Provably Correct Training of Neural Network Controllers

Yasser Shoukry

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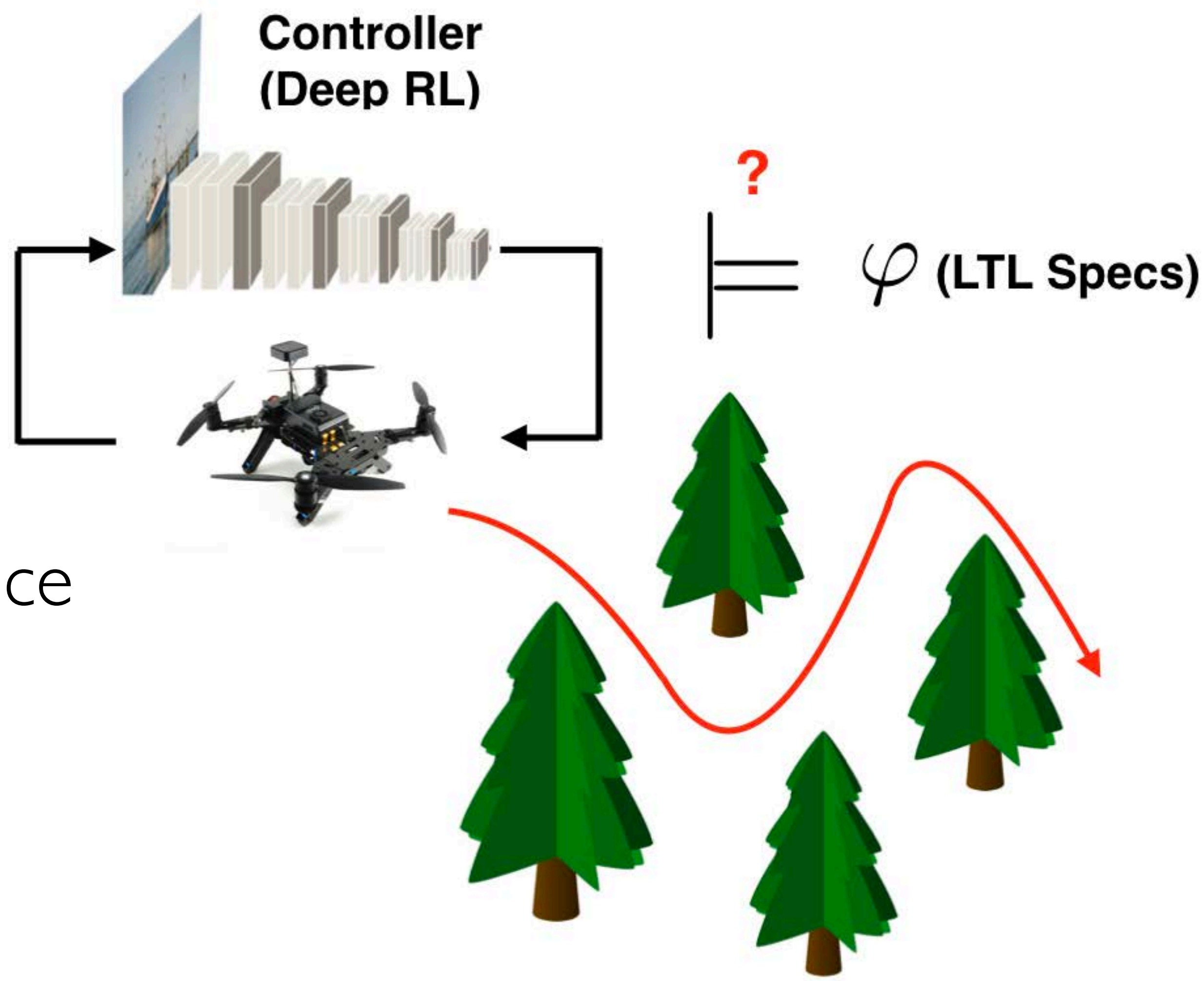
Resilient Cyber-Physical Systems Lab
Electrical Engineering and Computer Science
University of California, Irvine



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Video released of Uber self-driving crash that killed woman in Arizona

New footage of the crash that killed Elaine Herzberg raises fresh questions about why the self-driving car did not stop

▲ Uber dashcam footage shows lead up to fatal self-driving crash – video

The Guardian, Mar 22 2018

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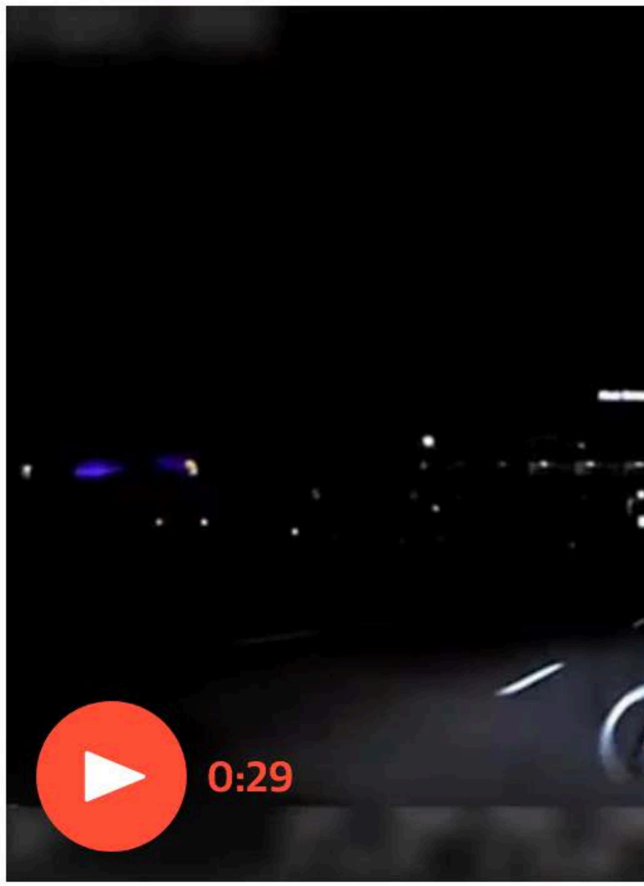

▲ Uber dashcam footage shows lead up to fatal

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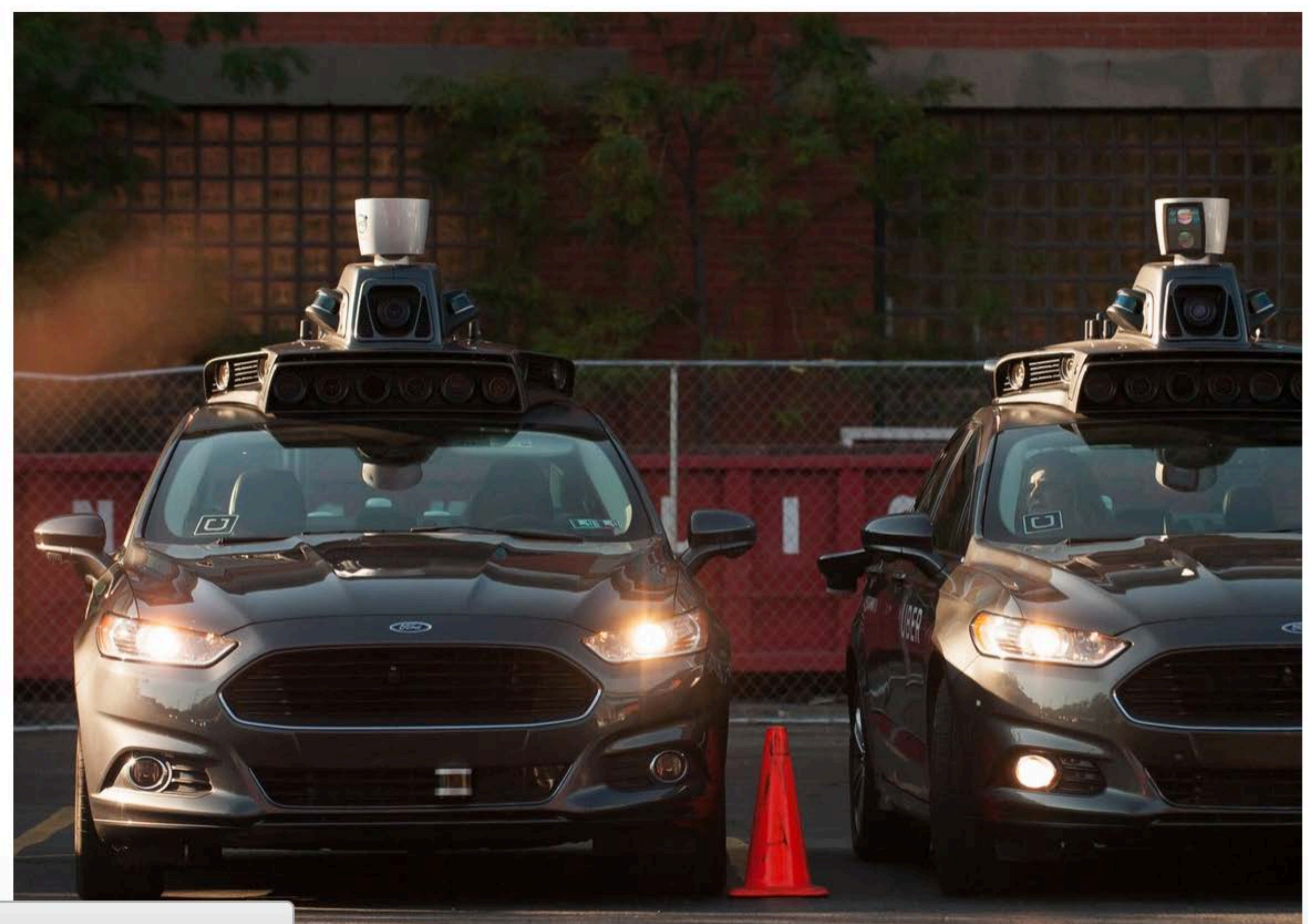

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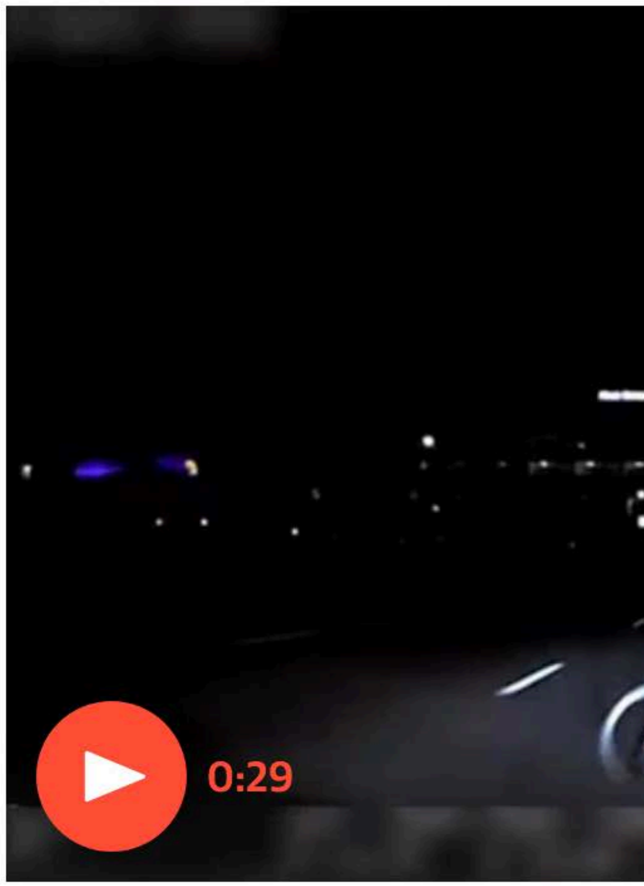

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The End of Starsky Robotics

Stefan Seltz-Axmacher Follow
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In 2015, I got obsessed with the idea of driverless trucks and started Starsky Robotics. In 2016, we became the first street-legal vehicle to be paid to do real work without a person behind the wheel. In 2018, we became the first street-legal truck to do a fully unmanned run, albeit on a closed road. In 2019, our truck became the first fully-unmanned truck to drive on a live highway.

And in 2020, we're shutting down.

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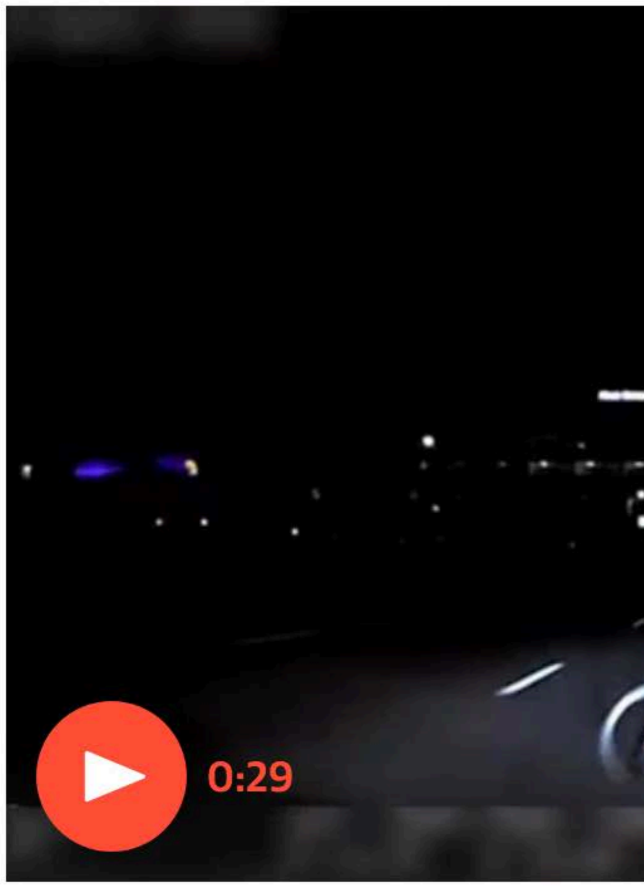

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Starsky Robotics
10–4 Labs
Starsky Robotics is a driverless truck startup which aims...

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It took me way too long to realize that VCs would rather a $1b business with a 90% margin than a $5b business with a 50% margin, even if capital requirements and growth were the same.

And growth would be the same. The biggest limiter of autonomous deployments isn't sales, it's safety.

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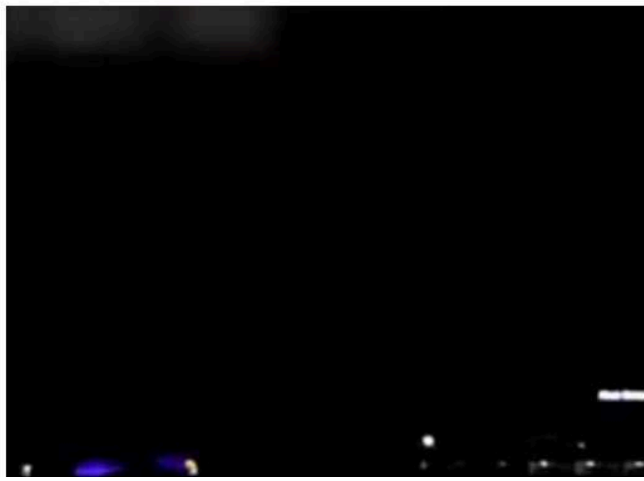

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The AV Space

There are too many problems with the AV industry to detail here: the professorial pace at which most teams work, the lack of tangible deployment milestones, the open secret that there isn't a robotaxi business model, etc. The biggest, however, is that supervised machine learning doesn't live up to the hype. It isn't actual artificial intelligence akin to C-3PO, it's a sophisticated pattern-matching tool.

'RE HIRING

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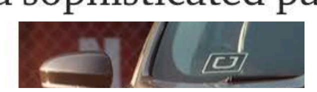

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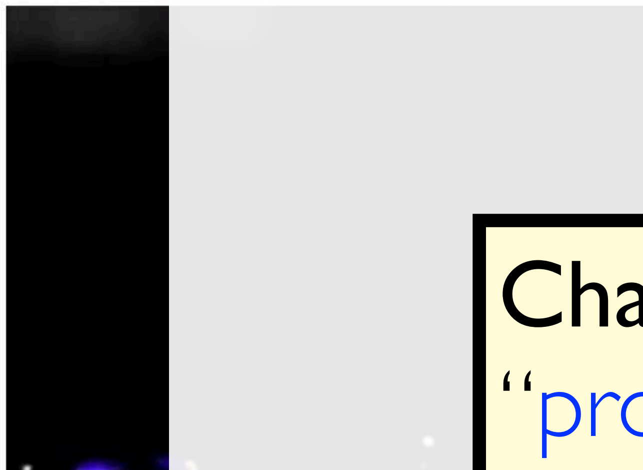

HOME FROM THE HONEYMOON,

Challenge: Can we systematically design "provably correct" deep neural networks?

- Theory
- Algorithms
- Implementation

Starsky Rob
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Starsky Robot
driverless tru
startup which
aims...

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3PO, it's a sophisticated pattern-matching tool.

Starsky Robotics
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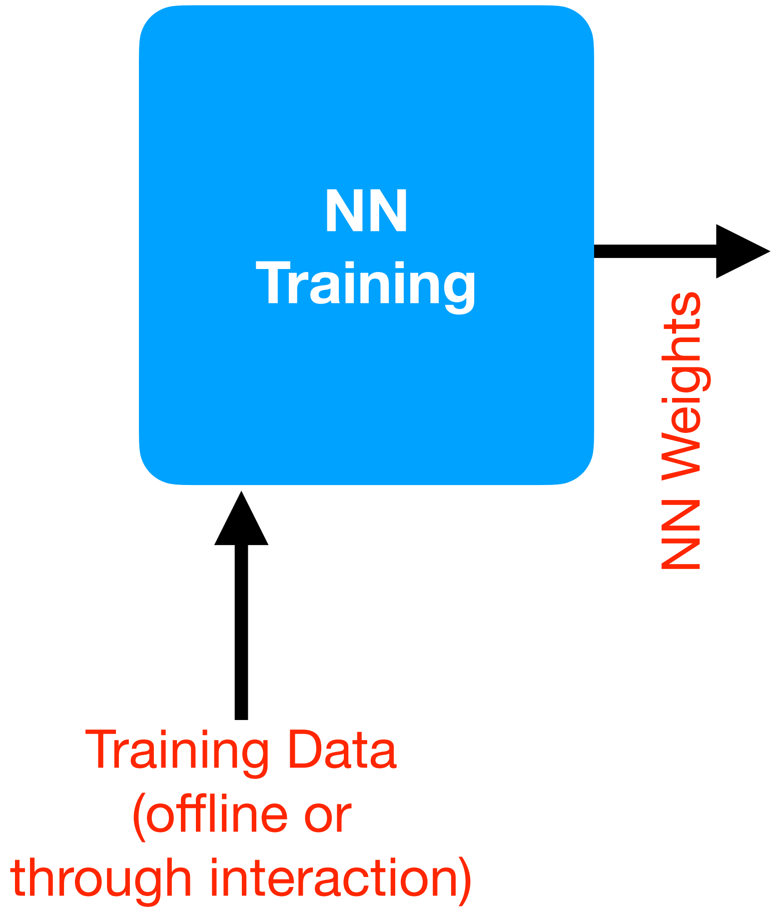
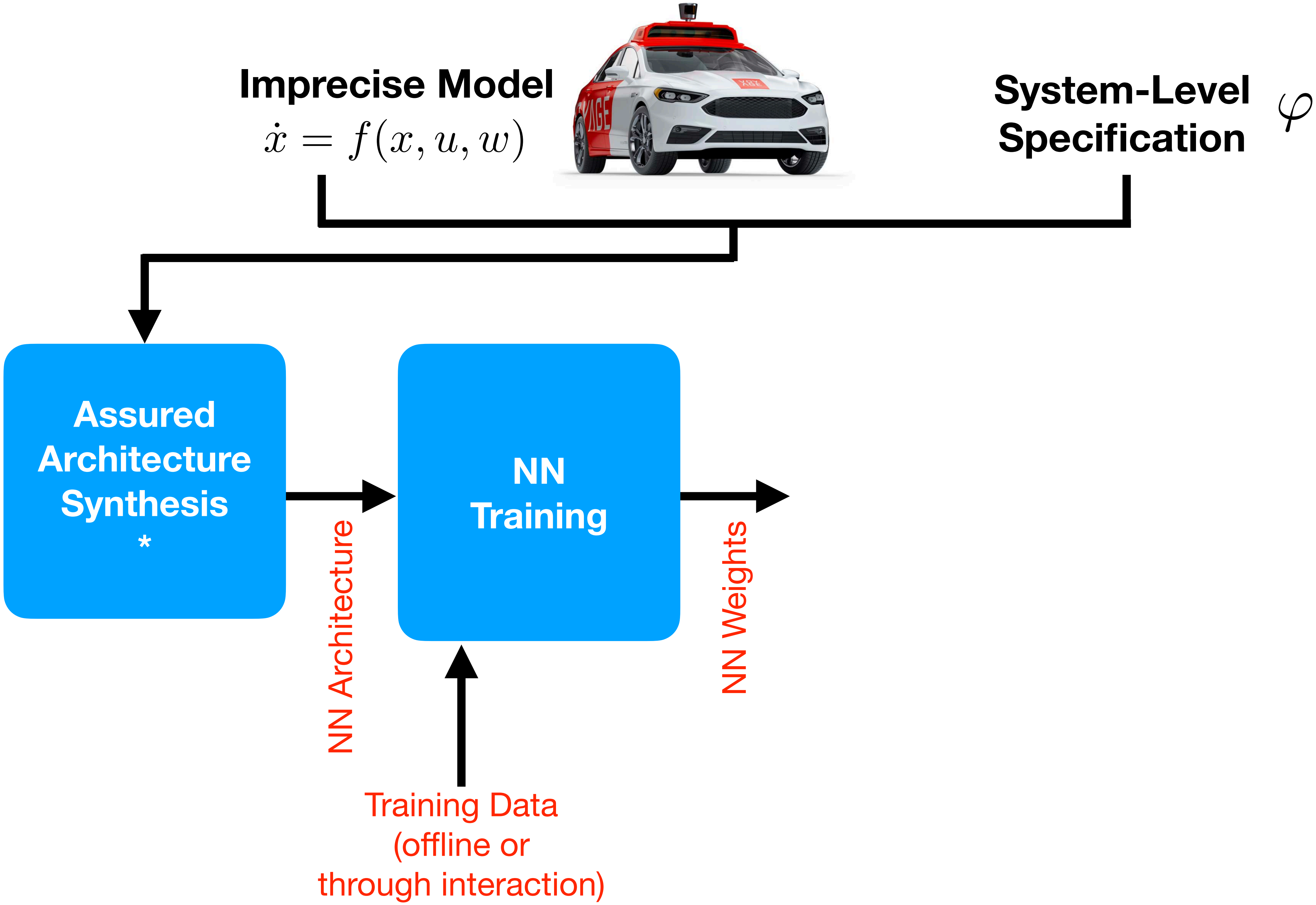

NN Training
NN Weights
Training Data (offline or through interaction)



* J. Ferlez, X. Sun, and Y. Shoukry, "Two-Level Lattice Neural Network Architectures for Control of Nonlinear Systems," CDC 2020.

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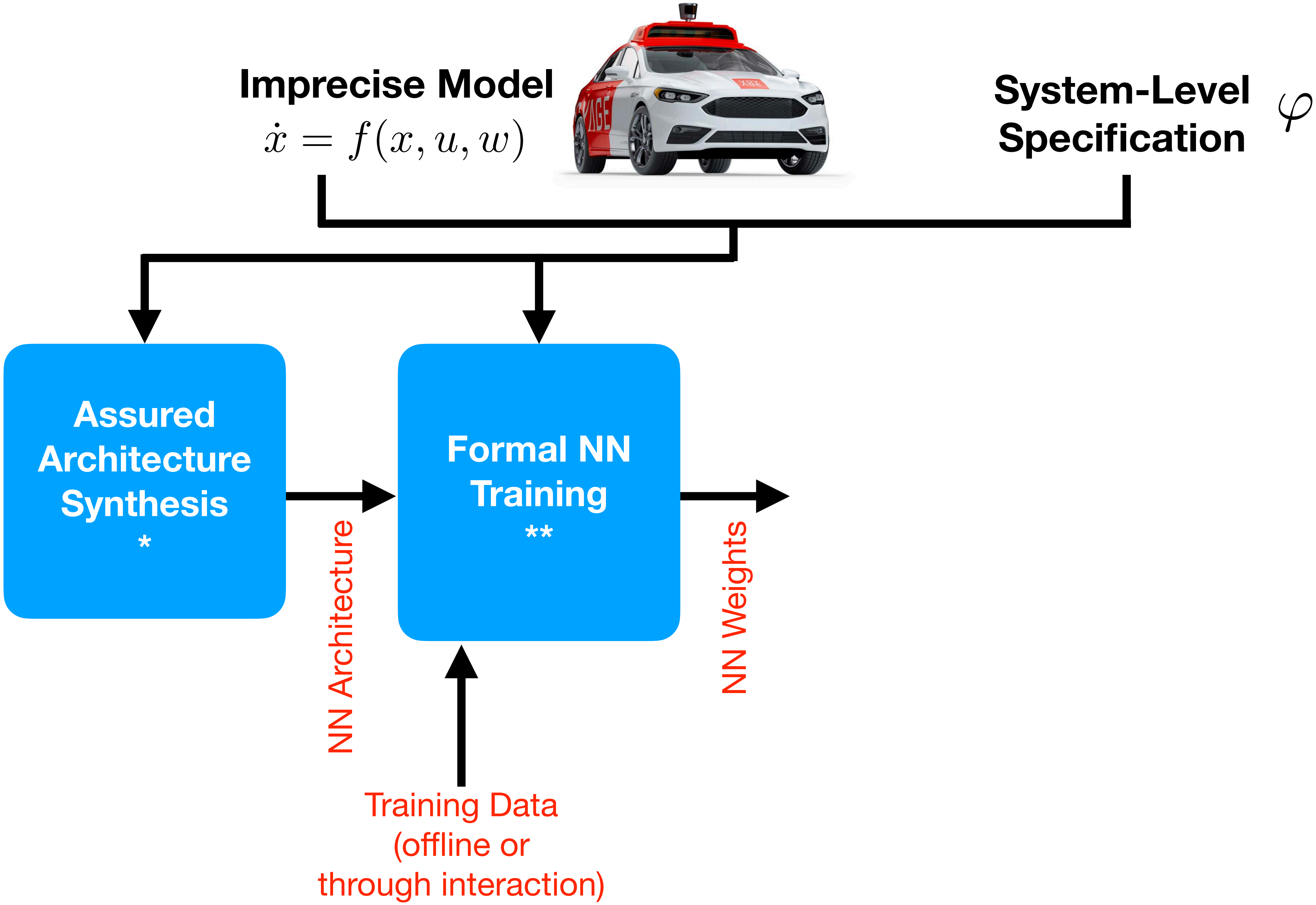

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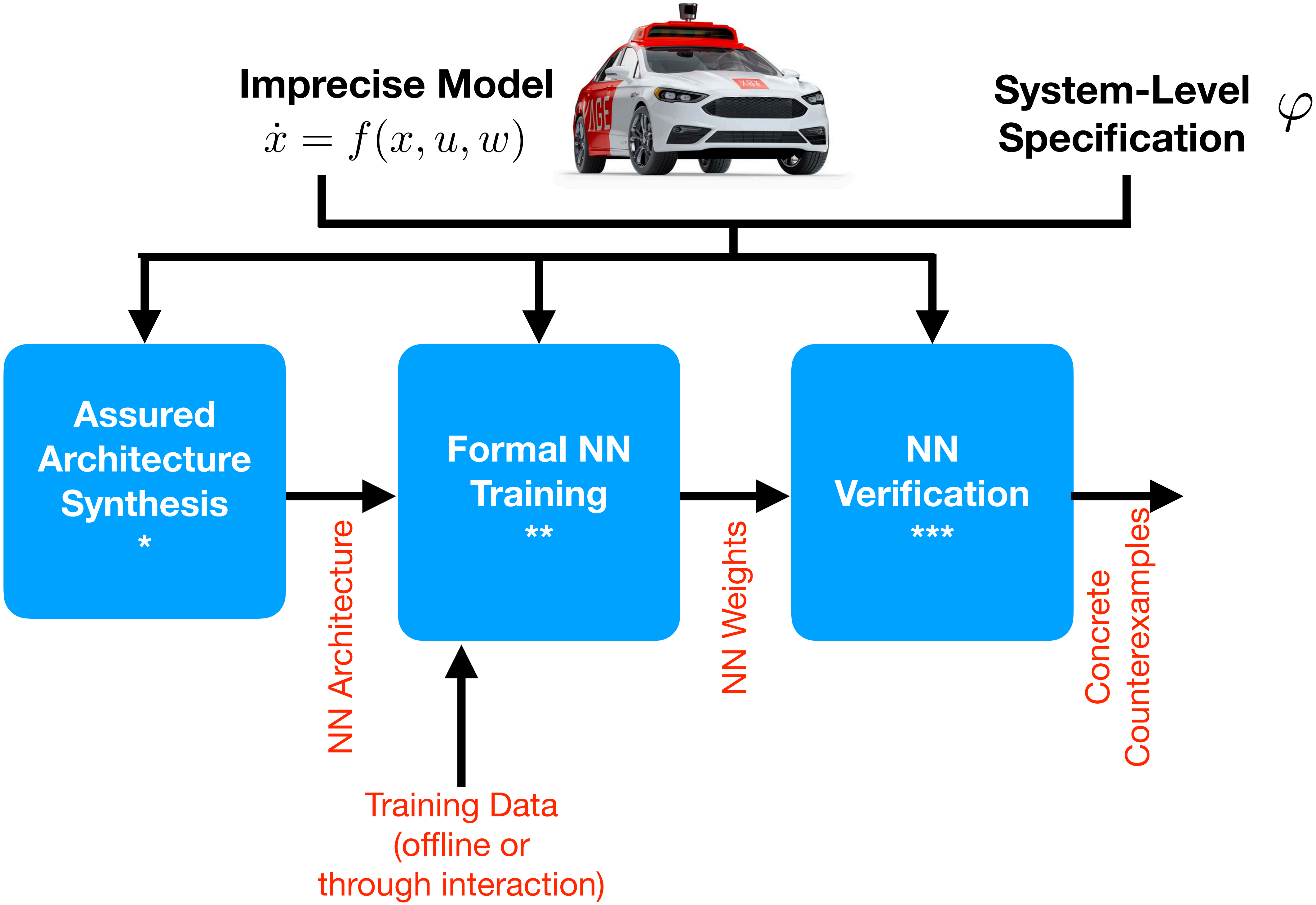

