Technische Universiteit
Eindhoven
University of Technology

Dual lattice attacks for closest vector problems (with preprocessing)

Thijs Laarhoven, Michael Walter

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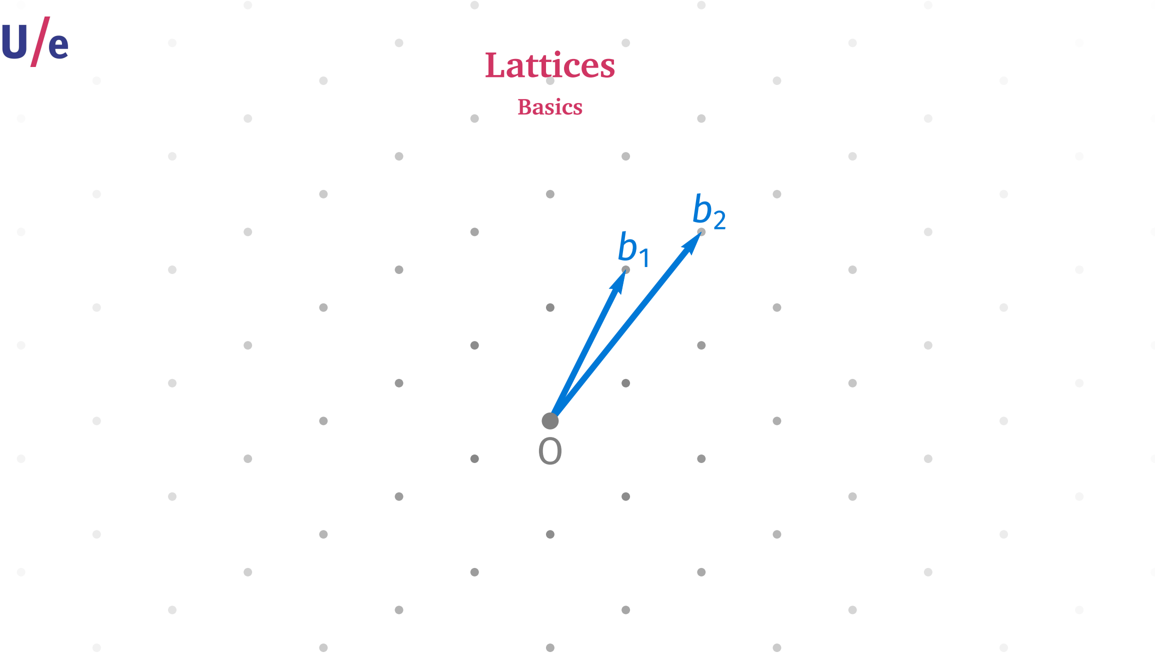
Simons Reunion, virtual
(June 15, 2021)

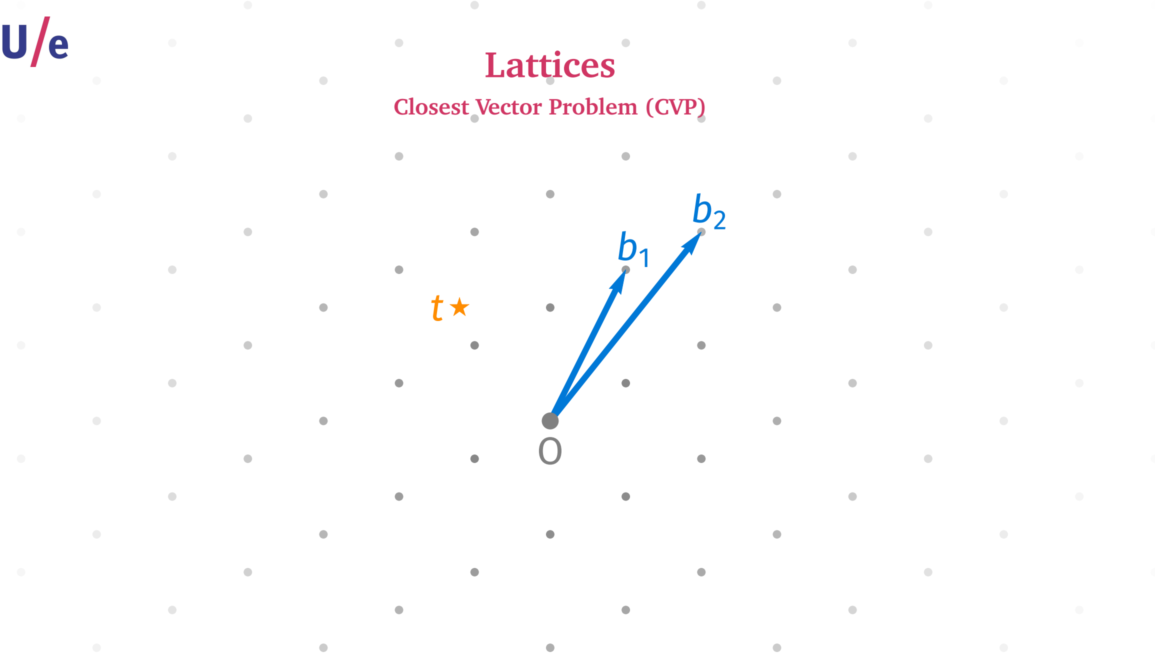
Lattices

Basics



Lattices

Closest Vector Problem (CVP)



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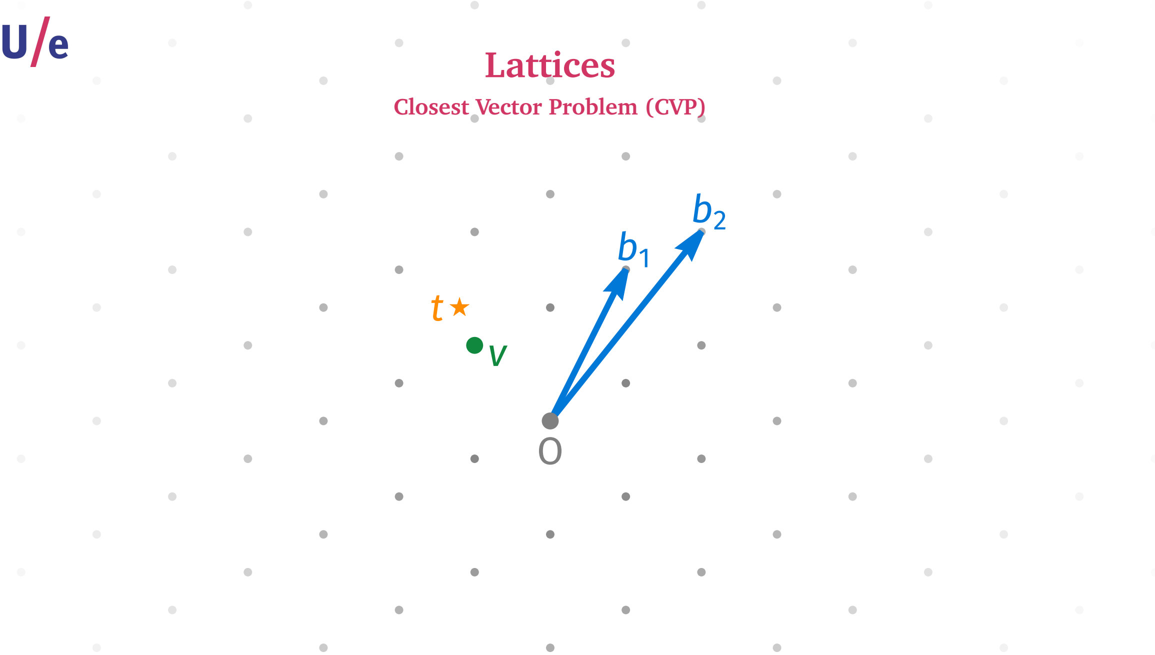

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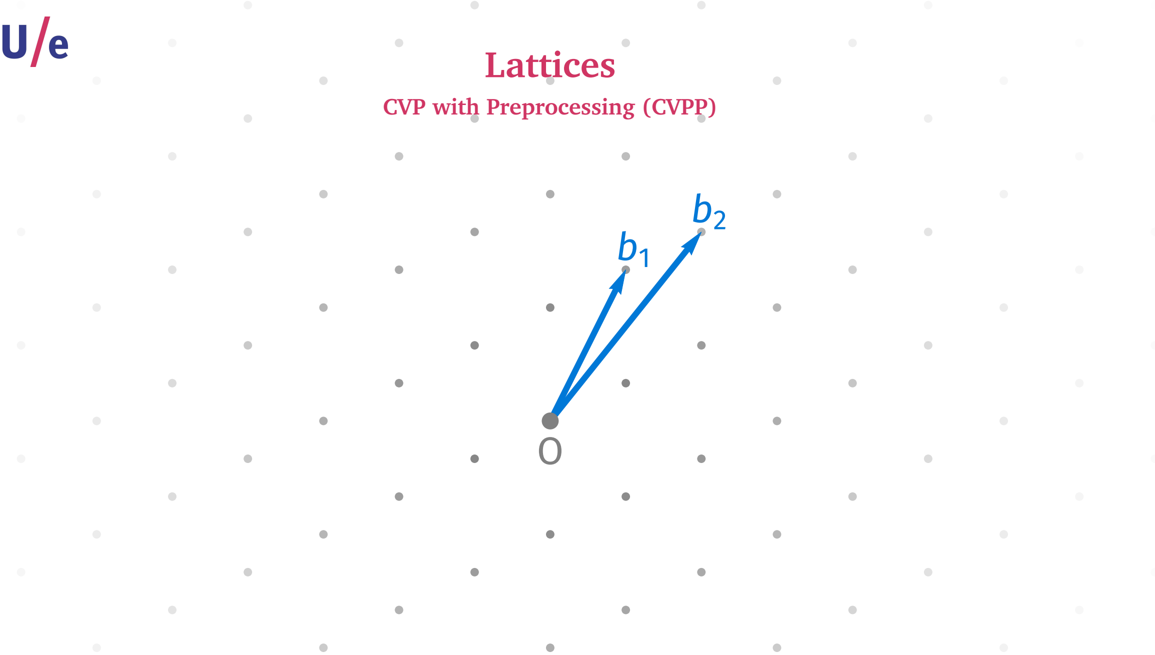
CVP with Preprocessing (CVPP)



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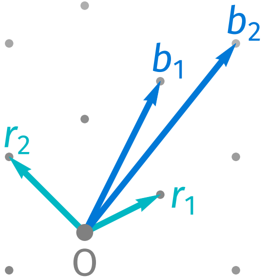

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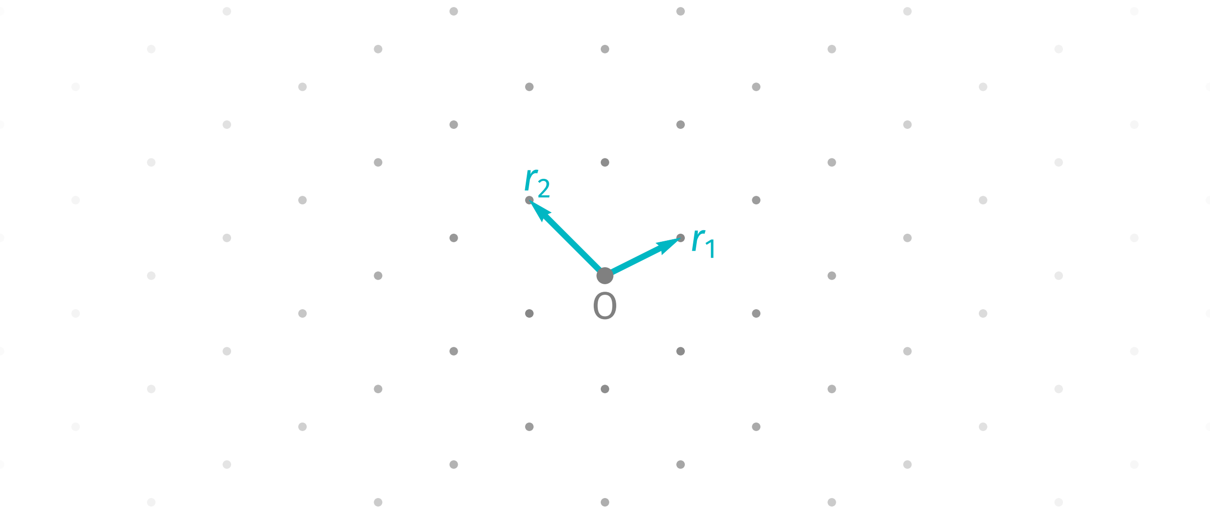

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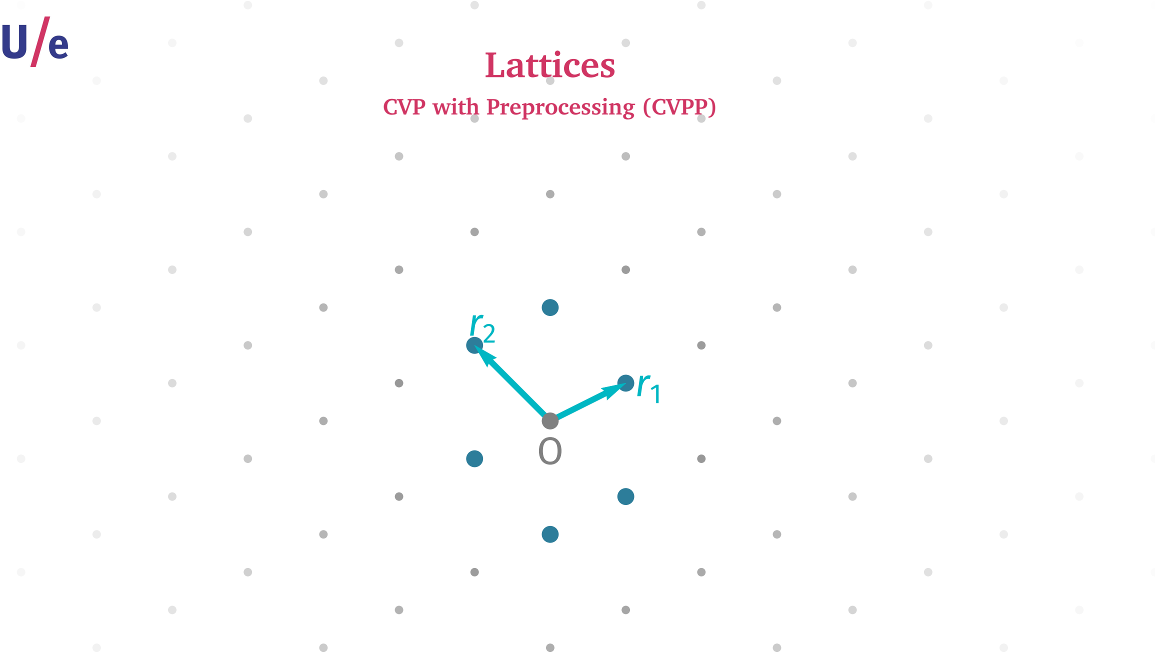

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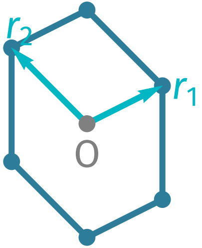

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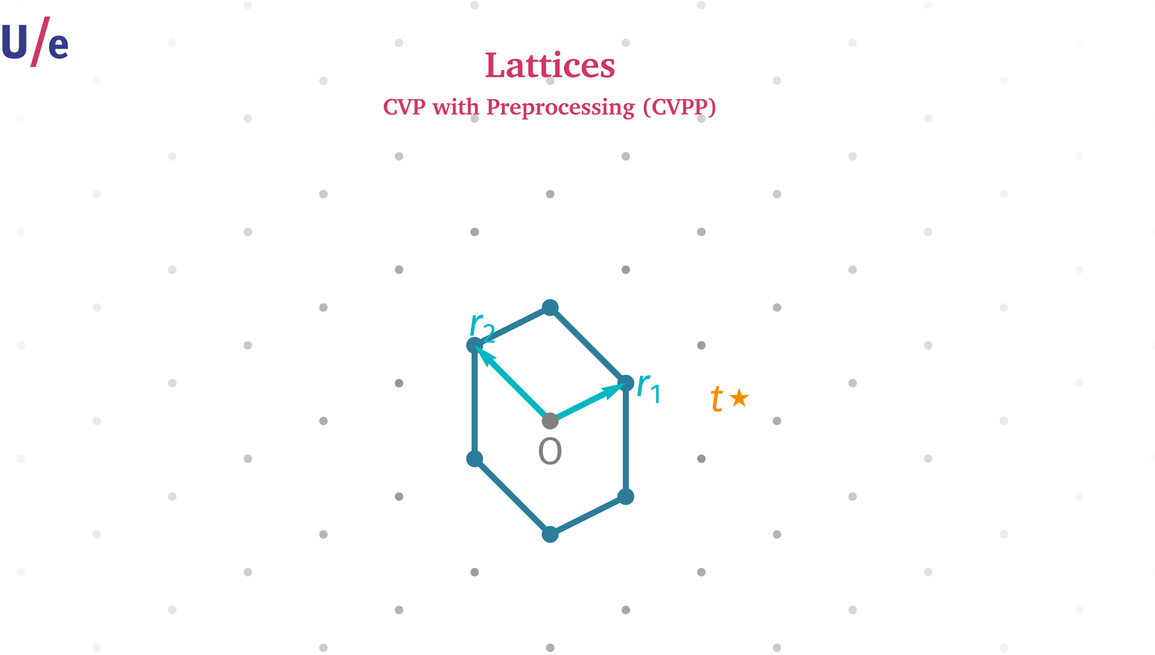

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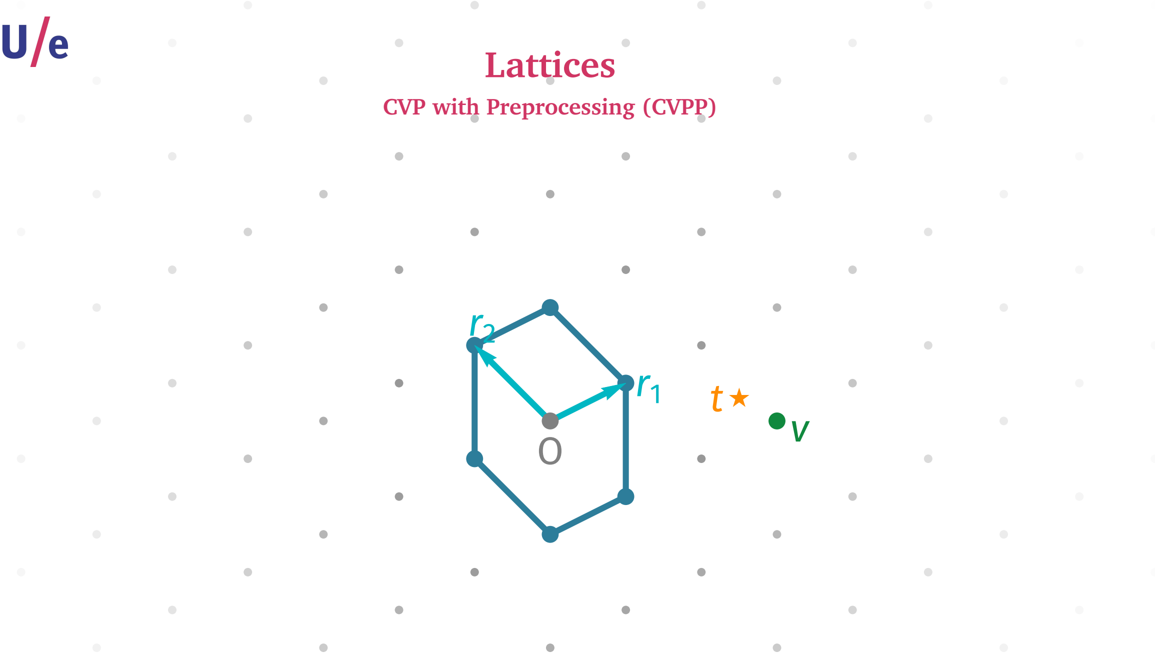
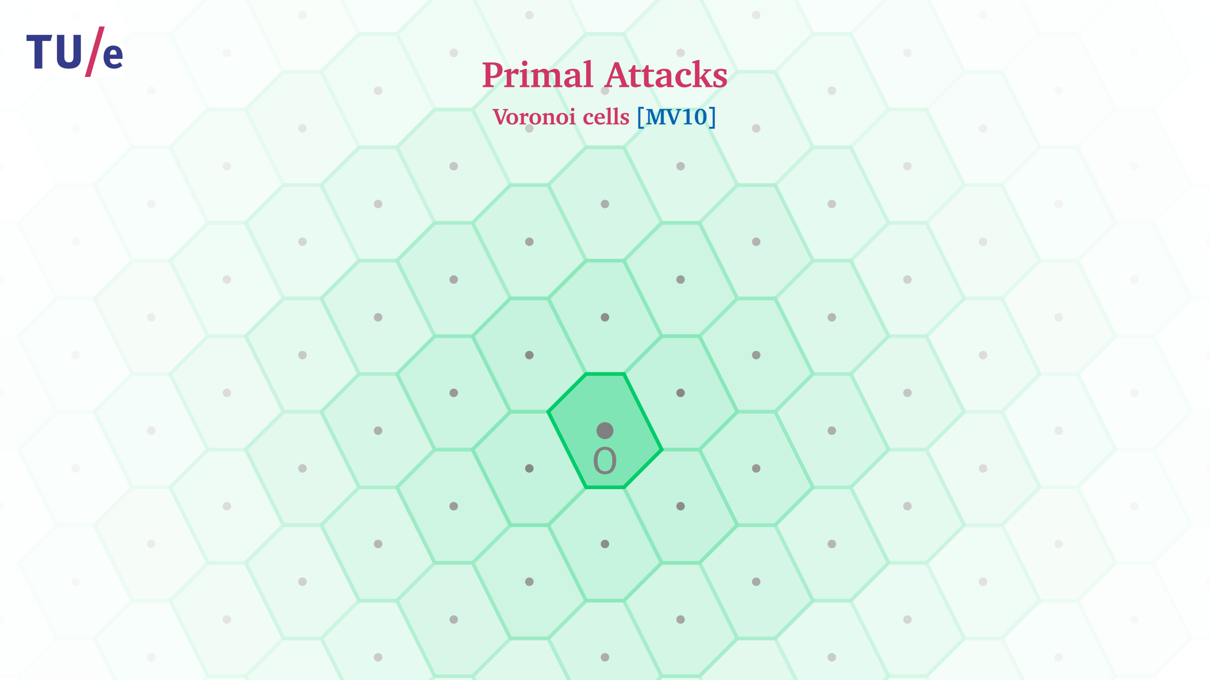


