

# Dreamcoder: Bootstrapping Inductive Program Synthesis With Wake-Sleep Library Learning

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2021

Synthesis of Models and Systems

# The premise of program induction

1. Represent knowledge as programs: as symbolic code

# The premise of program induction

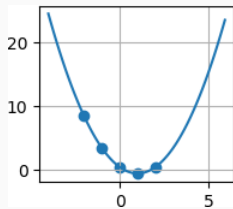
1. Represent knowledge as programs: as symbolic code
2. Learning=adding to that body of knowledge=  
making new programs=program synthesis

# Why program induction?



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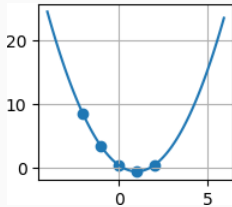
strong generalization  
+data efficiency



$$f(x) = (x-1)**2 - 0.5$$

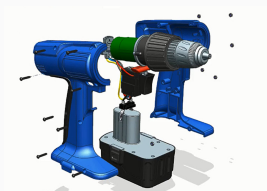
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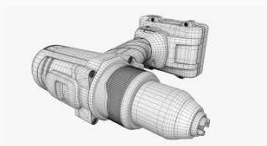


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interpretability

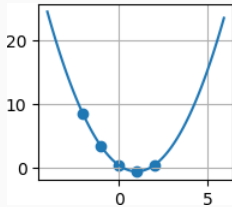


VS



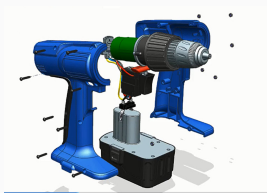
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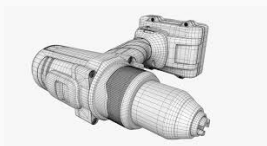


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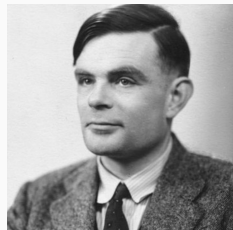
interpretability



VS



universal expressivity



# FlashFill (Gulwani 2012)

EXAMPLE 3 (Directory Name Extraction). Consider the following example taken from an excel online help forum.

Input $v_1$	Output
Company\Code\index.html	Company\Code\
Company\Docs\Spec\specs.doc	Company\Docs\Spec\

String Program:

$\text{SubStr}(v_1, \text{CPos}(0), \text{Pos}(\text{SlashTok}, \epsilon, -1))$

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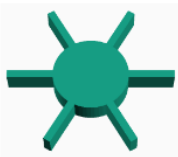
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# Szalinski (Nandi 2020)



(a) CAD model of ship's wheel

```
(Union  
(Cylinder [1, 5, 5])  
(Fold Union  
(Tabulate (i 6)  
(Rotate [0, 0, 60i]  
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```

(b) Caddy program

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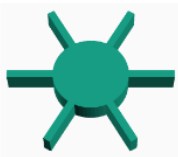
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Company\Docs\Spec\specs.doc	Company\Docs\Spec\

String Program:

$SubStr(v_1, CPos(0), Pos(SlashTok, \epsilon, -1))$

String expr  $P ::= Switch((b_1, e_1), \dots, (b_n, e_n))$   
 Bool  $b ::= d_1 \vee \dots \vee d_n$   
 Conjunction  $d ::= \pi_1 \wedge \dots \wedge \pi_n$   
 Predicate  $\pi ::= Match(v_i, r, k) \mid \neg Match(v_i, r, k)$   
 Trace expr  $e ::= Concatenate(f_1, \dots, f_n)$   
 Atomic expr  $f ::= SubStr(v_i, p_1, p_2)$   
                    $\mid ConstStr(s)$   
                    $\mid Loop(\lambda w : e)$   
 Position  $p ::= CPos(k) \mid Pos(r_1, r_2, c)$   
 Integer expr  $c ::= k \mid k_1 w + k_2$   
 Regular Expression  $r ::= TokenSeq(T_1, \dots, T_m)$   
 Token  $T ::= C + \mid [-C] +$   
                    $\mid SpecialToken$

# Szalinski (Nandi 2020)



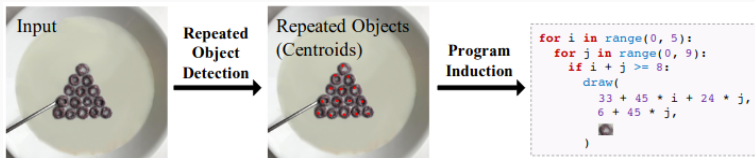
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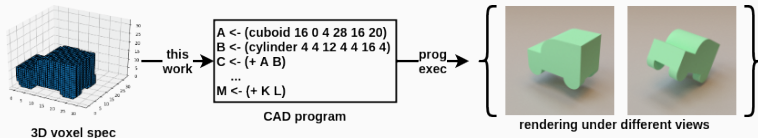
(b) Caddy program

op ::= + | - | × | /    num ::=  $\mathbb{R} \mid \langle var \rangle \mid \langle num \rangle \langle op \rangle \langle num \rangle$   
 vec2 ::= [ $\langle num \rangle$ ,  $\langle num \rangle$ ]    vec3 ::= [ $\langle num \rangle$ ,  $\langle num \rangle$ ,  $\langle num \rangle$ ]  
 affine ::= Translate | Rotate | Scale | **TranslateSpherical**  
 binop ::= Union | Difference | Intersection  
 cad ::= (Cuboid (vec3)) | (Sphere (num))  
           | (Cylinder (vec2)) | (HexPrism (vec2)) | ...  
           | ((affine) (vec3) (cad))  
           | ((binop) (cad) (cad))  
           | (**Fold** (binop) (cad-list))  
 cad-list ::= (List (cad)+)  
               | (Concat (cad-list)+)  
               | (Tabulate ((var) Z<sup>+</sup>) (cad))  
               | (Map2 (affine) (vec3-list) (cad-list))  
 vec3-list ::= (List (vec3)+)  
               | (Concat (vec3-list)+)  
               | (Tabulate ((var) Z<sup>+</sup>) (vec3))

# Visual programs



Mao\*, Zhang\*, et al 2019




Ellis\*, Nye\*, Pu\*, Sosa\*, et al 2019



partial image  $x_{part}$