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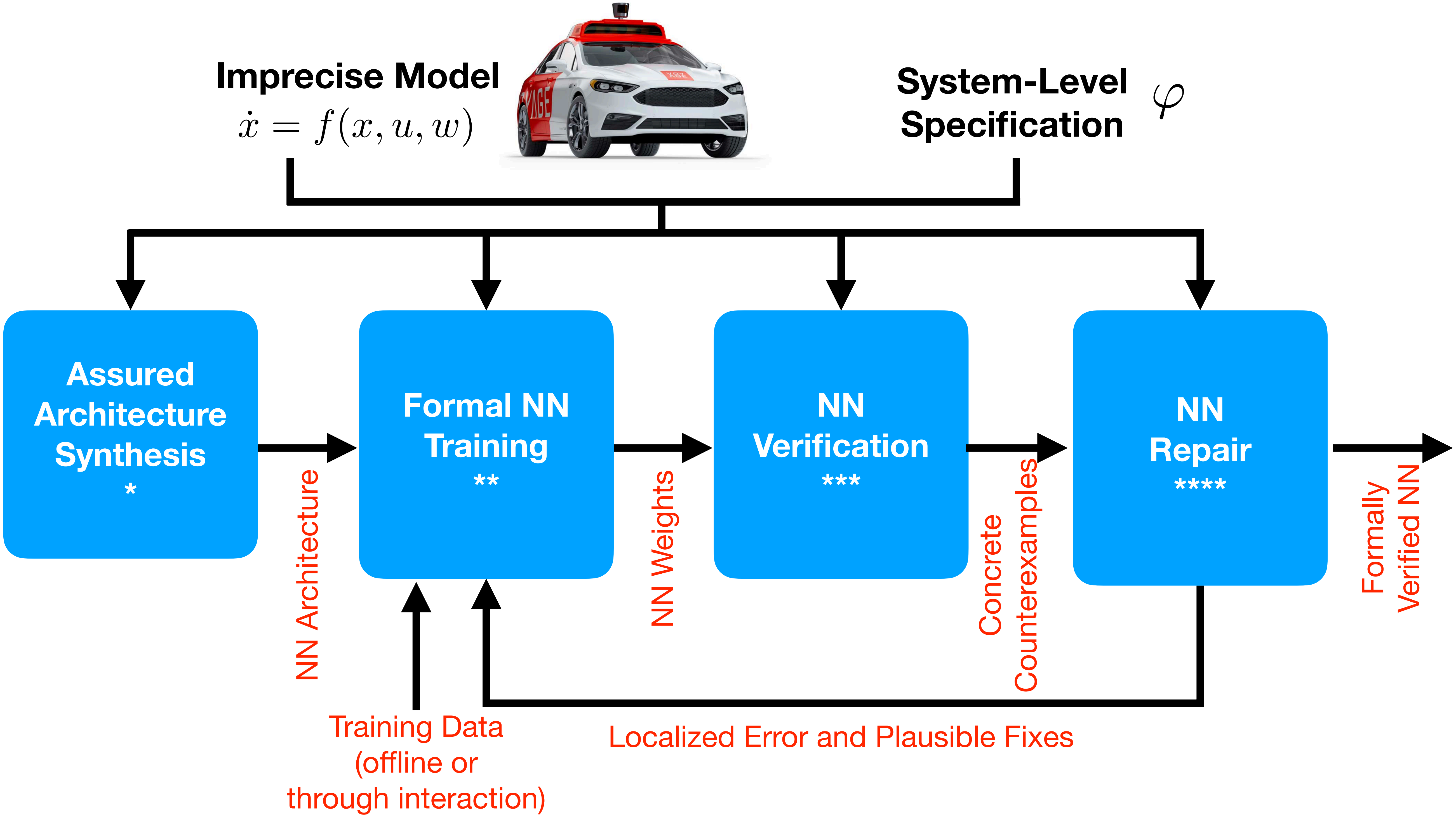

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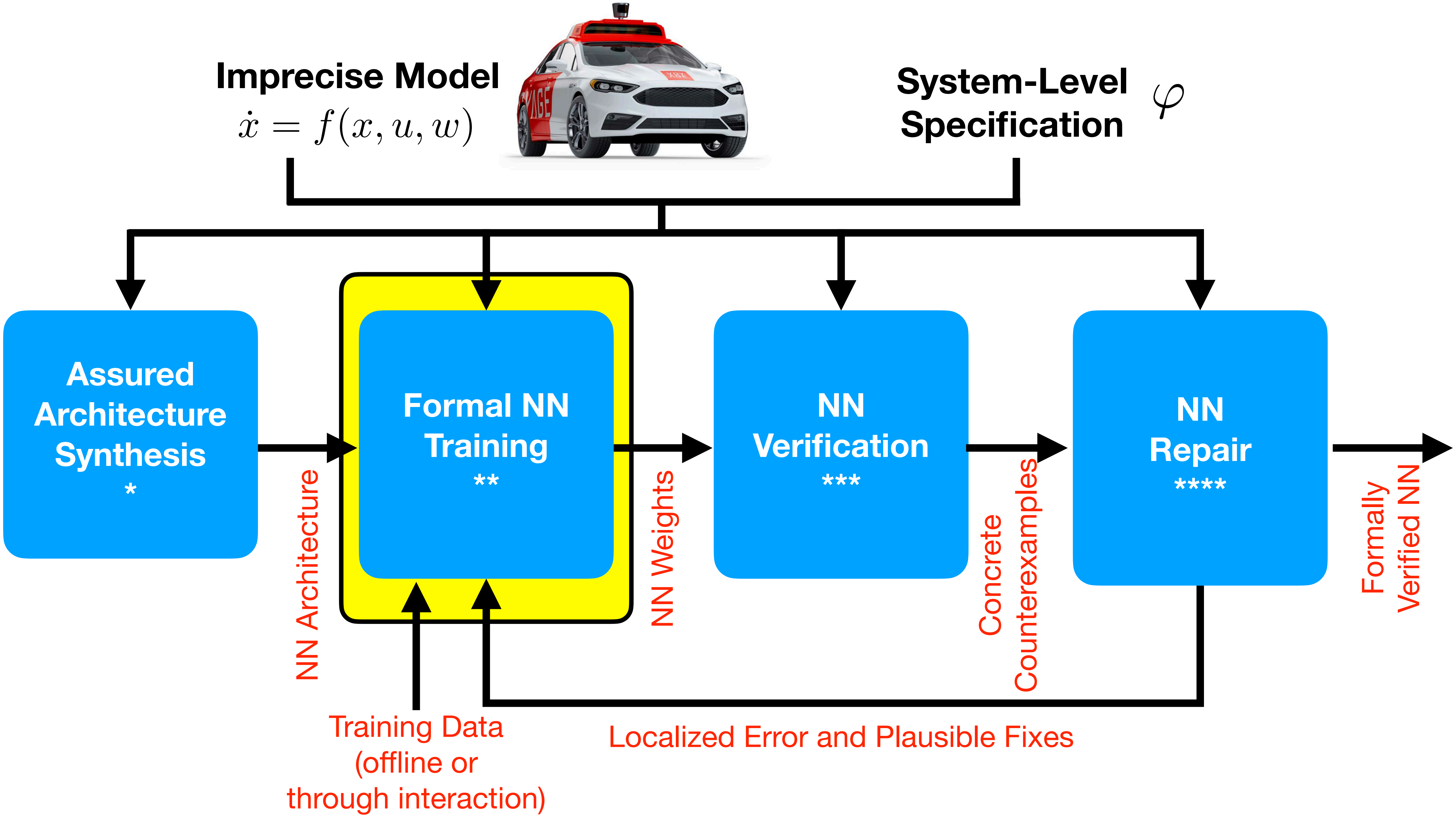

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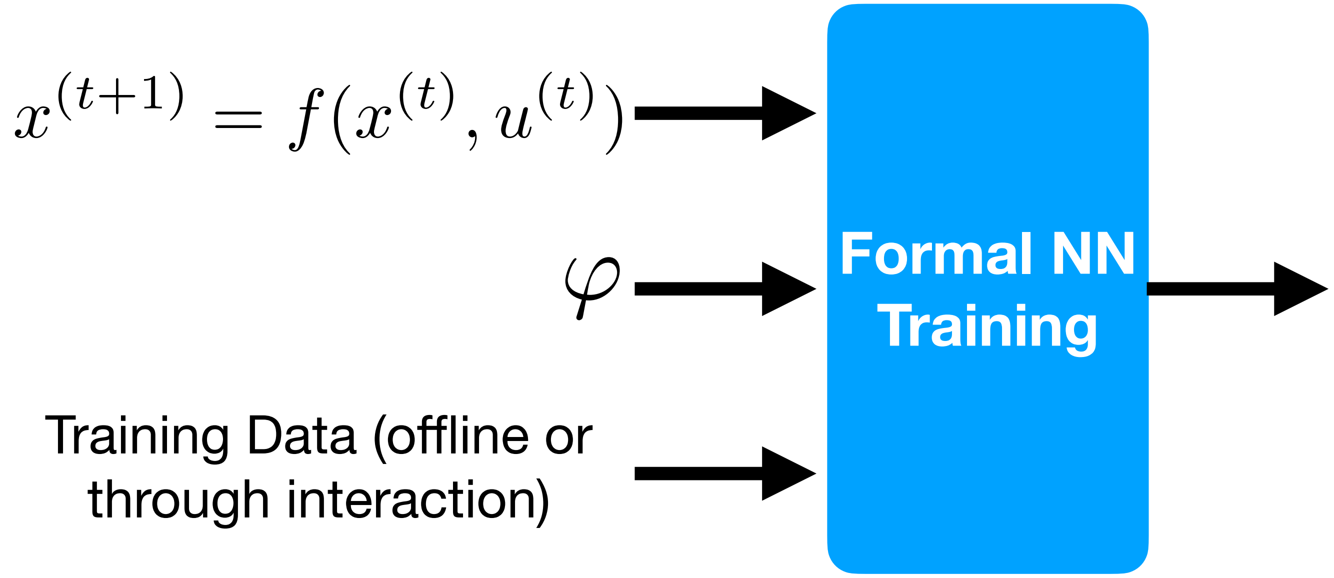
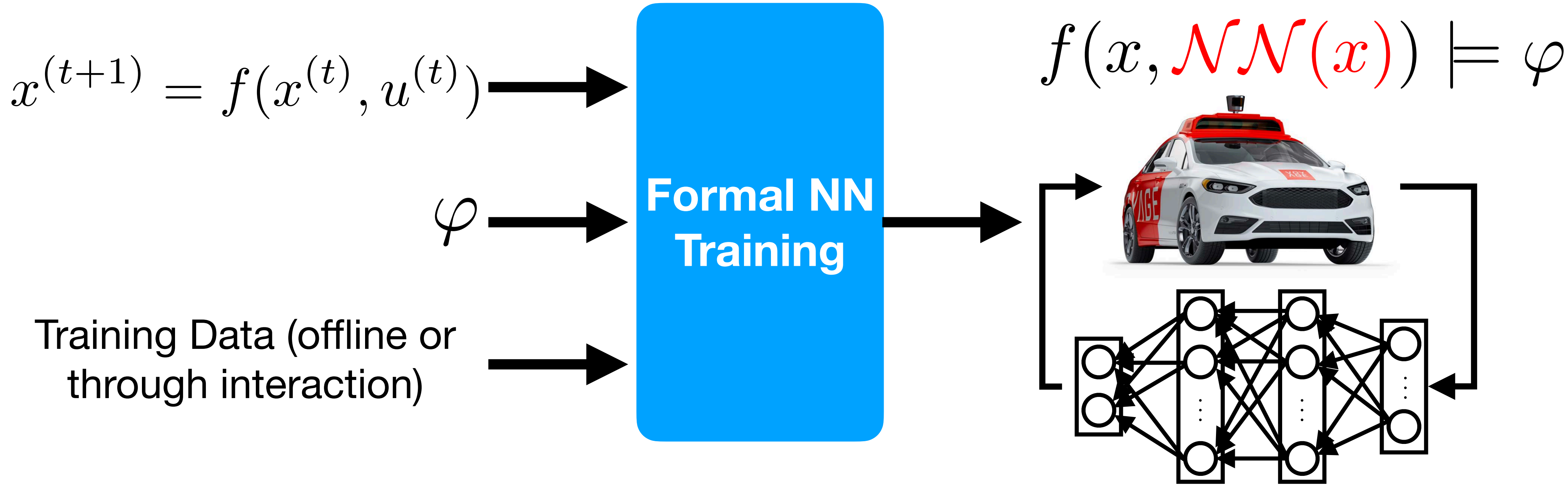

$x^{(t+1)} = f(x^{(t)}, u^{(t)})$
φ
Training Data (offline or through interaction)
Formal NN Training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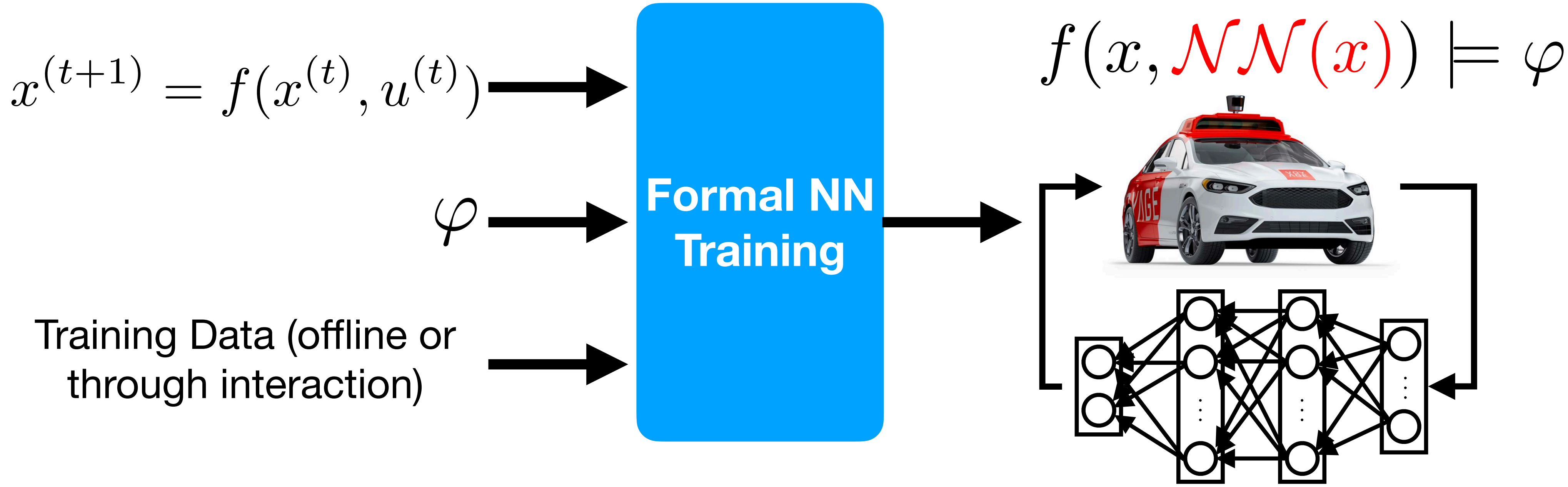
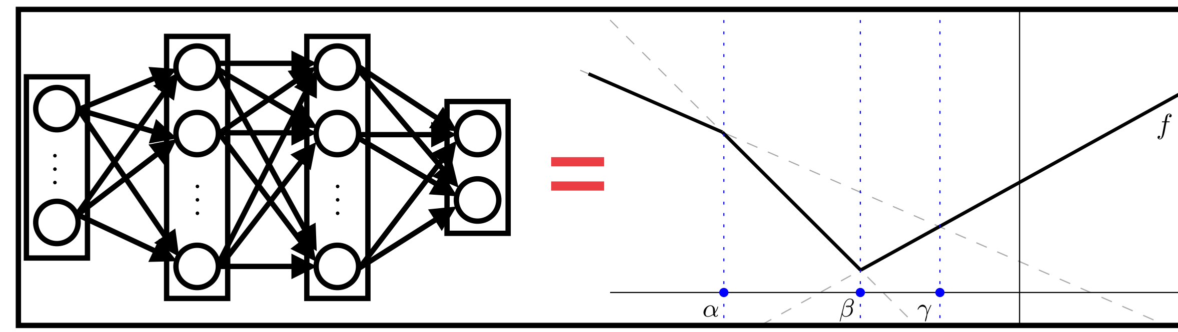
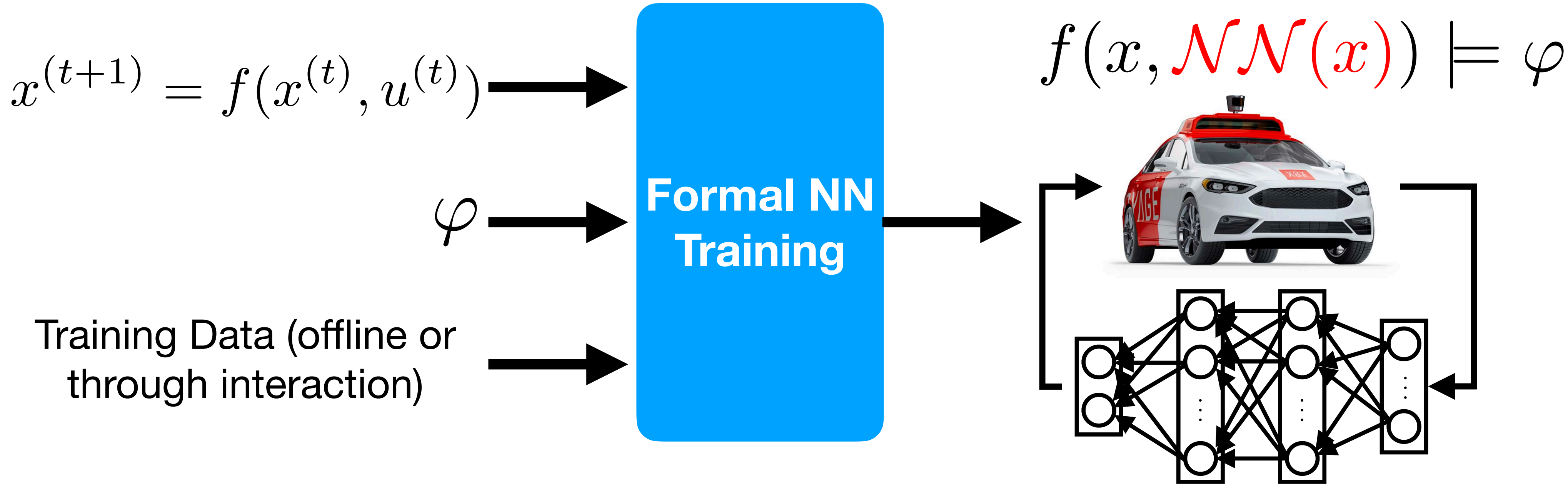
$f(x, \mathcal{NN}(x)) \models \varphi$



Core idea:

- Regression ReLU NN are Continuous Piece-Wise Affine (CPWA) functions





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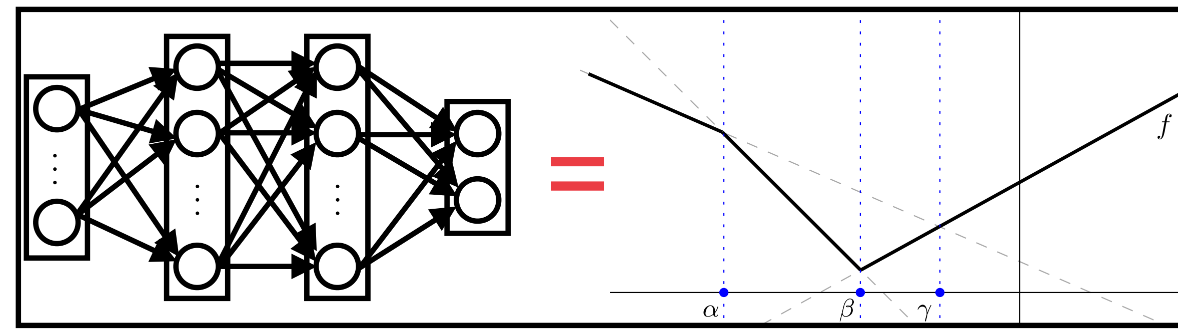

- Use reachability analysis to identify families of CPWA functions that satisfy the specs

$$f(x, K_{\mathrm{CPWA}}(x)) \models \varphi \quad \forall\, K_{\mathrm{CPWA}} \in \mathrm{CPWA}_{\varphi}$$

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$x^{(t+1)} = f(x^{(t)}, u^{(t)})$ →

φ →

Training Data (offline or through interaction) →

Formal NN Training →

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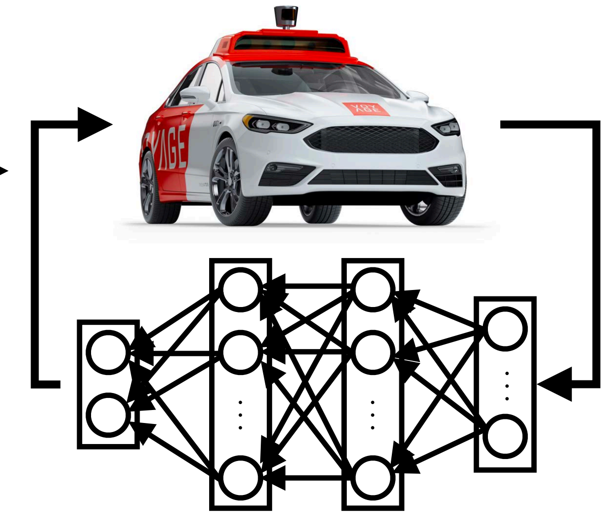

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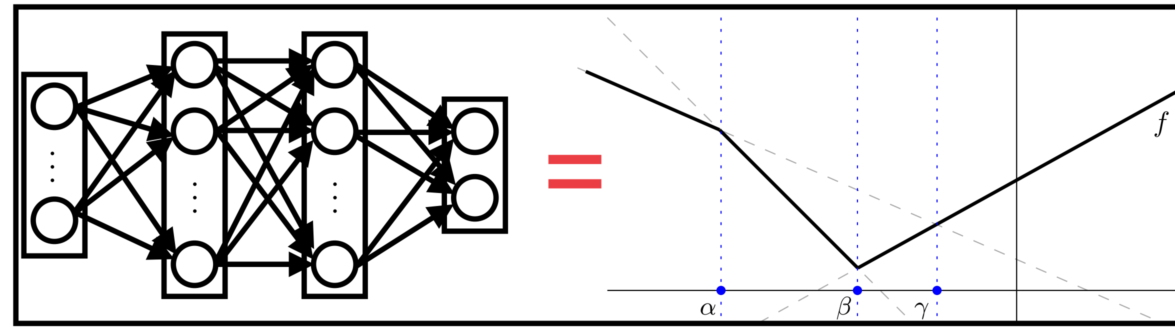

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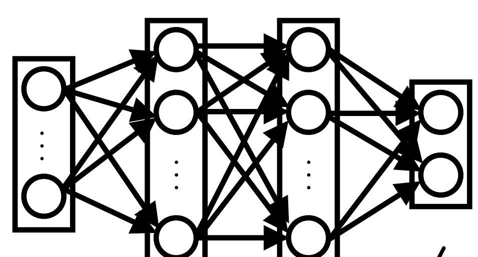
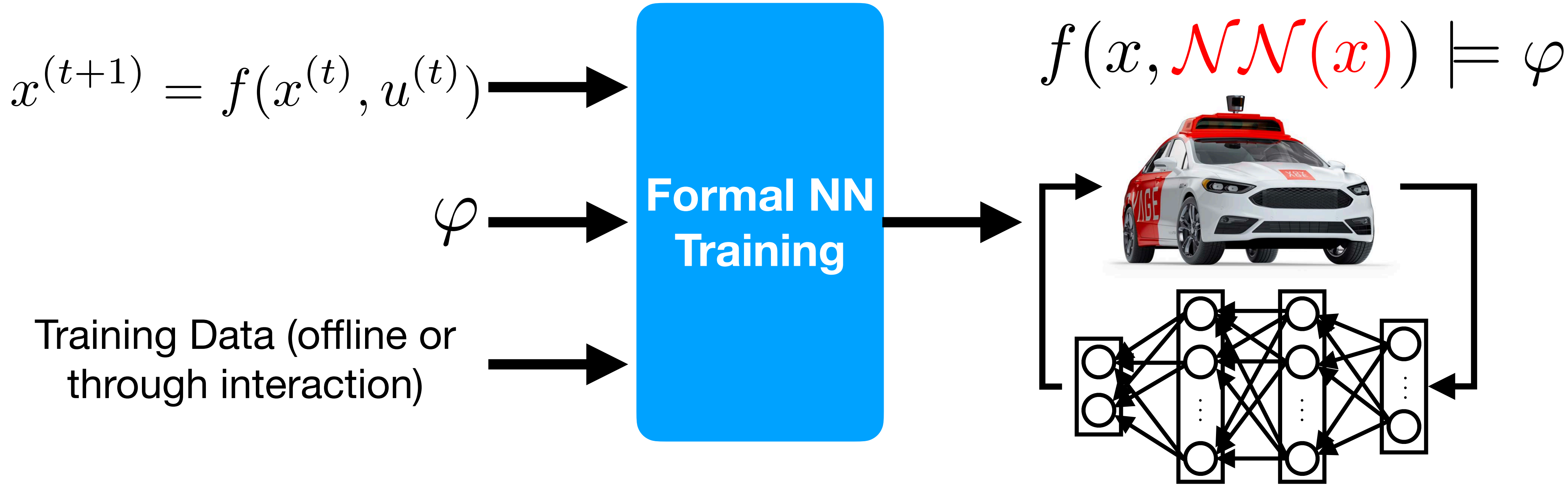
$$f(x, K_{\text{CPWA}}(x)) \models \varphi \quad \forall\, K_{\text{CPWA}} \in \text{CPWA}_\varphi$$

Training Data (offline or through interaction) $\xrightarrow{\text{Training}}$



$\notin \text{CPWA}_\varphi$



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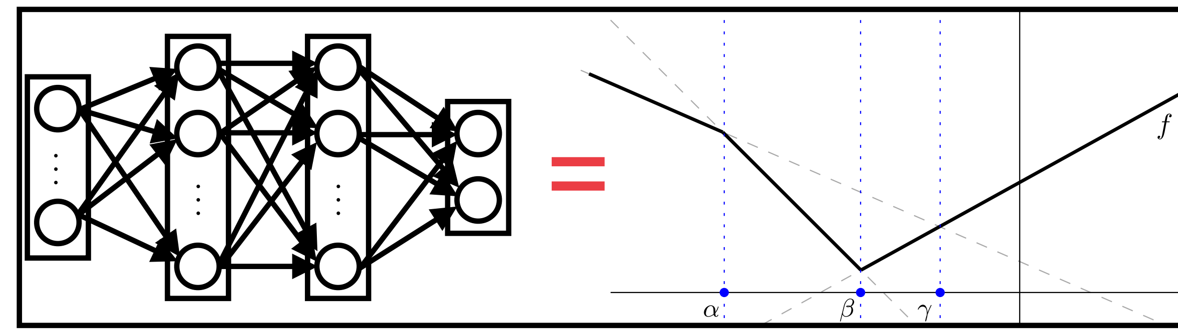

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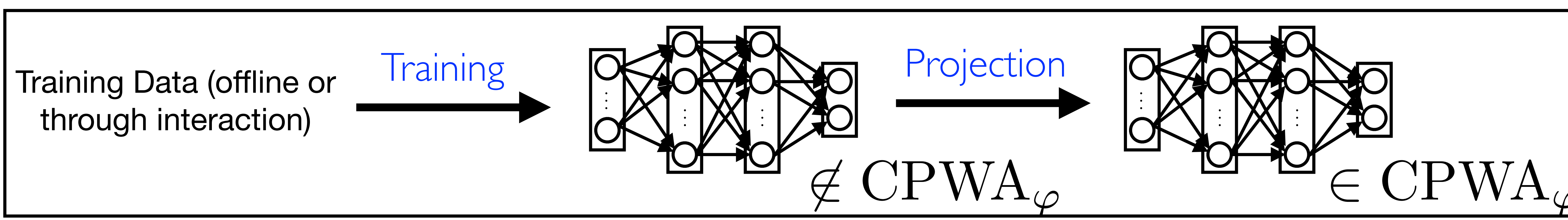