Primal Attacks

Voronoi cells [MV10]



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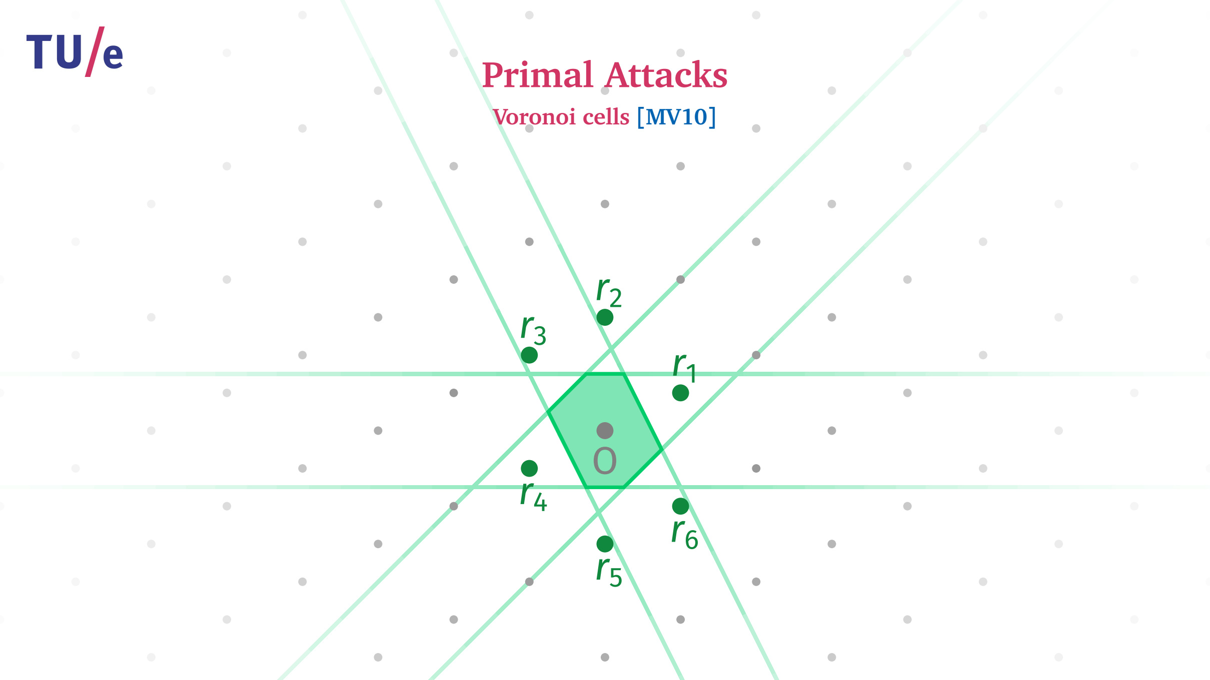
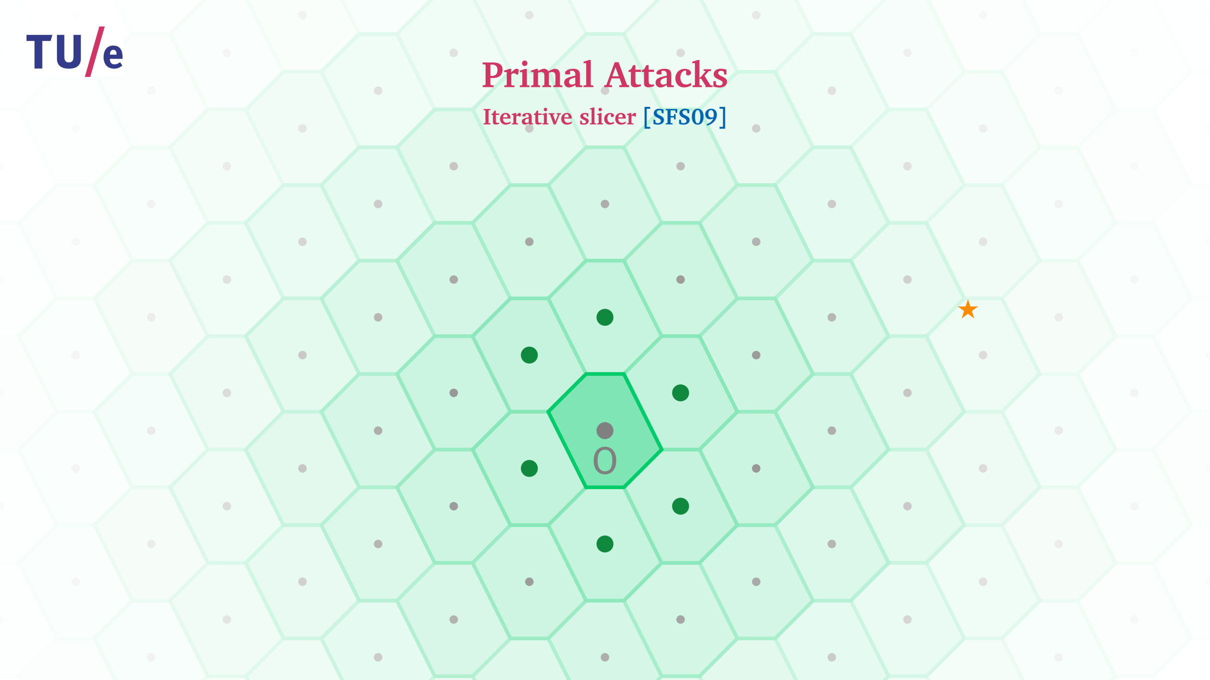


Primal Attacks

Iterative slicer [SFS09]



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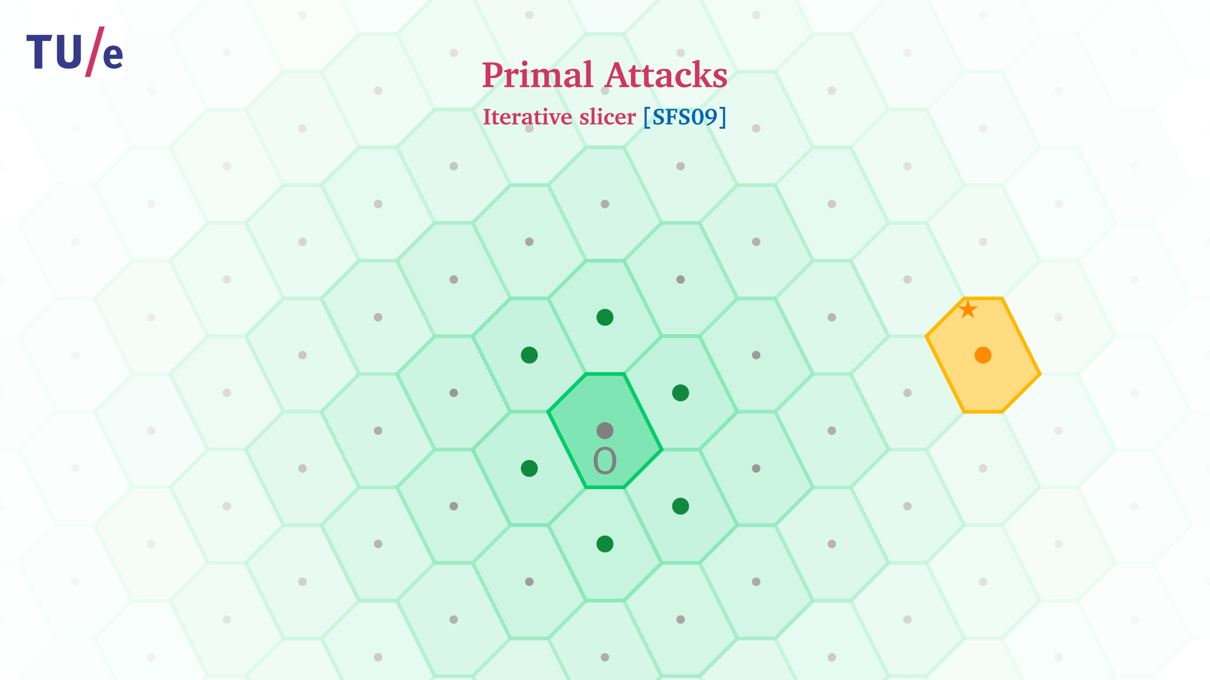

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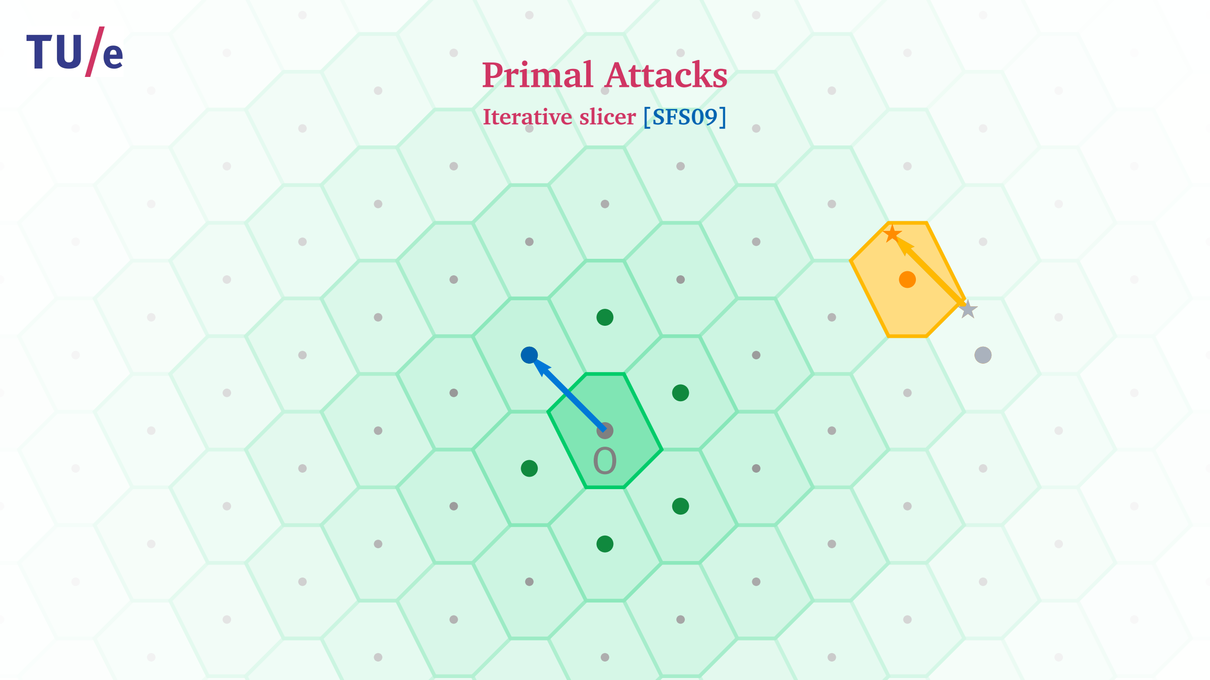

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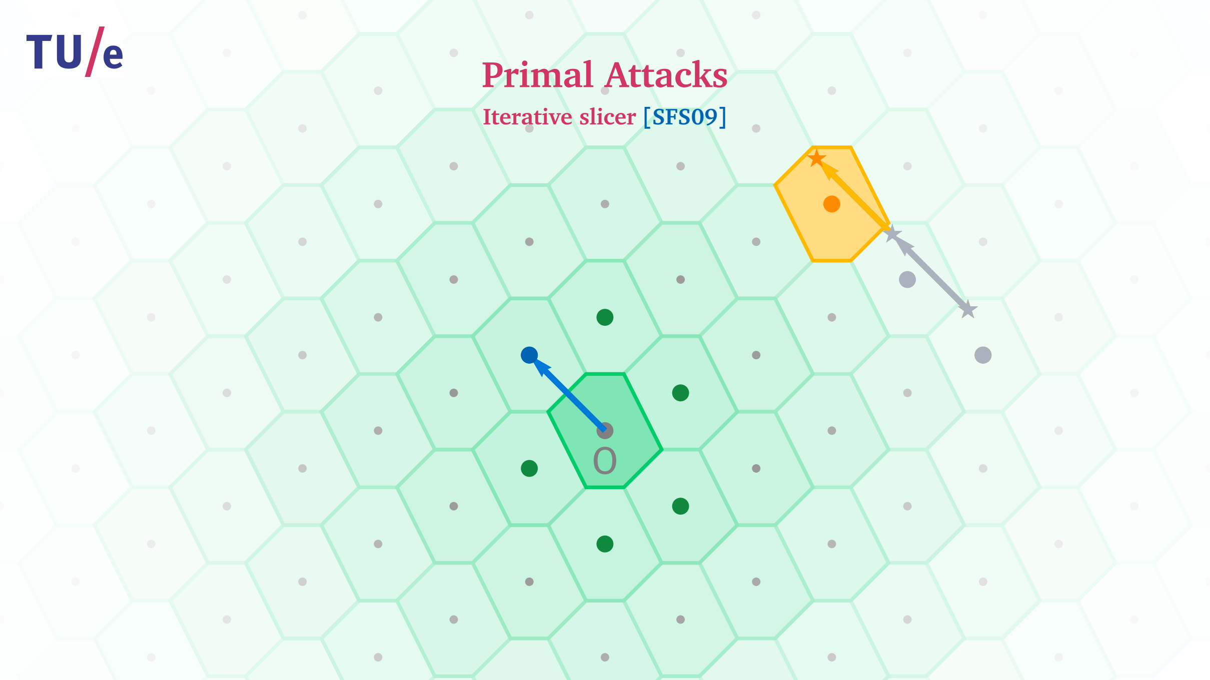

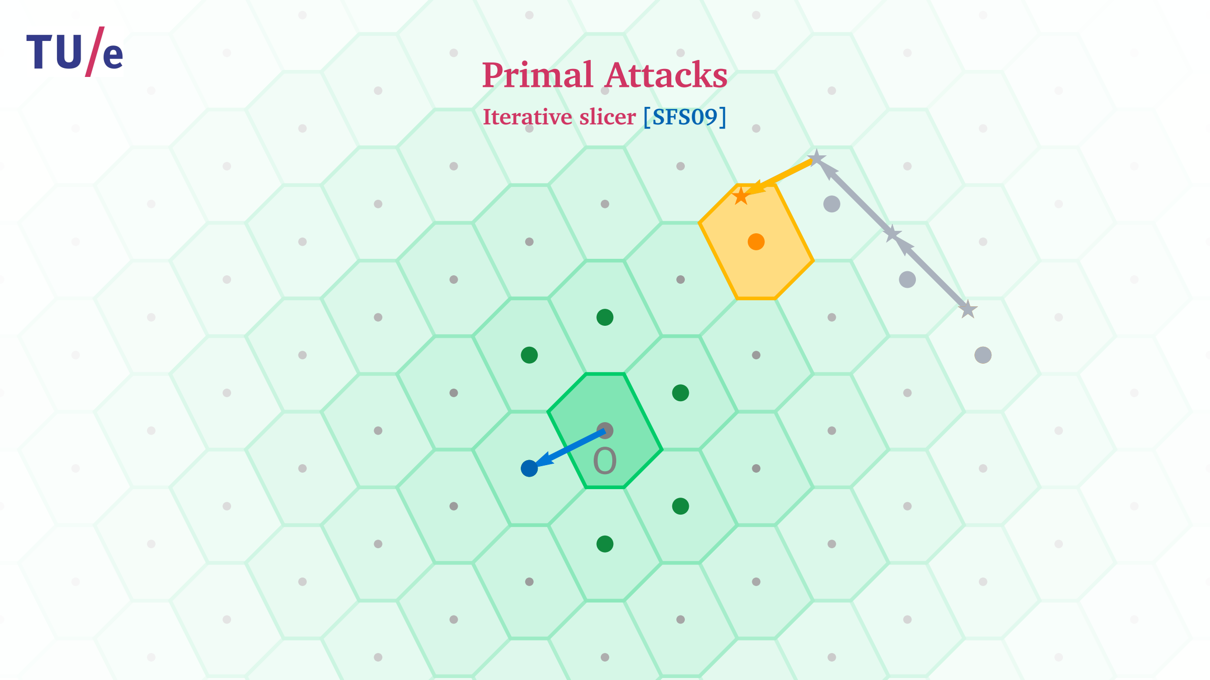

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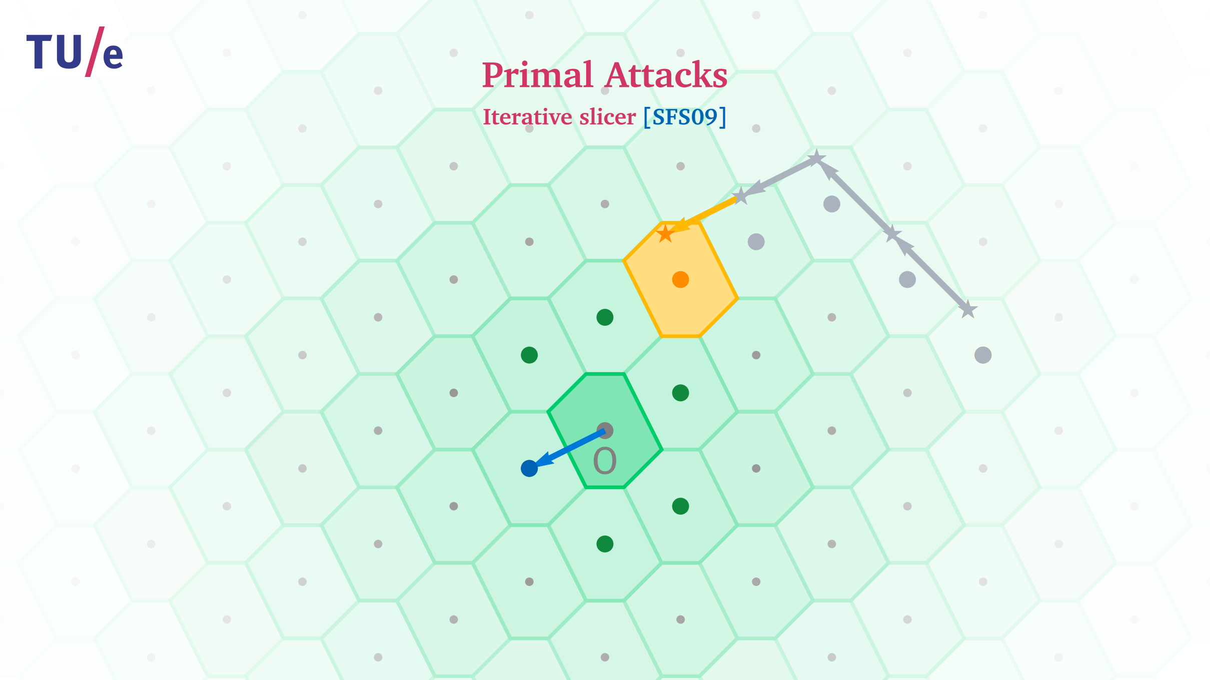

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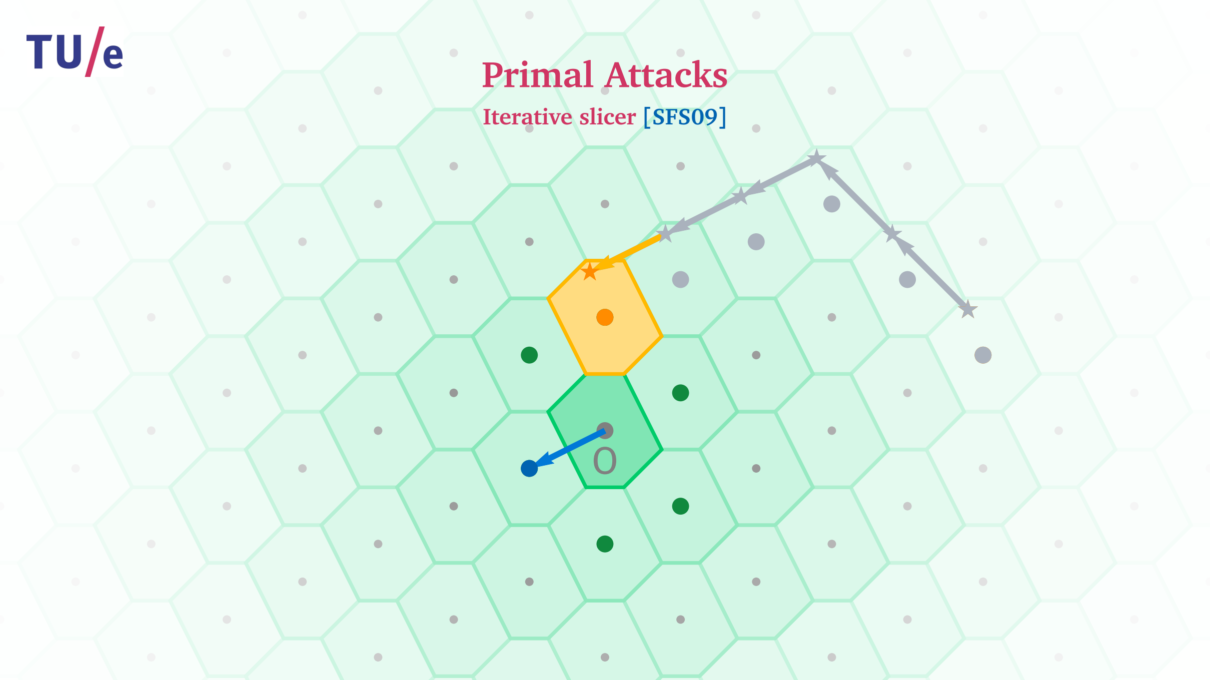