```

for i = 1..3
  for j = 1..1
    draw(i*2, j*1, )
    ...
    
```

synthesized program  $P_{part}$



```

Draw("Top", "Circle", position, geometry)

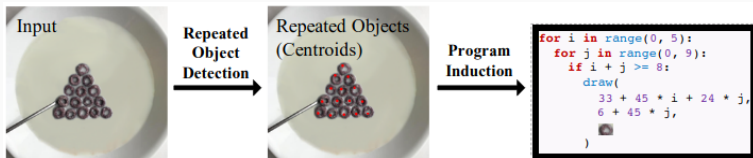
for(i < 2, "translation", a)
  for(j < 2, "translation", b)
    Draw("Leg", "Cub", position + i*a + j*b, c)

for(i < 2, "translation", c)
  Draw("Layer", "Rec", position + i*c, geomet
    
```

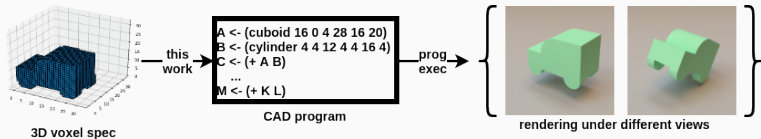
Young et al 2019

Tian et al 2019

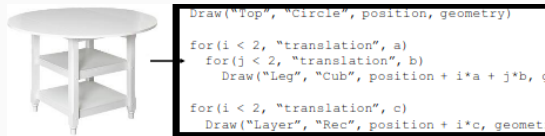
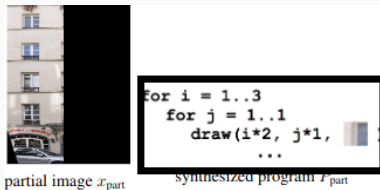
# Where does this language come from?



Mao\*, Zhang\*, et al 2019



Ellis\*, Nye\*, Pu\*, Sosa\*, et al 2019



Young et al 2019

Tian et al 2019



## Program Induction and learning to learn

learning a DSL

learning to synthesize

synergy between DSL+learned synthesizer

Goal: acquire domain-specific knowledge needed to induce a class of programs

- Library of abstractions (domain specific language)
- Inference strategy (synthesis algorithm)

# Library learning

## Initial Primitives

·  
·  
map  
fold  
if  
cons  
>  
·  
·

## Sample Problem: sort list

[9 2 7 1] → [1 2 7 9]  
[3 8 9 4 2] → [2 3 4 8 9]  
[6 2 2 3 8 5] → [2 2 3 5 6 8]  
...

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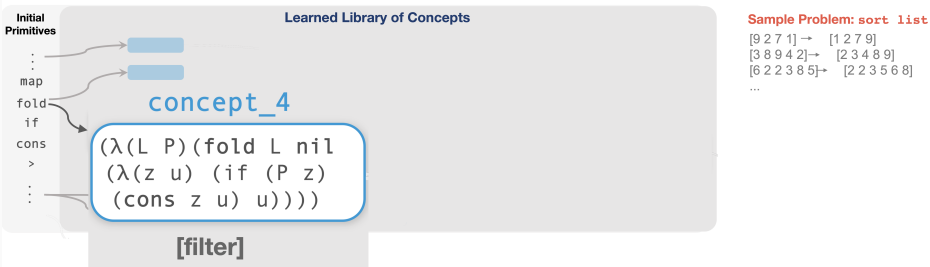