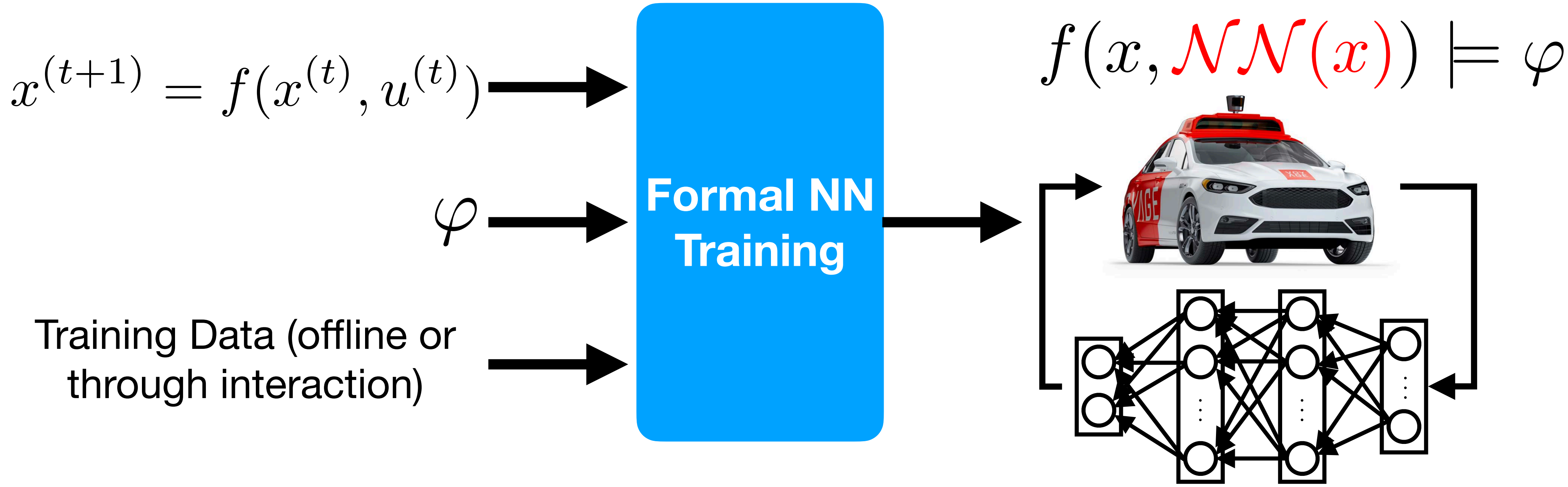

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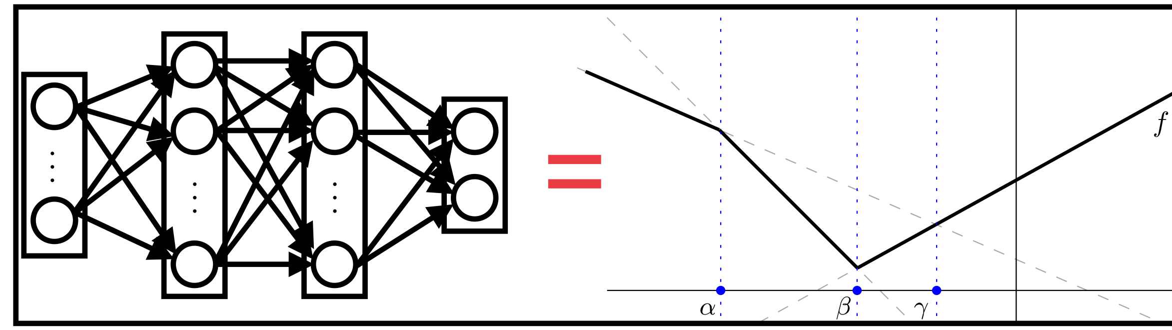
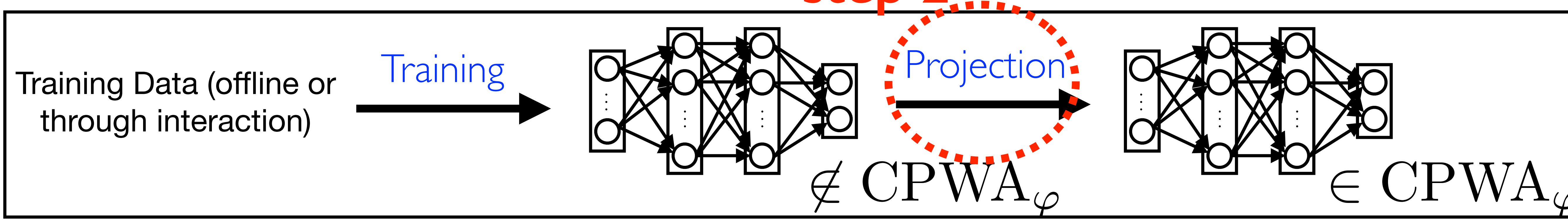


step 1

- Use reachability analysis to identify families of CPWA functions that satisfy the specs

$$f(x, K_{\mathrm{CPWA}}(x)) \models \varphi$$
$$\forall\, K_{\mathrm{CPWA}} \in \mathrm{CPWA}_{\varphi}$$

step 2



step 1

- Use reachability analysis to identify families of CPWA functions that satisfy the specs

step 1

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$$x^{(t+1)} = f(x^{(t)}, u^{(t)})$$

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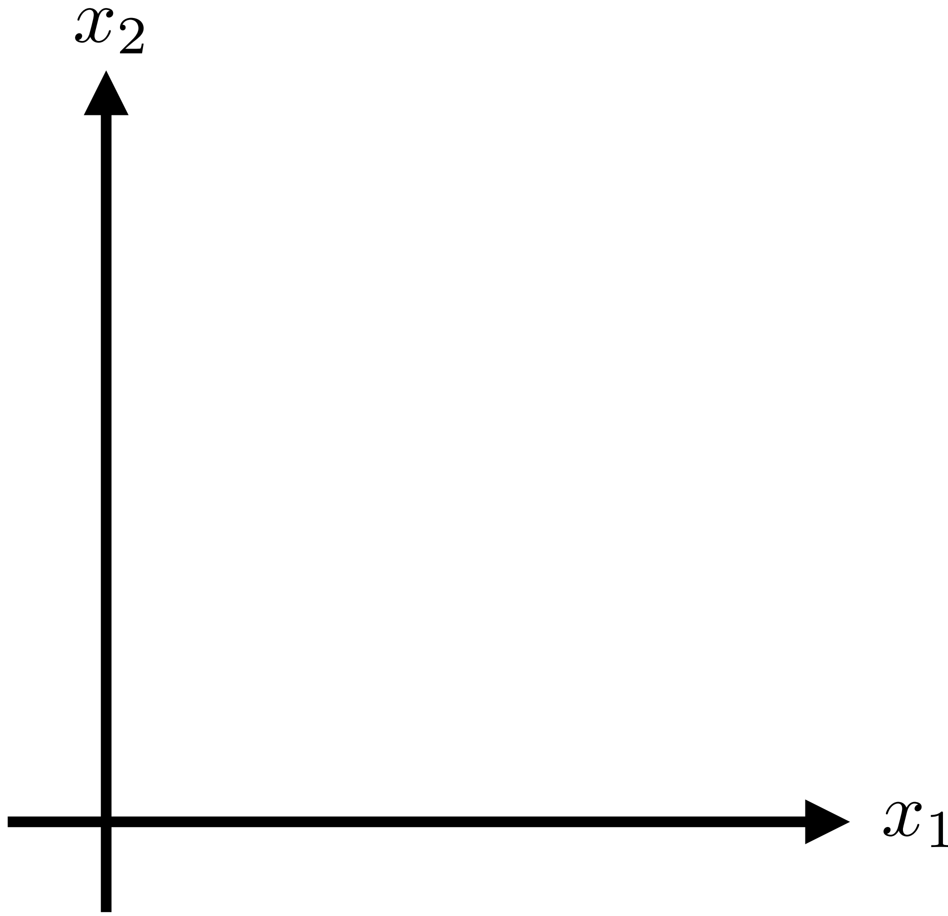

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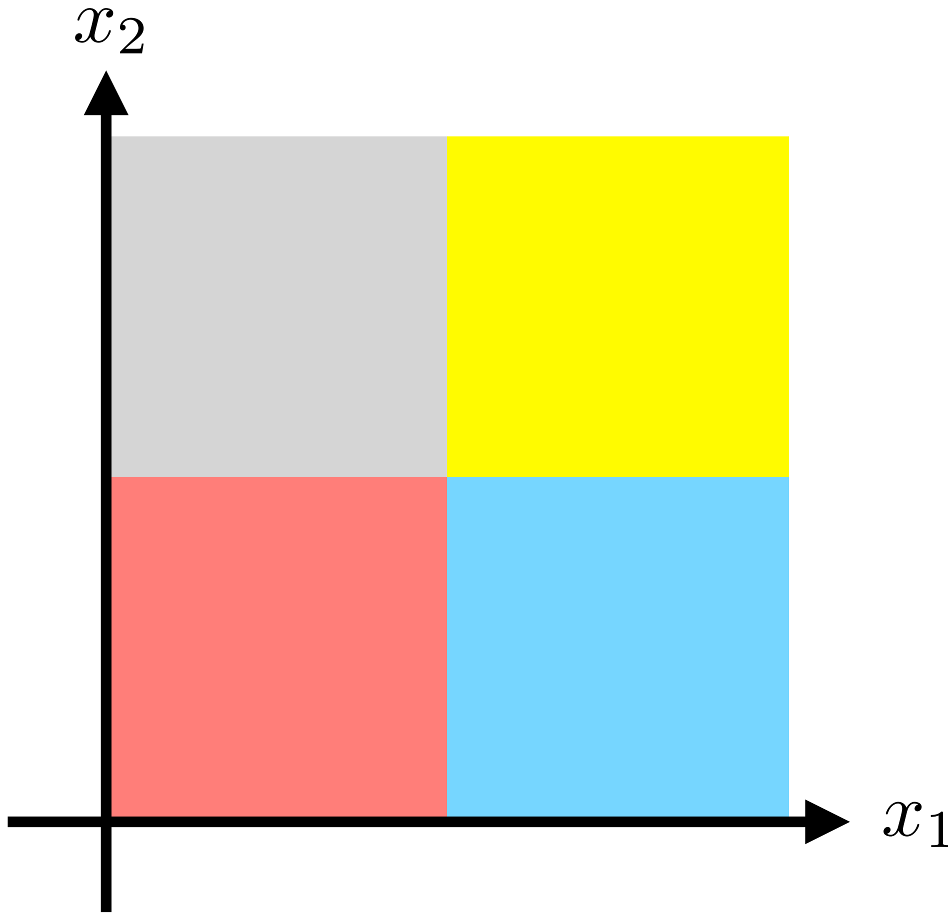

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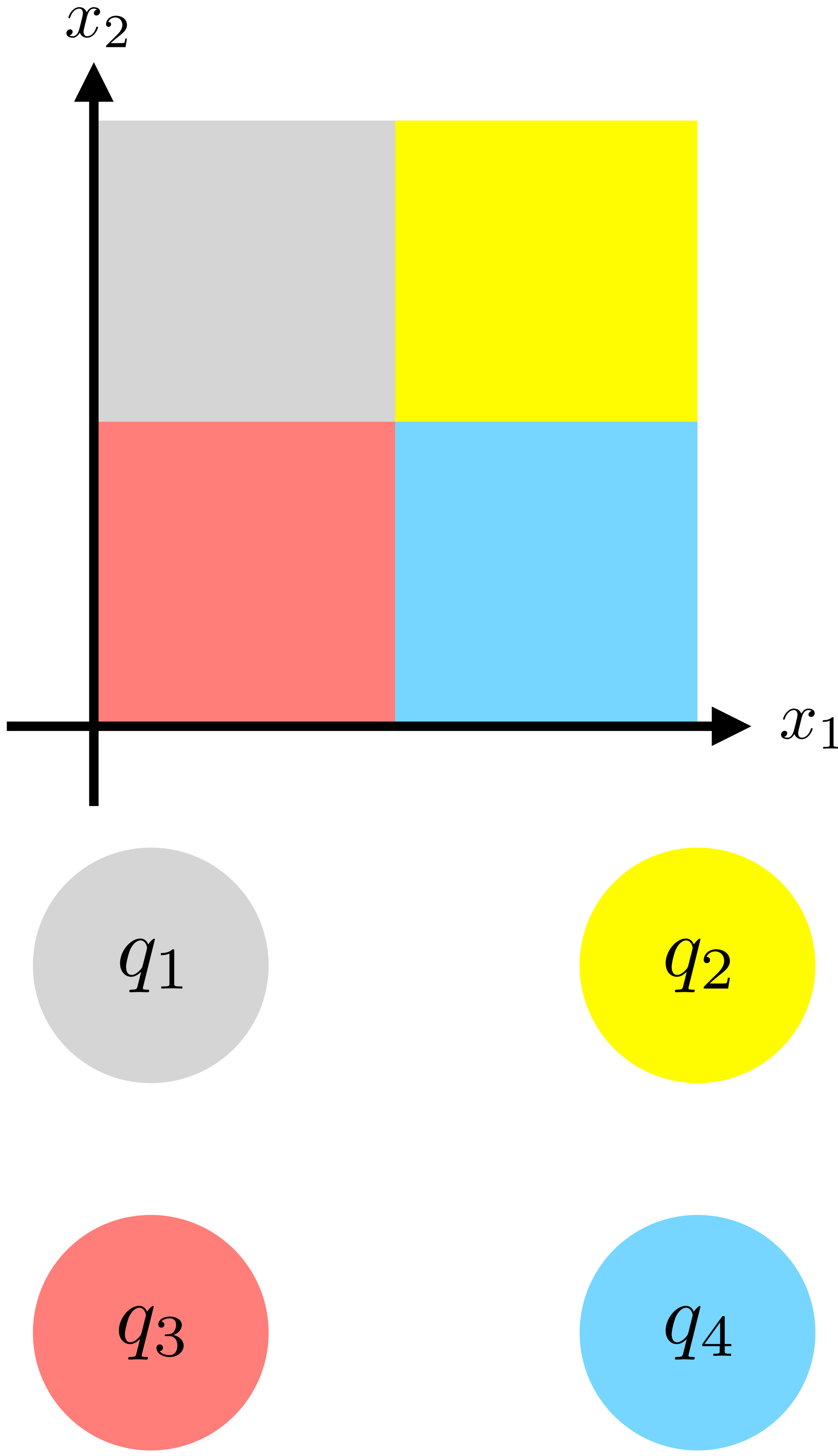

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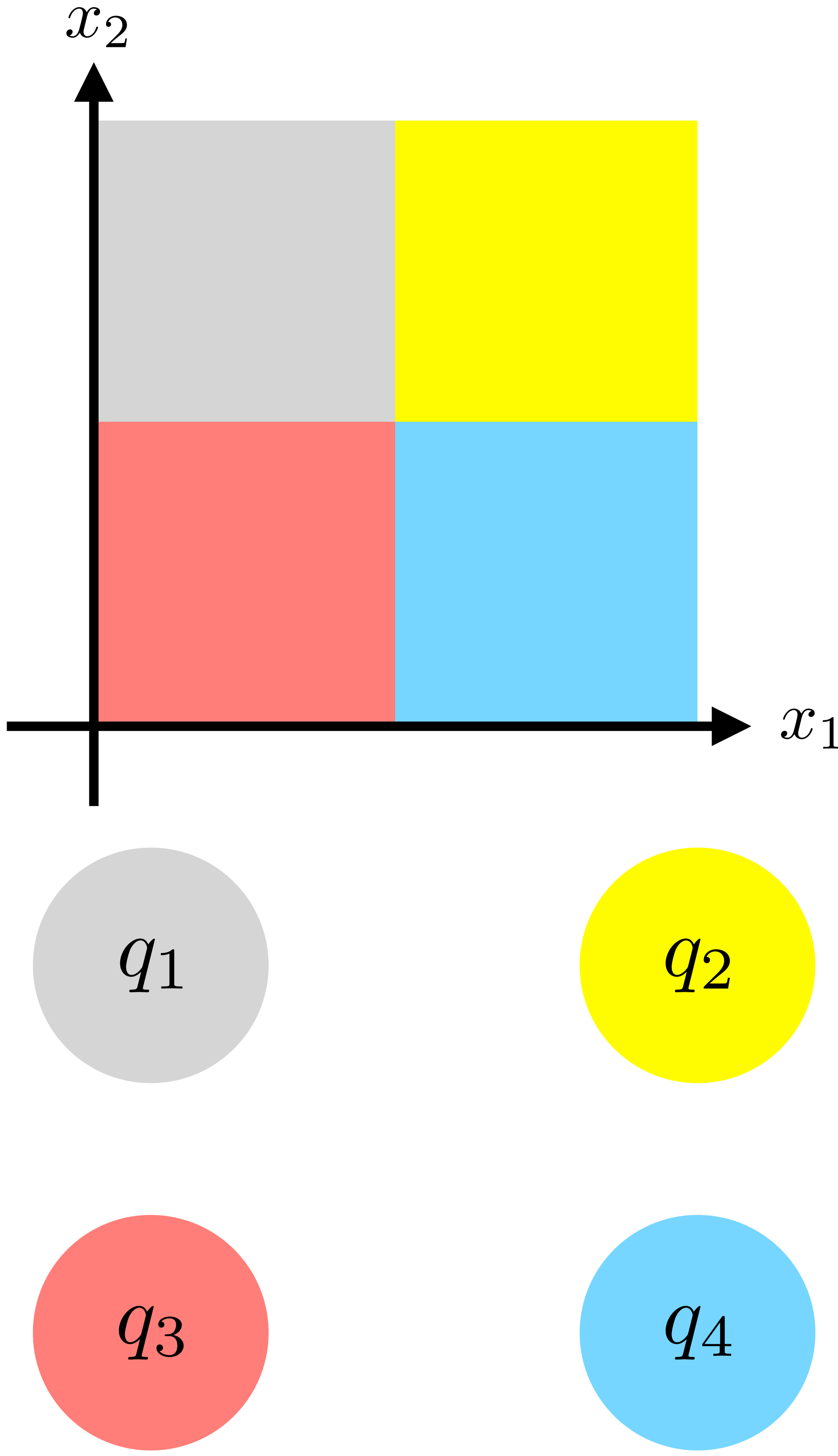
step 1

- Use reachability analysis to identify families of CPWA functions that satisfy the specs

$$x^{(t+1)} = f(x^{(t)}, u^{(t)})$$

Abstract states:	$\mathbb{X} = \{q_1, q_2, \ldots, q_n\}$



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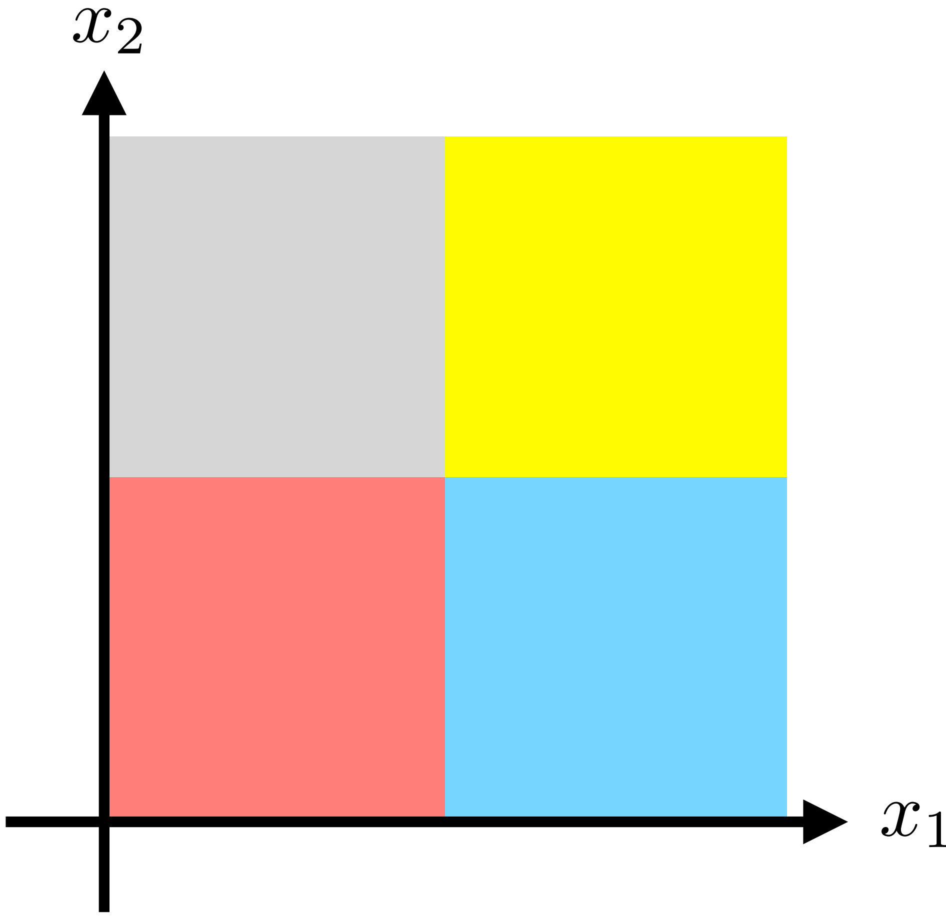
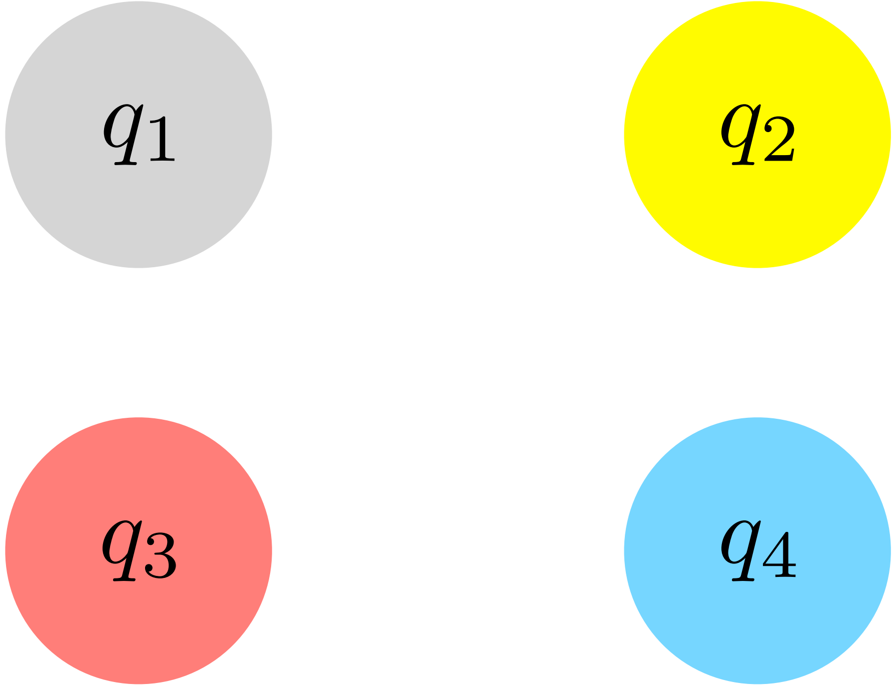
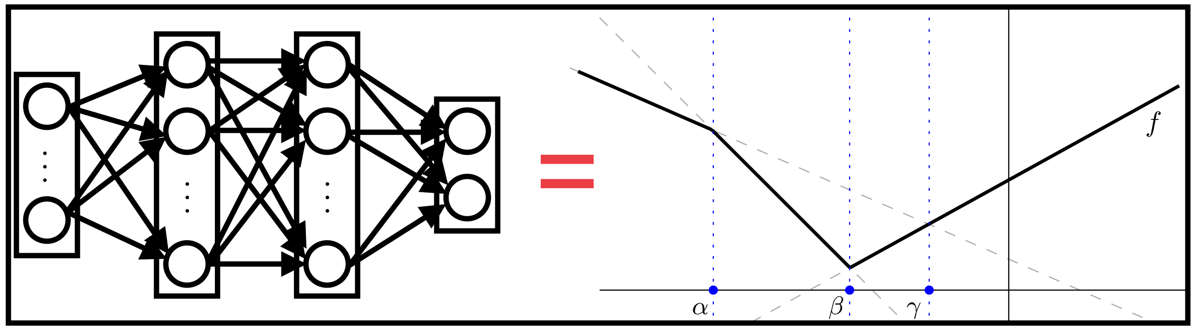




Recall:



NN = Continuous Piece-Wise Affine (CPWA) functions

CPS Lab Resilient

step 1

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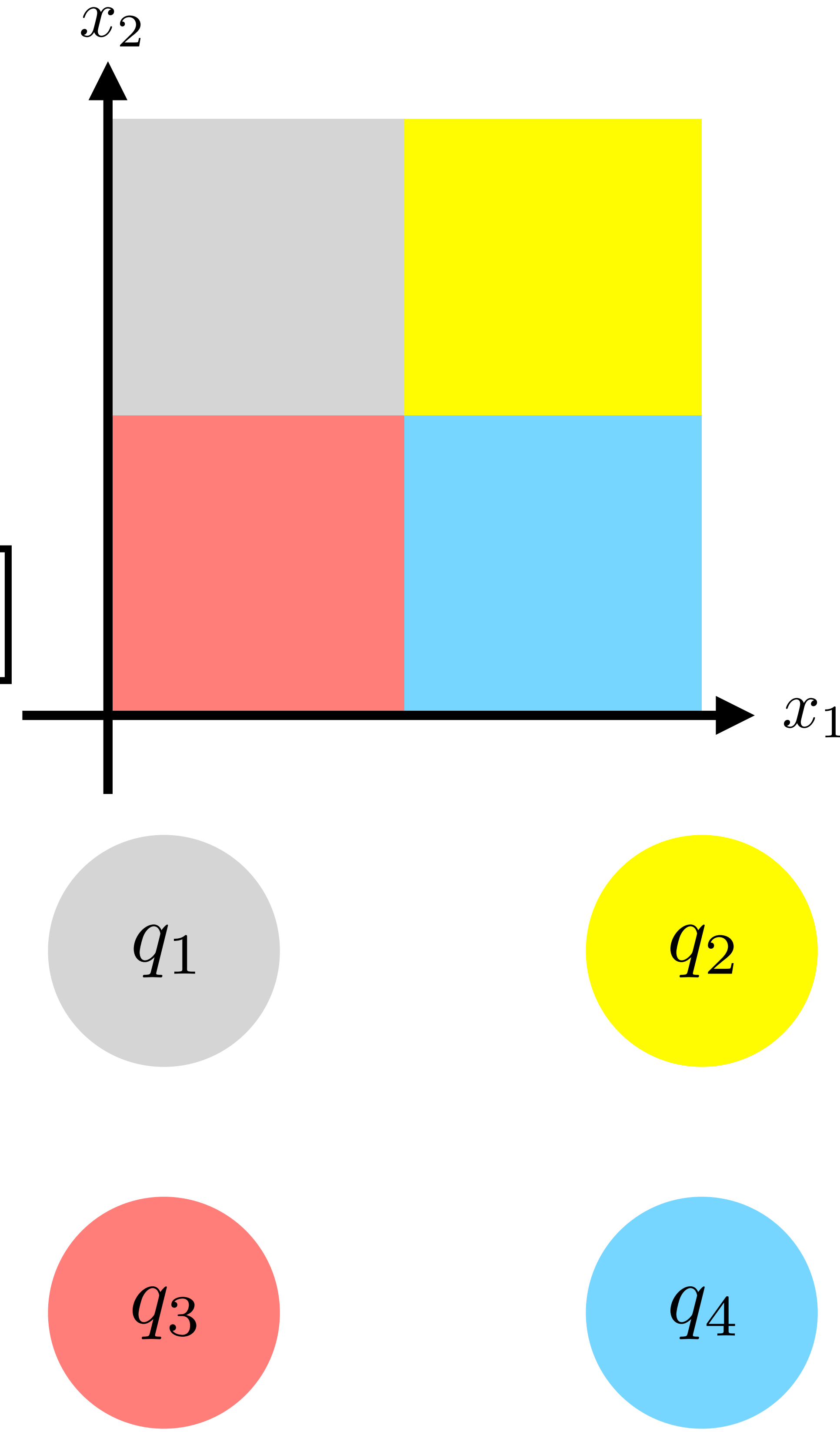
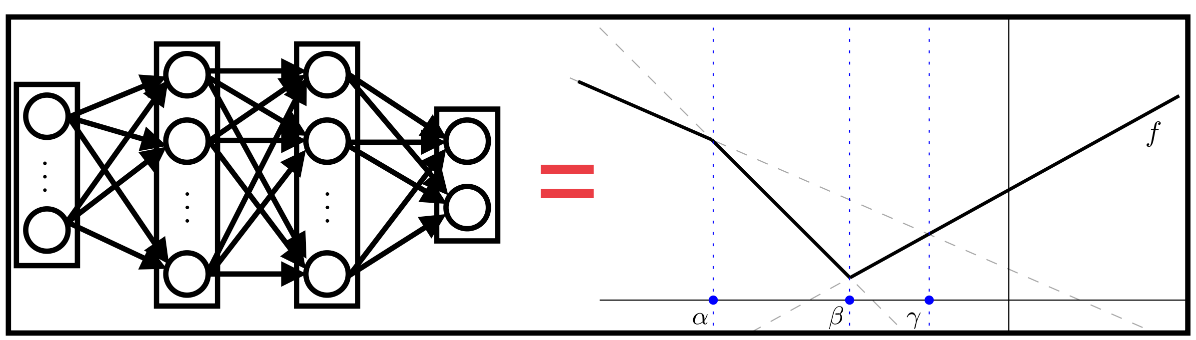
$$x^{(t+1)} = f(x^{(t)}, u^{(t)})$$

Abstract states: $\mathbb{X} = \{q_1, q_2, \ldots, q_n\}$

$$u^{(t)} = K_i x^{(t)} + b_i$$



Recall:



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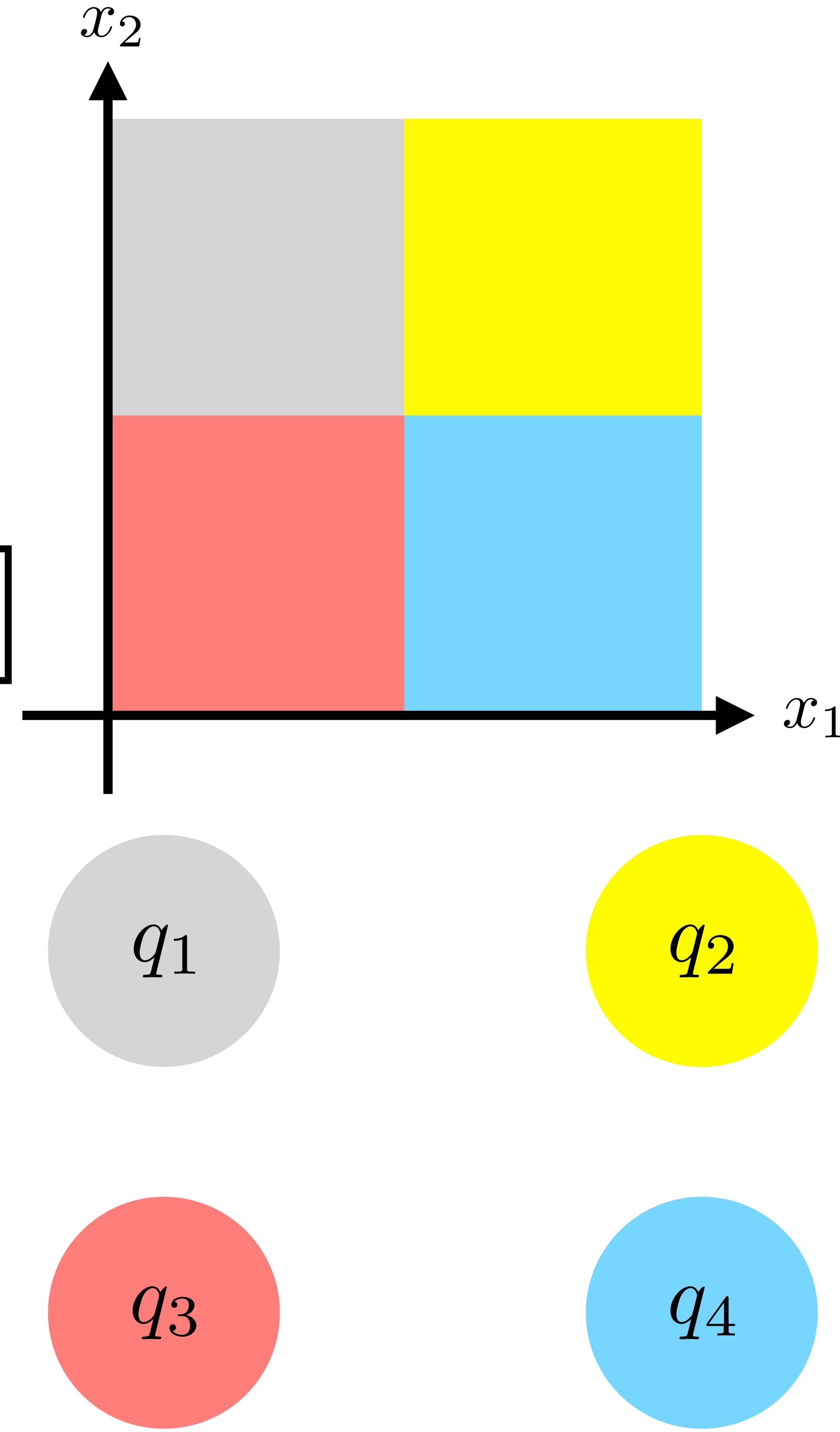
$$\mathcal{P} = \{(K, b) \mid K \in \underbrace{\mathcal{K}}_{\text{polytopic}}, b \in \underbrace{\mathcal{B}}_{\text{polytopic}}\}$$



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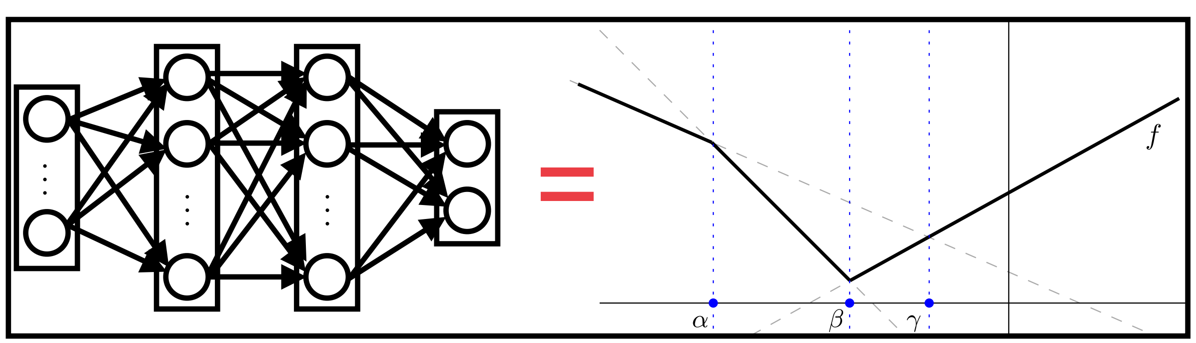

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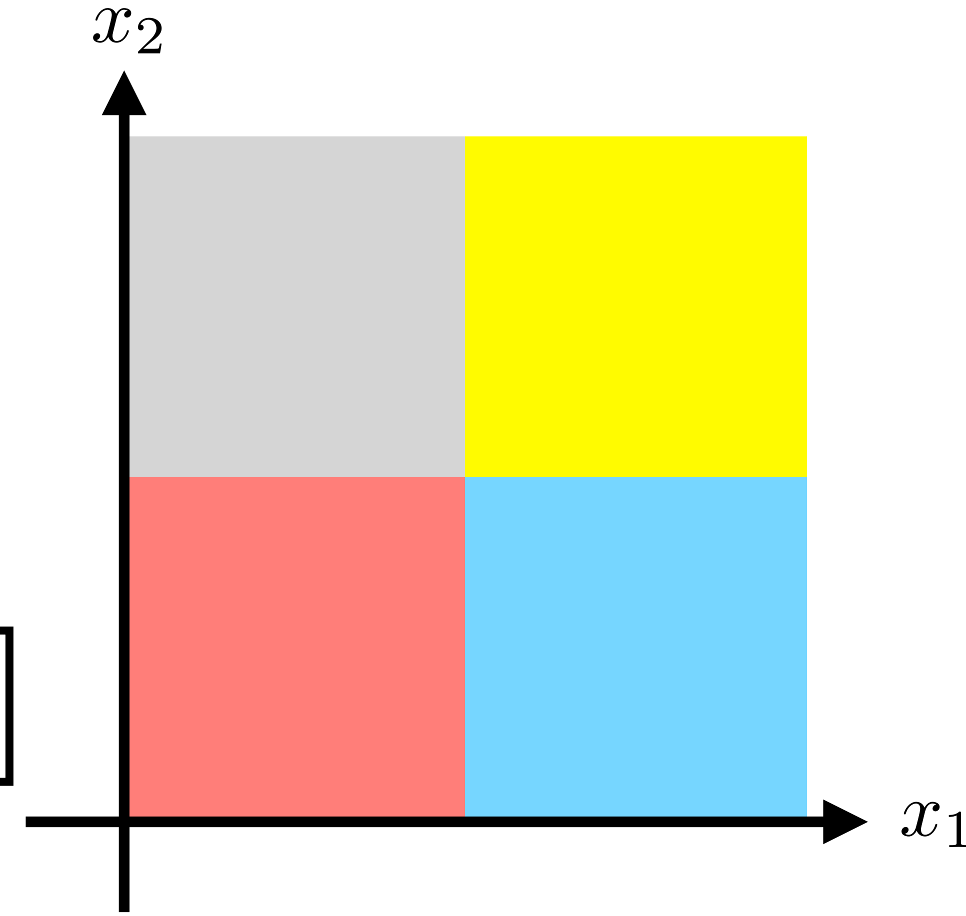
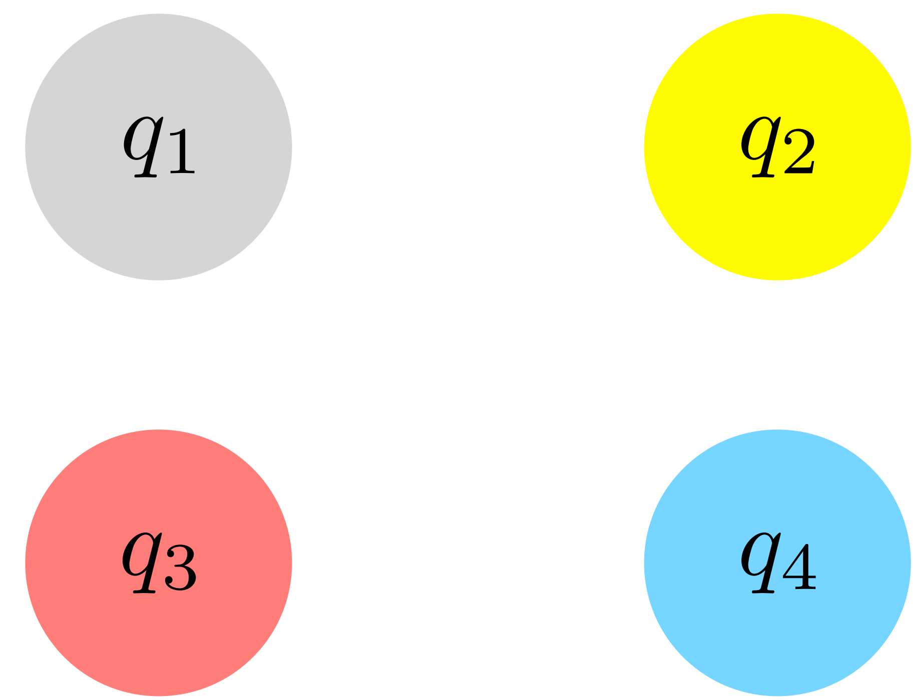
Controller Partitions:	$\mathbb{P} = \{P_1, P_2, \ldots, P_m\}$





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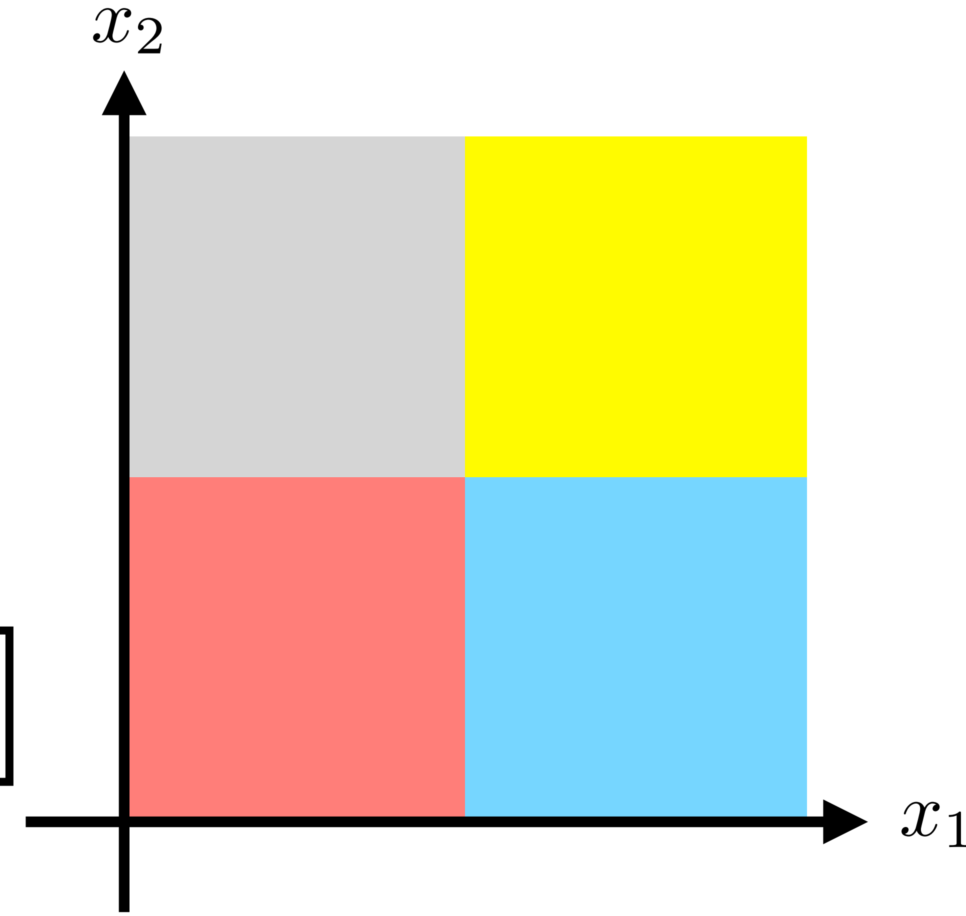
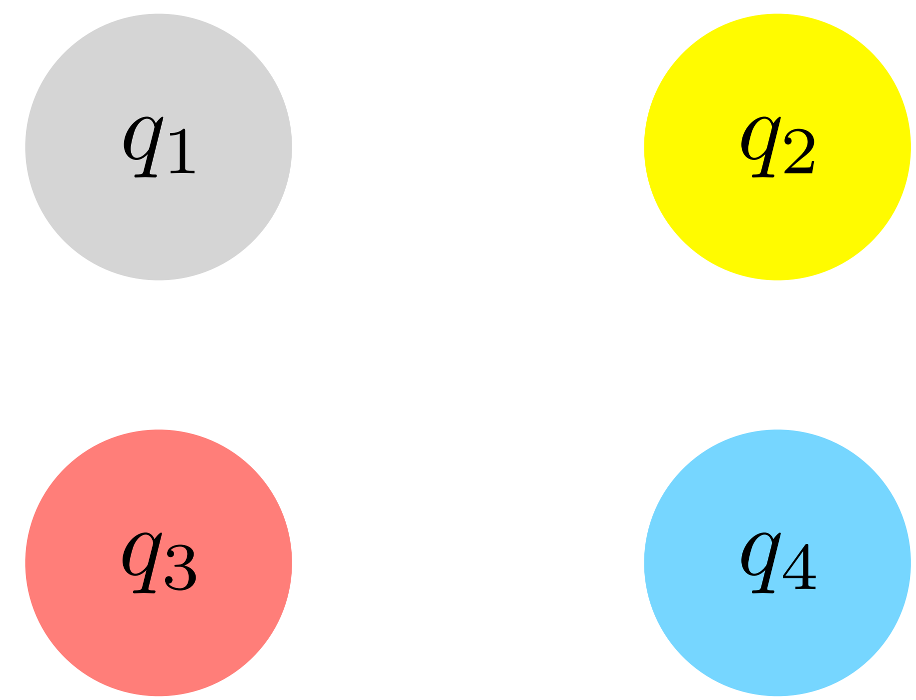
$$\mathrm{Post}(q_1, P_1) \; ?$$





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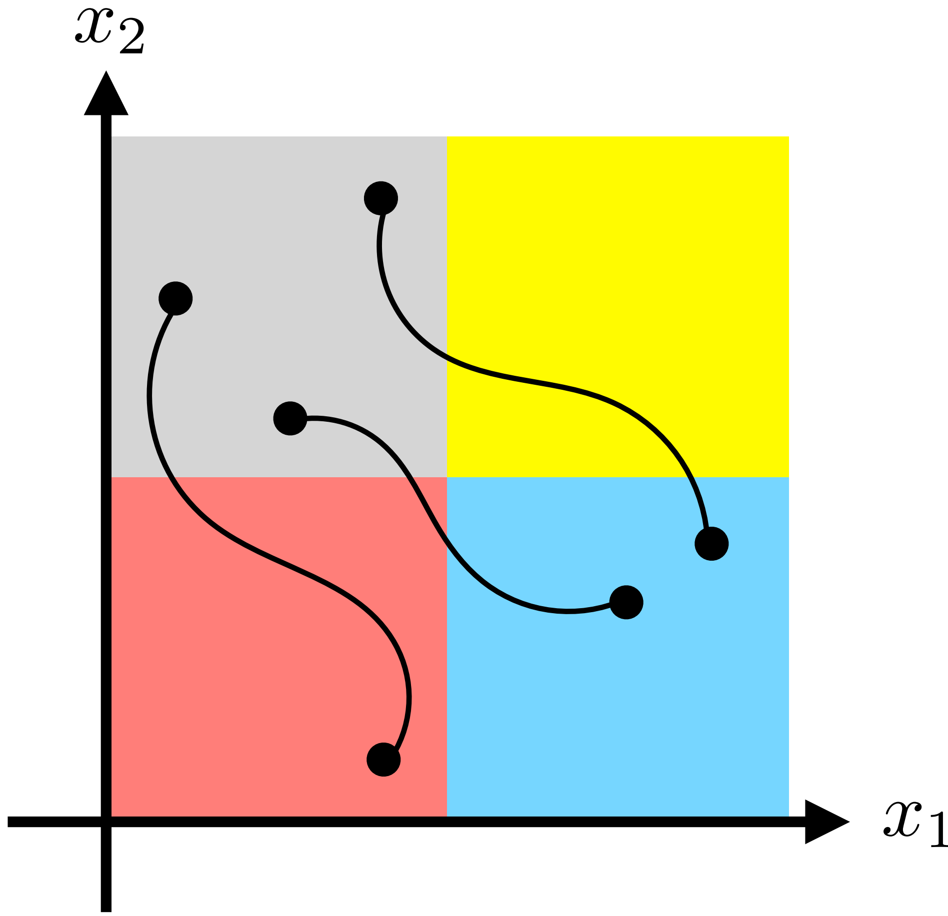

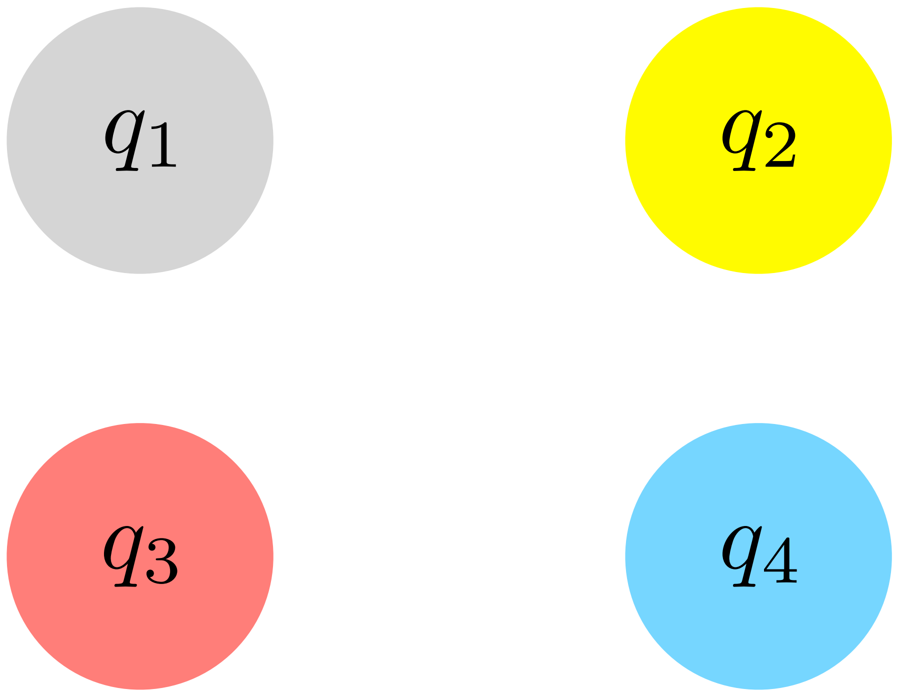

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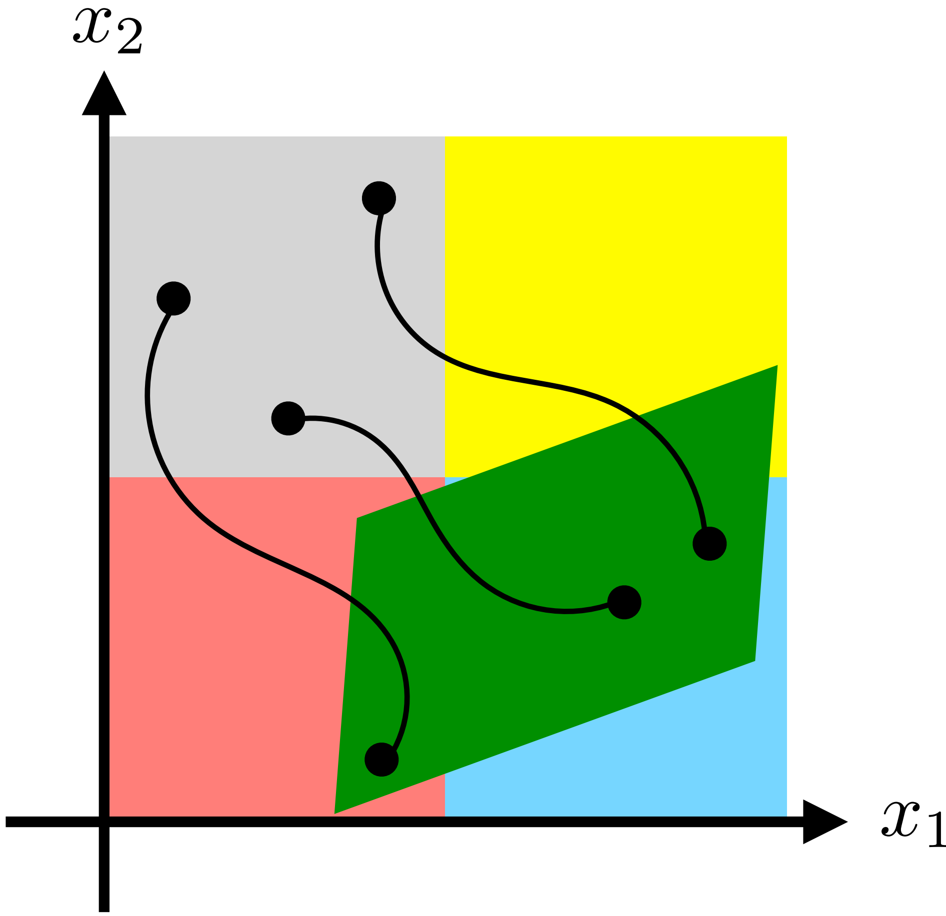

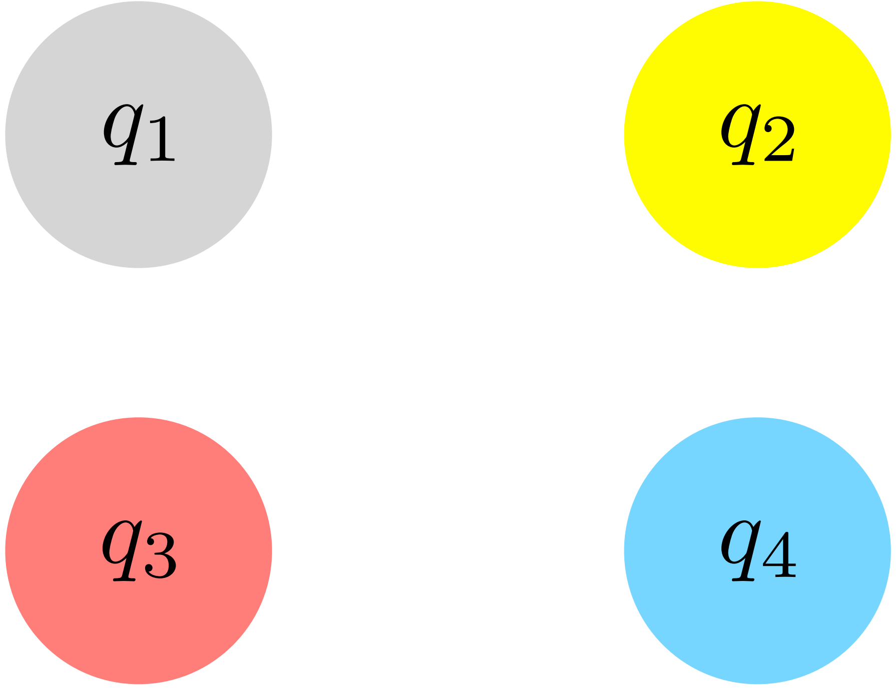

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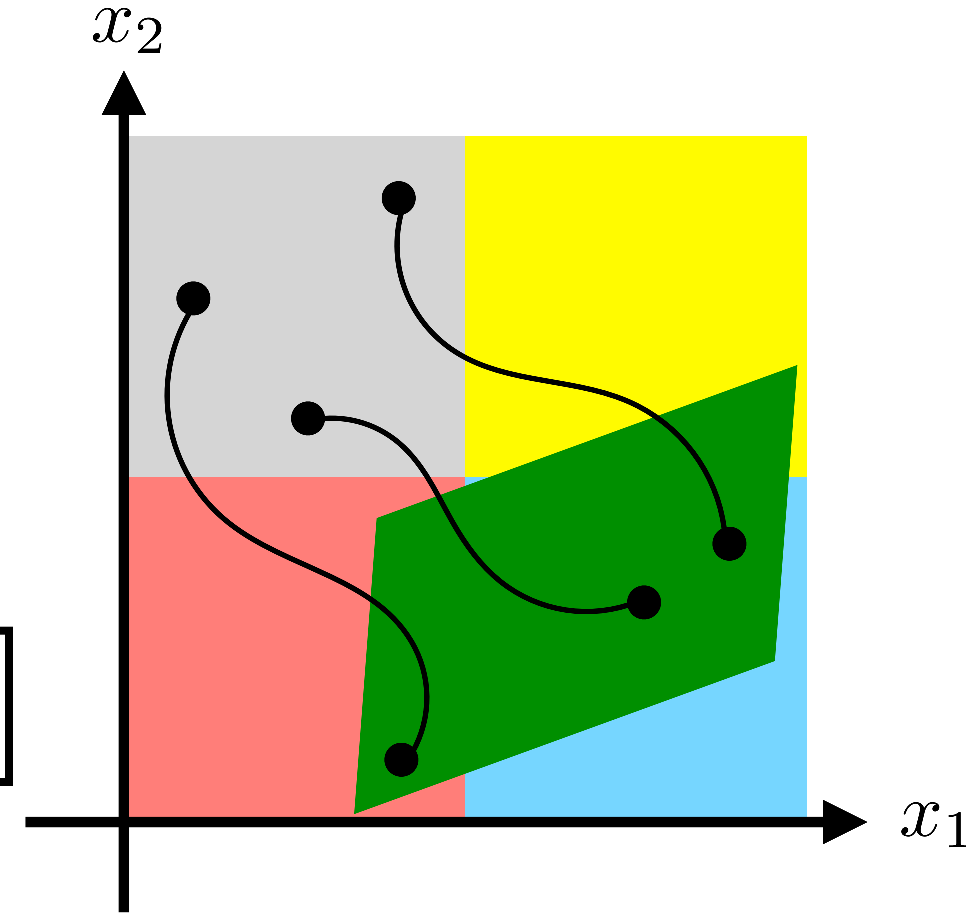

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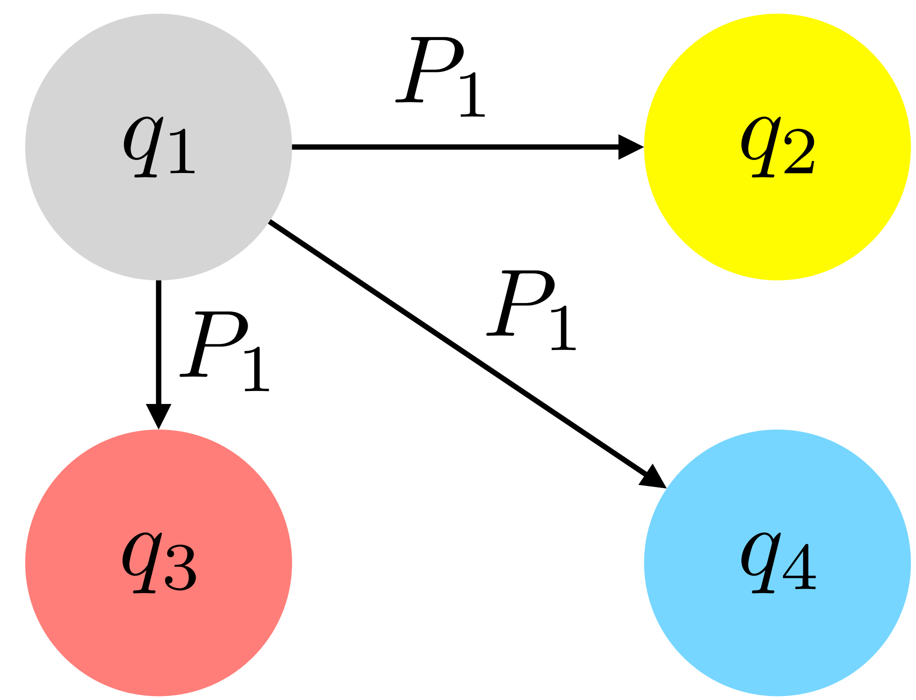