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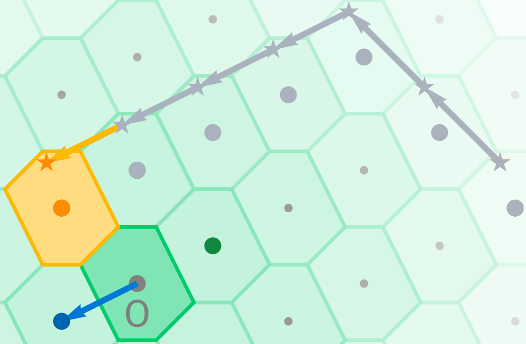

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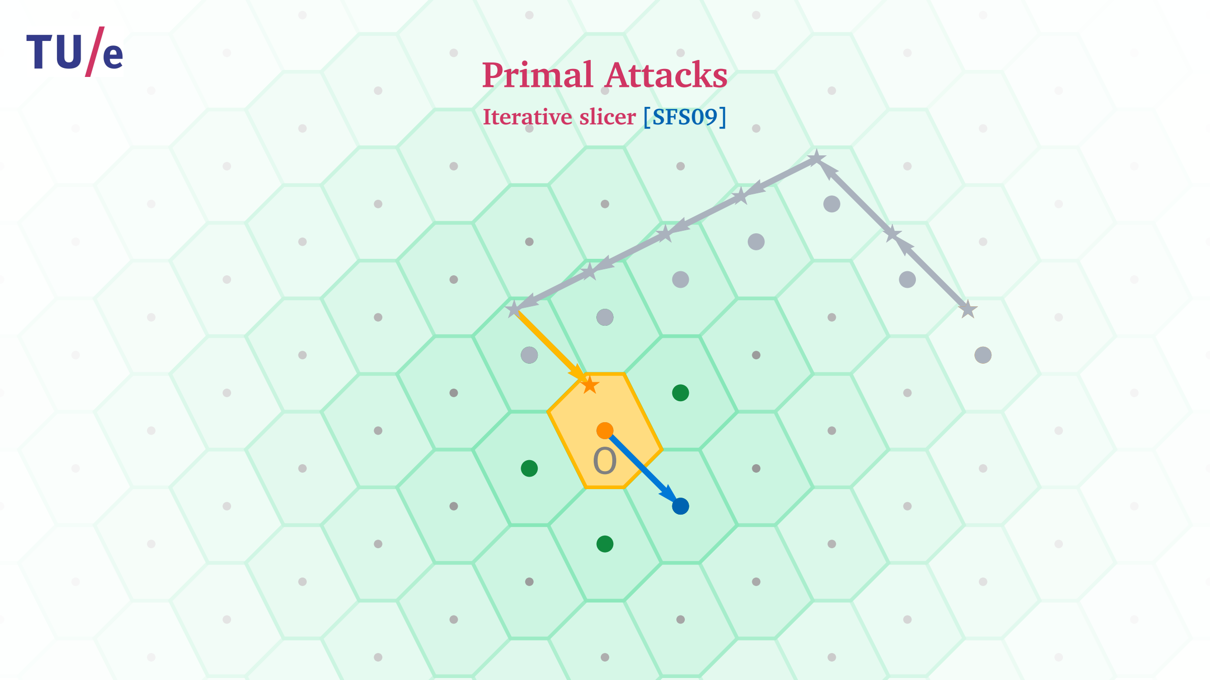

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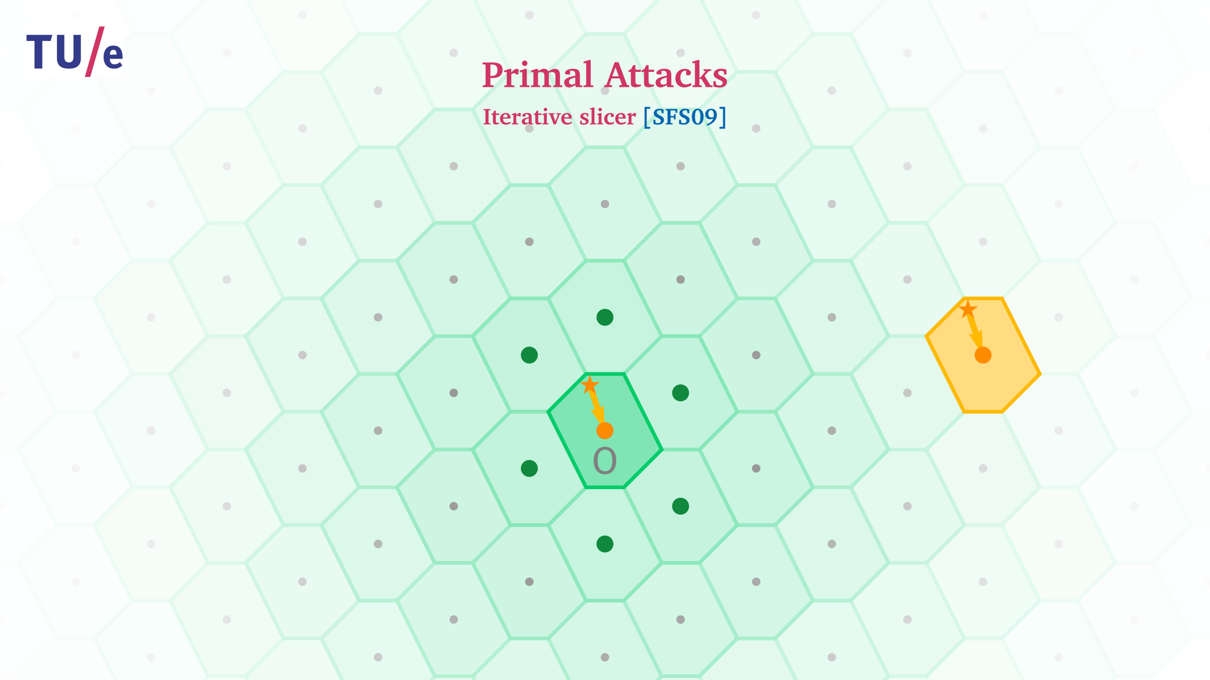

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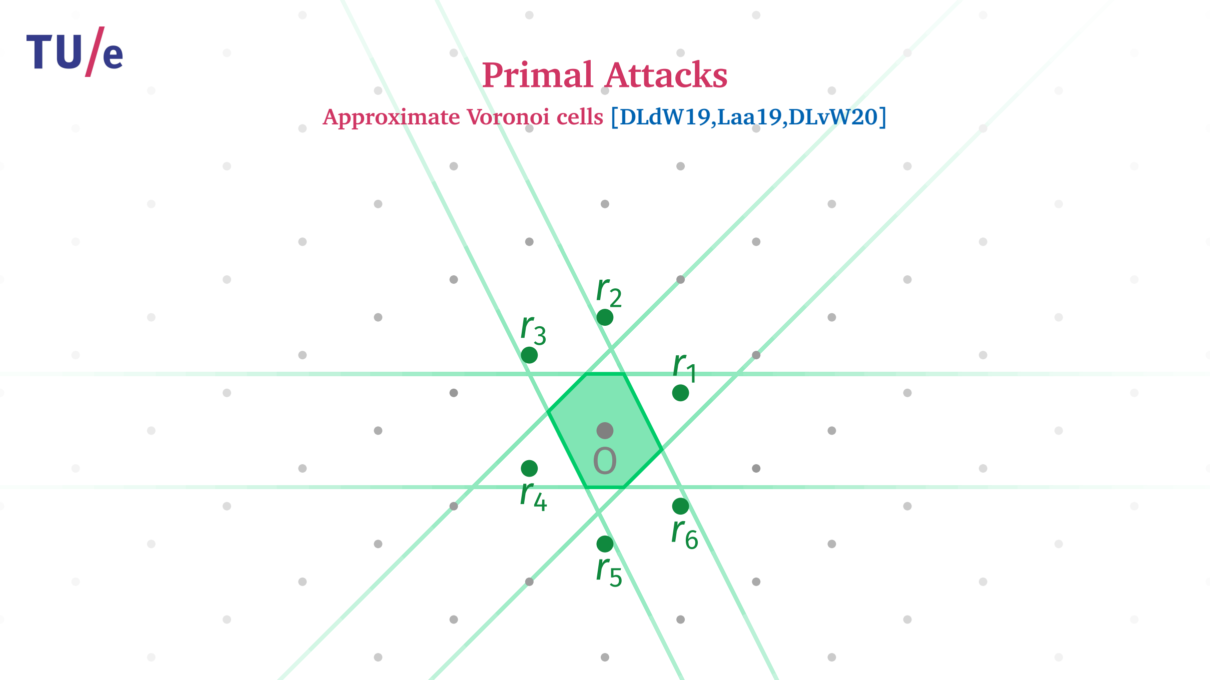
Approximate Voronoi cells [DLdW19,Laa19,DLvW20]



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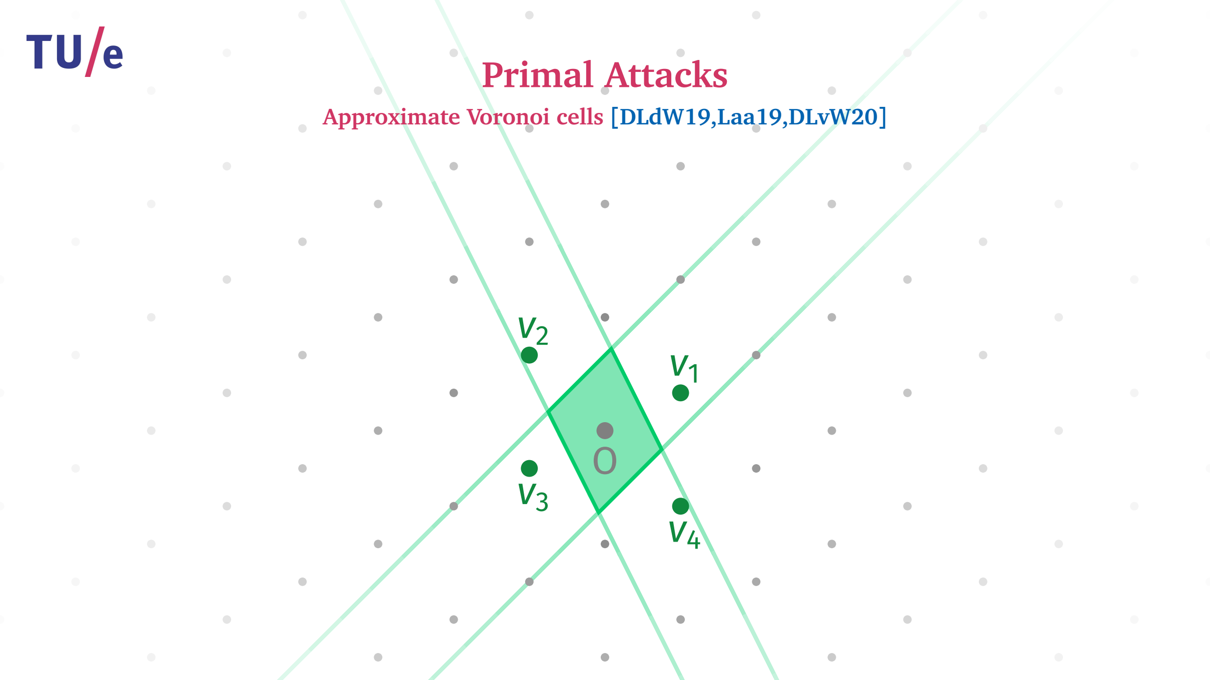

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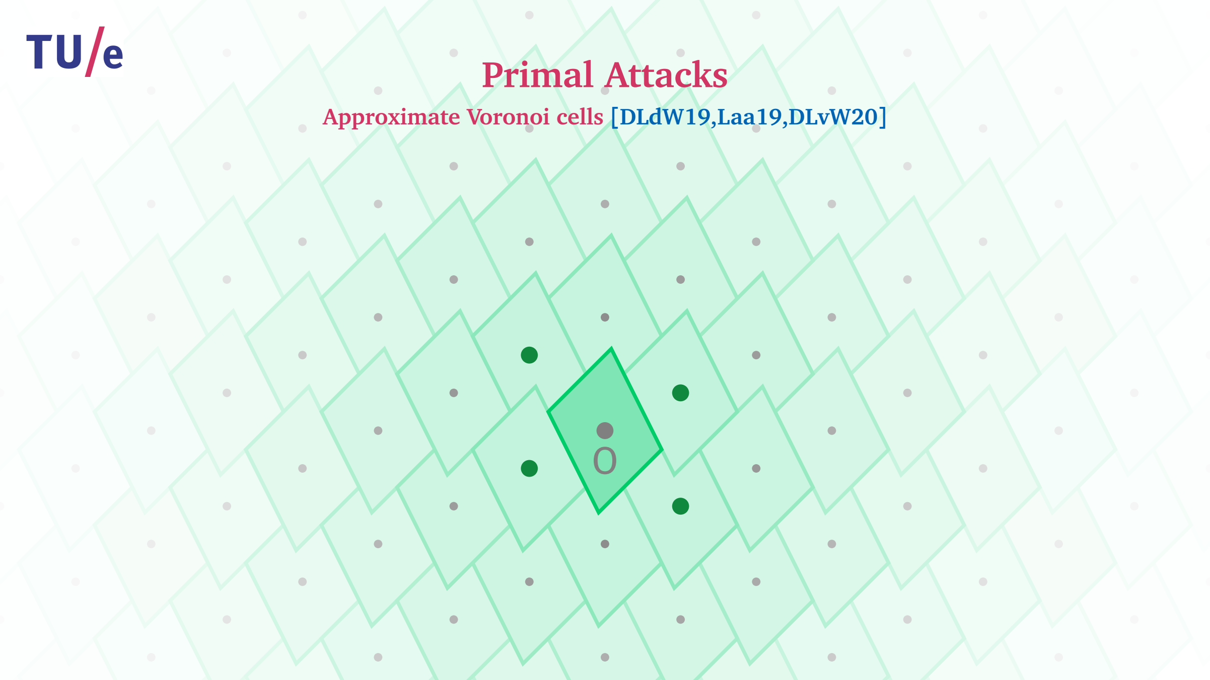

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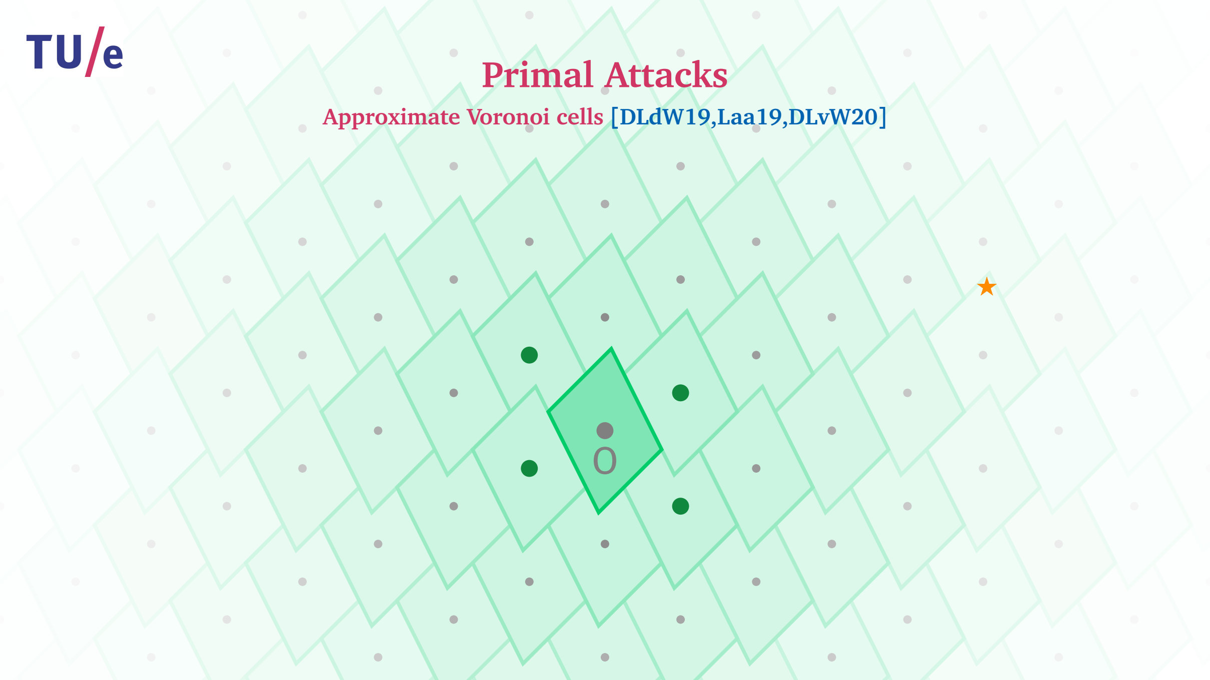

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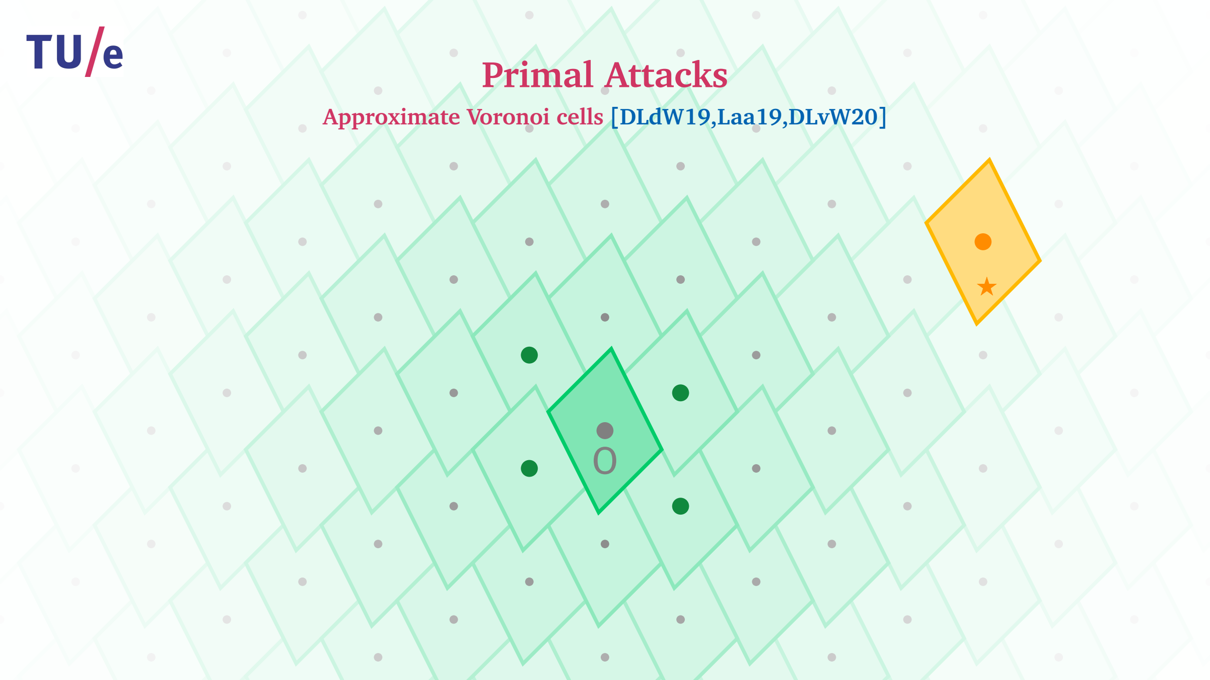

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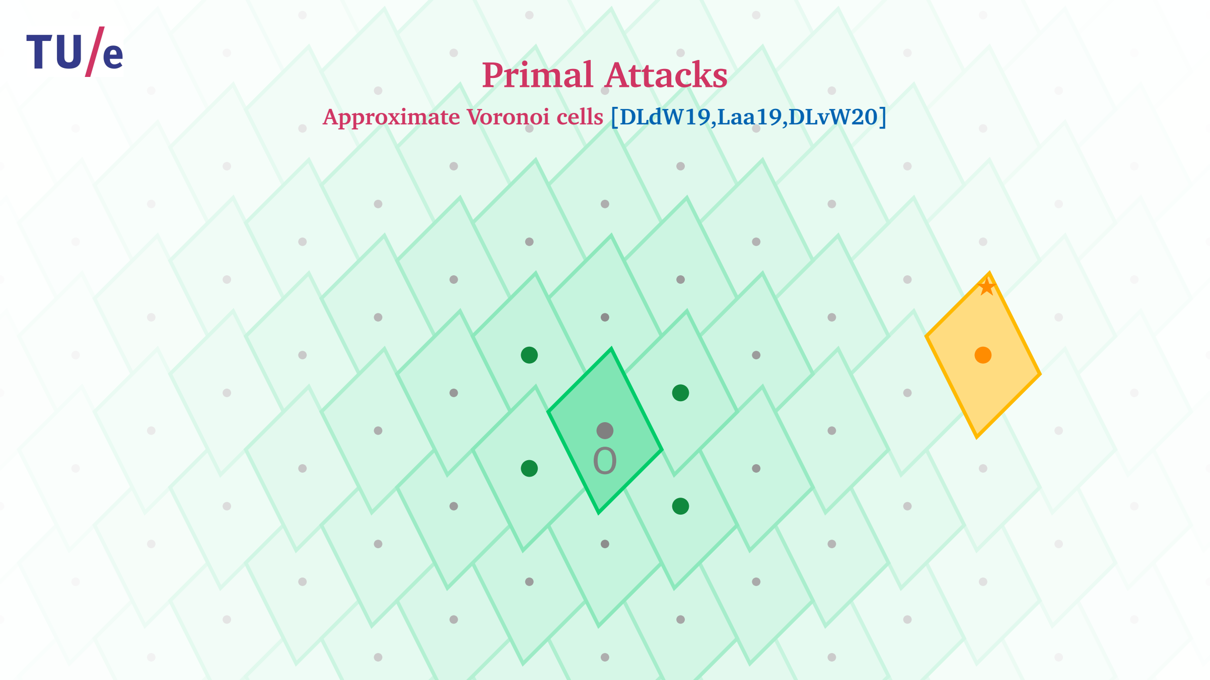