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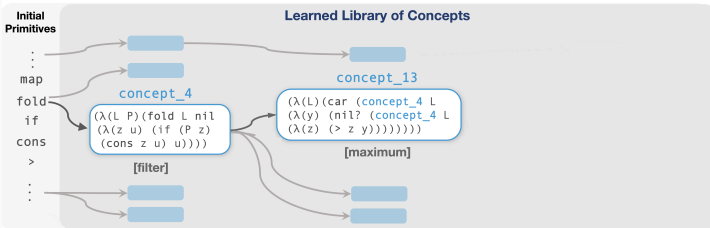


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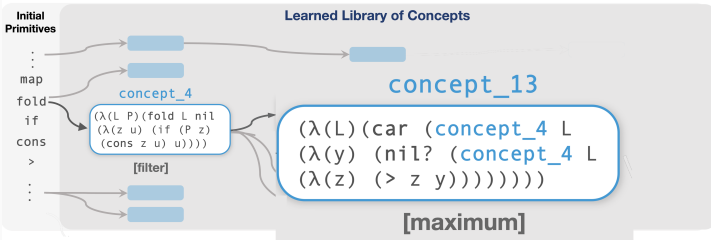
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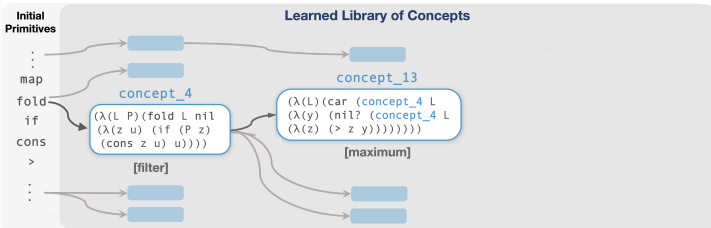
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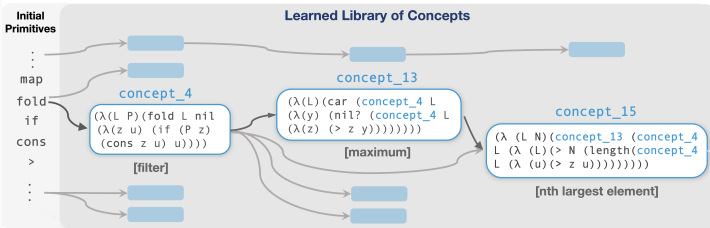
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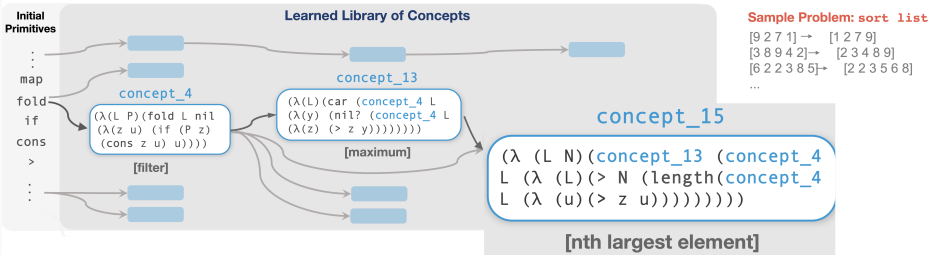
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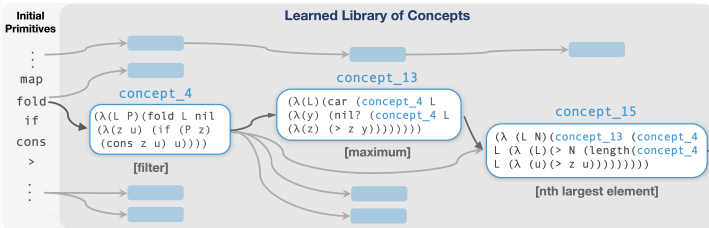
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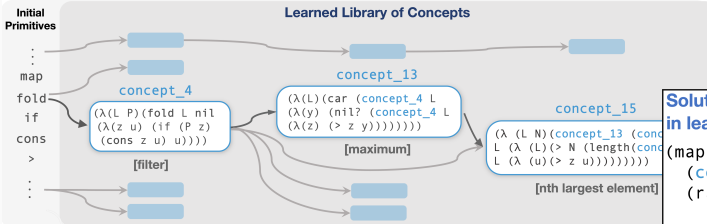
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**Solution to sort list discovered in learned language:**

```
(map (λ (n)
      (concept_15 L (+ 1 n)))
     (range (length L)))
```

# Library learning



**Sample Problem: sort list**

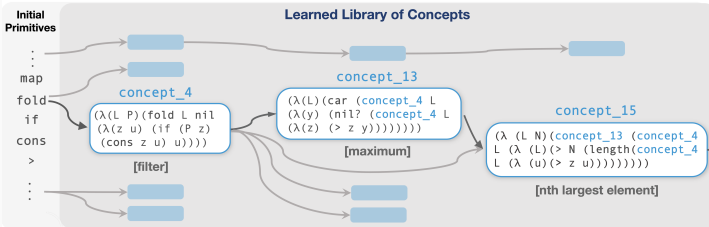
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get Nth largest element,  
where N is 1, 2, 3, ...

# Library learning



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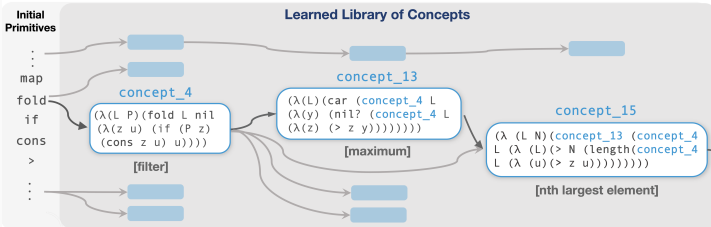
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## Solution rewritten in initial primitives:

```
(lambda (x) (map (lambda (y) (car (fold (fold x nil (lambda (z u) (if (gt? (+ y 1) (length (fold x nil (lambda (v w) (if (gt? z v) (cons v w) w)))) (cons z u) u)) nil (lambda (a b) (if (nil? (fold (fold x nil (lambda (c d) (if (gt? (+ y 1) (length (fold x nil (lambda (e f) (if (gt? c e) (cons e f) f)))) (cons c d) d)) nil (lambda (g h) (if (gt? g a) (cons g h) h))) (cons a b) b)))))) (range (length x))))
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# Library learning



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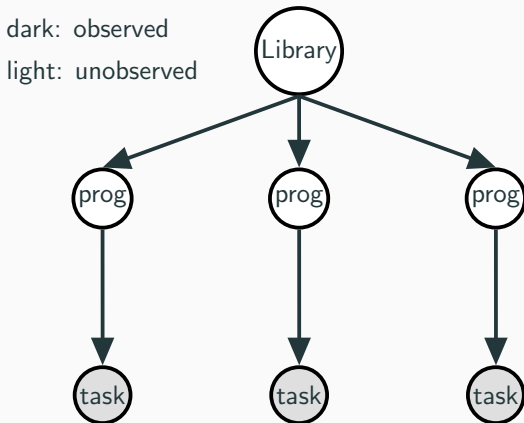
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```

induced sort program found in  $\leq 10$ min. Brute-force search without learned library would take  $\approx 10^{73}$  years

- **Wake:** Solve problems by writing programs
- **Sleep:** Improve library and neural recognition model:
  - **Abstraction sleep:** Improve library
  - **Dream sleep:** Improve neural recognition model

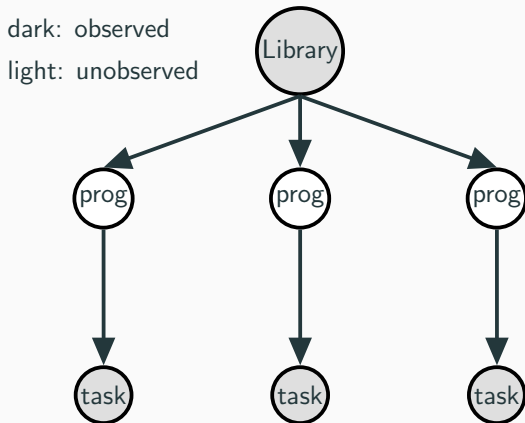
cf. Helmholtz machine, wake/sleep neural network training algorithms

# Library learning as Bayesian inference



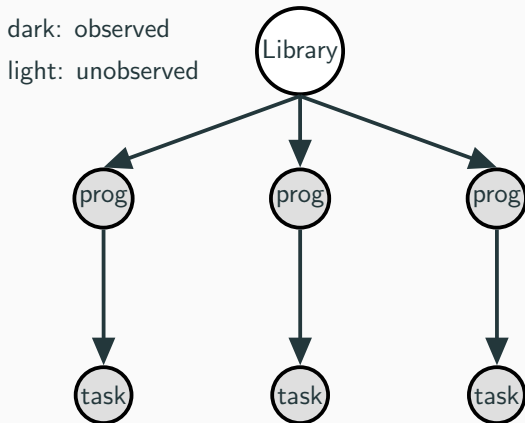
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# Library learning as Bayesian inference



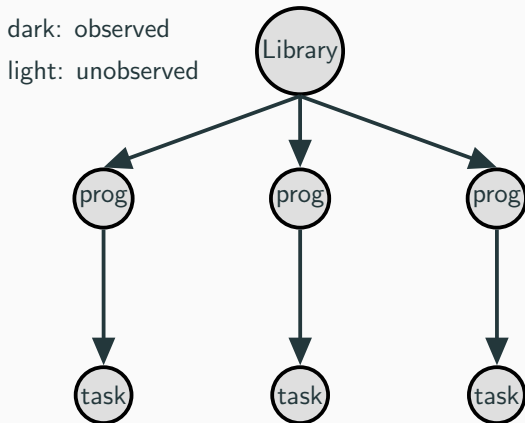
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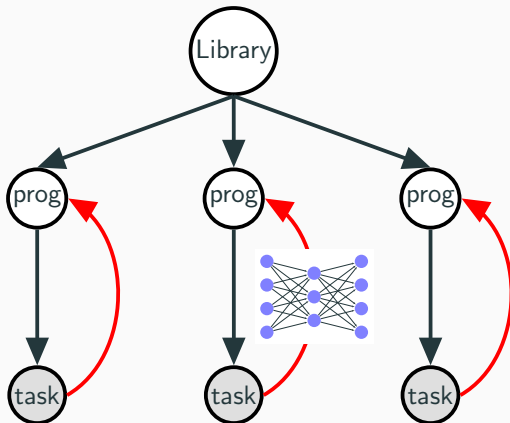
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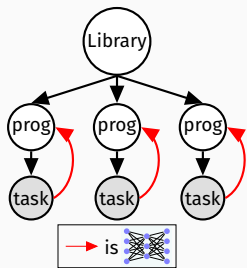


