{b}$, output $rand$
2. Sample $k \leftarrow [n]$ s.t. $x_k = 1$
3. Set $x'_k = 0$ and $R'_k = Enc(pk, 1; r_k)$
4. For $i \neq k$, set $R'_i = R_i$ and $r'_i = r_i$
5. Output $rand' = (x'_1, \ldots, x'_n, \{R'_i\}_{x'_i=0}, \{r'_i\}_{x'_i=1})$

Pseudorandom Ciphertext

By pretending one ciphertext enc(1) is random, parity flipped!

Statistical distance from honest dist is 1/poly(n)

Construction

$Eval(pk, f, ct_1, \ldots, ct_k)$:

1. Interpret ct_i as special FHE ciphertext ct_i
2. Output $Eval(pk, f, ct_1, \ldots, ct_k)$

$Dec(dsk, ct)$:

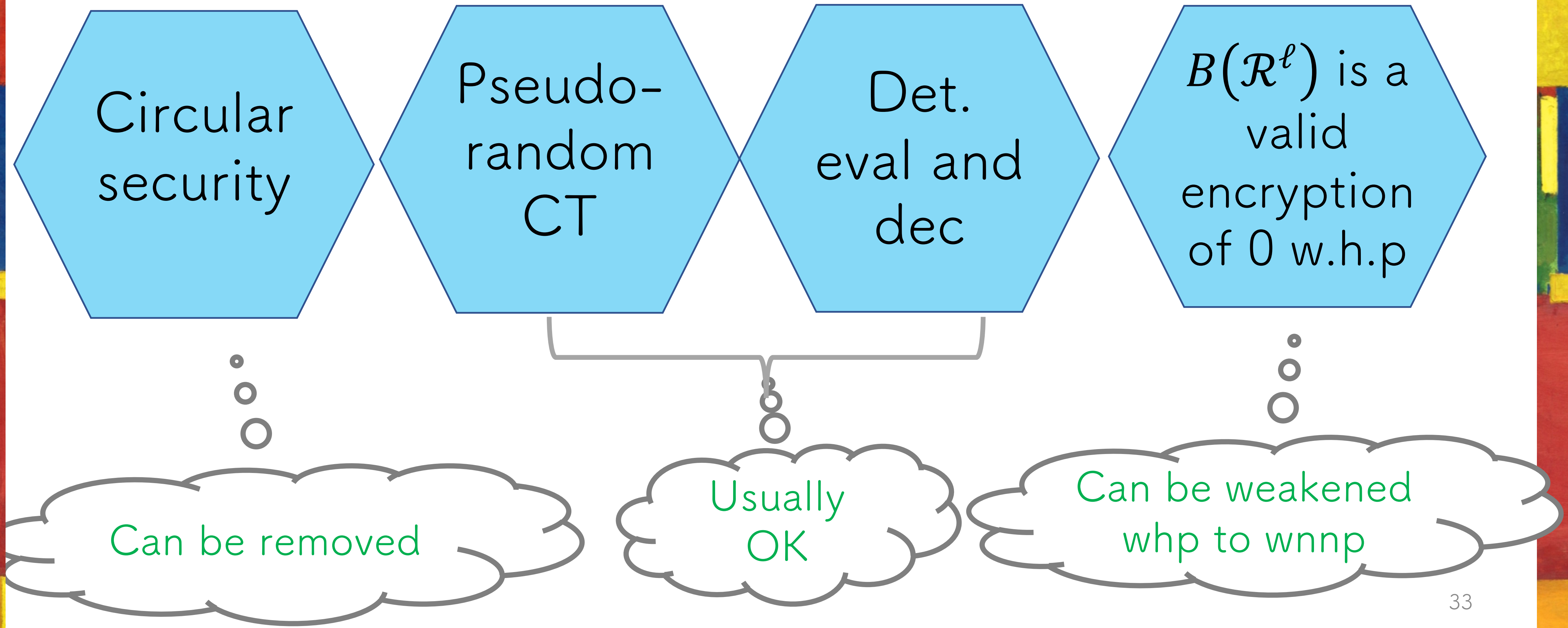
1. Interpret ct as special FHE ciphertext ct
2. Output $Dec(sk, ct)$

As before!

Special FHE

Definition and Instantiation

Special FHE


Circular security
Pseudo-random CT
Det. eval and dec
$B(\mathcal{R}^{\ell})$ is a valid encryption of 0 w.h.p
Can be removed
Usually OK
Can be weakened whp to wnnp

Weaker Special FHE

1. Pseudorandom Ciphertext
2. Deterministic evaluation and decry
3. Decryption always outputs a valid mess and

Almost always the case

$$\mathbf{Pr}[\boldsymbol{Dec}(\boldsymbol{sk}, \boldsymbol{R}) = \mathbf{0}] = \mathbf{1}/\boldsymbol{poly}$$

where $\boldsymbol{R} \leftarrow \mathcal{R}^{\ell}$ and $(\boldsymbol{pk}, \boldsymbol{sk}) \leftarrow \boldsymbol{Gen}$

[BGV14] FHE satisfies all properties!

Online-Offline Encryption

Bulk of the computation is independent of the message, and may be performed in an offline pre-processing phase.

$Enc(dpk, b)$:

1. Select $x_1, \ldots, x_n \leftarrow \{0,1\}$ s.t. $\sum_i x_i = b \ (mod\ 2)$
2. For $x_i = 0$, select $R_i \leftarrow \mathcal{R}^\ell$
3. For $x_i = 1$, select $r_i \leftarrow \{0,1\}^{\ell'}$ and set $R_i = Enc(pk, 1; r_i)$
4. Output $dct = B(R_1) \oplus \cdots \oplus B(R_n)$

n-1 computations of $B(R_i)$ can be done offline:
choose R_n depending on b and compute $B(R_n)$ online

Main Takeaway:
Evaluation compactness in FHE implies deniability compactness in DE!

Going Forward

- Compact CT ➔ compact encryption runtime?
  - Analogy to FE [LPST16,GKPVZ13]
- Technical barrier: unidirectional cheating
- Need: Invertible oblivious sampling with bias
  - SW construction may be viewed through this lens
- From LWE: can have oblivious sampling with bias (this work) or oblivious sampling with inversion but not both (so far).

Thank You

Images Credit: Hans Hoffman