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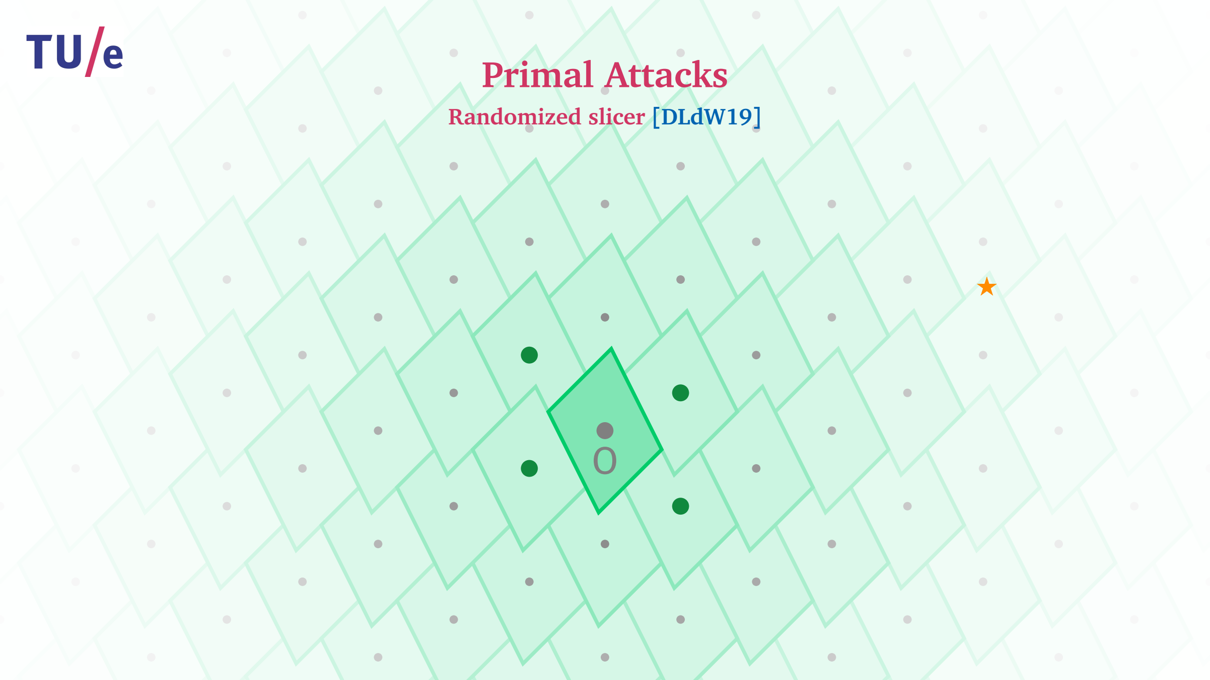

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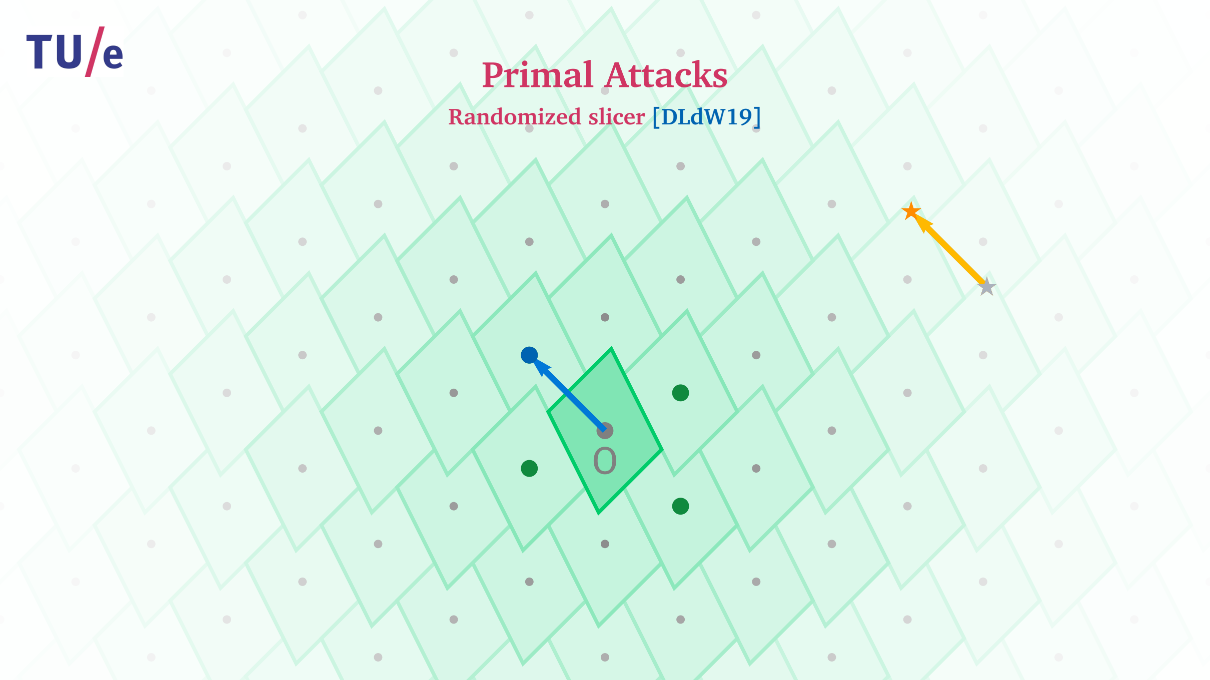

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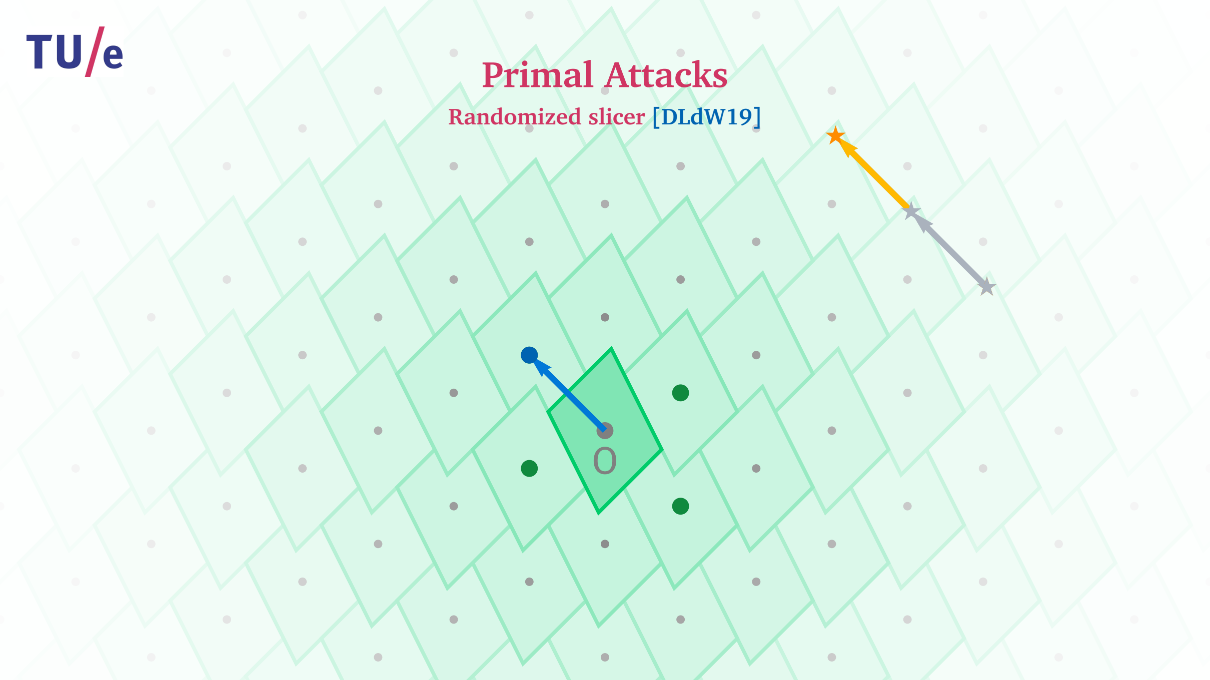

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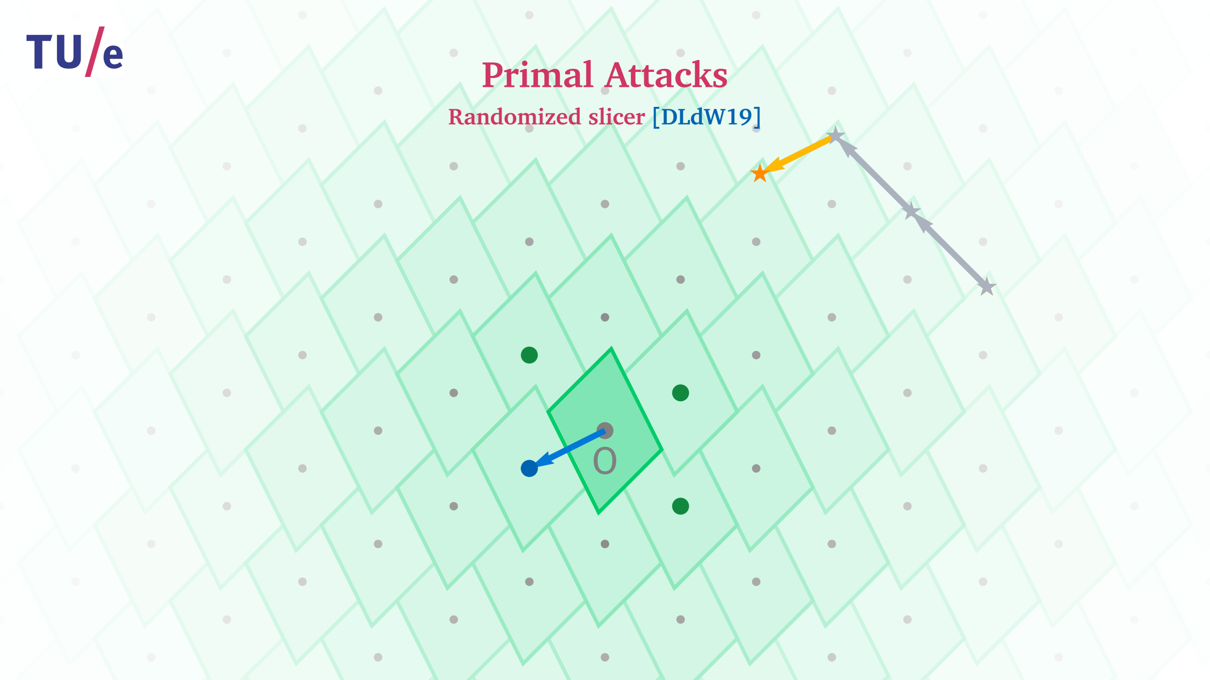

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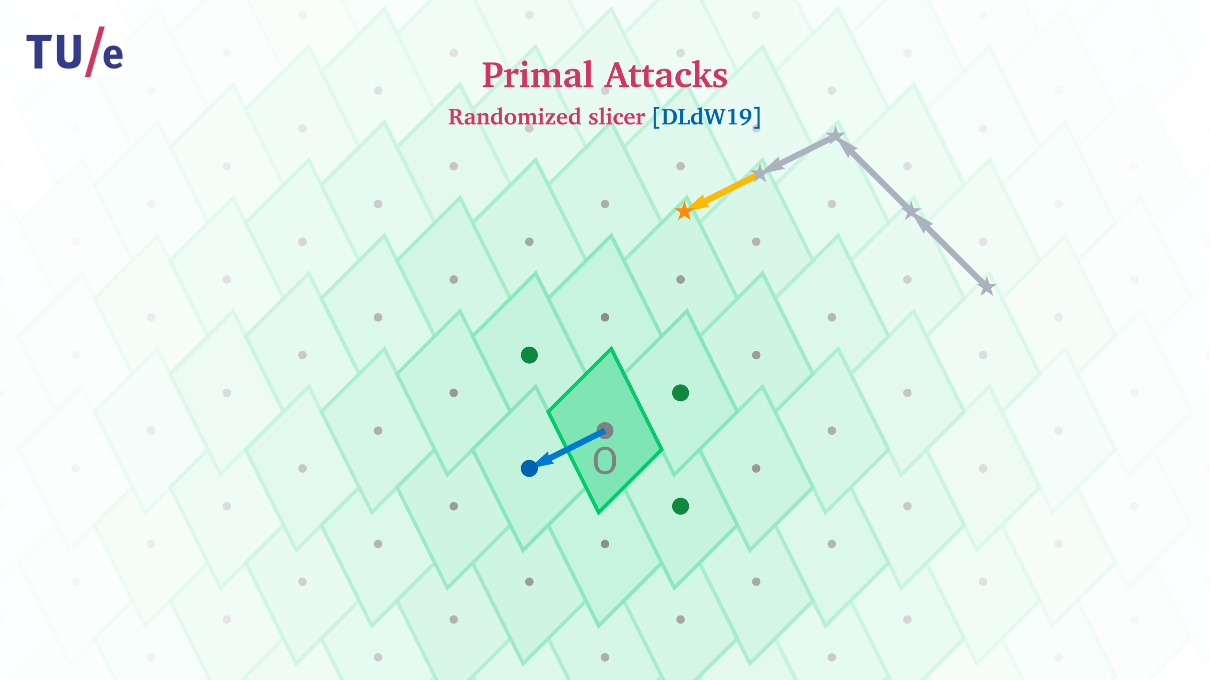

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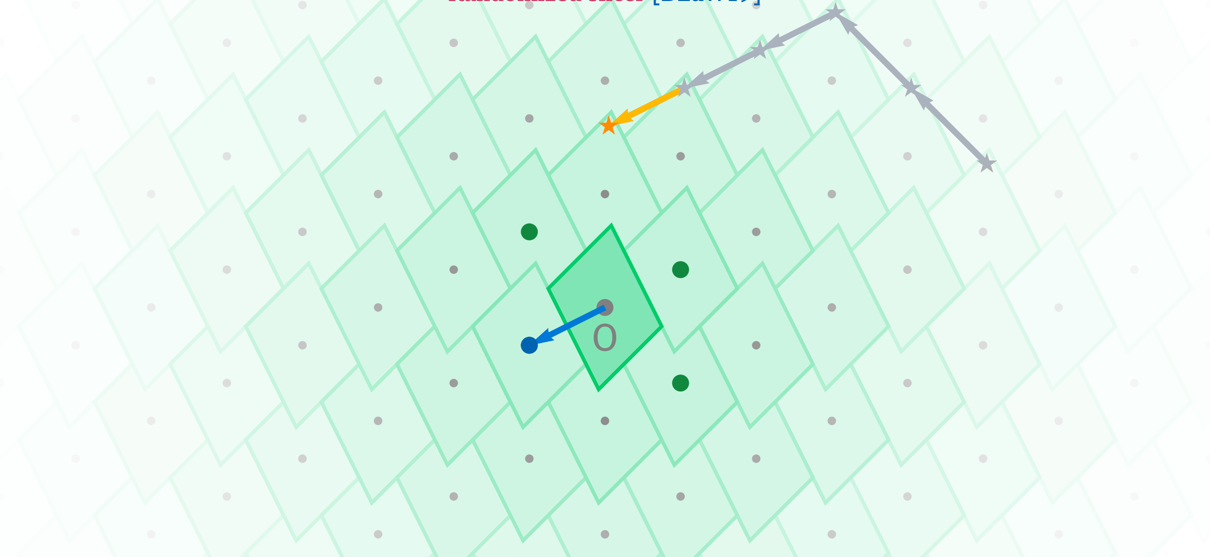

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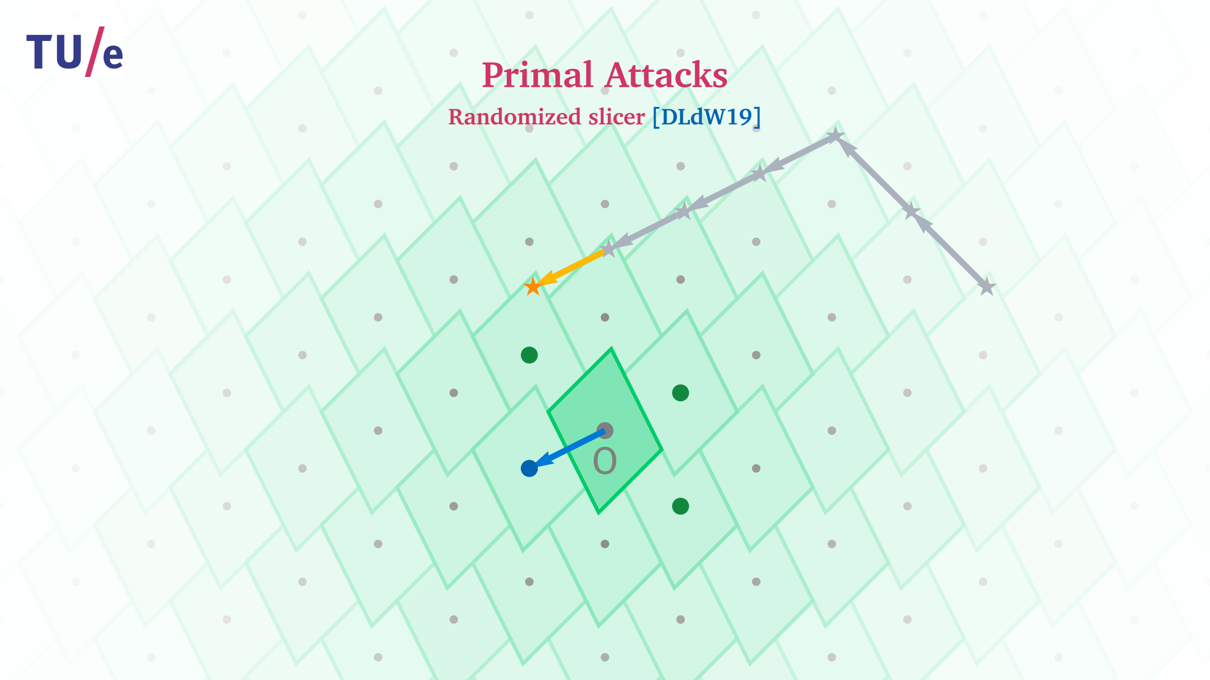

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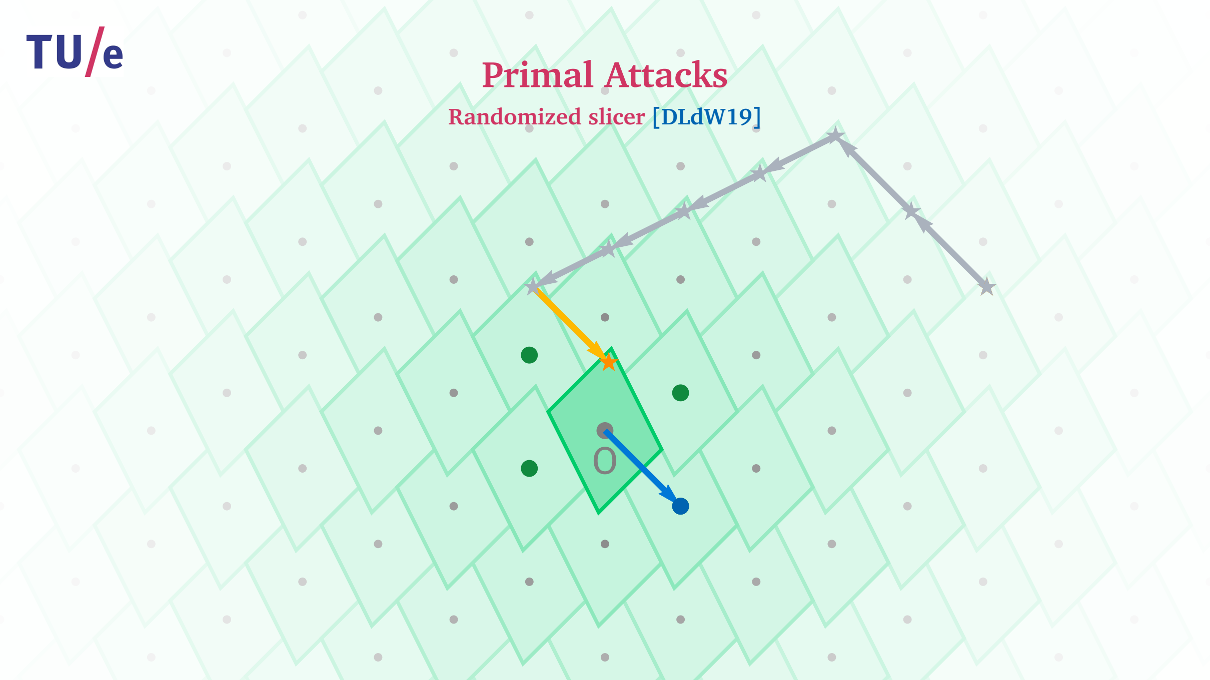

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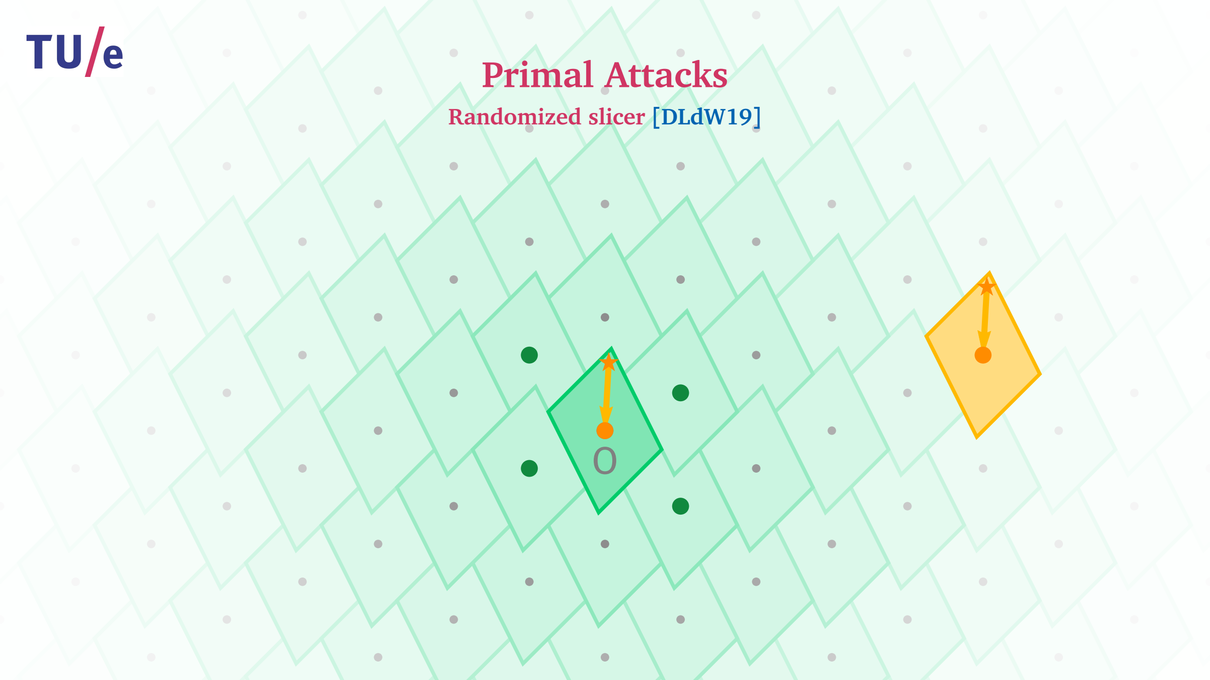