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# Library learning as **neurally-guided** Bayesian inference

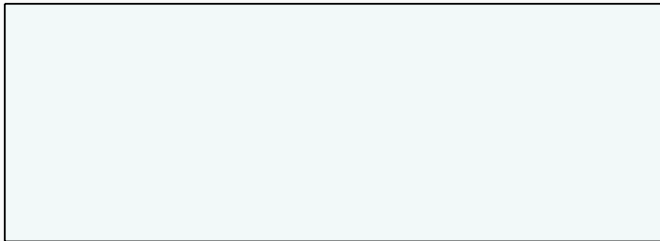


library learning via program analysis +  
new neural inference network for program synthesis +  
better program representation (Lisp+polymorphic types [Milner 1978])

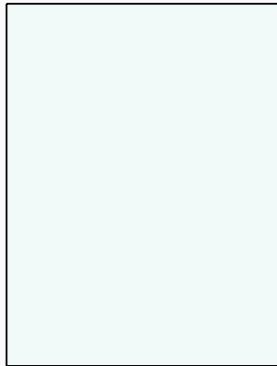




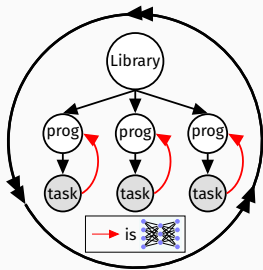
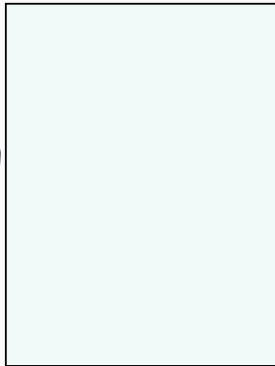
WAKE

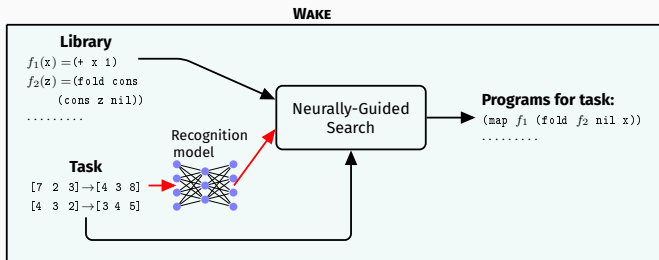


SLEEP: ABSTRACTION



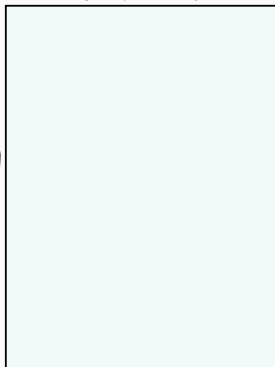
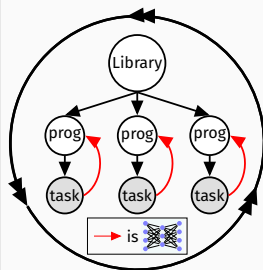
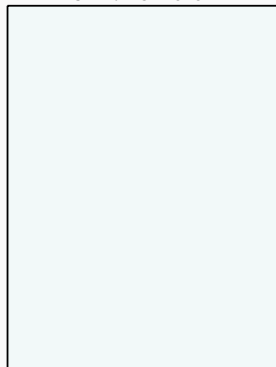
SLEEP: DREAMING

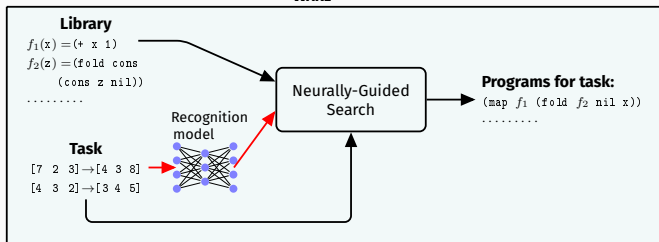
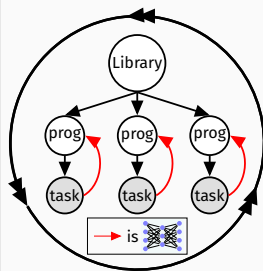
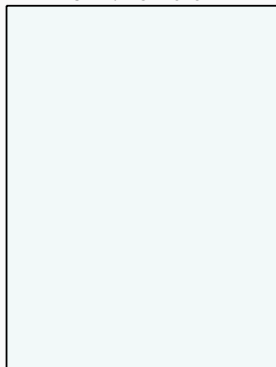
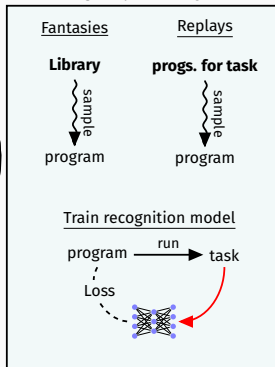