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Resilient CPS Lab

step 1

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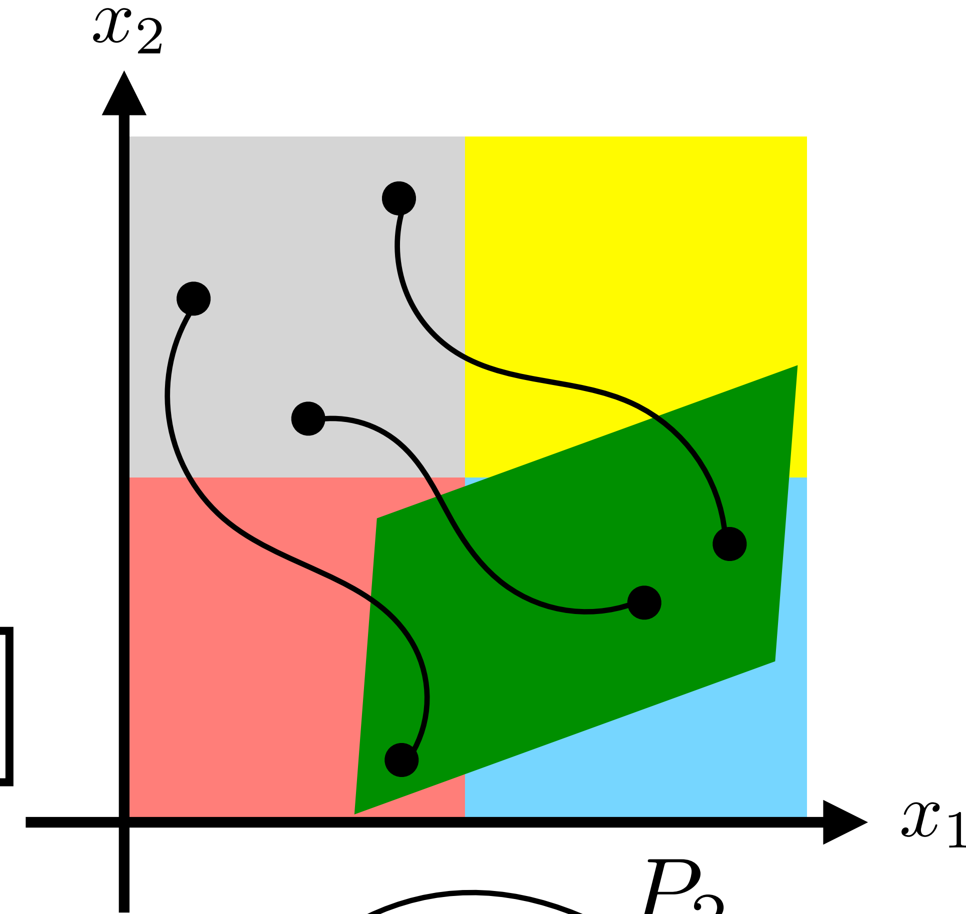

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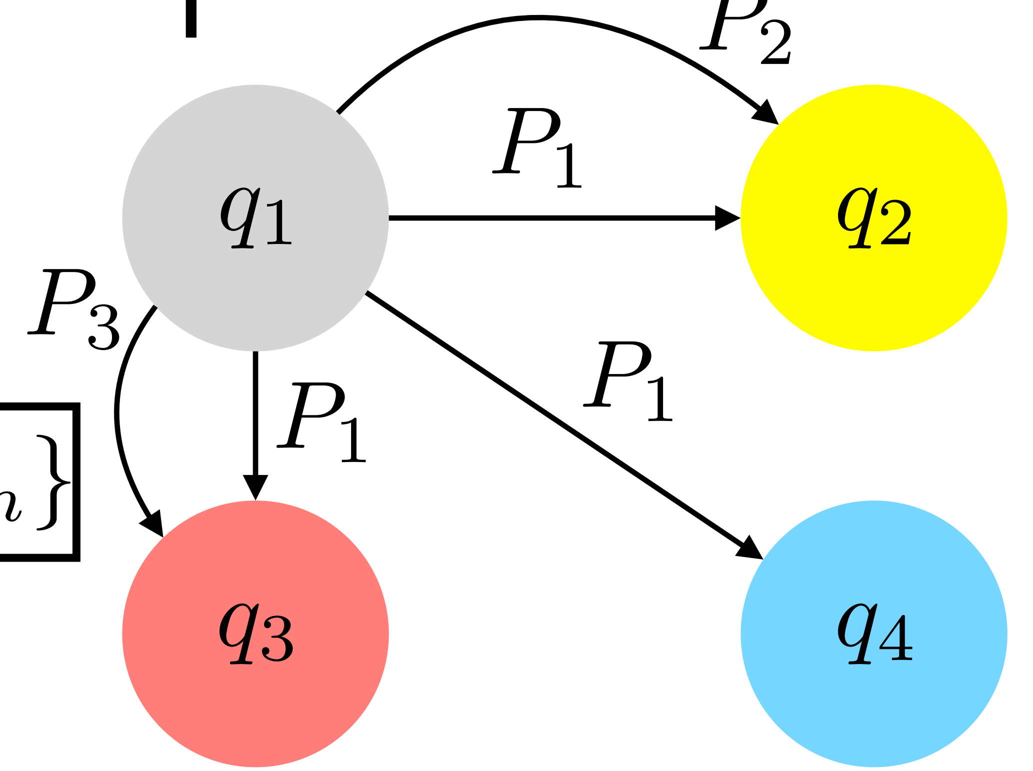

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Resilient CPS Lab

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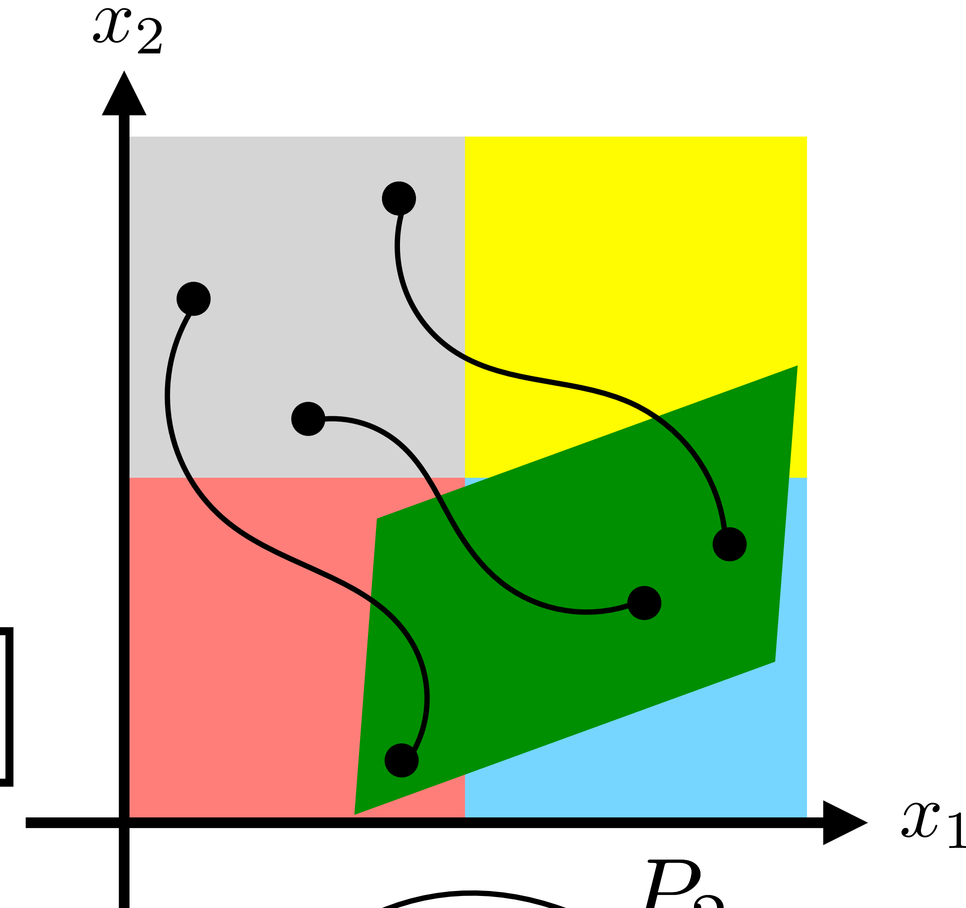

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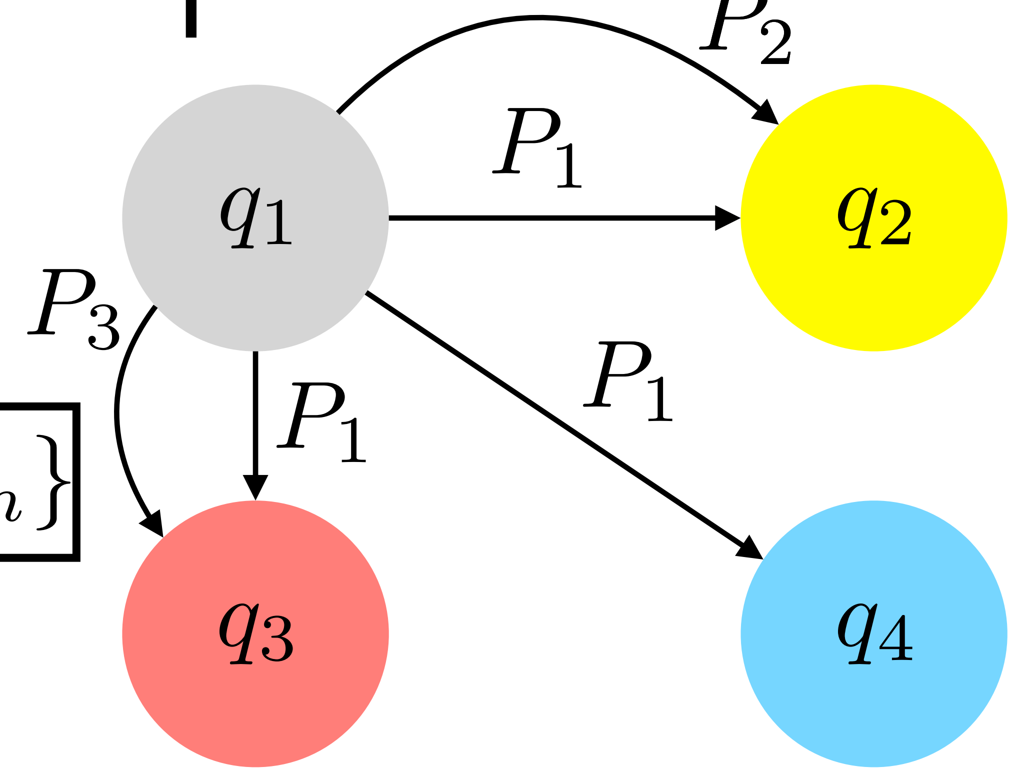

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Note: Computing the **Post** operator can be done using existing techniques for reachability analysis of nonlinear systems (with the caveat that existing tools focus on partitioning the "**input**" space instead of the "**controller**" space).

step 1

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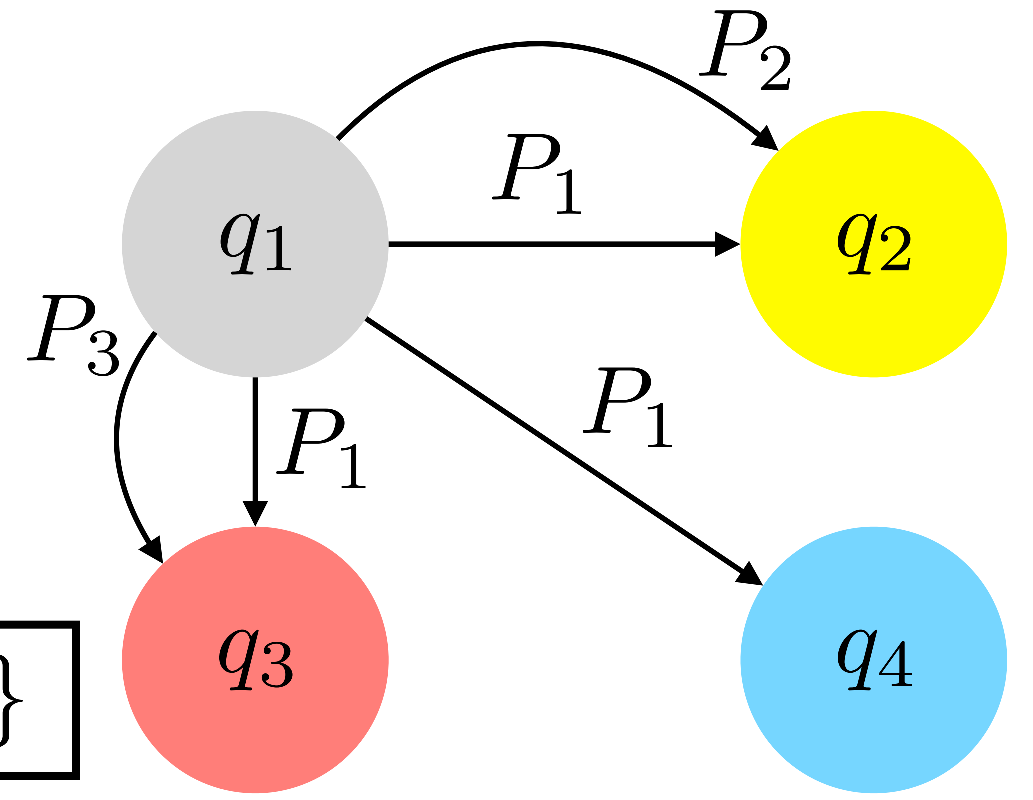

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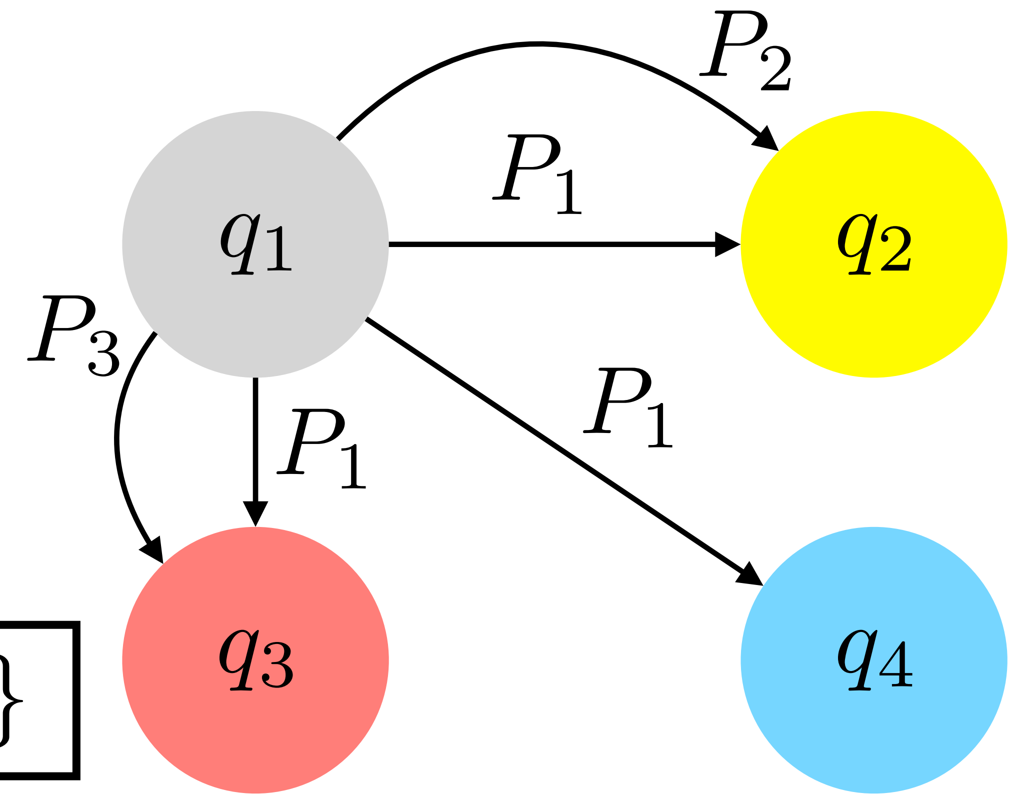

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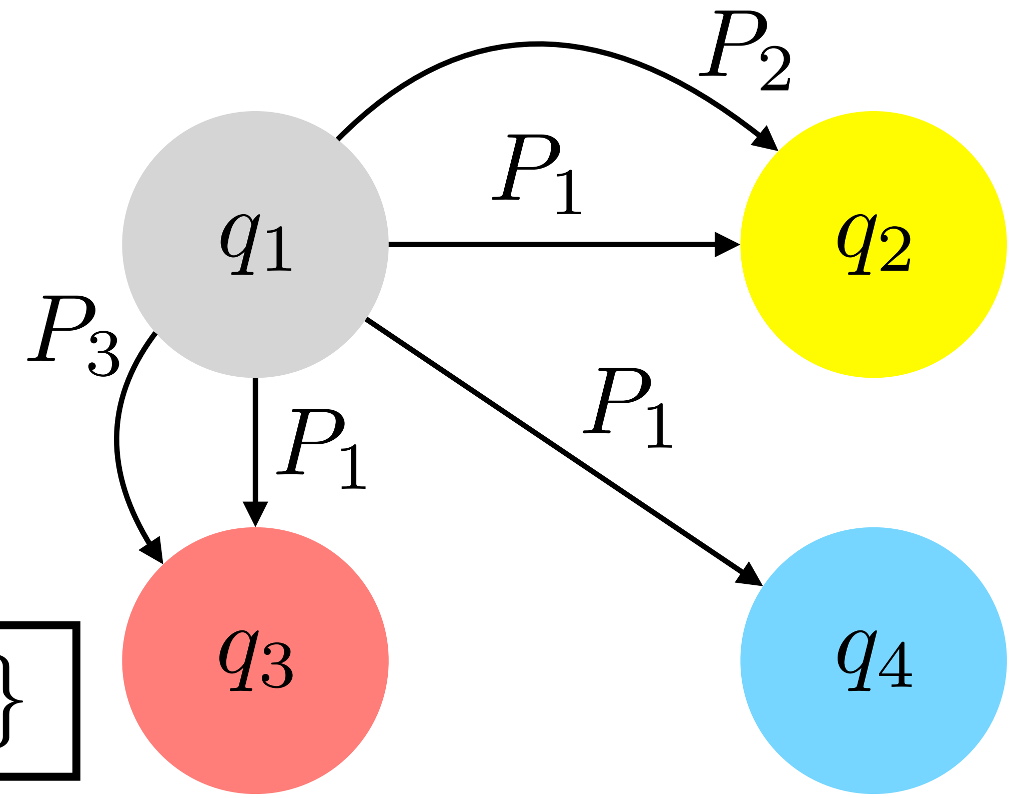

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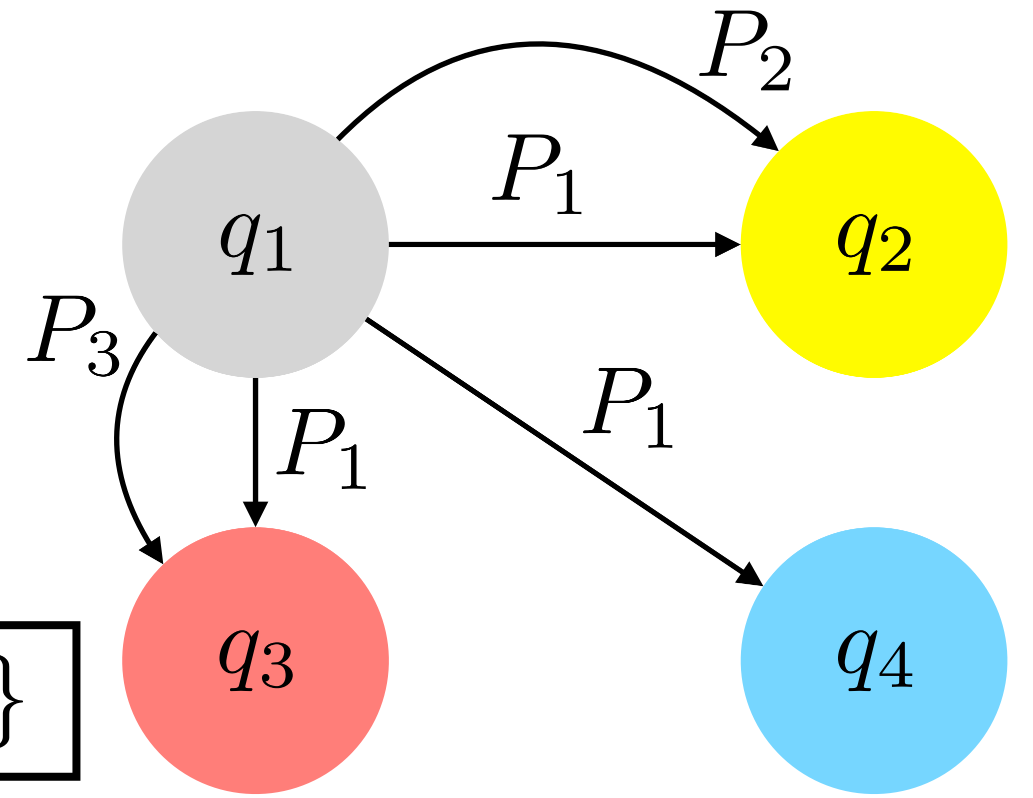

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CPS Lab
Resilient

step 1

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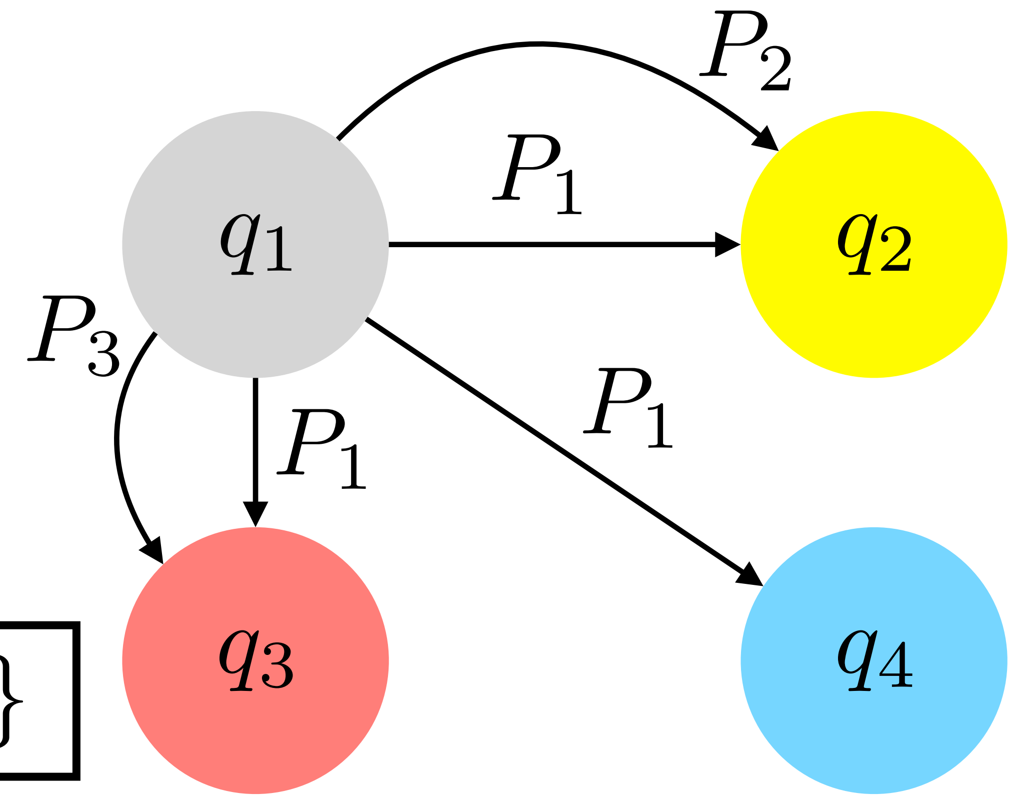

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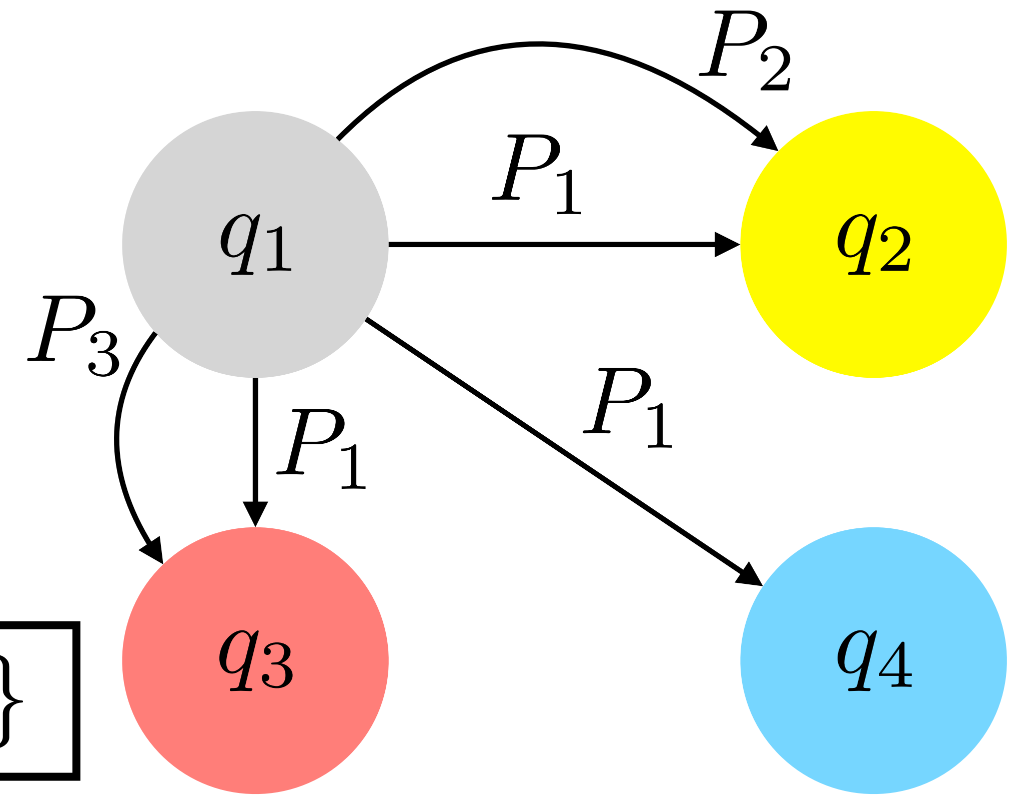

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CPS Lab Resilient

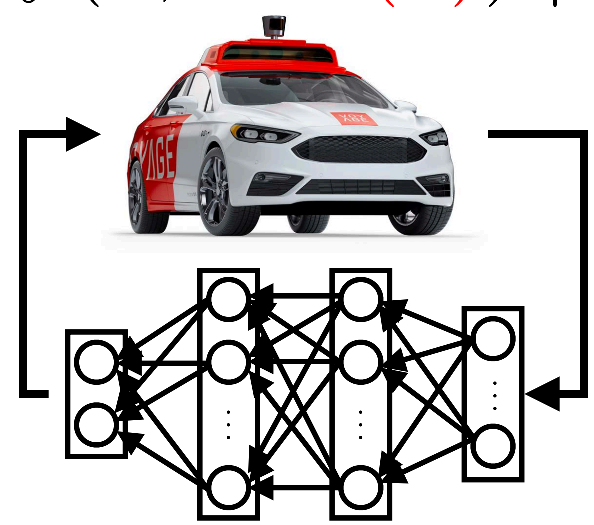
$$x^{(t+1)} = f(x^{(t)}, u^{(t)}) \rightarrow$$

$$\varphi \rightarrow$$

Training Data (offline or through interaction) →

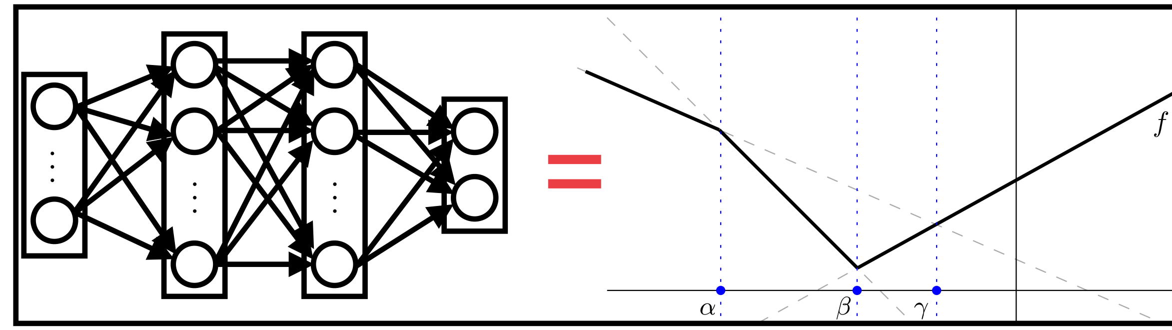
Formal NN Training →

$$f(x, \color{red}{\mathcal{NN}(x)}) \models \varphi$$



Core idea:

- Regression ReLU NN are Continuous Piece-Wise Affine (CPWA) functions



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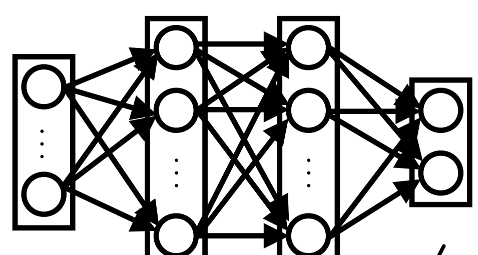
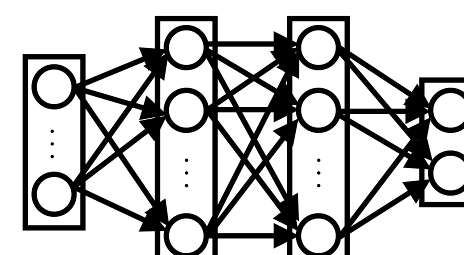
$$f(x, \color{blue}{K_{\text{CPWA}}(x)}) \models \varphi \quad \forall\, K_{\text{CPWA}} \in \text{CPWA}_{\varphi}$$