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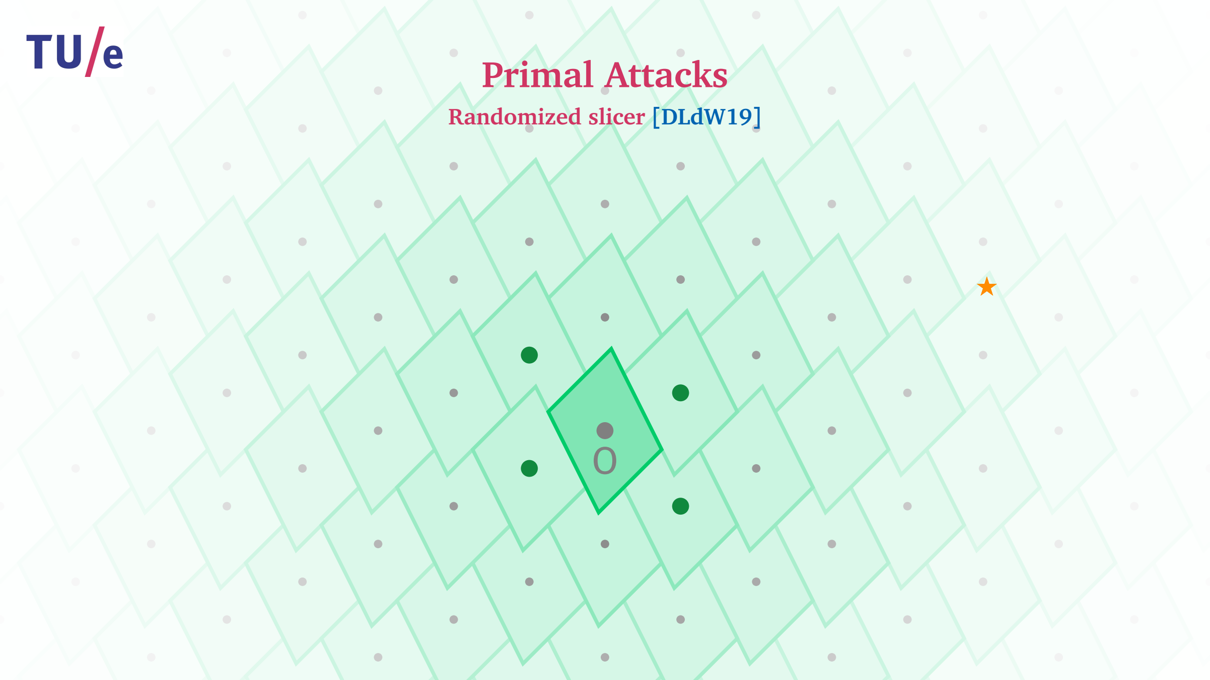

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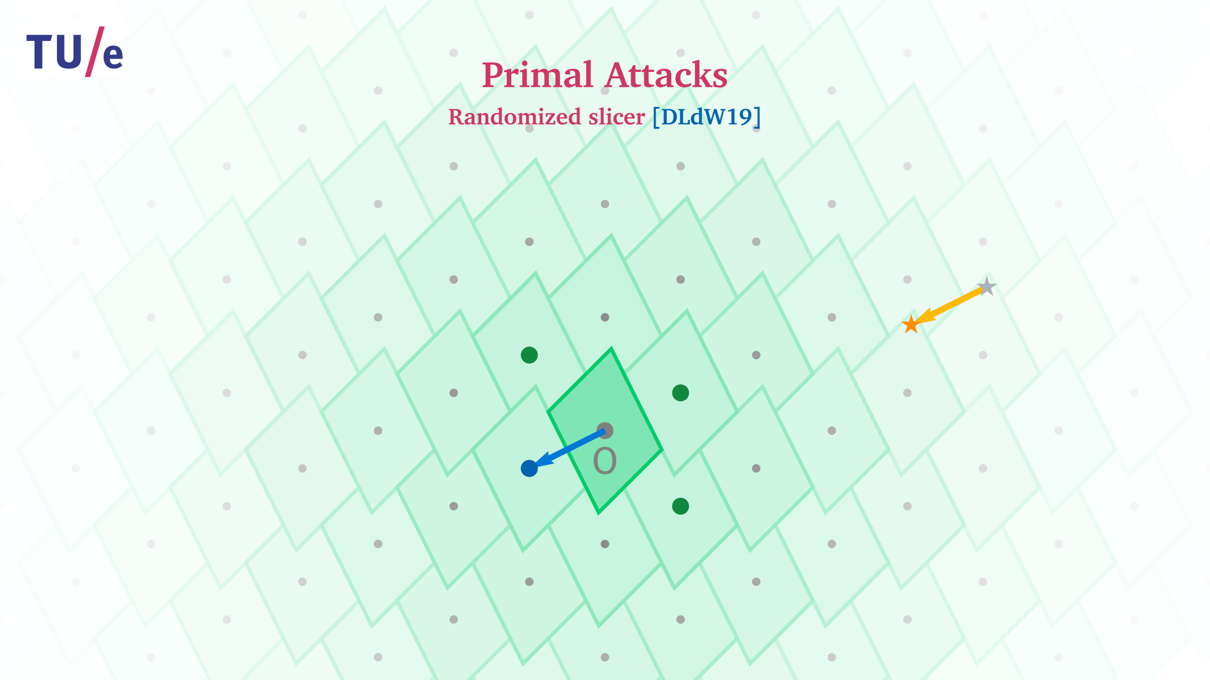

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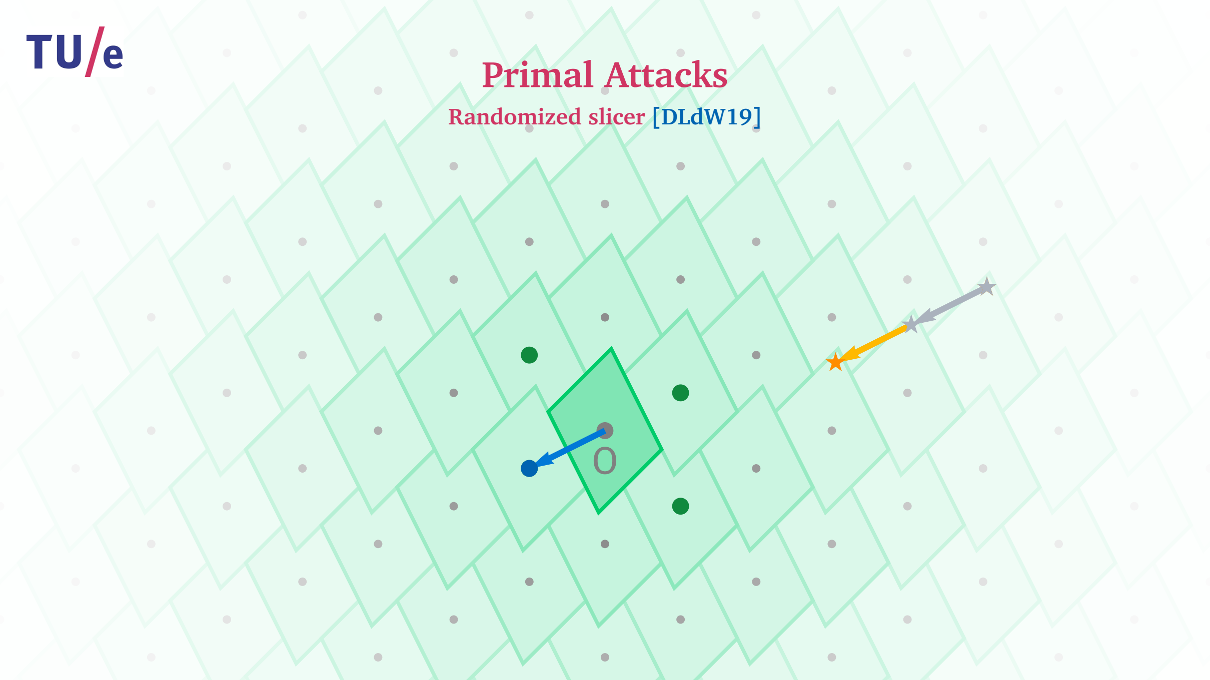

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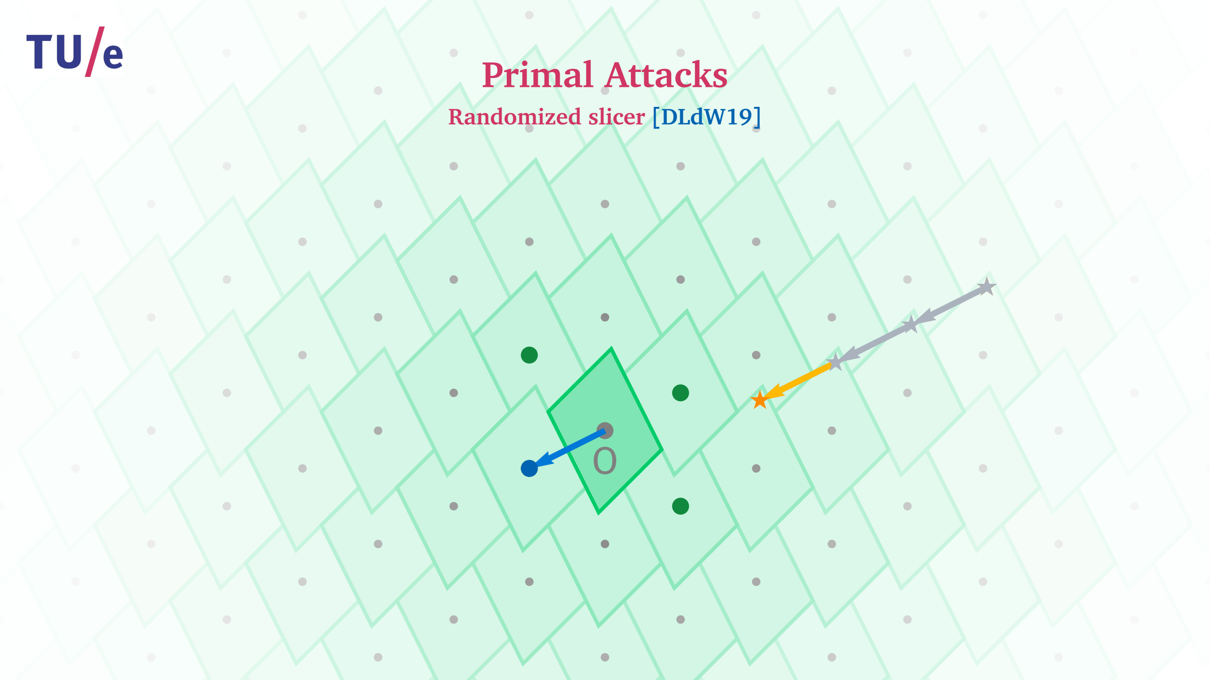

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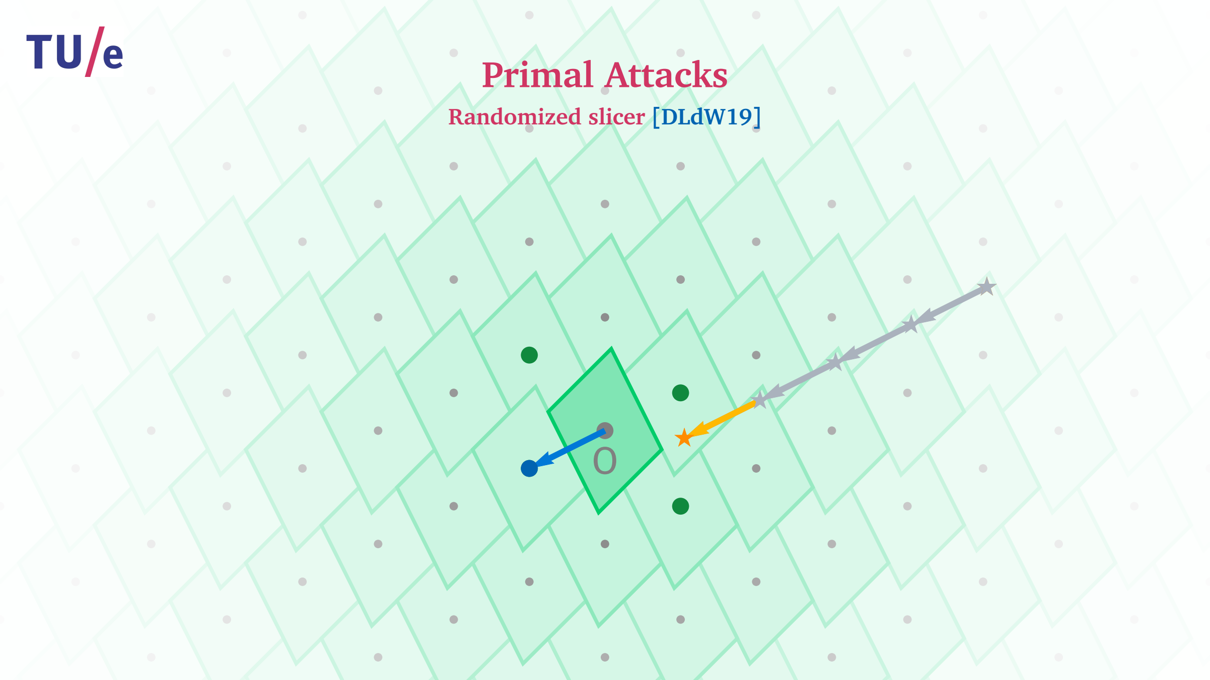

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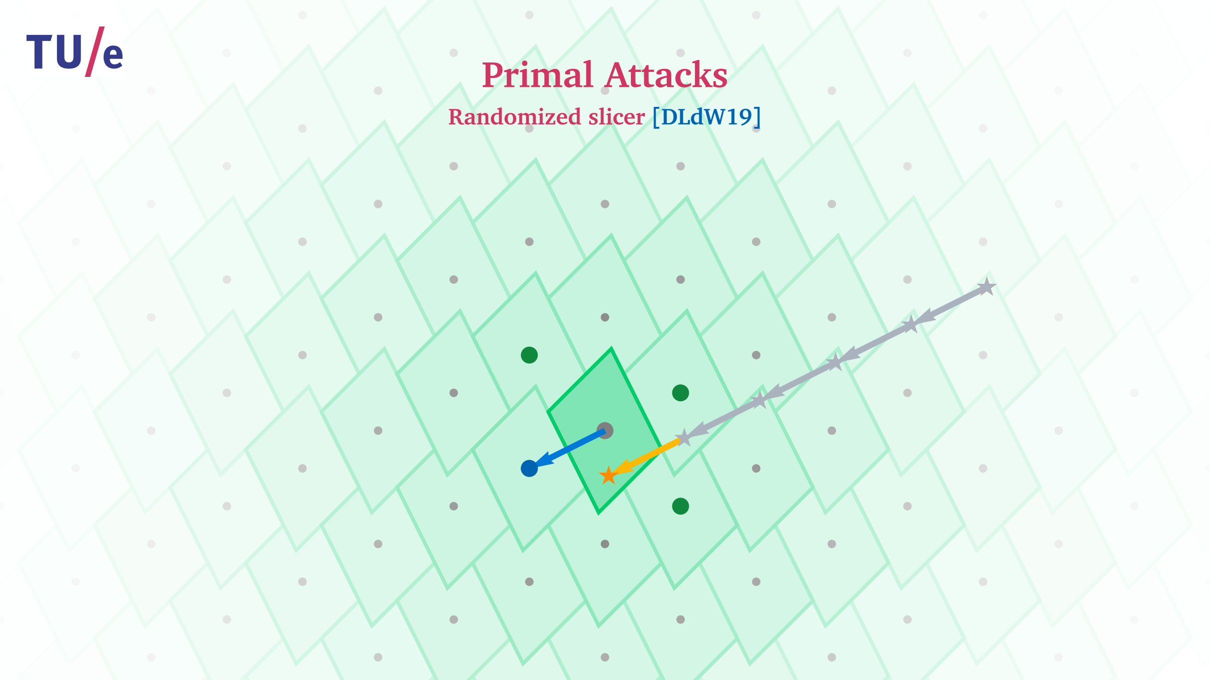

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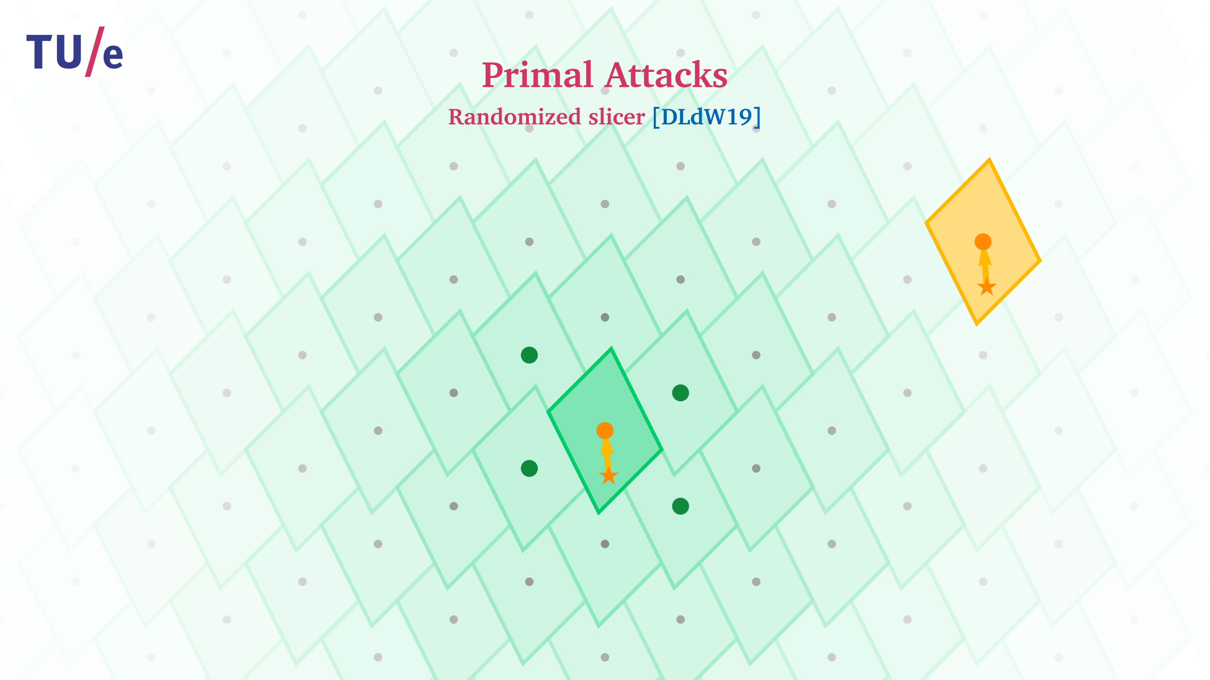

Primal Attacks

Overview

- *Preprocessing*: Find short primal lattice vectors ($2^{O(d)}$ time, space)

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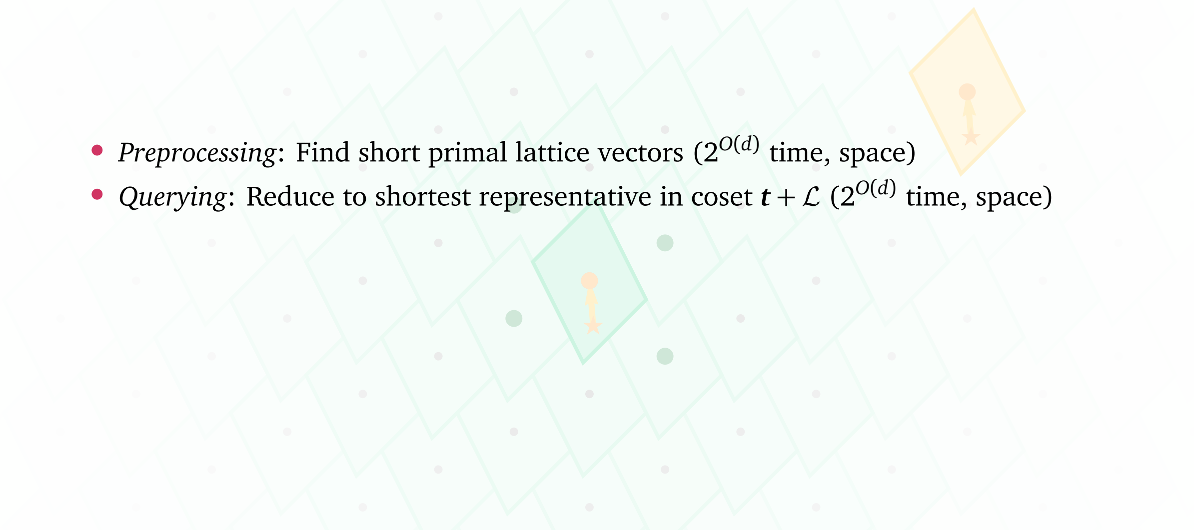