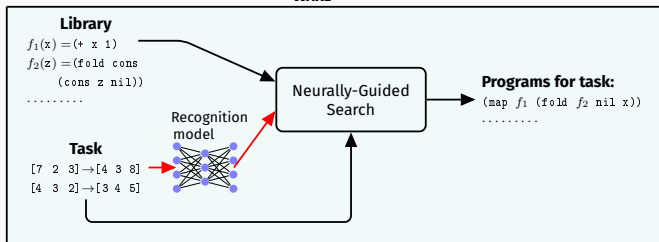
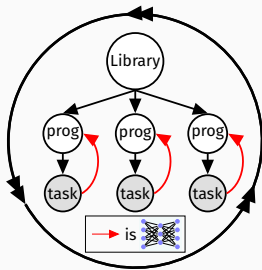
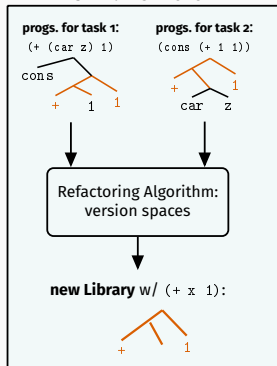
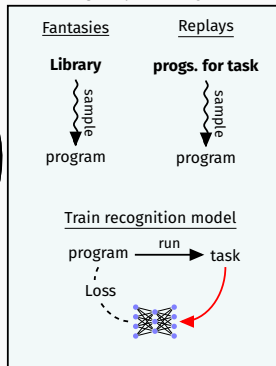


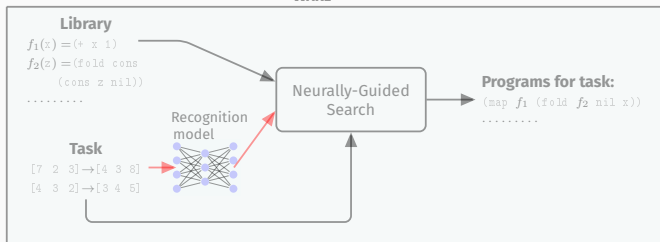
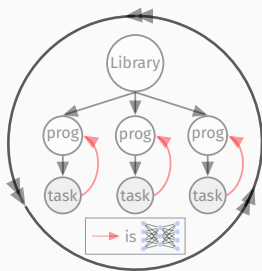
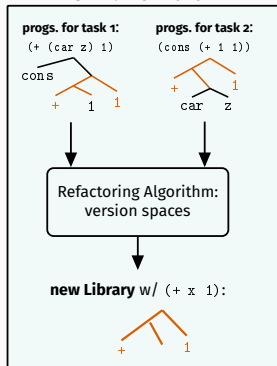
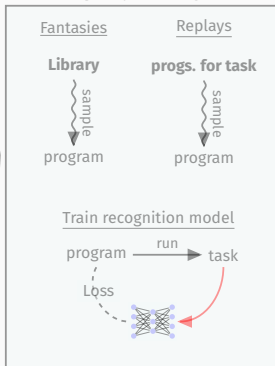
**SLEEP: ABSTRACTION**

**SLEEP: DREAMING**



**WAKE****SLEEP: ABSTRACTION****SLEEP: DREAMING**

**WAKE****SLEEP: ABSTRACTION****SLEEP: DREAMING**

**SLEEP: ABSTRACTION****SLEEP: DREAMING**

Program Induction and learning to learn  
learning a DSL  
learning to synthesize  
synergy between DSL+learned synthesizer

# Abstraction Sleep: Growing the library via refactoring

Task: [1 2 3] → [2 4 6]  
[4 3 4] → [8 6 8]

Task: [1 2 3] → [0 1 2]  
[4 3 4] → [3 2 3]

# Abstraction Sleep: Growing the library via refactoring

Task: [1 2 3] → [2 4 6]  
[4 3 4] → [8 6 8]

Wake: program search

```
(Y (λ (r l) (if (nil? l) nil  
  (cons (+ (car l) (car l))  
    (r (cdr l))))))
```

Task: [1 2 3] → [0 1 2]  
[4 3 4] → [3 2 3]

Wake: program search

```
(Y (λ (r l) (if (nil? l) nil  
  (cons (- (car l) 1)  
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```



# Abstraction Sleep: Growing the library via refactoring

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```

refactor  
(10<sup>14</sup> refactorings)

```
((λ (f) (Y (λ (r l) (if (nil? l)  
  nil  
  (cons (f (car l))  
        (r (cdr l)))))))  
(λ (z) (+ z z)))
```

Task: [1 2 3]→[0 1 2]  
[4 3 4]→[3 2 3]

Wake: program search

```
(Y (λ (r l) (if (nil? l) nil  
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(10<sup>14</sup> refactorings)

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Sleep: Abstraction

# Abstraction Sleep: Growing the library via refactoring

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refactor

(10<sup>14</sup> refactorings)

```
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  nil  
  (cons (f (car l))  
    (r (cdr l)))))))  
(λ (z) (- z 1)))
```