Training Data (offline or through interaction) $\xrightarrow{\text{Training}}$



$\notin \text{CPWA}_{\varphi}$ $\xrightarrow{\text{Projection}}$



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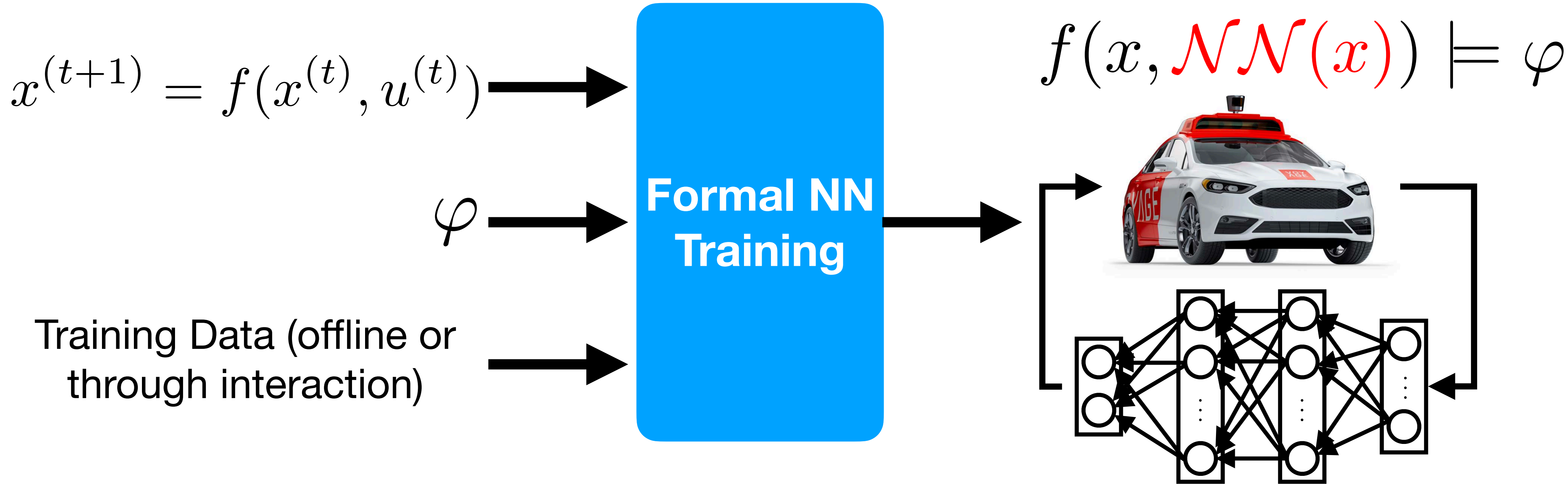

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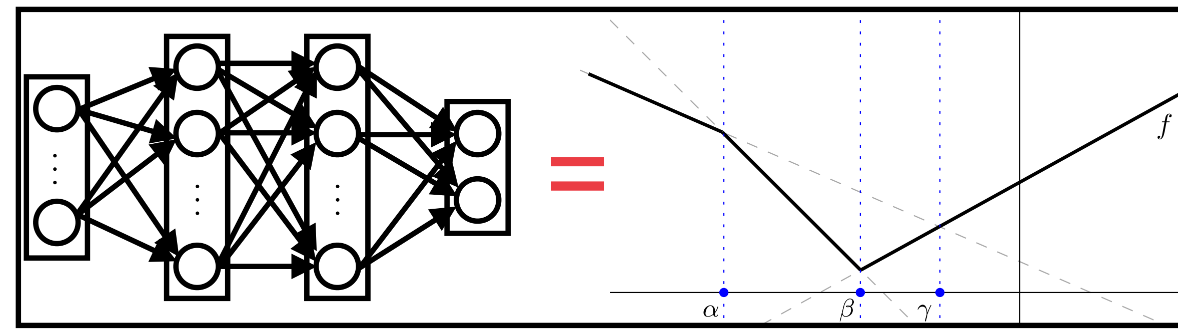

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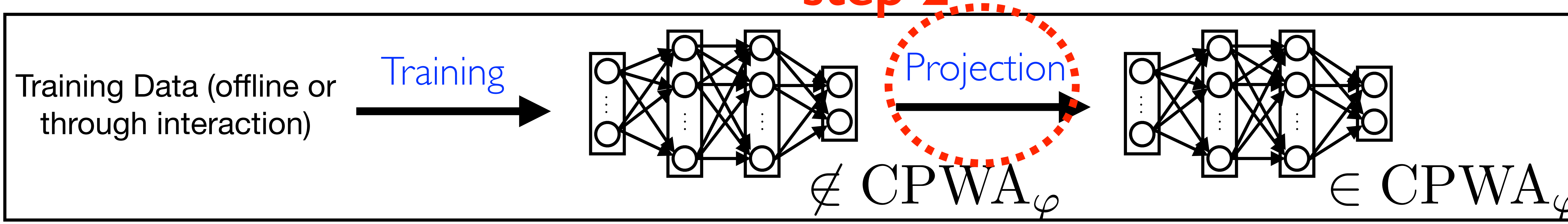
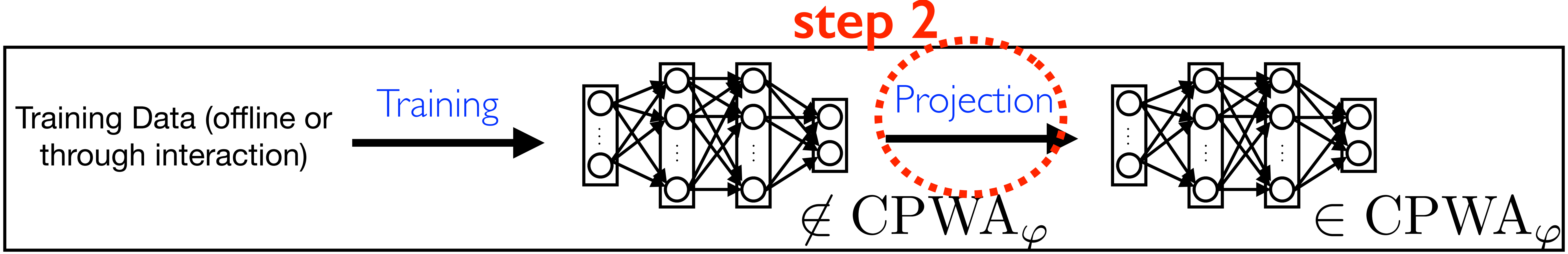
step 2





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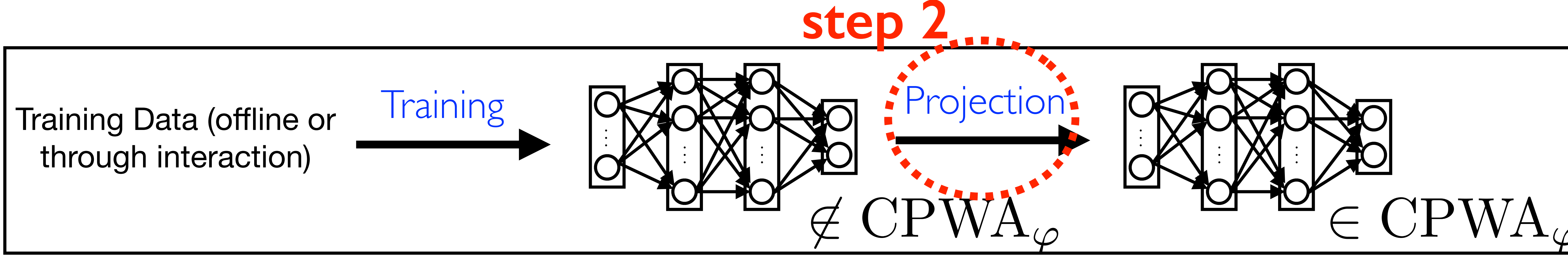
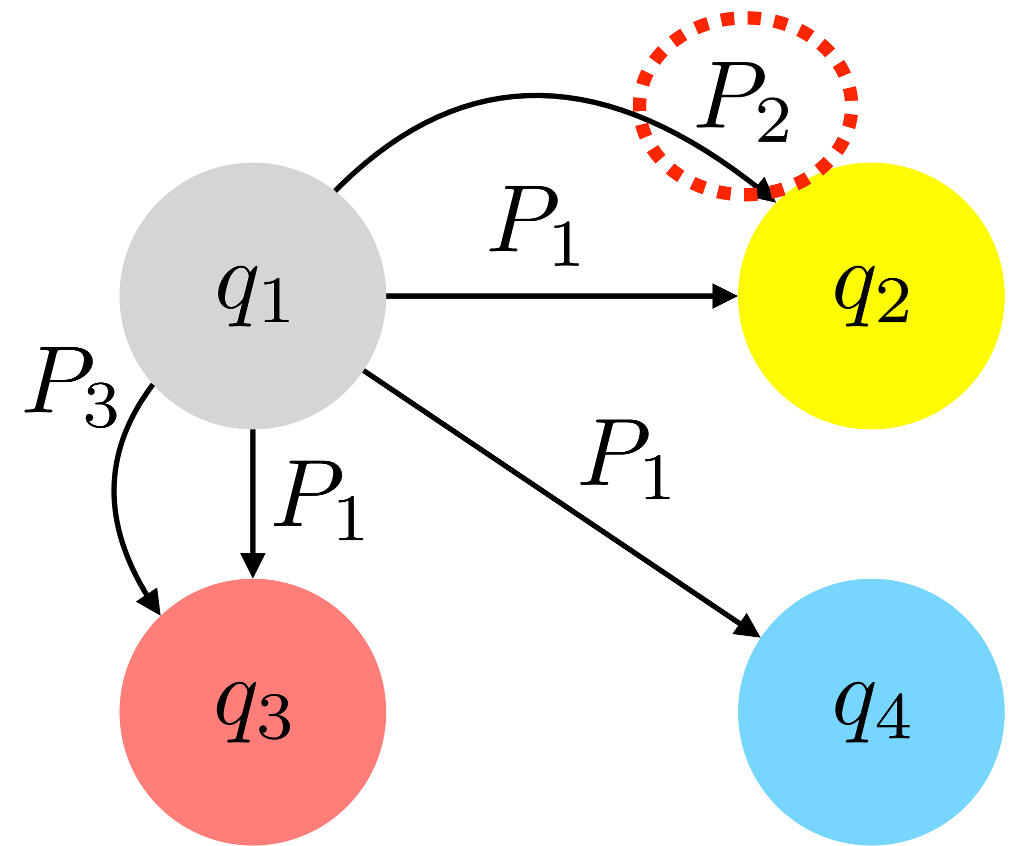
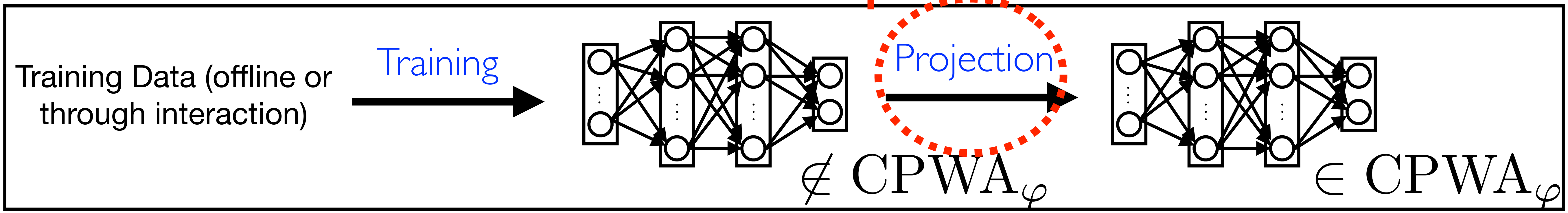
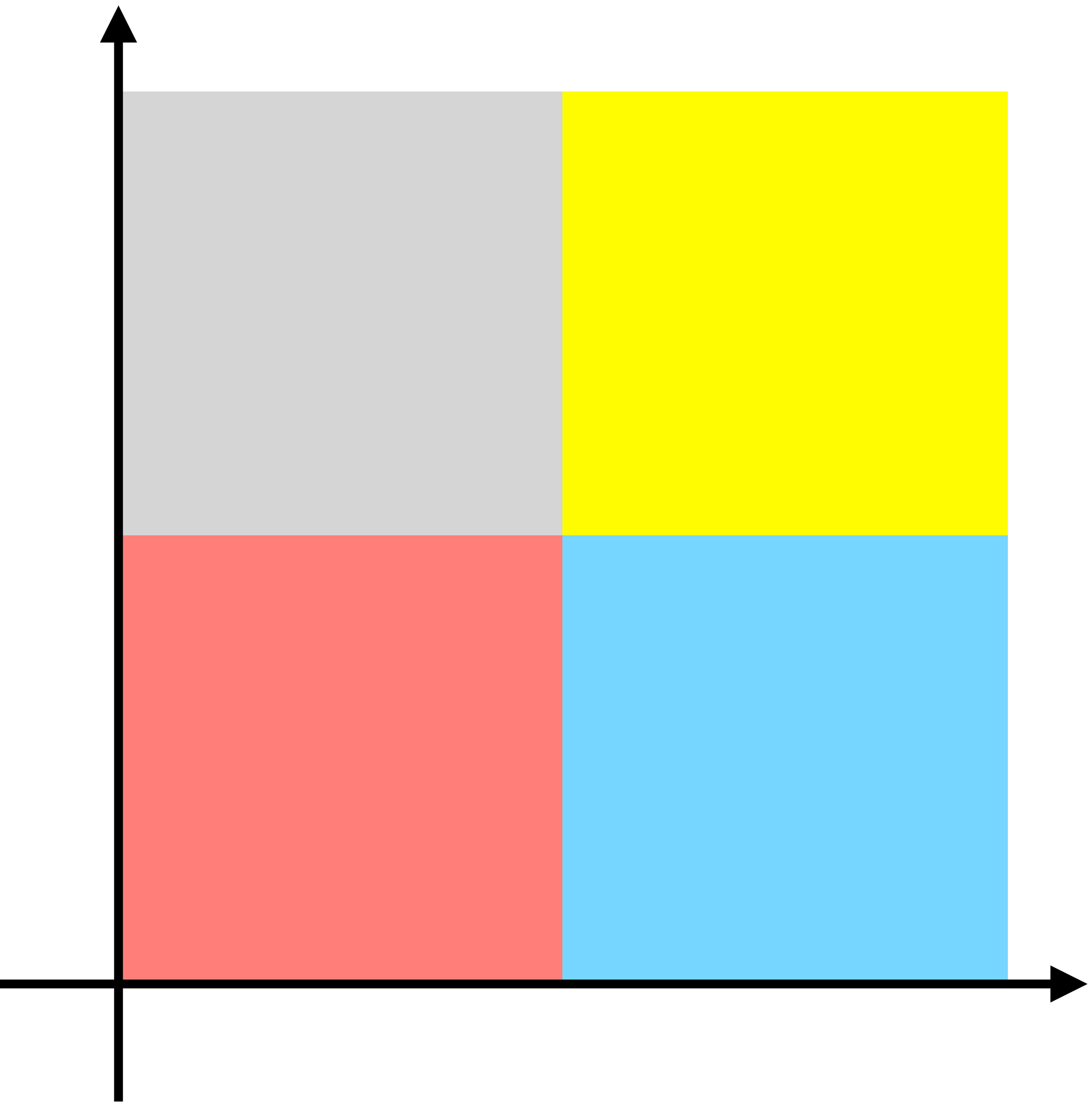


- For each abstract state, select one controller partition $P^\star$ from CPWA_φ



step 2





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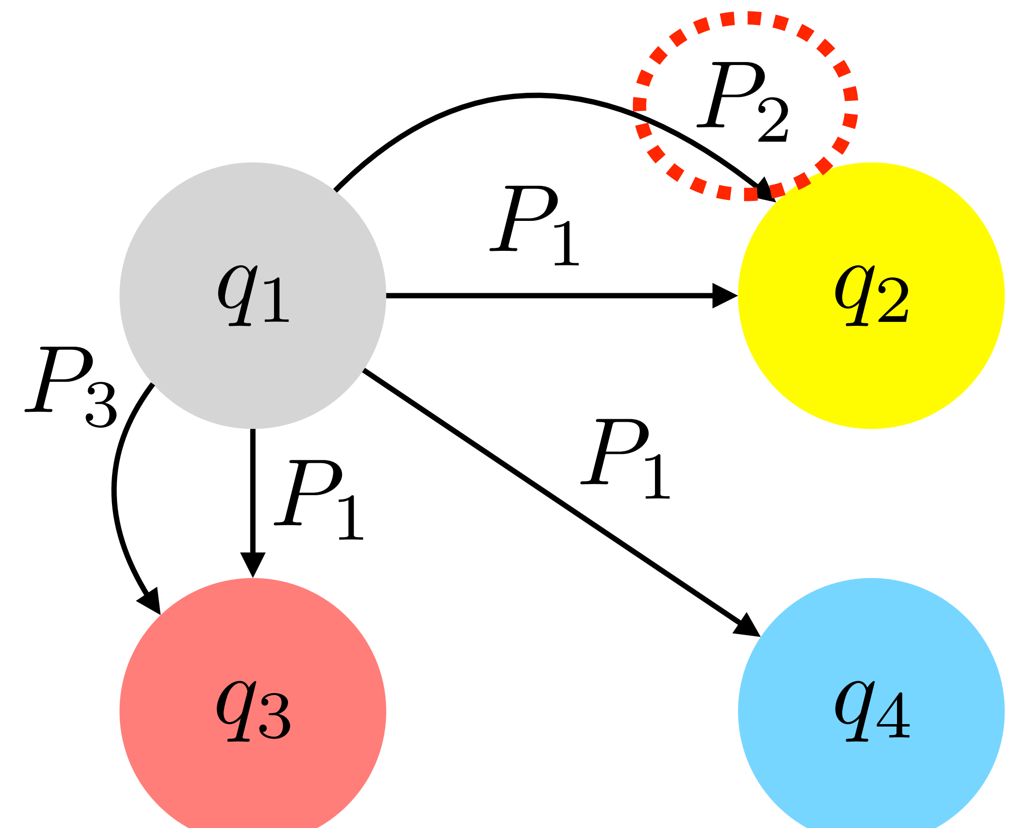
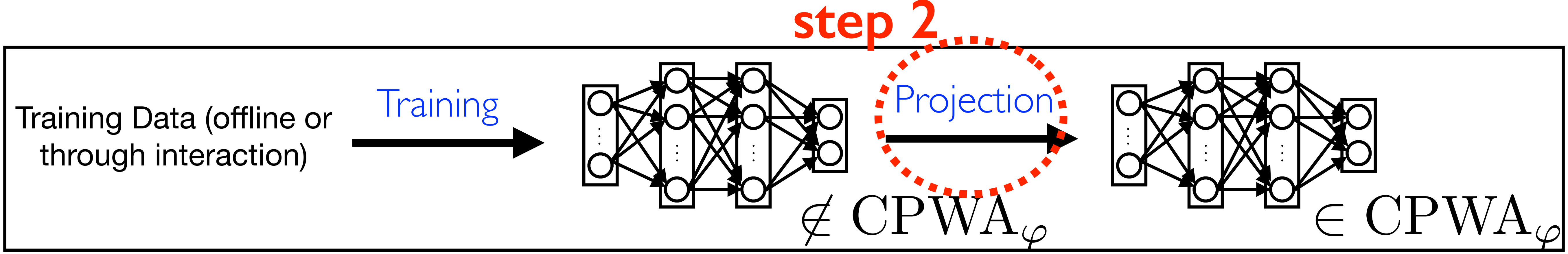
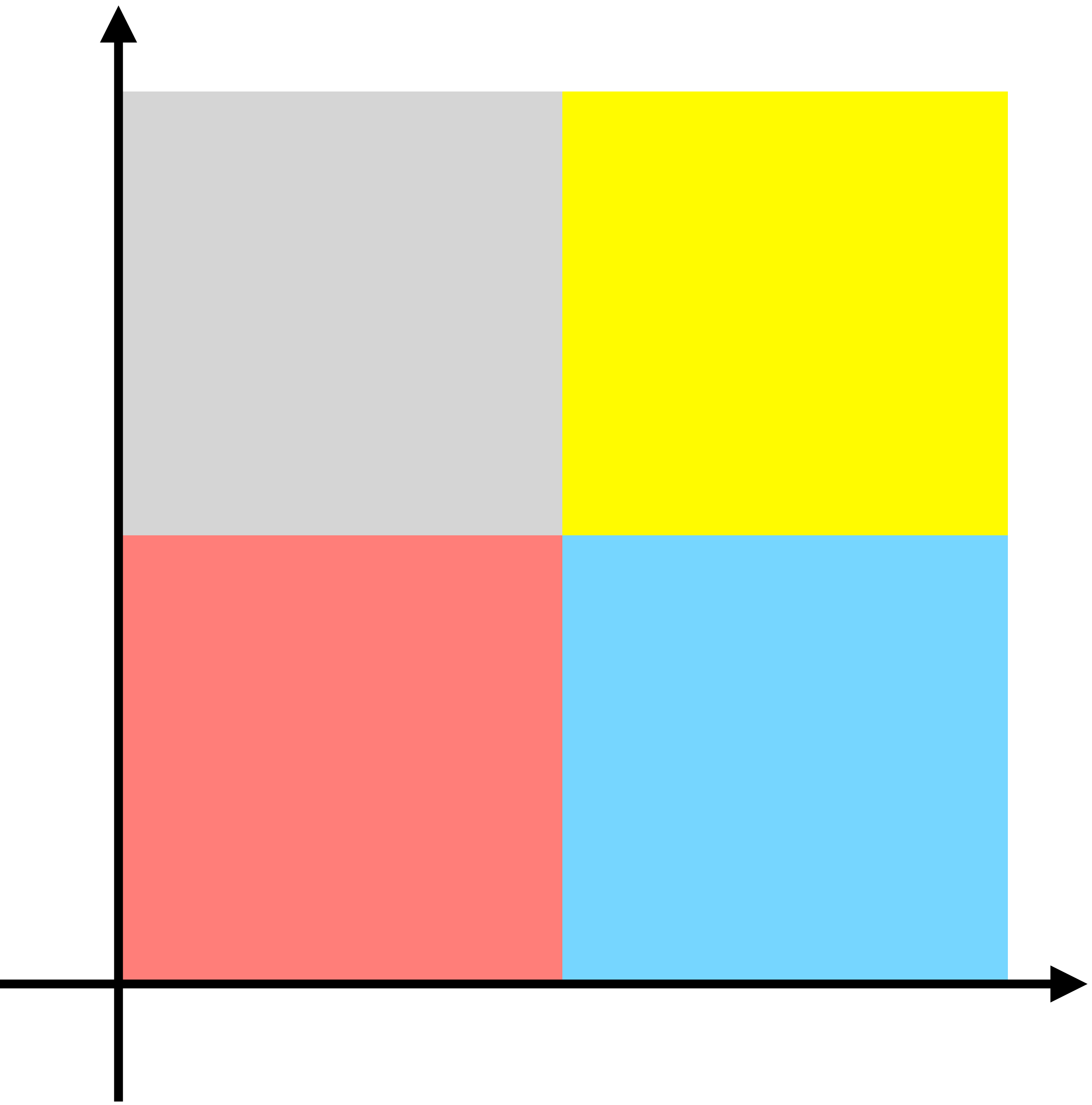


Resilient CPS Lab





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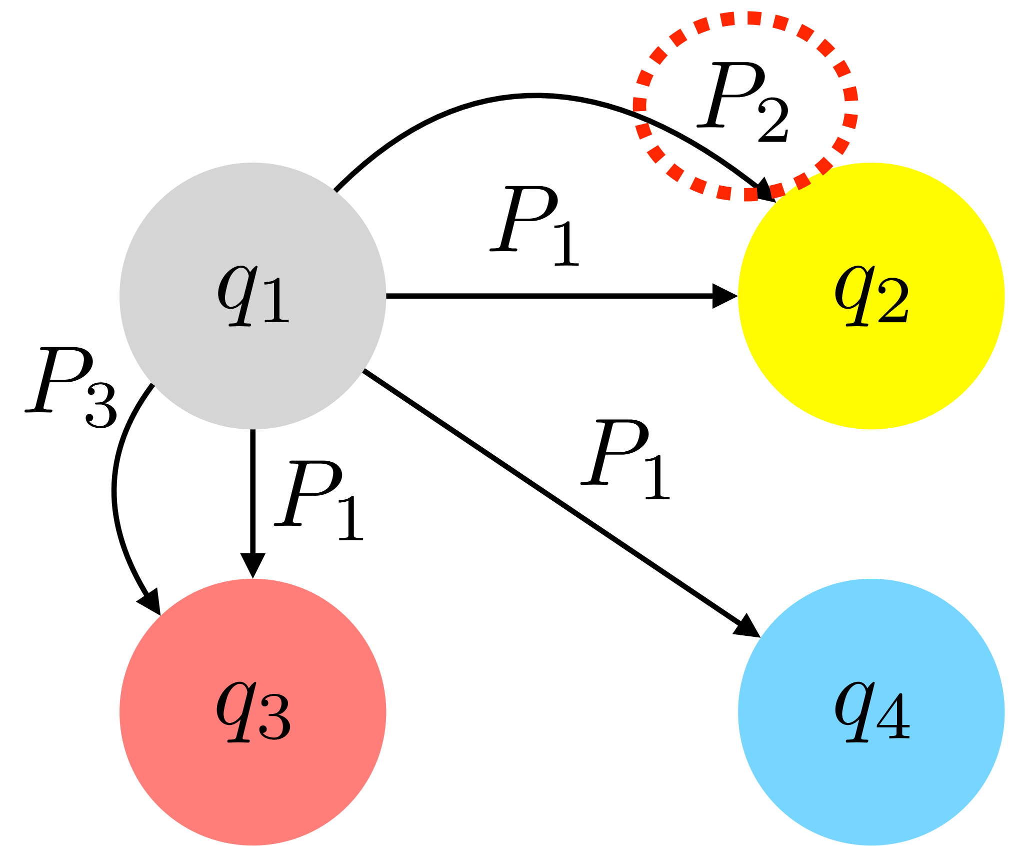

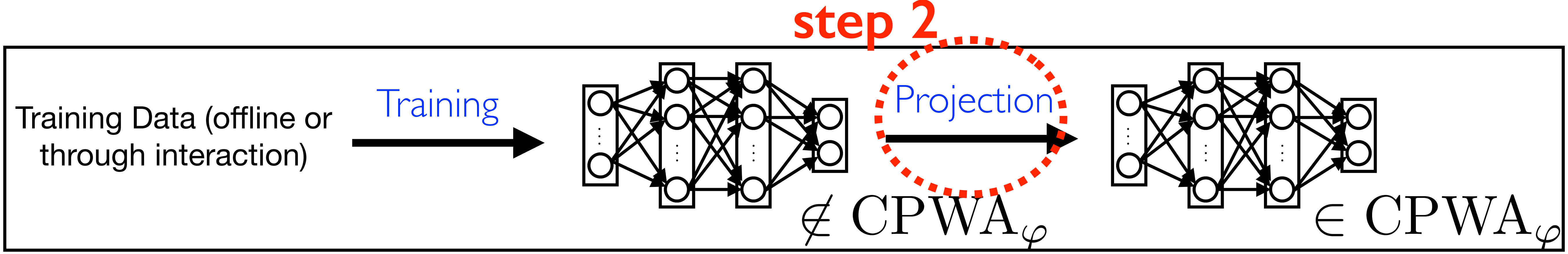

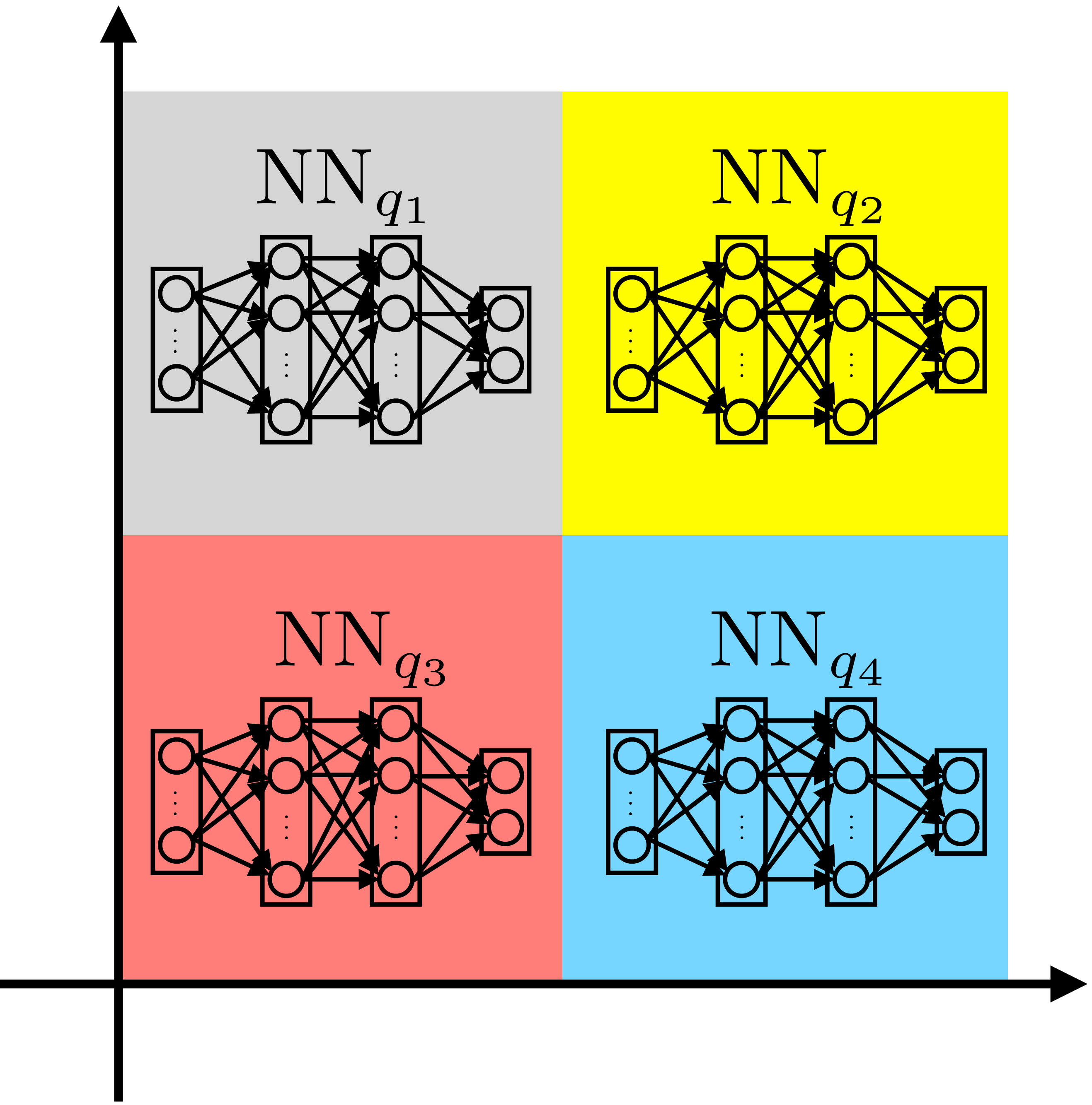

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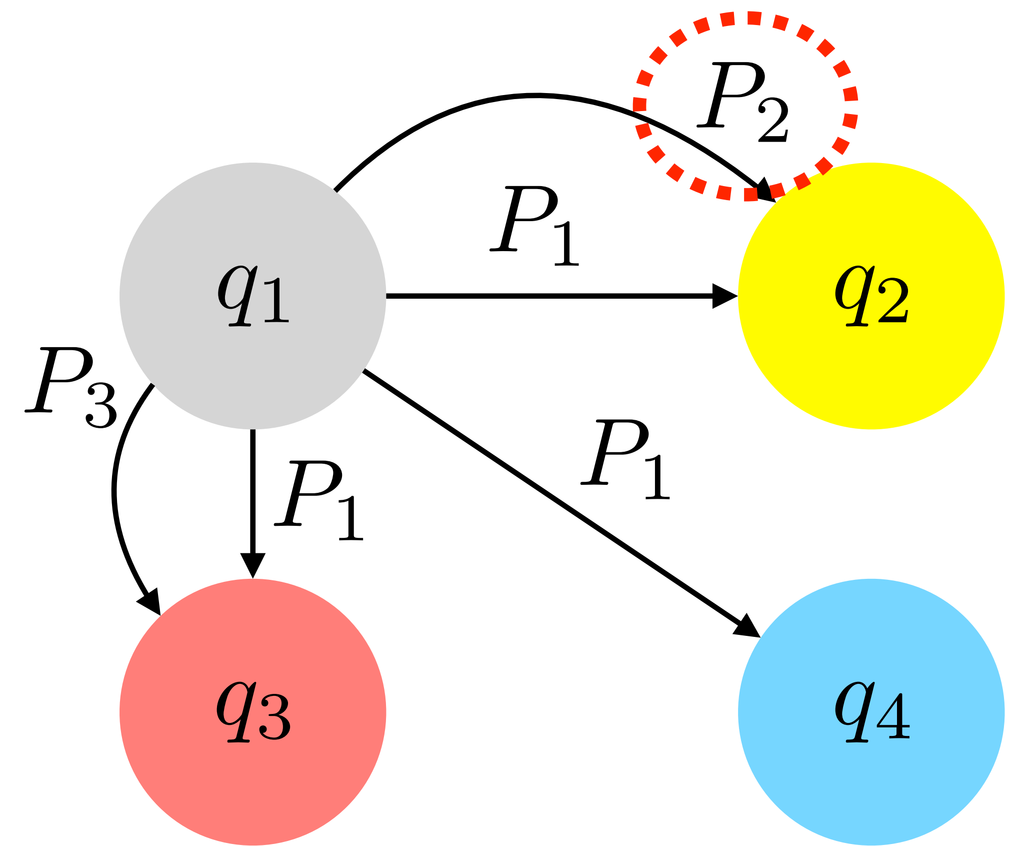