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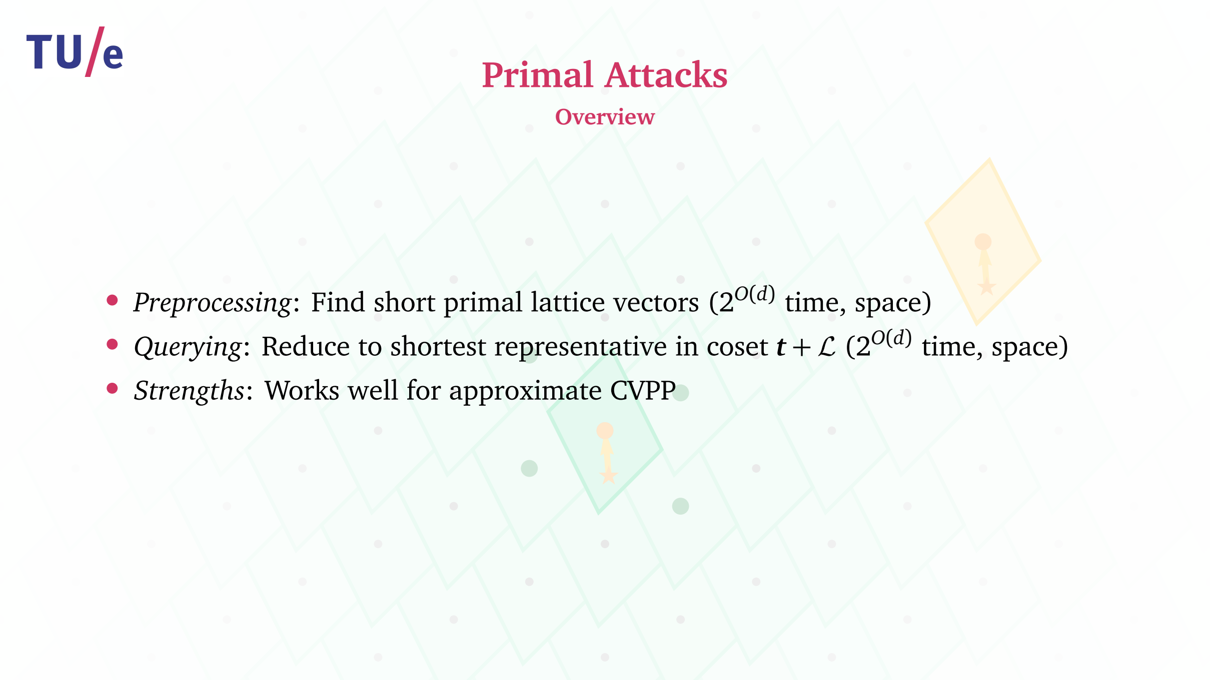

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- *Querying*: Reduce to shortest representative in coset $\boldsymbol{t} + \mathcal{L}$ ($2^{O(d)}$ time, space)
- *Strengths*: Works well for approximate CVPP
- *Limitations*: Does not scale well for BDDP

Dual Attacks

Dual: $\mathcal{L}^* = \{\boldsymbol{w} \in \mathbb{R}^d : \langle \boldsymbol{v}, \boldsymbol{w} \rangle \in \mathbb{Z}, \forall\, \boldsymbol{v} \in \mathcal{L}\}$

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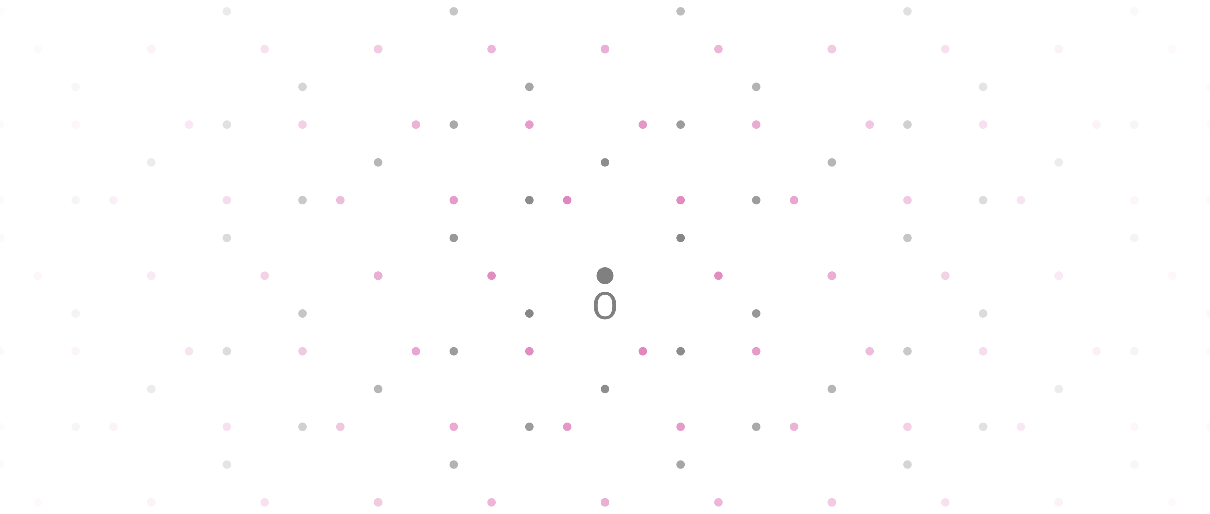

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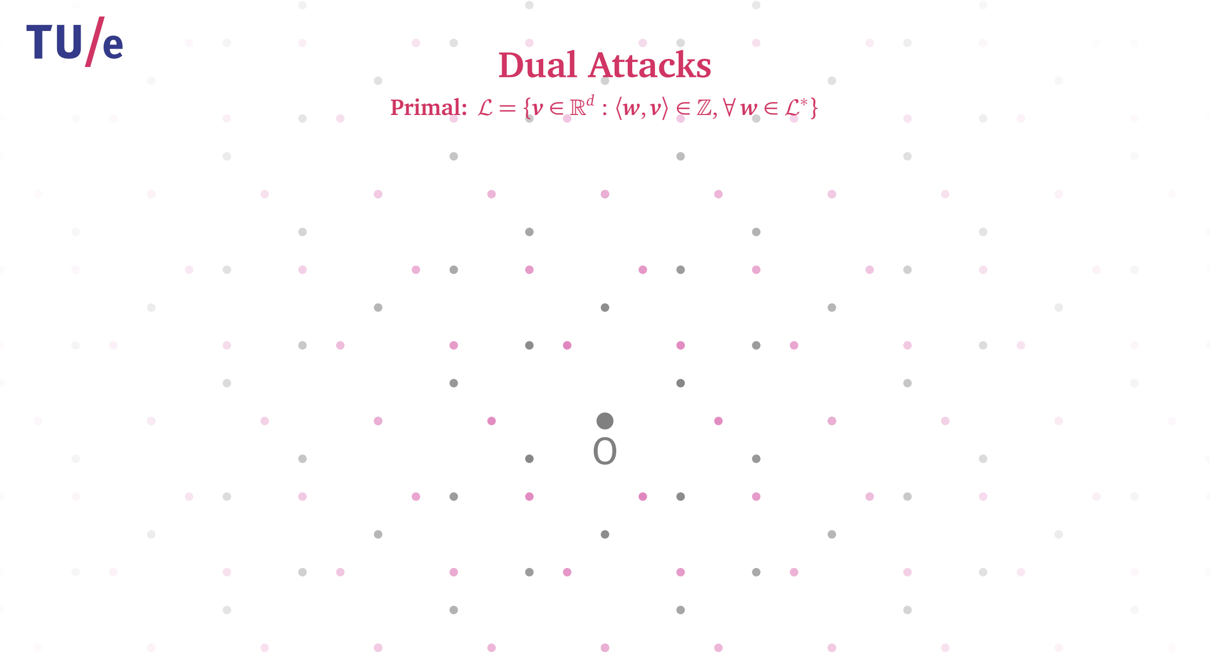

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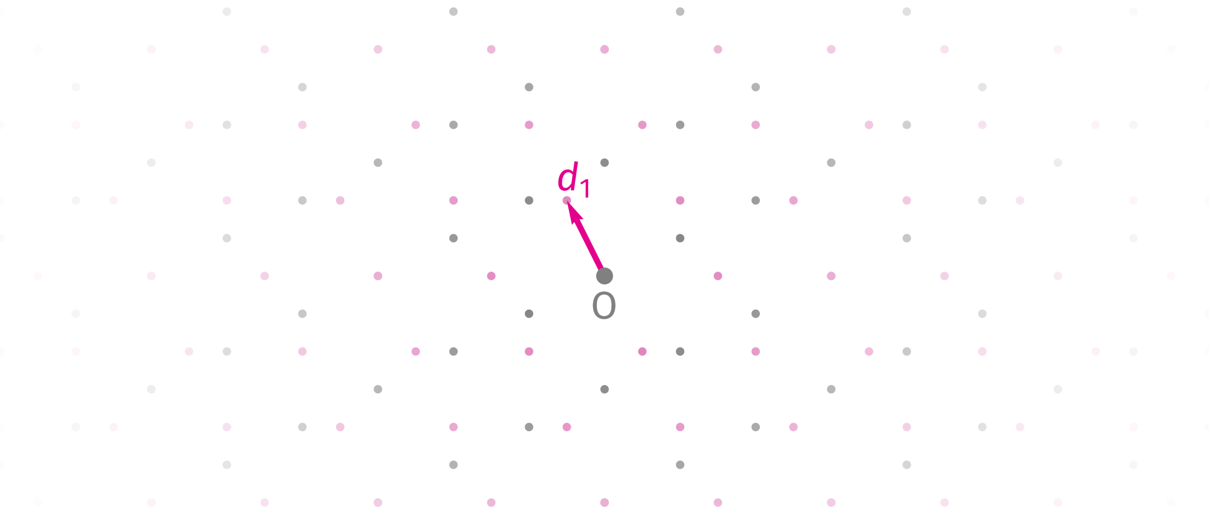

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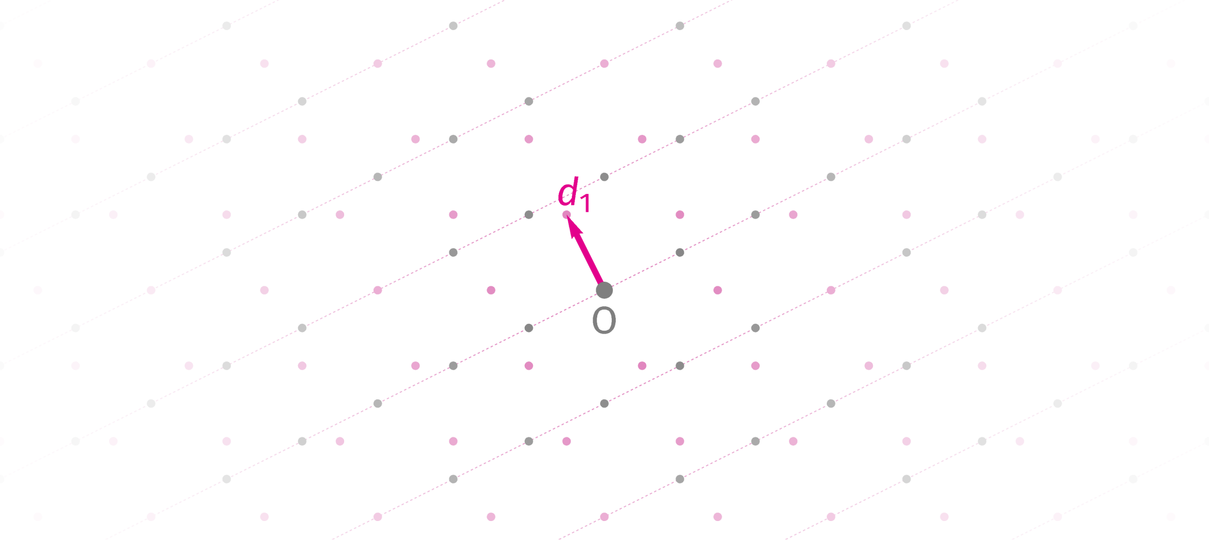

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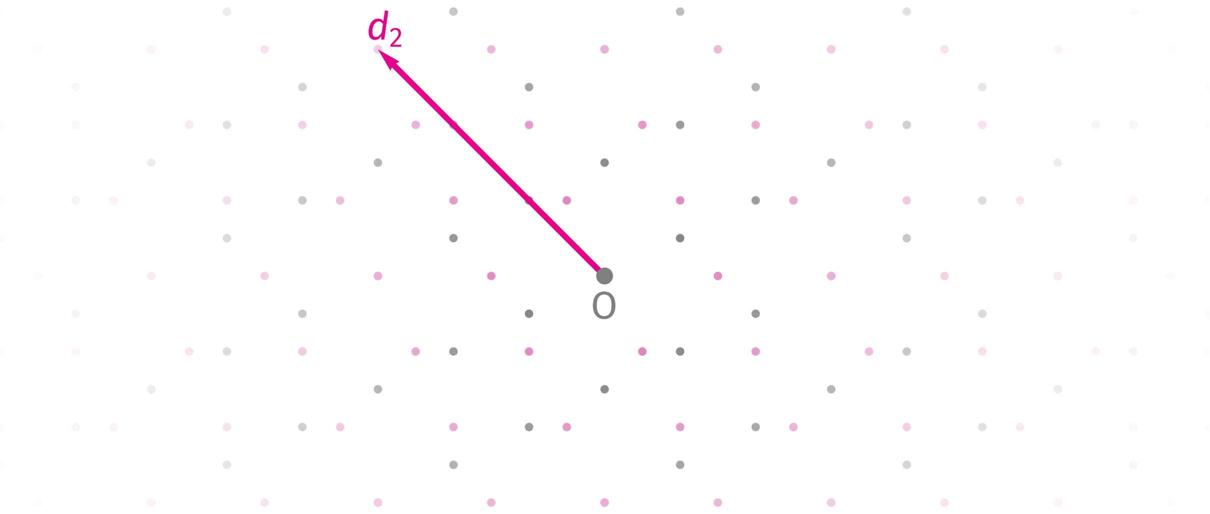

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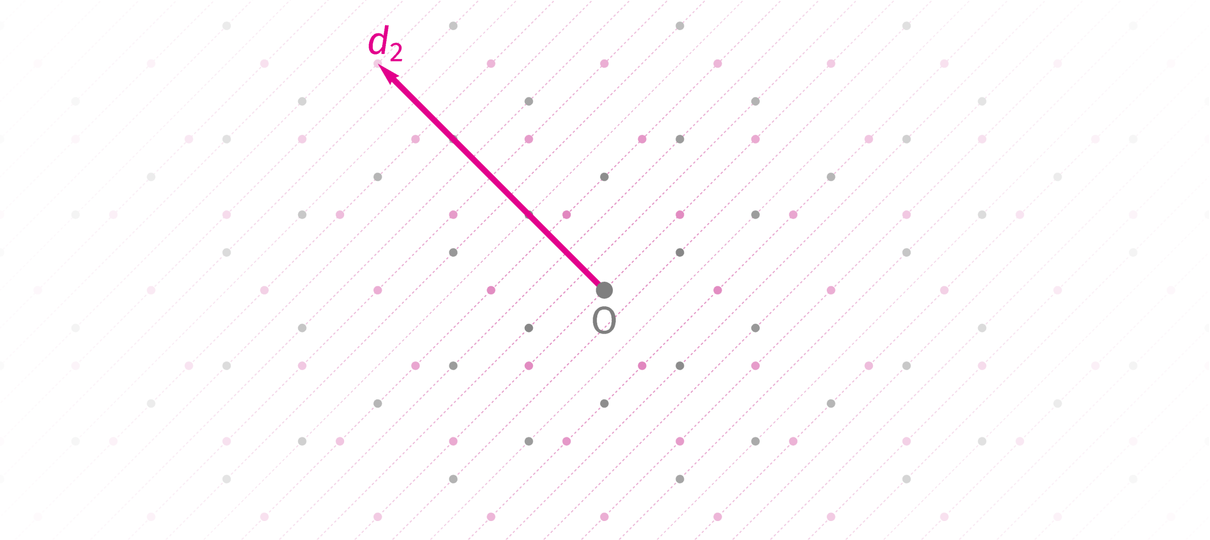
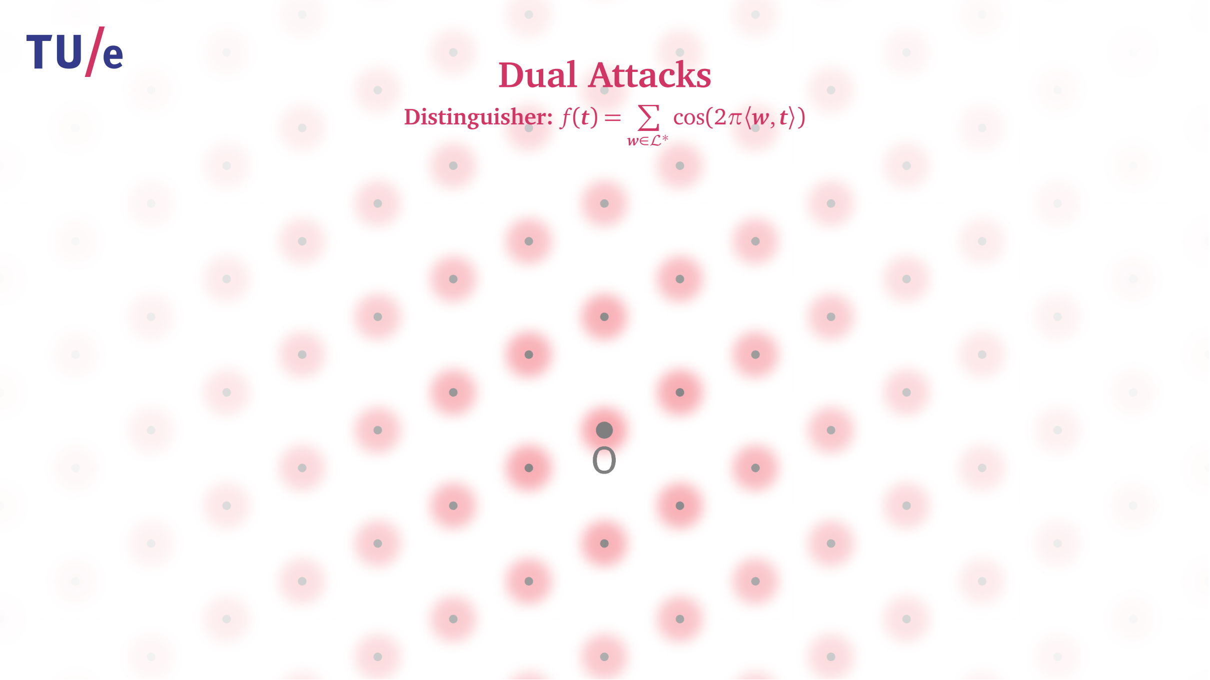


Dual Attacks

Distinguisher: $f(\boldsymbol{t}) = \sum_{\boldsymbol{w}\in\mathcal{L}^*} \cos(2\pi\langle \boldsymbol{w}, \boldsymbol{t}\rangle)$



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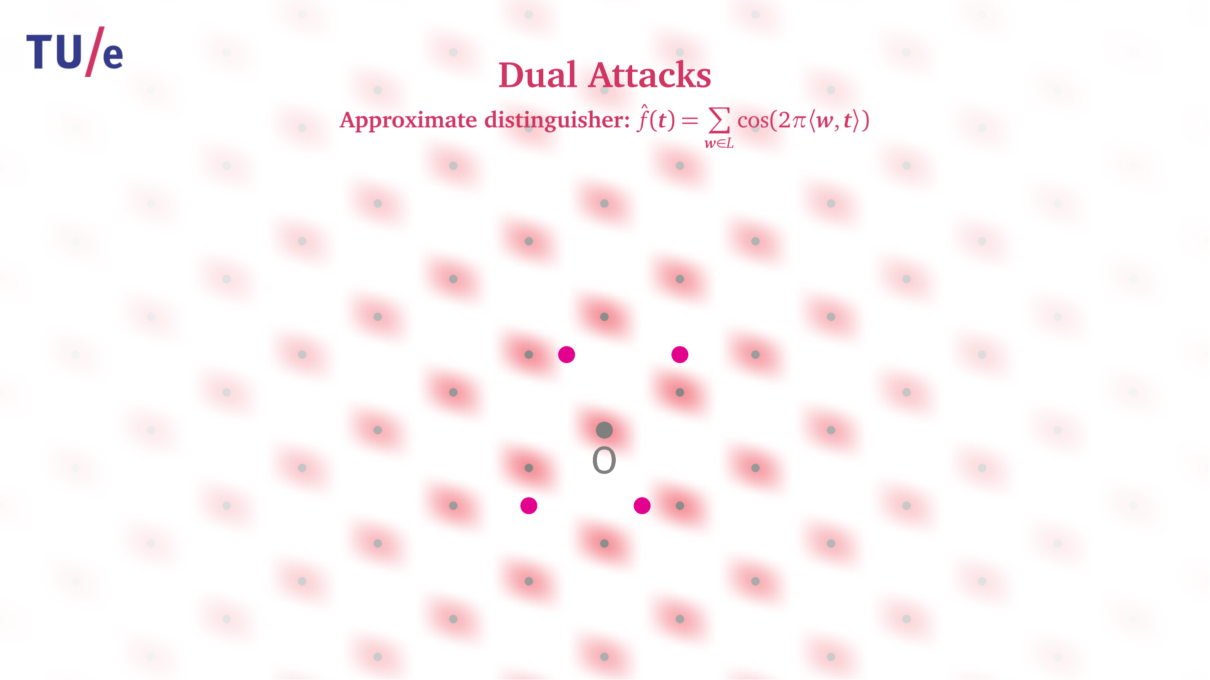
Approximate distinguisher: $\hat{f}(t) = \sum_{w \in L} \cos(2\pi \langle w, t \rangle)$