Sleep: Abstraction

Compress (MDL/Bayes objective)

```
(MAP (λ (z) (+ z z))) (MAP (λ (z) (- z 1)))
```

```
MAP = (λ (f) (Y (λ (r l) (if (nil? l)  
  nil  
  (cons (f (car l))  
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```

# Abstraction Sleep: Growing the library via refactoring

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Wake: program search

```
(Y (λ (r l) (if (nil? l) nil  
  (cons (- (car l) 1)  
        (r (cdr l))))))
```

these  $10^{14}$  refactorings represented in exponentially more efficient refactoring data structure:

equivalence graphs+version spaces using  $10^6$  nodes, calculated in under 5min

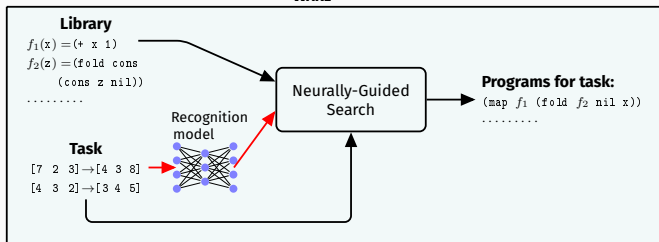
c.f. [Tate et al 2009], [Gulwani 2012]

Compress (MDL/Bayes objective)

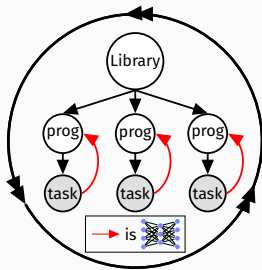
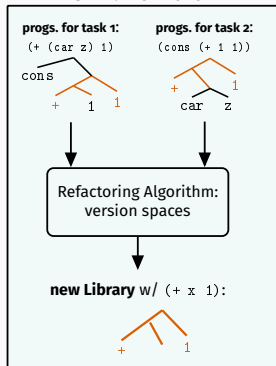
```
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MAP = (λ (f) (Y (λ (r l) (if (nil? l) nil  
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```

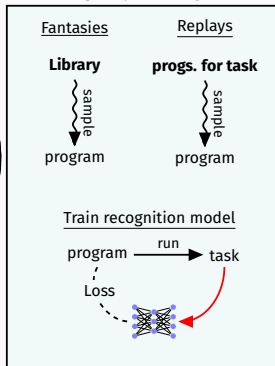
Program Induction and learning to learn  
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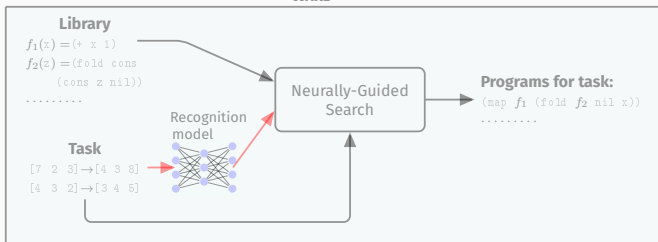
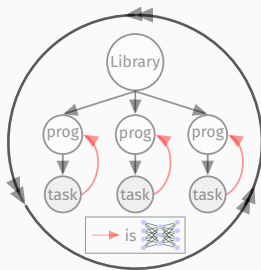
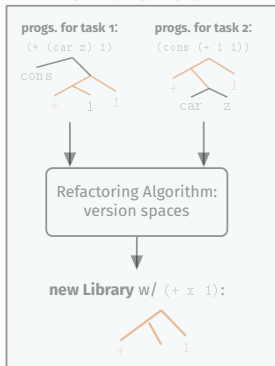
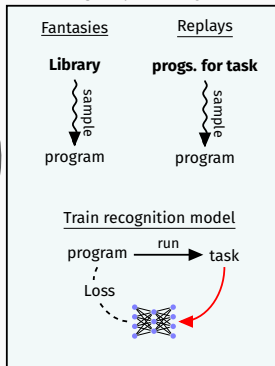


## SLEEP: ABSTRACTION

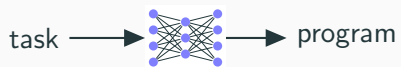


## SLEEP: DREAMING

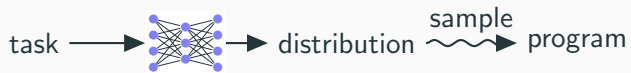


**SLEEP: ABSTRACTION****SLEEP: DREAMING**

# Neural recognition model guides search

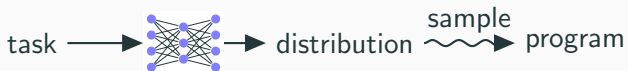


# Neural recognition model guides search





# Neural recognition model guides search

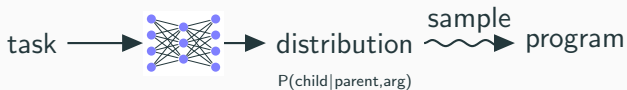


is a...

recurrent network (Devlin et al 2017)

unigram model (Menon et al 2013; Balog et al 2016)