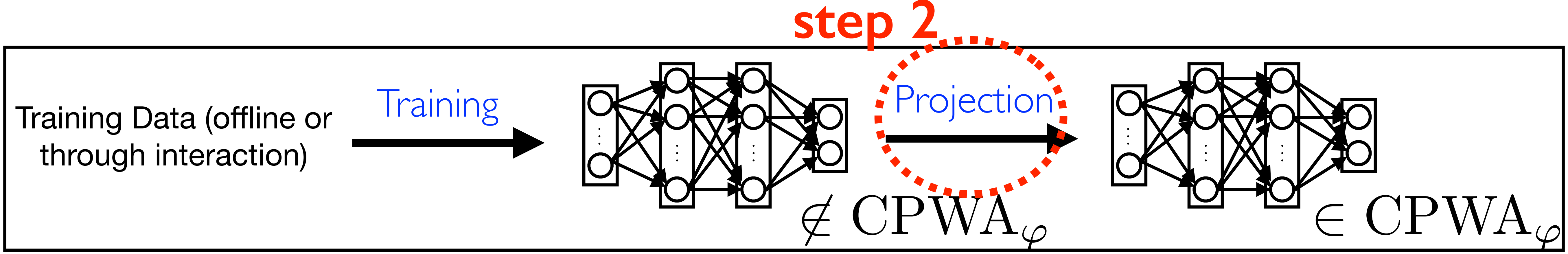

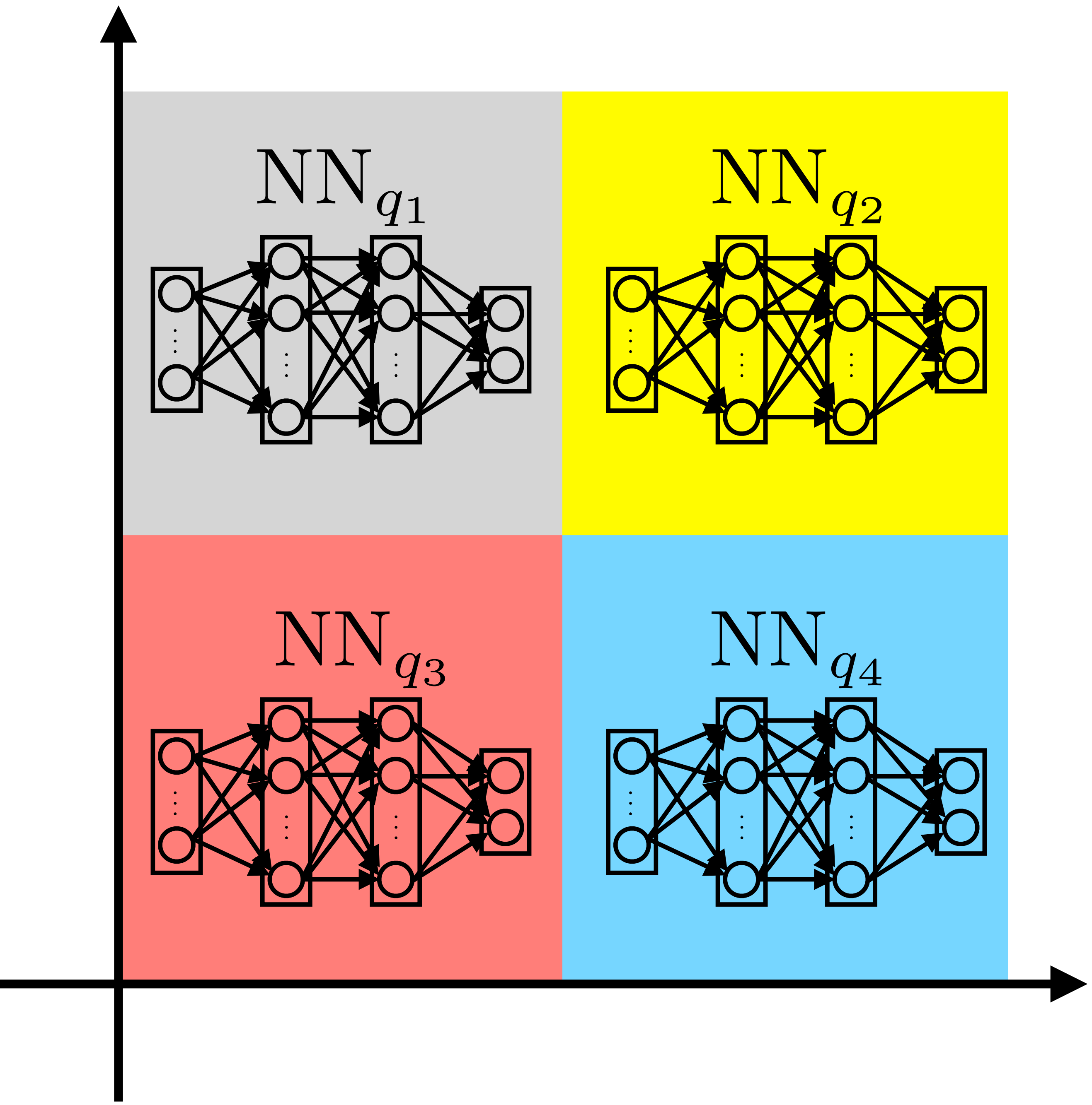

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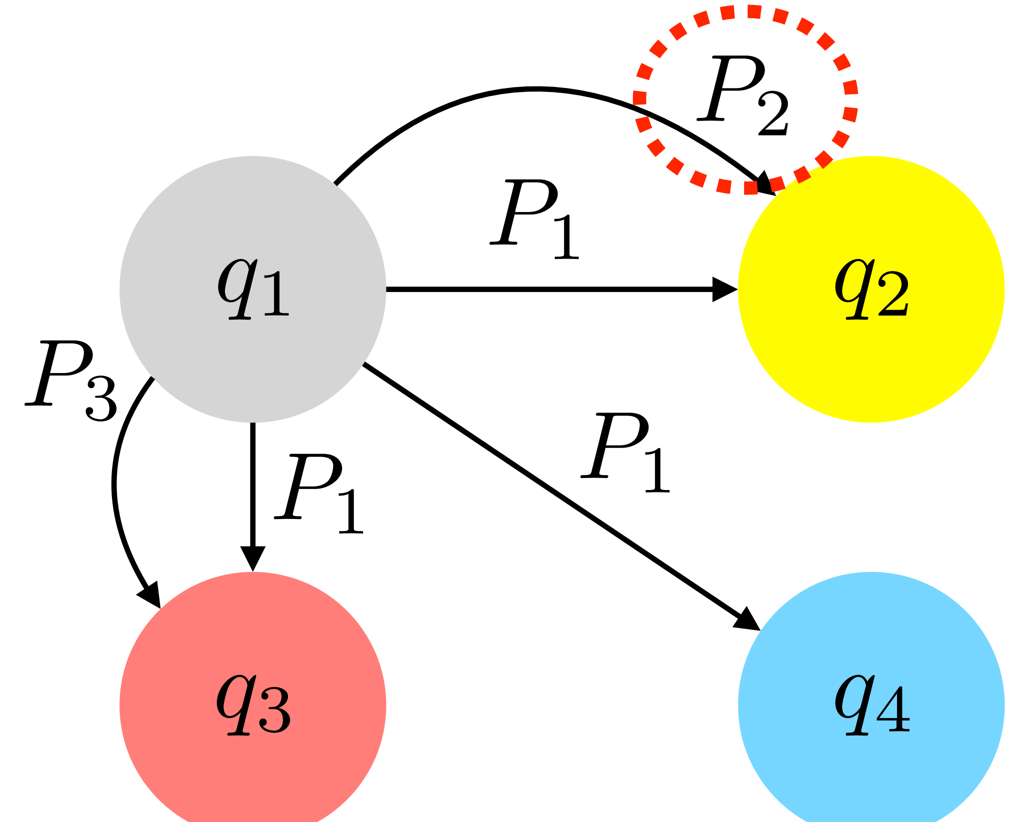
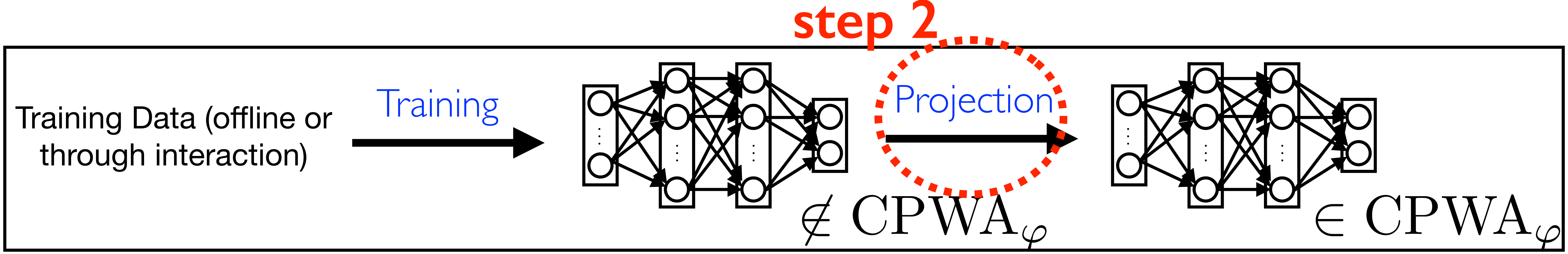
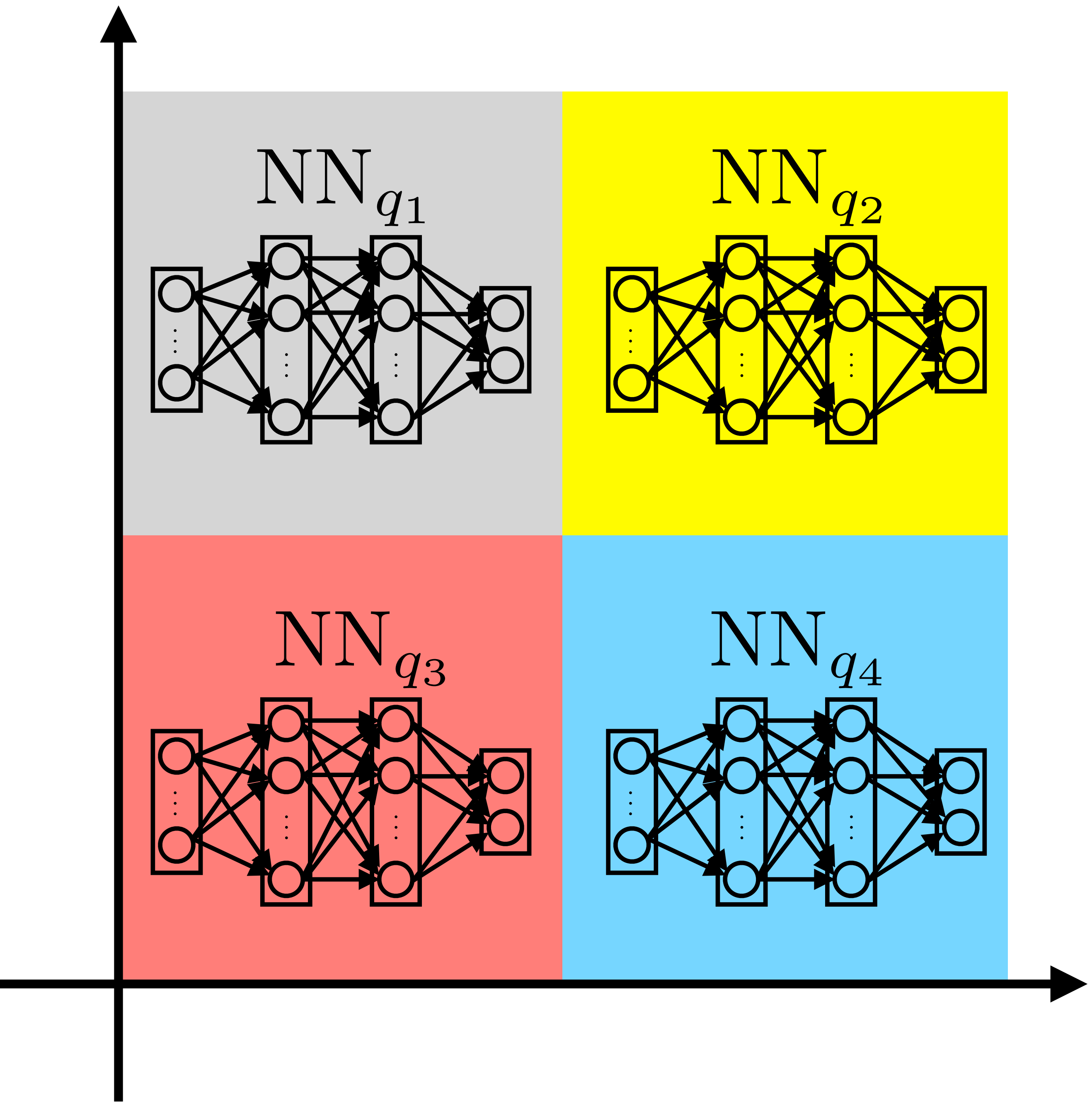


Resilient CPS Lab





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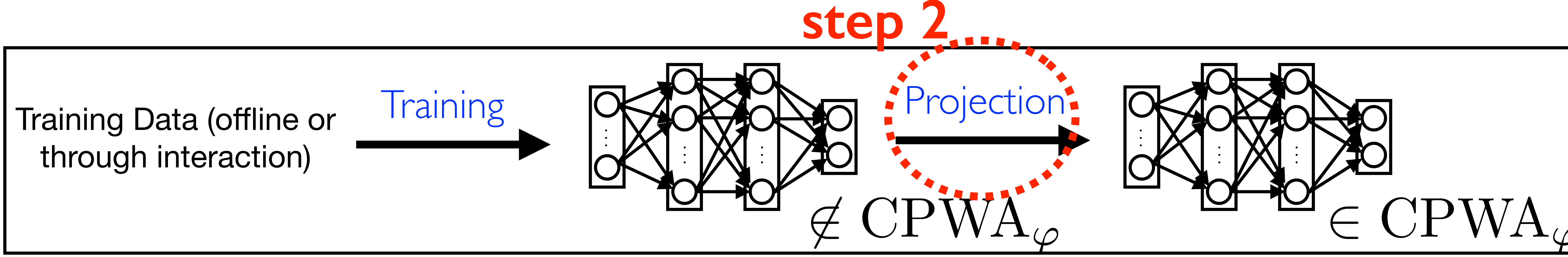

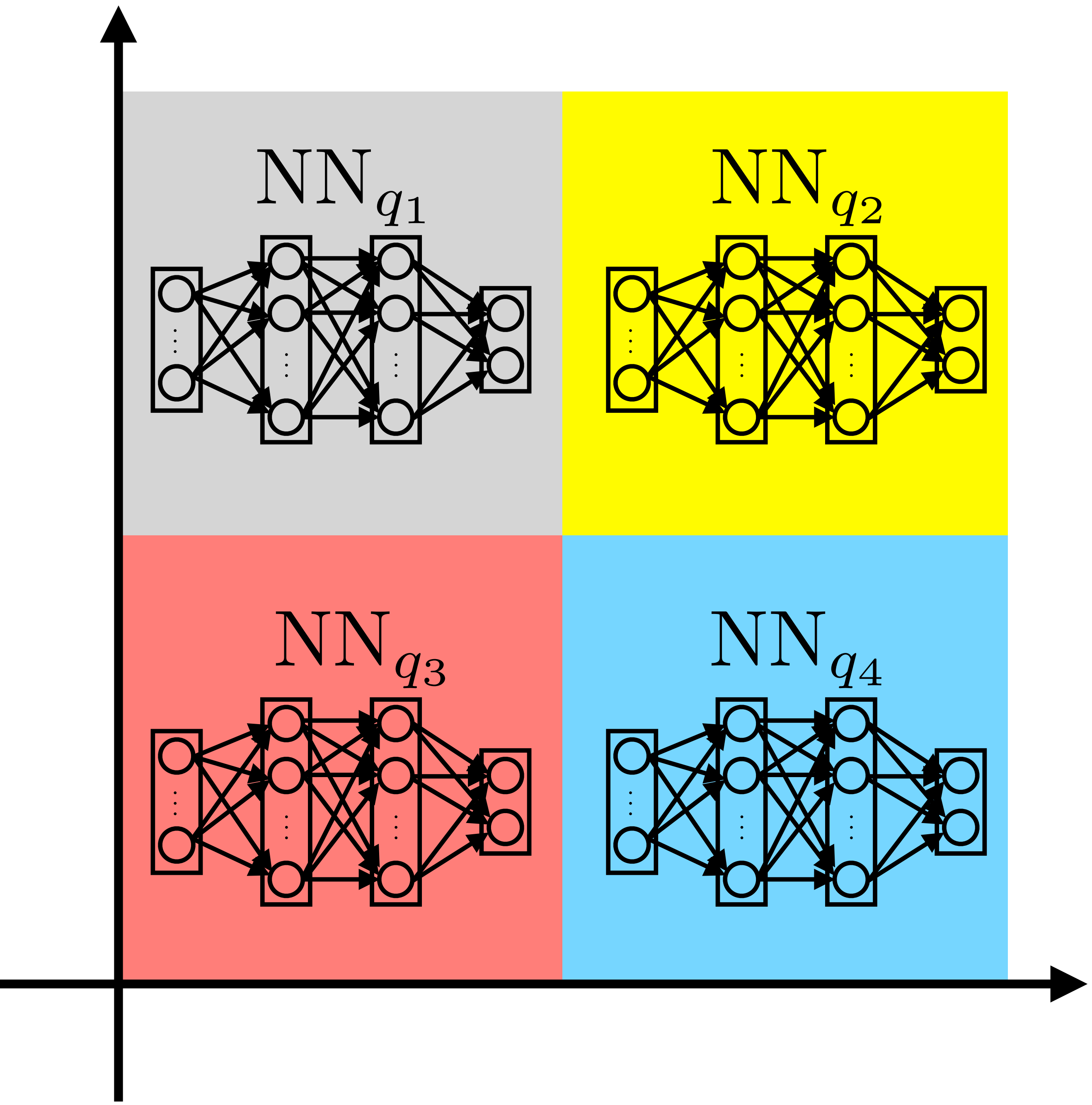

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- Projection:

$$\min_{\widehat{W}} \|W - \widehat{W}\|$$

$$\text{s.t.} \quad (K_i, b_i) \in P^\star \quad \forall\ (K_i, b_i) \in \mathrm{NN}_q$$

(convex optimization problem if done layer-by-layer)

CPS Lab Resilient

Theorem (informal):

Consider the nonlinear system $x^+ = f(x, u)$ and a safety specification φ. Define a "global" neural network controller as the composition of "local" neural network controllers:

$$\mathrm{NN} = \mathrm{NN}_{q_1} || \mathrm{NN}_{q_2} || \dots \mathrm{NN}_{q_n}$$

Then:

$$f(x, \mathrm{NN}(x)) \models \varphi$$

X. Sun and Y. Shoukry, "Provably Correct Training of Neural Network Controllers Using Reachability Analysis," arXiv 2021.

Theorem (informal):

Consider the nonlinear system $x^+ = f(x,u)$ and a safety specification φ. Define a “global” neural network controller as the composition of “local” neural network controllers:

$$\mathrm{NN} = \mathrm{NN}_{q_1} || \mathrm{NN}_{q_2} || \ldots \mathrm{NN}_{q_n}$$

Then:

$$f(x, \mathrm{NN}(x)) \models \varphi$$

X. Sun and Y. Shoukry, "Provably Correct Training of Neural Network Controllers Using Reachability Analysis," arXiv 2021.

The result holds for liveness specifications under an additional assumption.

$$\zeta_x^{(t+\Delta t)} = \zeta_x^{(t)} + \Delta t \ v \ \cos(\theta^{(t)})$$
$$\zeta_y^{(t+\Delta t)} = \zeta_y^{(t)} + \Delta t \ v \ \sin(\theta^{(t)})$$
$$\theta^{(t+\Delta t)} = \theta^{(t)} + \Delta t \ u^{(t)}$$

$$\zeta_x^{(t+\Delta t)} = \zeta_x^{(t)} + \Delta t \ v \ \cos(\theta^{(t)})$$
$$\zeta_y^{(t+\Delta t)} = \zeta_y^{(t)} + \Delta t \ v \ \sin(\theta^{(t)})$$
$$\theta^{(t+\Delta t)} = \theta^{(t)} + \Delta t \ u^{(t)}$$

- Safe data collected and used for training
- Same data used in both experiments

$$\zeta_x^{(t+\Delta t)} = \zeta_x^{(t)} + \Delta t \; v \; \cos(\theta^{(t)})$$
$$\zeta_y^{(t+\Delta t)} = \zeta_y^{(t)} + \Delta t \; v \; \sin(\theta^{(t)})$$
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Formal NN Training

NN Training

$$\zeta_x^{(t+\Delta t)} = \zeta_x^{(t)} + \Delta t \ v \ \cos(\theta^{(t)})$$

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$$\theta^{(t+\Delta t)} = \theta^{(t)} + \Delta t \ u^{(t)}$$

- Safe data collected and used for training
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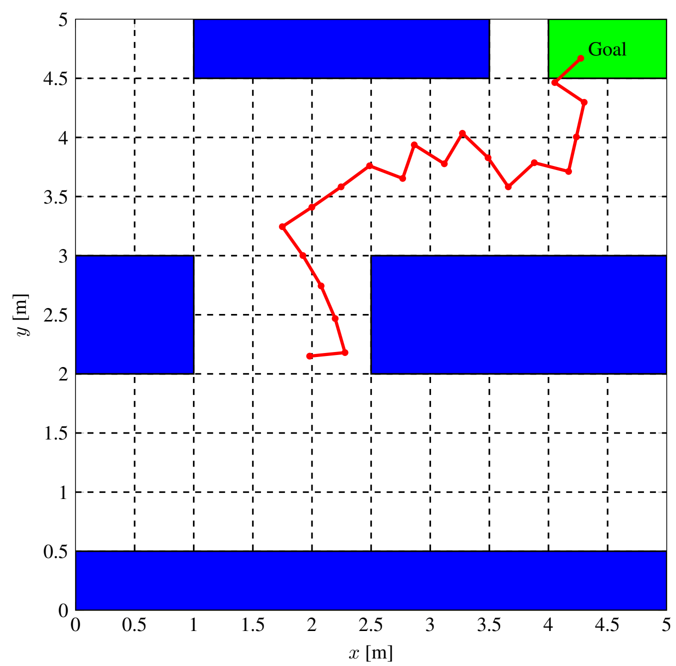
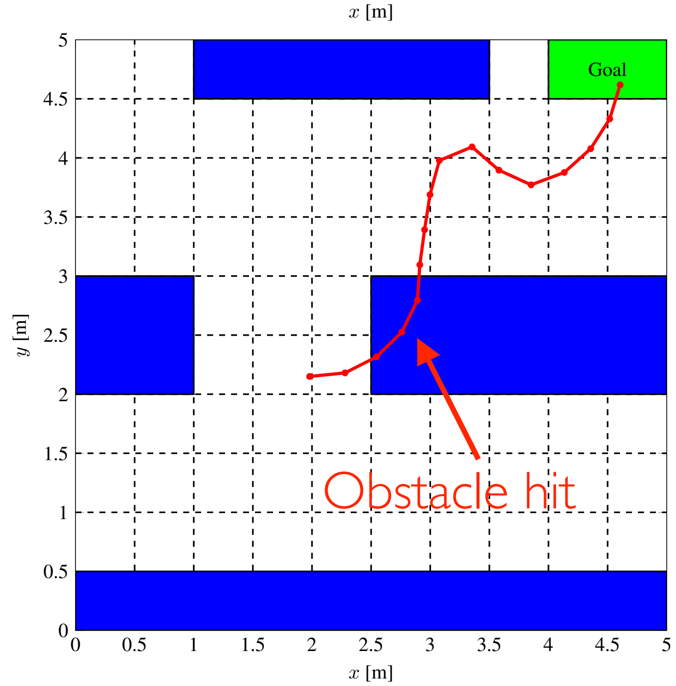
Formal NN Training



NN Training



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$$\zeta_x^{(t+\Delta t)} = \zeta_x^{(t)} + \Delta t \; v \; \cos(\theta^{(t)})$$

$$\zeta_y^{(t+\Delta t)} = \zeta_y^{(t)} + \Delta t \; v \; \sin(\theta^{(t)})$$

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- Safe data collected and used for training
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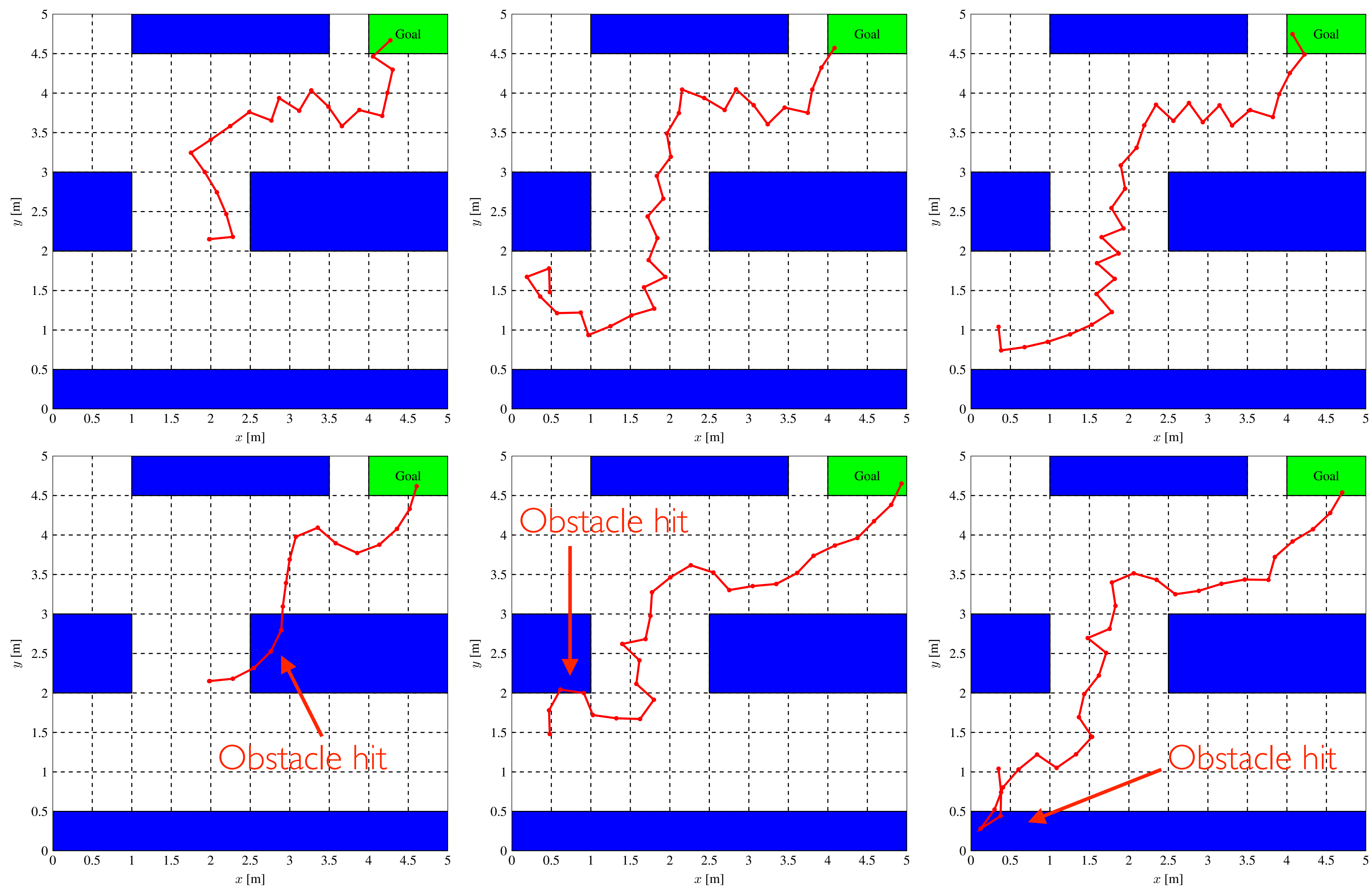
Formal NN Training