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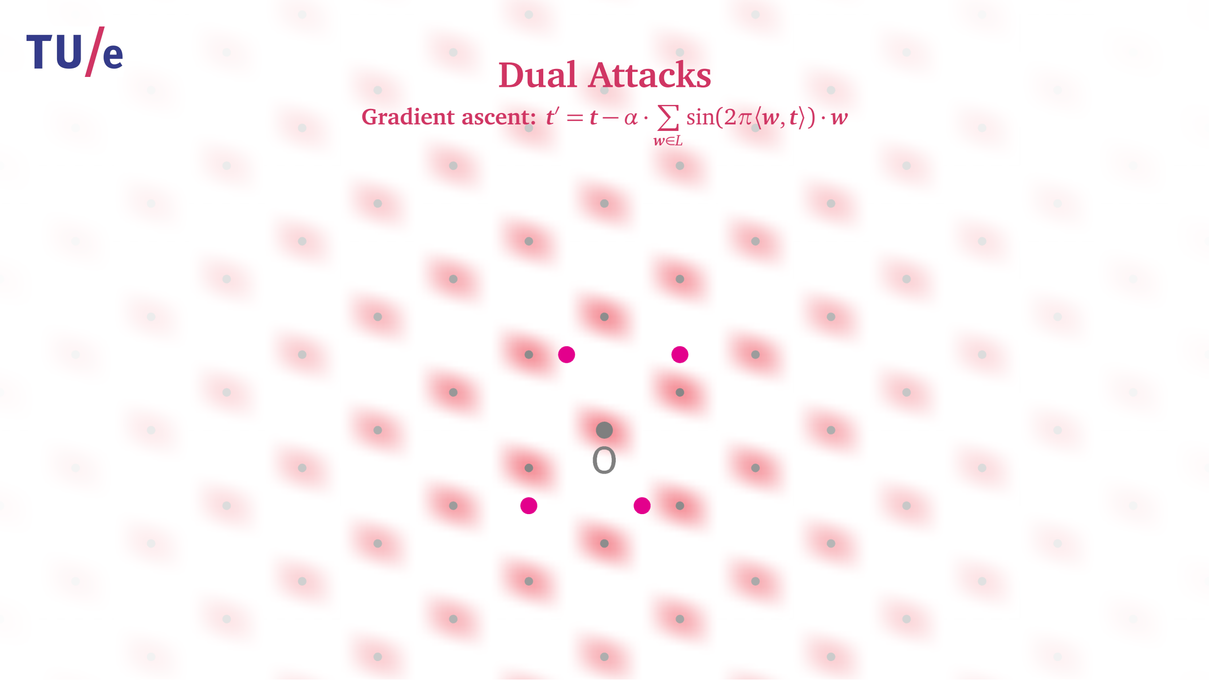
Gradient ascent: $t' = t - \alpha \cdot \sum_{w \in L} \sin(2\pi \langle w, t \rangle) \cdot w$



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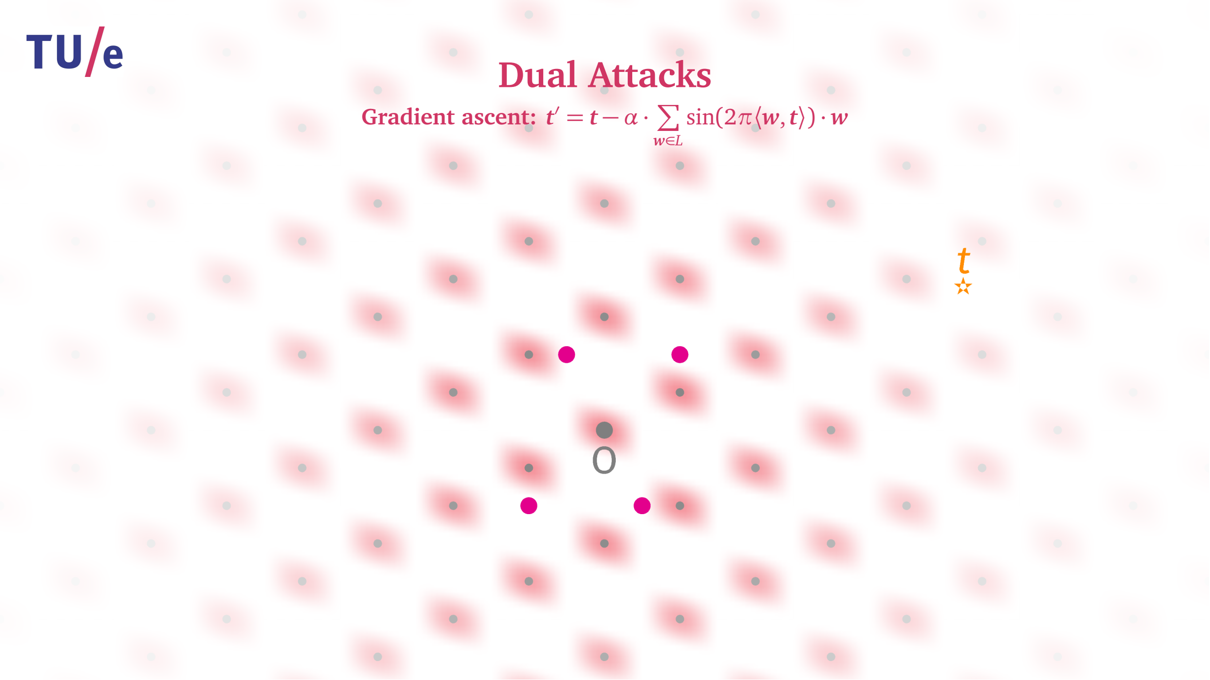

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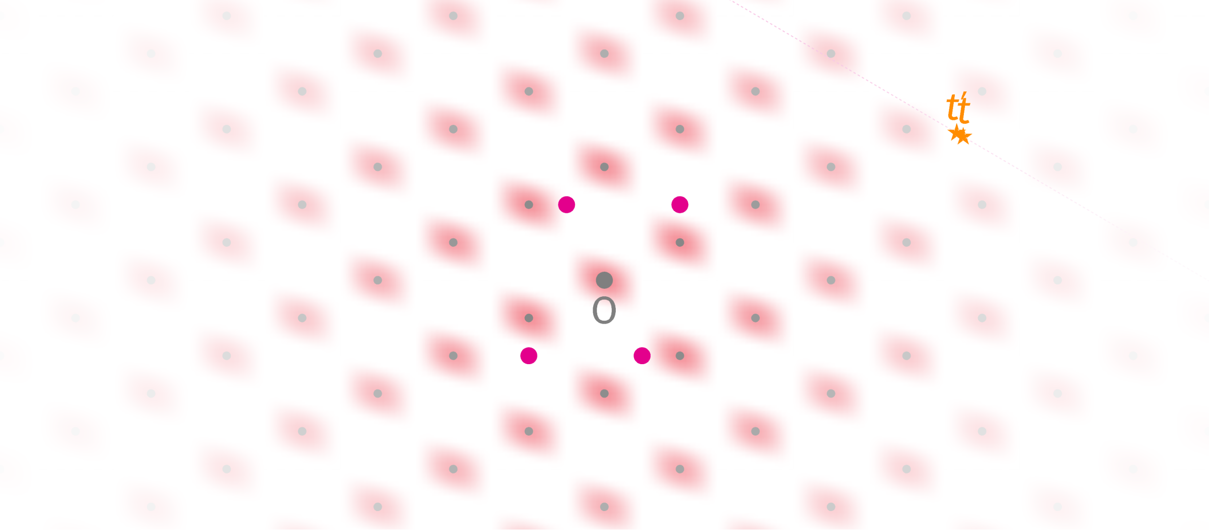

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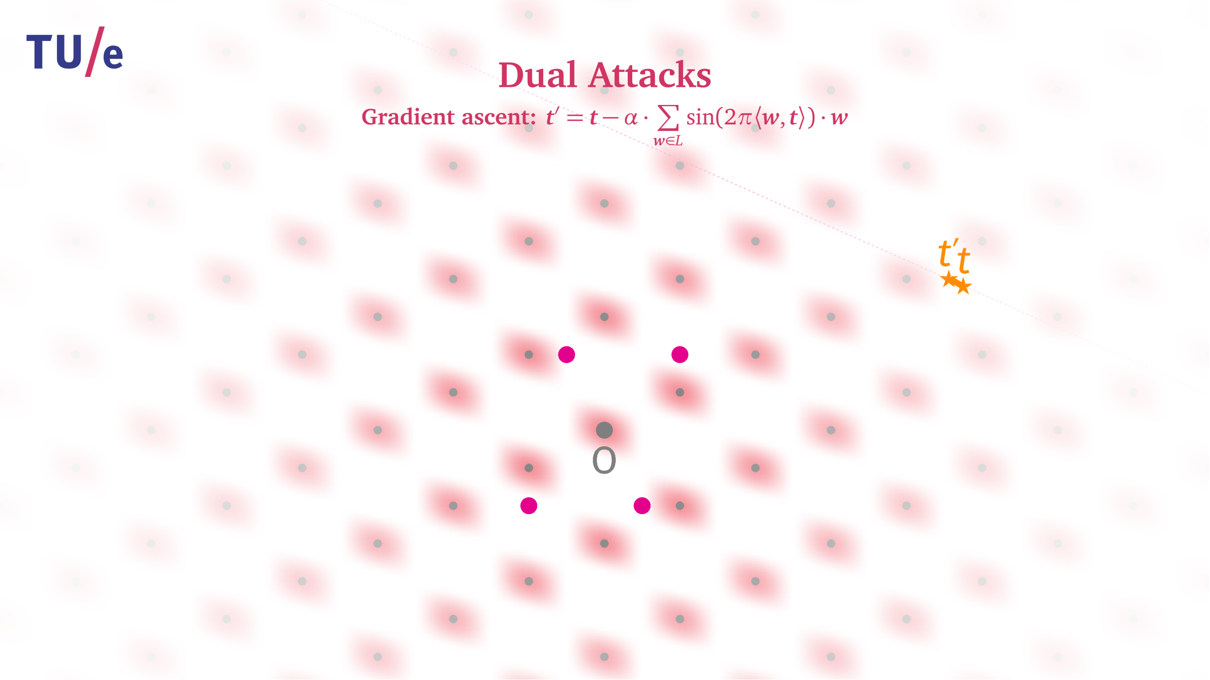

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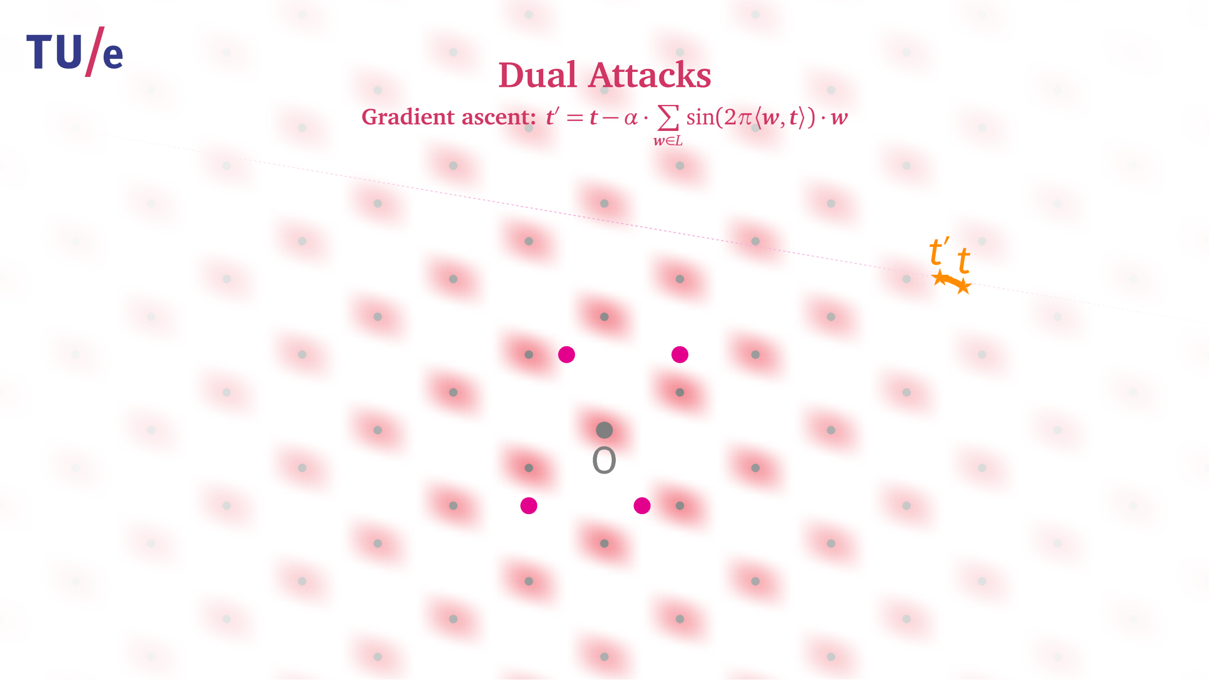
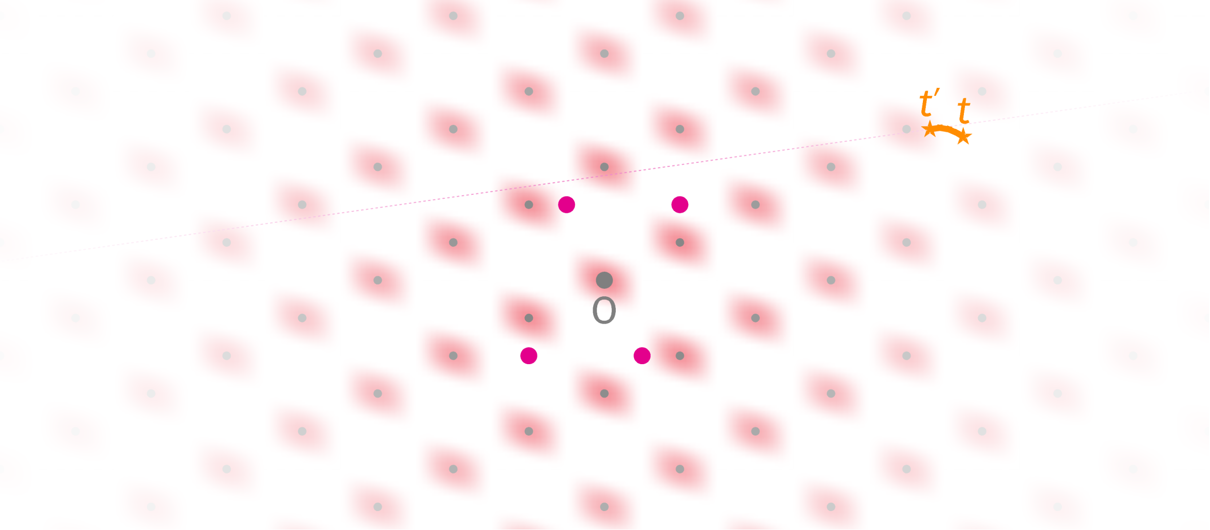


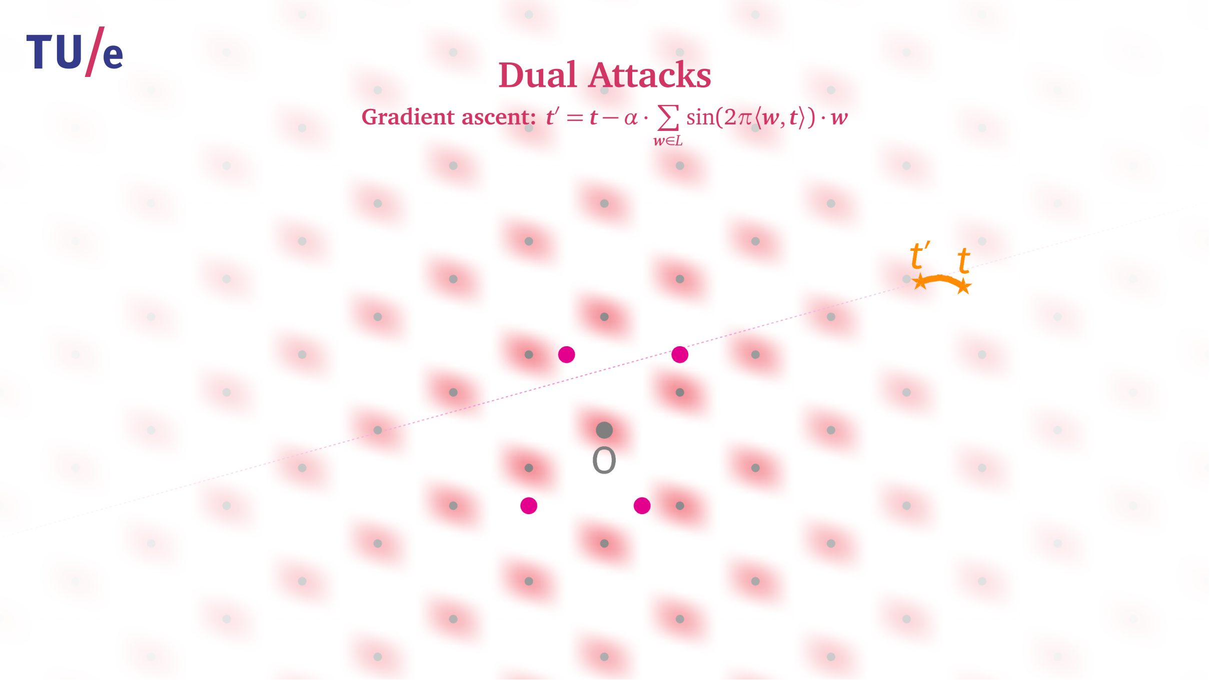
Dual Attacks

Gradient ascent: $t' = t - \alpha \cdot \sum_{w \in L} \sin(2\pi\langle w, t\rangle) \cdot w$



Dual Attacks

Gradient ascent: $t' = t - \alpha \cdot \sum_{w \in L} \sin(2\pi \langle w, t \rangle) \cdot w$



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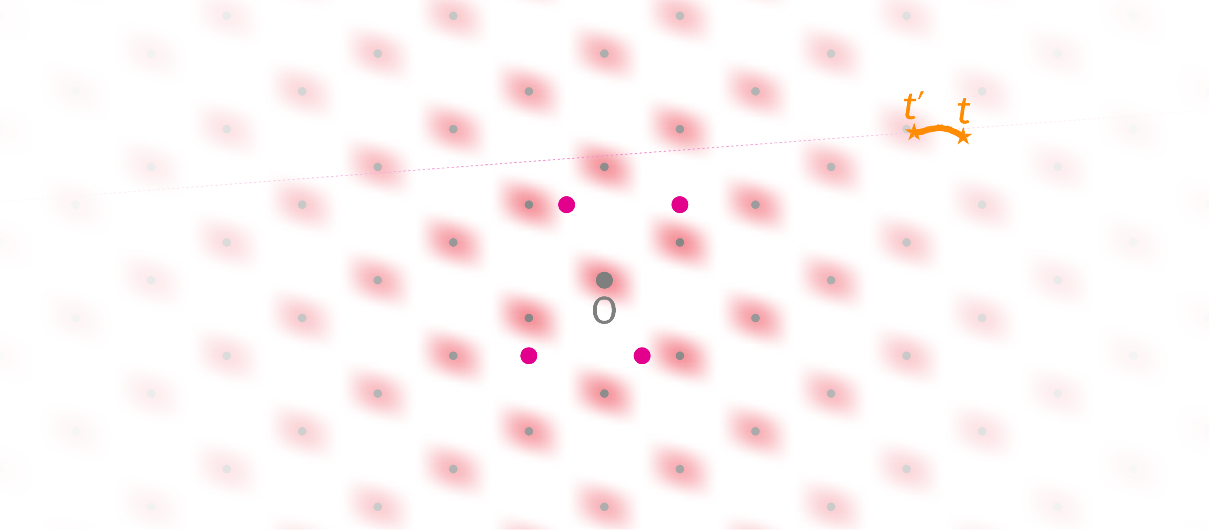

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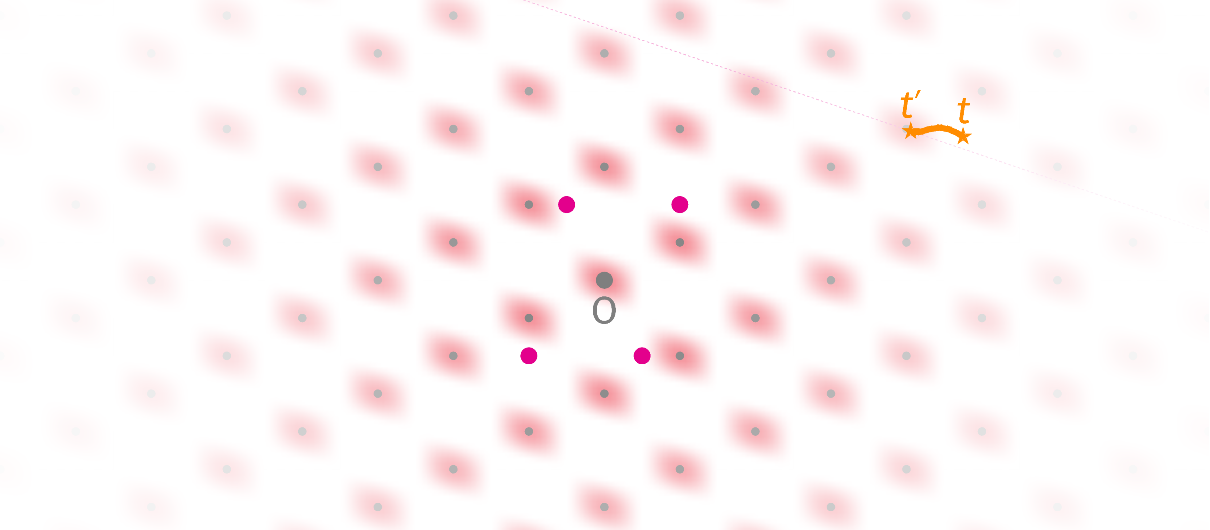

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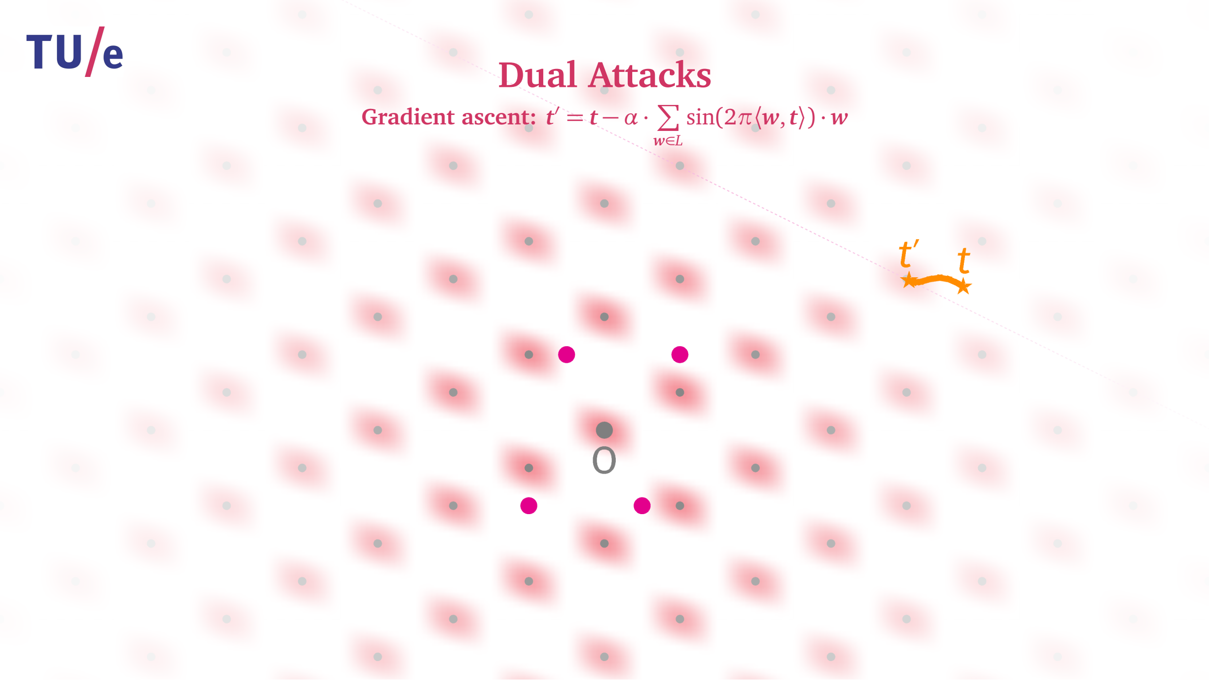

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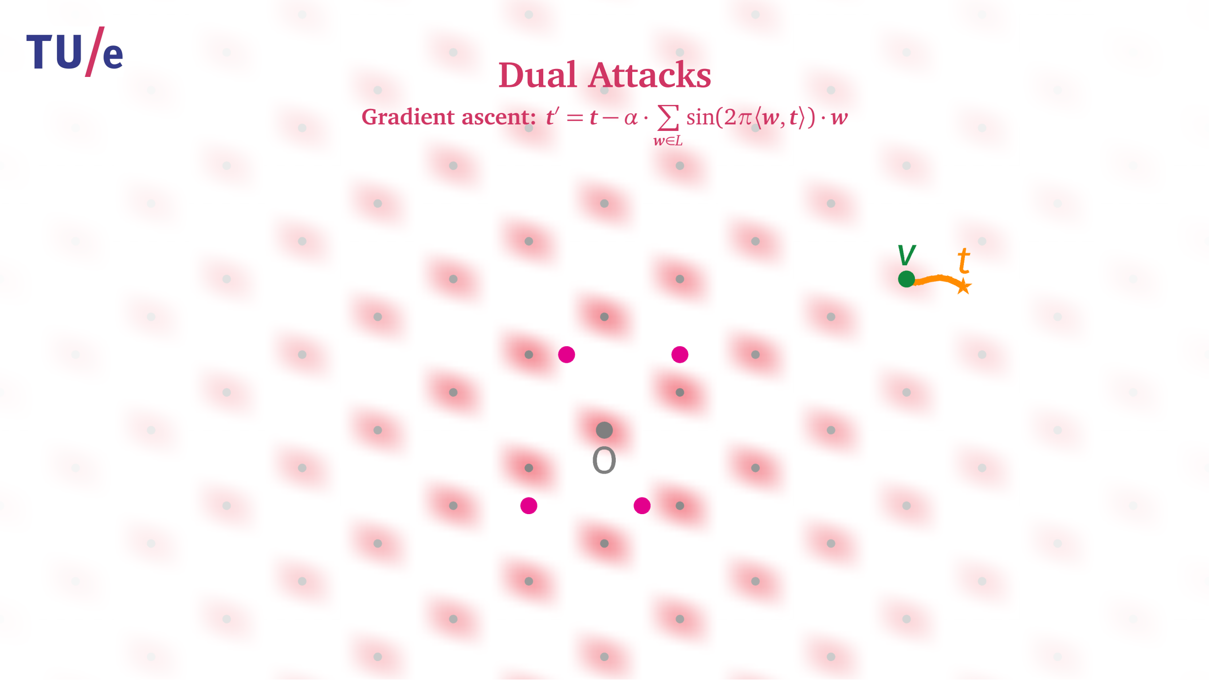
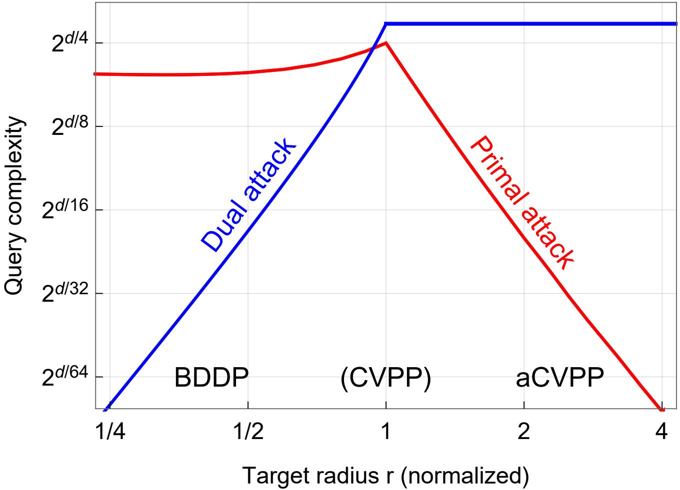


Dual Attacks

Asymptotics (with preprocessing)



Dual Attacks

Experiments ($d = 80$)

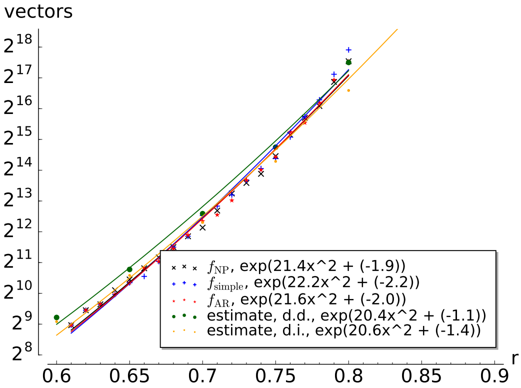


Figure: Complexity of distinguishing from random at radius r ($p = 0.90$).

Dual Attacks

Experiments ($d = 80$)

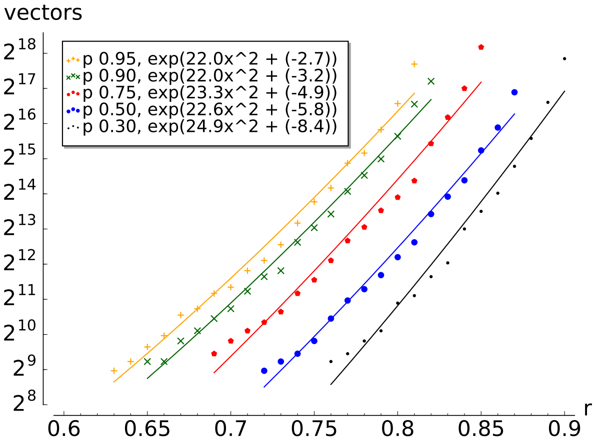


Figure: Complexity of decoding a target at distance r with probability p in dimension 80.

Dual Attacks

Experiments ($d = 80$)

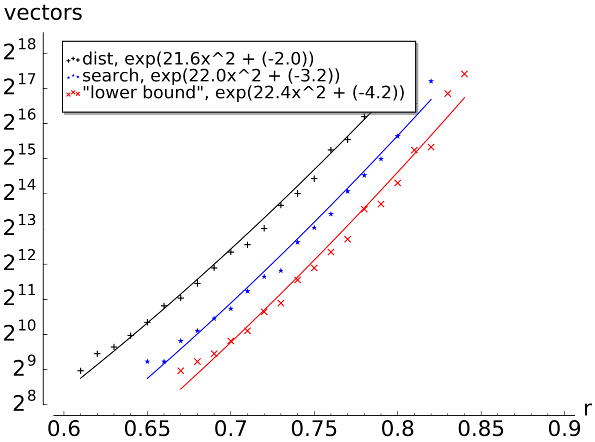


Figure: Experimental complexities for distinguishing/searching and a heuristic lower bound.

Dual Attacks

Experiments (variable d)

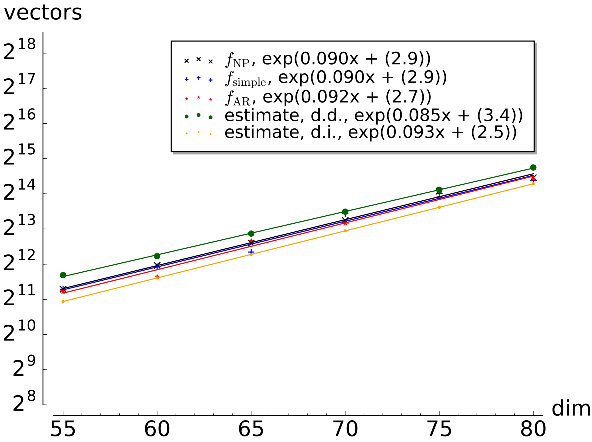


Figure: Complexity of distinguishing a planted target at radius 0.75 from random ($p = 0.9$).

Dual Attacks

Experiments ($d = 80$)

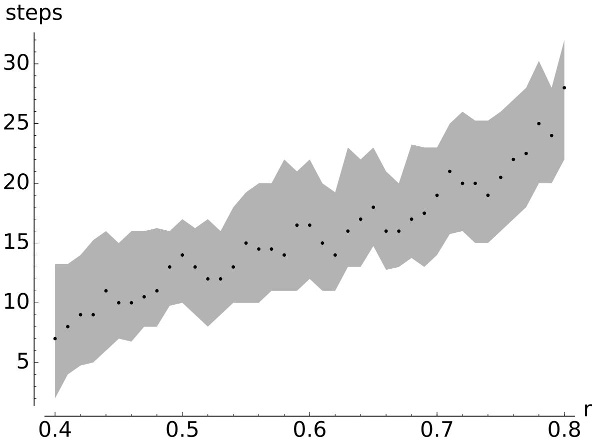
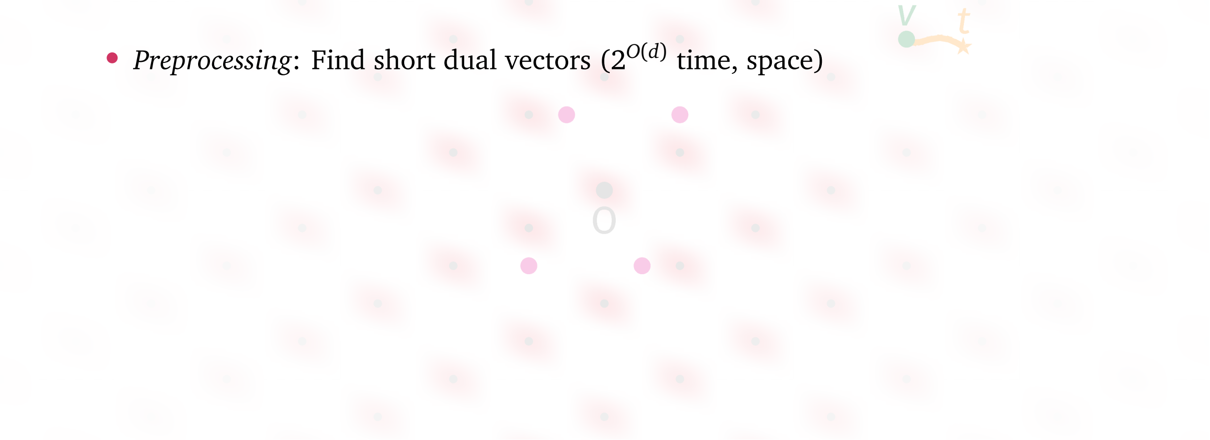


Figure: Steps required to decode target at radius r using 2^{14} vectors ($p = 0.9$).

Dual Attacks

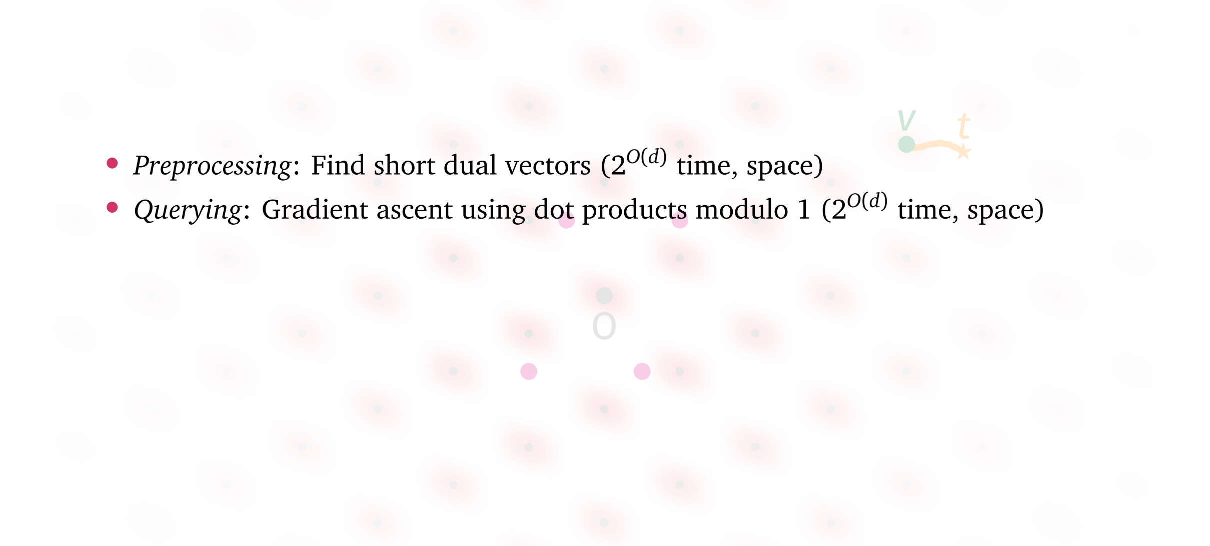
Overview

- *Preprocessing*: Find short dual vectors ($2^{O(d)}$ time, space)



Dual Attacks

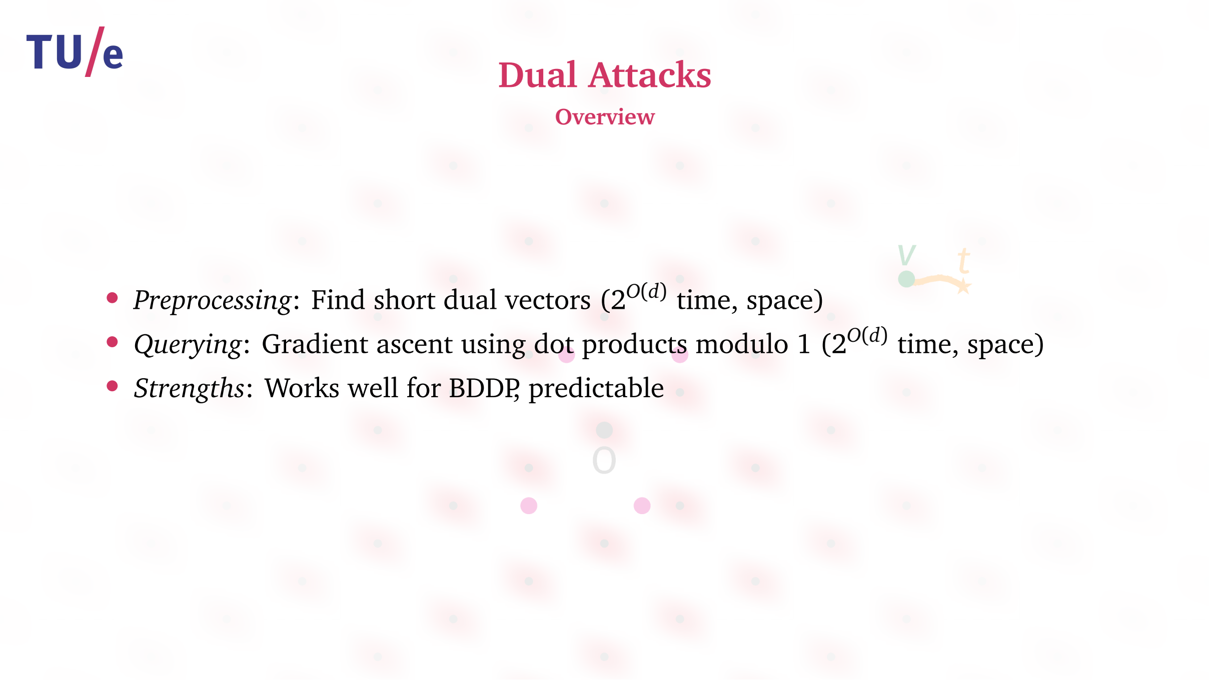
Overview



- *Preprocessing*: Find short dual vectors ($2^{O(d)}$ time, space)
- *Querying*: Gradient ascent using dot products modulo 1 ($2^{O(d)}$ time, space)

Dual Attacks

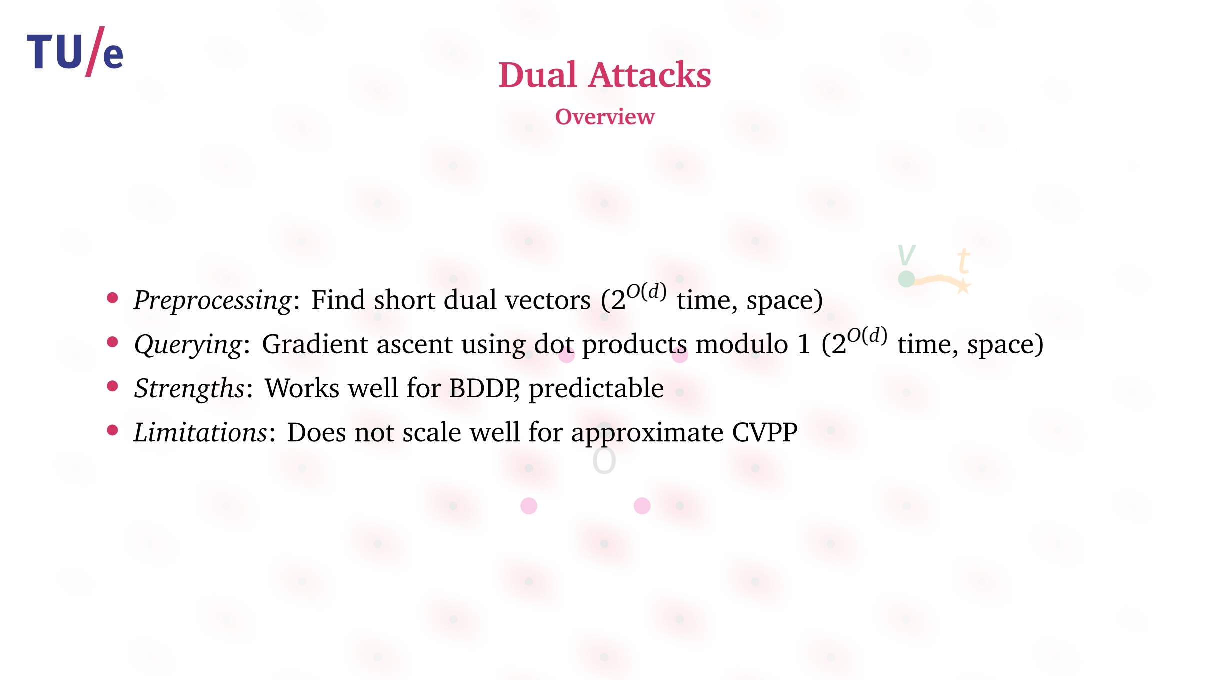
Overview



- *Preprocessing*: Find short dual vectors ($2^{O(d)}$ time, space)
- *Querying*: Gradient ascent using dot products modulo 1 ($2^{O(d)}$ time, space)
- *Strengths*: Works well for BDDP, predictable

Dual Attacks

Overview



- *Preprocessing*: Find short dual vectors ($2^{O(d)}$ time, space)
- *Querying*: Gradient ascent using dot products modulo 1 ($2^{O(d)}$ time, space)
- *Strengths*: Works well for BDDP, predictable
- *Limitations*: Does not scale well for approximate CVPP

Conclusion

Summary

Primal Attacks

- Using list of short primal lattice vectors
- Works well for approximate CVP(P), not for BDD(P)

Conclusion

Summary

Primal Attacks

- Using list of short primal lattice vectors
- Works well for approximate CVP(P), not for BDD(P)

Dual Attacks

- Using list of short dual lattice vectors
- Works well for BDD(P), not for approximate CVP(P)
- *Contribution*: Complete heuristic average-case analysis
- *Contribution*: Experiments, closely matching heuristic predictions

Conclusion

Open problems

Combining both approaches?

- Short primal vectors → Efficient approximate CVPP algorithm
- Short dual vectors → Efficient BDDP algorithm
- Short primal *and* dual vectors → ???

Conclusion

Open problems

Combining both approaches?

- Short primal vectors → Efficient approximate CVPP algorithm
- Short dual vectors → Efficient BDDP algorithm
- Short primal *and* dual vectors → ???

Applications?

- Dual attack: Faster algorithm for huge BDD batches
- Sieving-enumeration hybrid [DLdW20]: not so promising
- Other applications?