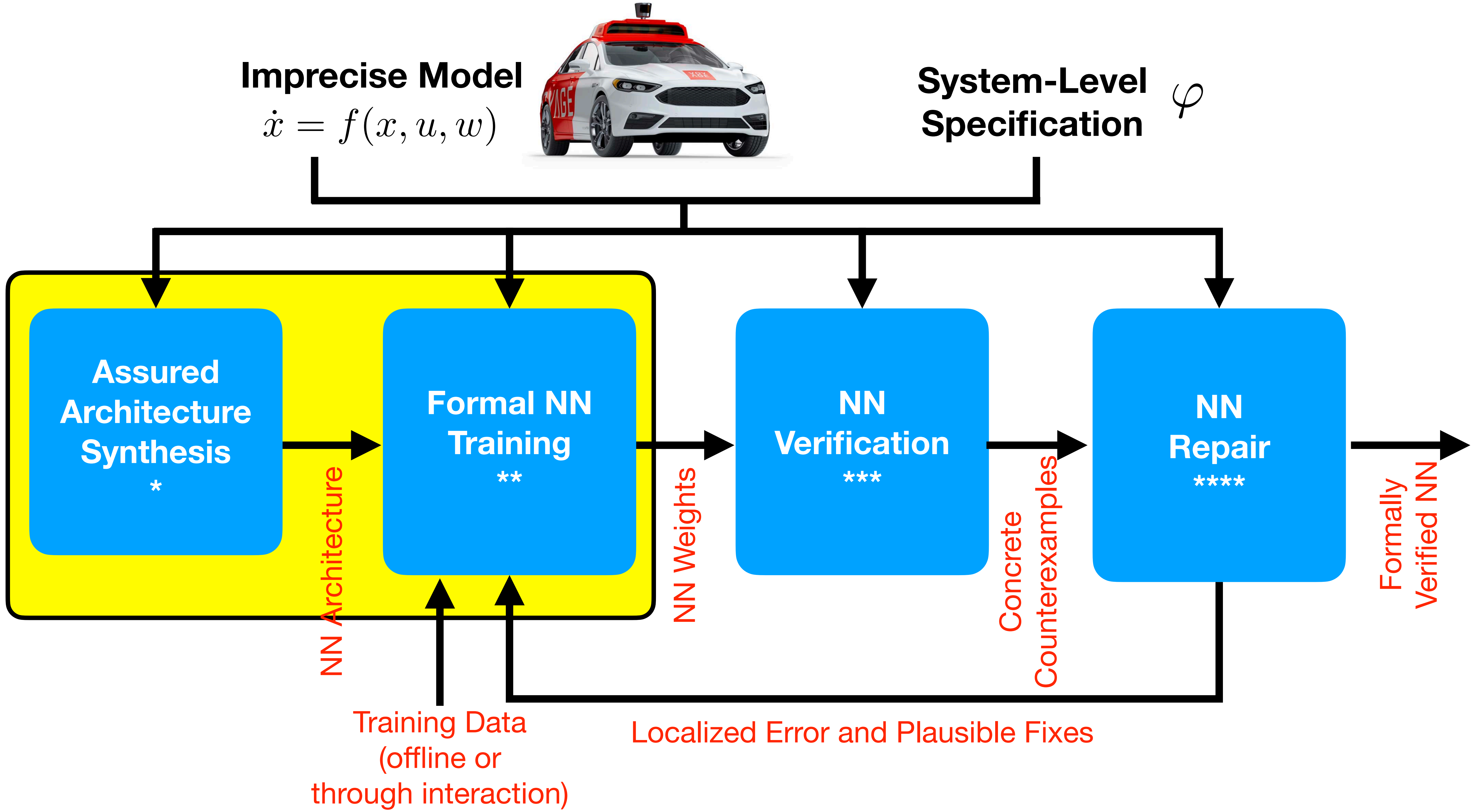
NN Training



Resilient CPS Lab

Workspace Index	Number of Abstract States	Number of Controller Partitions	Number of Safe & Reachable Abstract States	Compute Reachable Sets [s]	Construct Posterior Graph [s]	Compute Function P_{safe} [s]	Assign Controller Partitions [s]
1	552	160	400	52.6	82.3	0.06	0.7
1	552	320	400	107.5	160.3	0.1	0.9
1	552	640	400	223.1	329.6	0.2	1.7
1	1104	160	800	108.2	333.0	0.2	2.3
1	1104	320	800	219.6	684.2	0.4	2.7
1	1104	640	800	451.5	1297.4	0.6	4.2
2	904	160	632	88.1	159.1	0.1	1.0
2	904	320	632	203.6	313.2	0.2	1.1
2	904	640	632	393.2	660.8	0.3	1.7
2	1808	160	1264	202.1	634.6	0.3	3.4
2	1808	320	1264	388.6	1298.1	0.6	4.0
2	1808	640	1264	778.2	2564.4	0.9	5.9

System Dimension n	Number of Abstract States	Compute Reachable Sets [s]	Construct Posterior Graph [s]
2	69	0.6	0.7
4	276	2.7	2.6
6	1104	11.7	34.2
8	4416	57.1	521.0
10	17664	258.1	9840.4



* J. Ferlez, X. Sun, and Y. Shoukry, "Two-Level Lattice Neural Network Architectures for Control of Nonlinear Systems," CDC 2020.

* J. Ferlez and Y. Shoukry, "AReN: Assured ReLU NN Architecture for Model Predictive Control of LTI Systems," HSCC, 2020.

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*** H. Khedr, J. Ferlez, and Y. Shoukry, "PEREGRiNN: Penalized-Relaxation Greedy Neural Network Verifier,"CAV, 2021.

*** J. Ferlez and Y. Shoukry, "Bounding the Complexity of Formally Verifying Neural Networks: A Geometric Approach," arXiv 2020.

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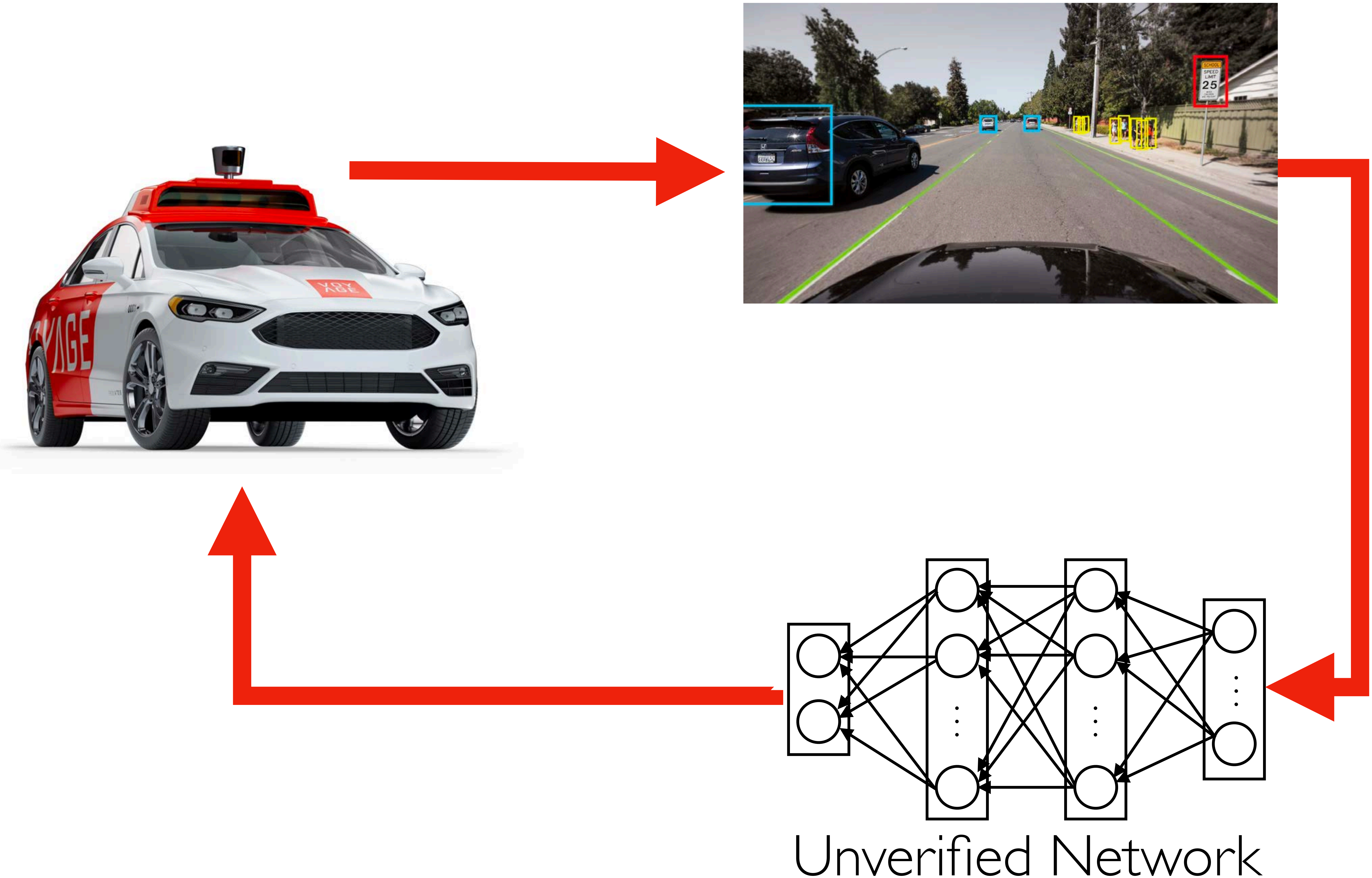
Synthesis of NN-based Safety Filters



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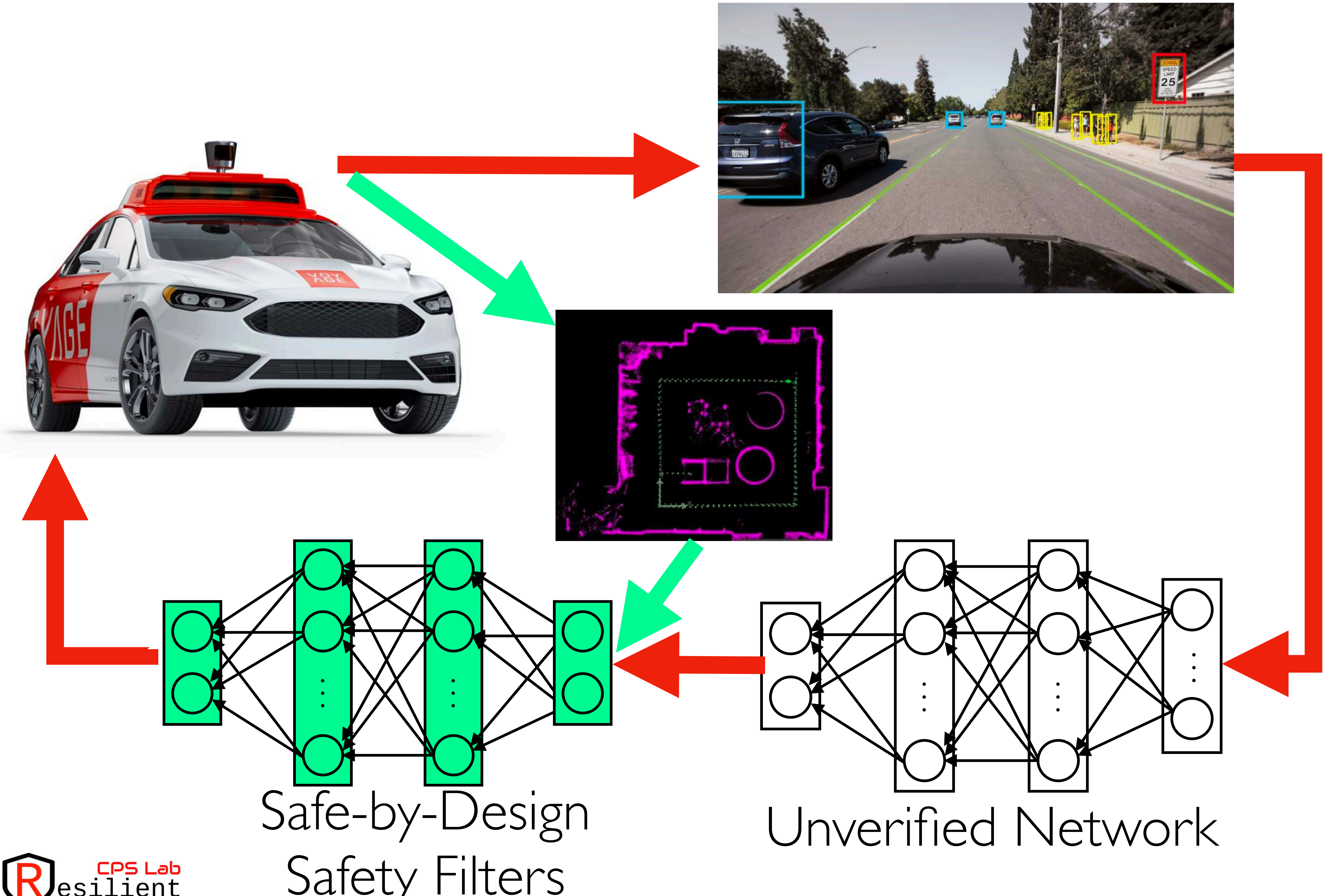
Synthesis of NN-based Safety Filters



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Synthesis of NN-based Safety Filters

Synthesis of NN-based Safety Filters

ROBUSTNESS DEMO #1

Agent #2

ShieldNN ON

Agent #3

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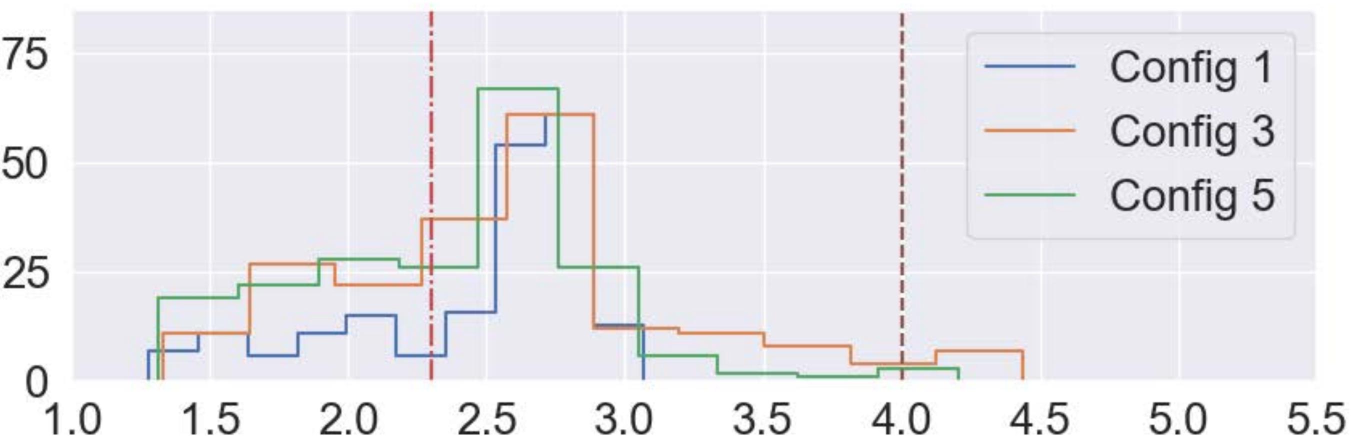
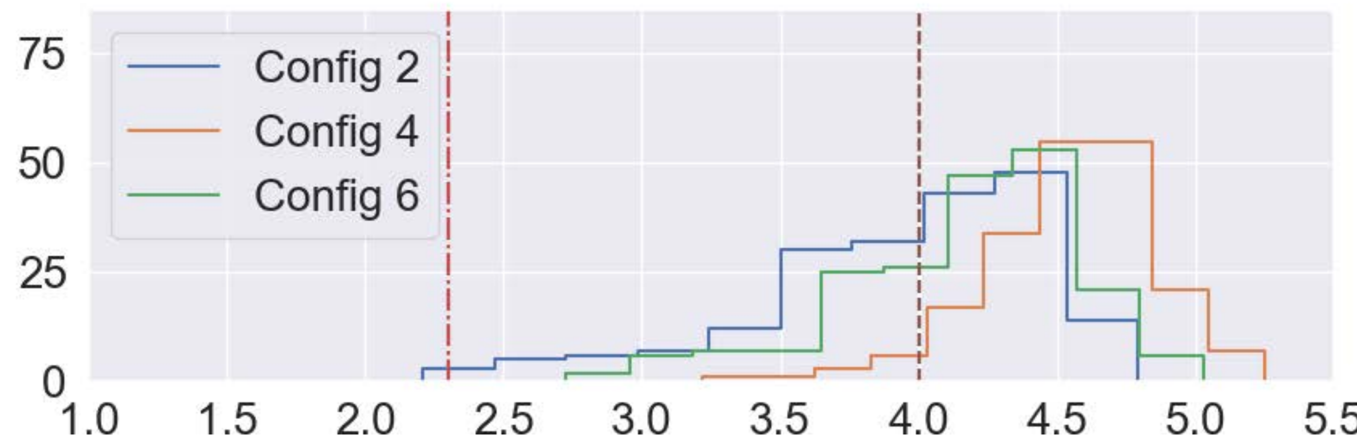
Synthesis of NN-based Safety Filters



Without Root-of-Trust Network



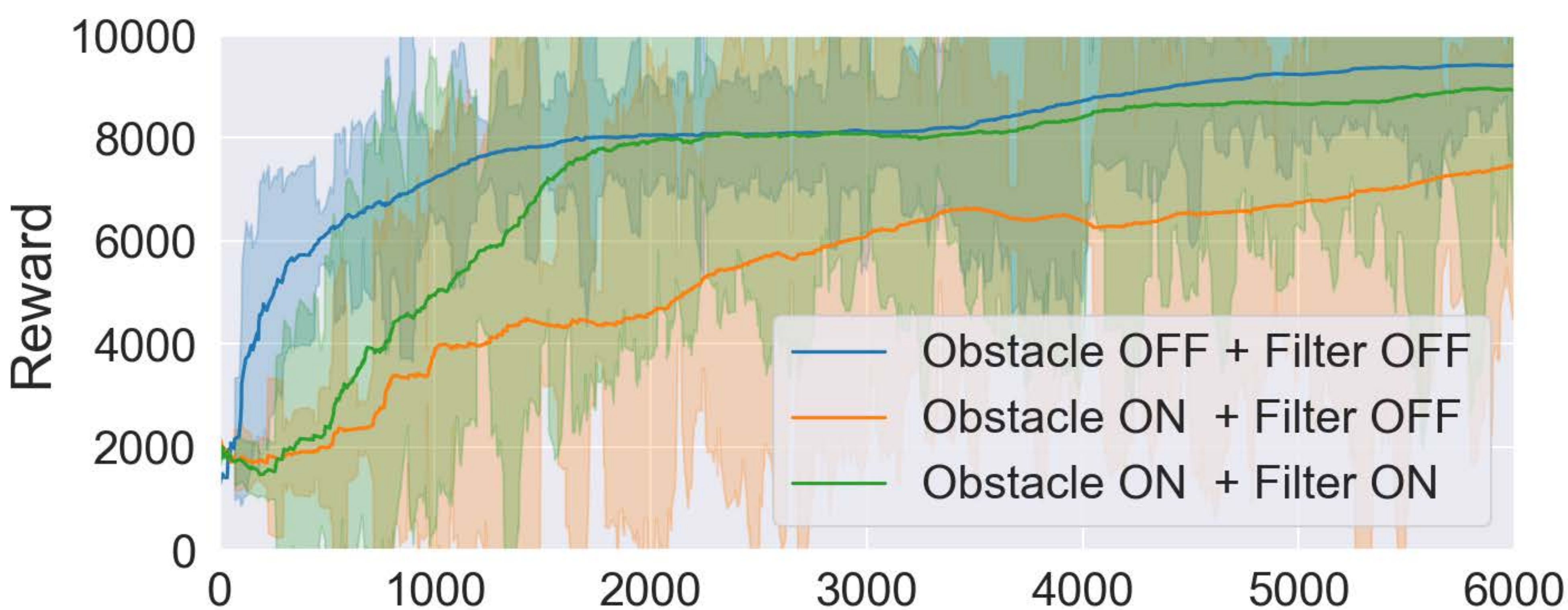
With Root-of-Trust Network

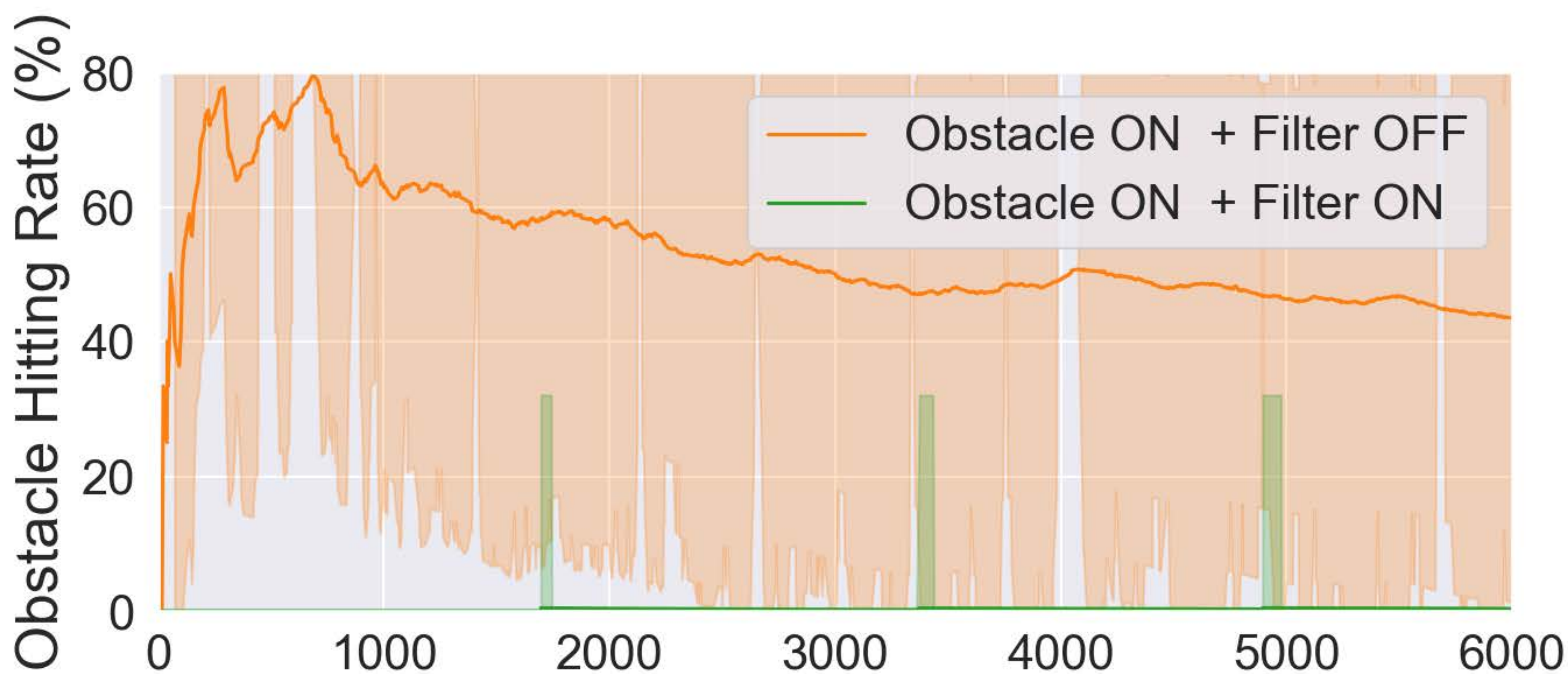
	Training		Testing	Experiment 1		Experiment 2	
Config	**Obstacle**	**Filter**	**Filter**	**TC%**[1]	**OHR%**[2]	**TC%**[1]	**OHR%**[2]
1	OFF	OFF	OFF	7.59	99.5	27.53	79.5
2	OFF	OFF	ON	98.82	0.5	98.73	0.5
3	ON	OFF	OFF	94.82	8.5	71.88	34
4	ON	OFF	ON	100	0	100	0
5	ON	ON	OFF	62.43	44	50.03	60
6	ON	ON	ON	100	0	100	0

[1] TC% := Track Completion % [2] OHR% := Obstacle Hit Rate %

- J. Ferlez, M. Raffat, C. Fleming, and Y. Shoukry, "ShieldNN: A Provably Safe NN Filter for Unsafe NNControllers", ArXiv 2020.

Synthesis of NN-based Safety Filters





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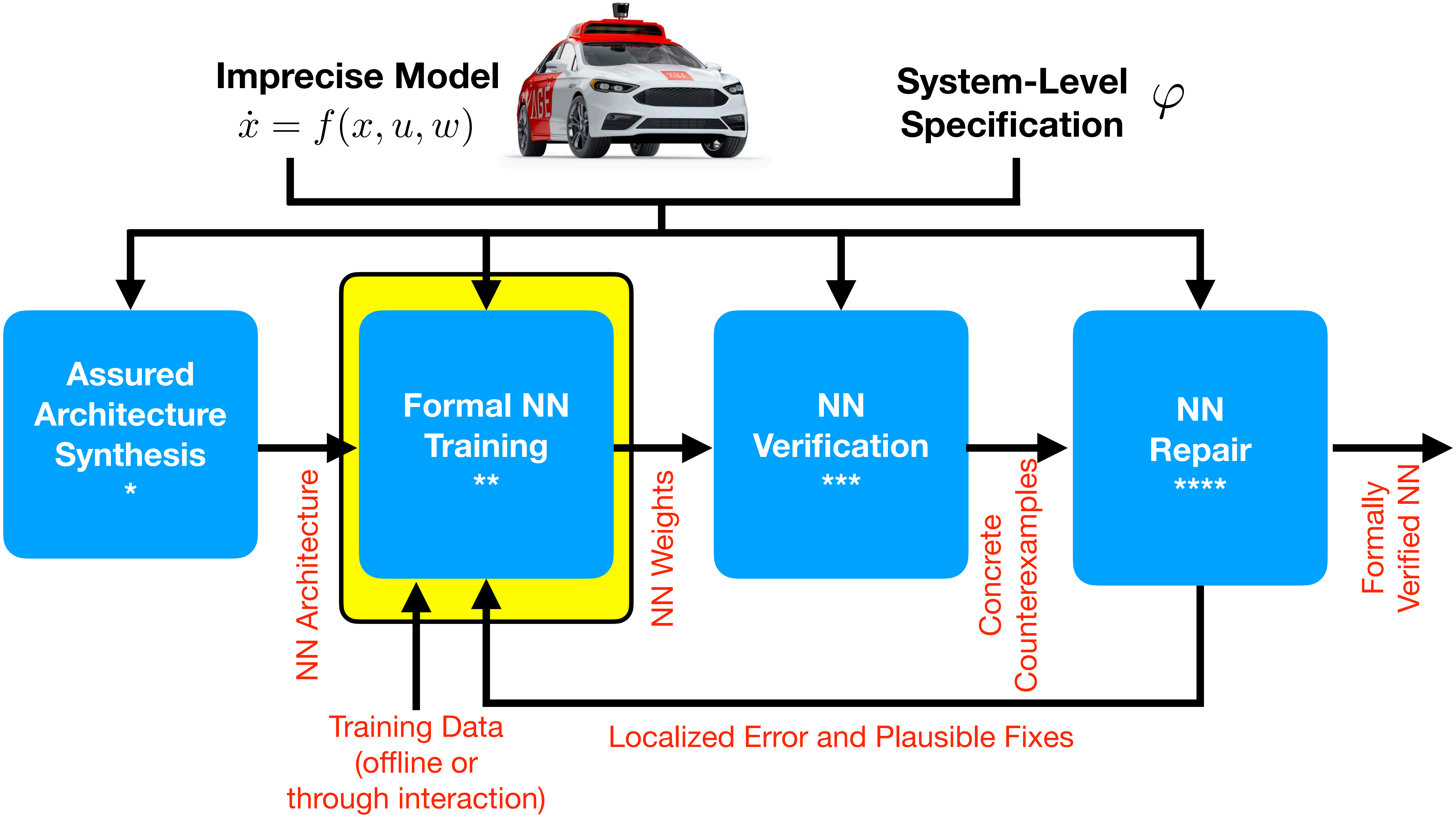

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Thanks !

Xiaowu Sun

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Wael Fatnassi

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Momina Sajid

Dr. James Ferlez

NSF

NORTHROP GRUMMAN

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