# Neural recognition model guides search

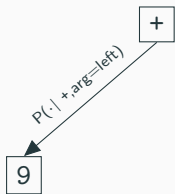


is a **“bigram”** model over syntax trees

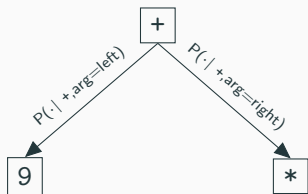
# Neural recognition model guides search



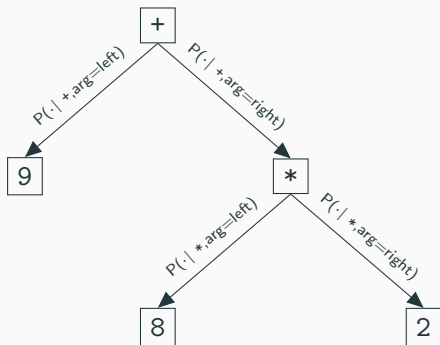
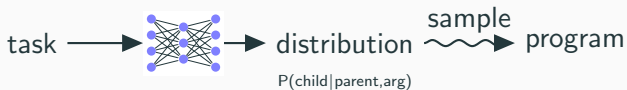
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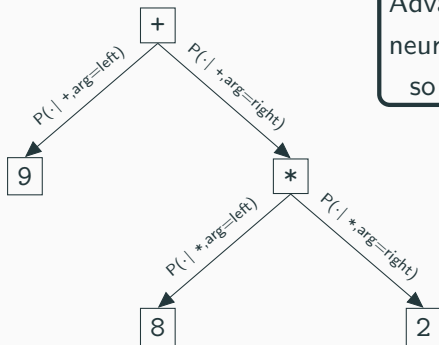
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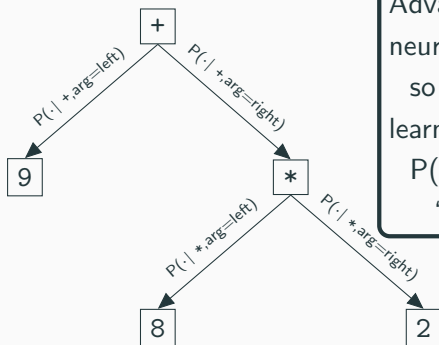
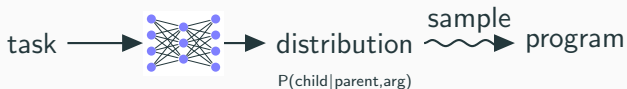
# Neural recognition model guides search



Advantages:

neural net runs once per task,  
so CPU bottlenecks instead of GPU

# Neural recognition model guides search



## Advantages:

neural net runs once per task,  
so CPU bottlenecks instead of GPU  
learns to break syntactic symmetries:

$$P(1|*,\text{arg}=\text{left})=0.0$$

“do not multiply by one”



Program Induction and learning to learn  
learning a DSL  
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synergy between DSL+learned synthesizer

# DreamCoder Domains

## List Processing

### Sum List

[1 2 3] → 6  
[4 6 8 1] → 17

### Double

[1 2 3] → [2 4 6]  
[4 5 1] → [8 10 2]

## Text Editing

### Abbreviate

Allen Newell → A.N.  
Herb Simon → H.S.

### Drop Last Three

shrdlu → shr  
shakey → sha

## Regexes

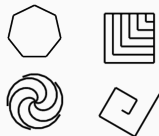
### Phone numbers

(555) 867-5309  
(650) 555-2368

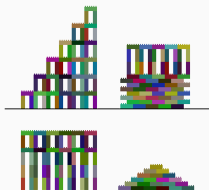
### Currency

\$100.25  
\$4.50

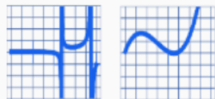
## LOGO Graphics



## Block Towers



## Symbolic Regression



$$y = f(x)$$

## Recursive Programming

### Filter Red

[■ ■ ■ ■ ■] → [■ ■]  
[■ ■ ■ ■ ■] → [■ ■ ■ ■ ■]  
[■ ■ ■ ■ ■] → [■ ■ ■ ■ ■]

## Physical Laws

$$\vec{a} = \frac{1}{m} \sum_i \vec{F}_i$$

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r}|^2} \hat{r}$$

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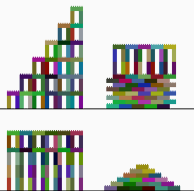
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\$4.50

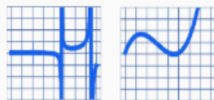
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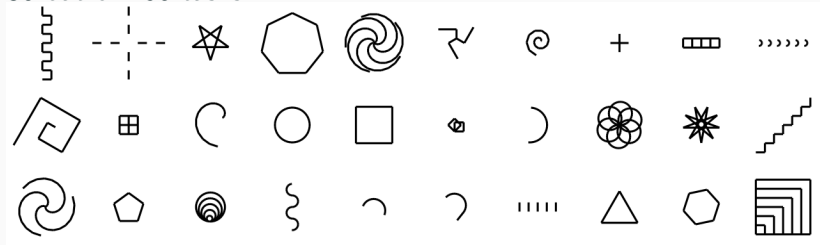
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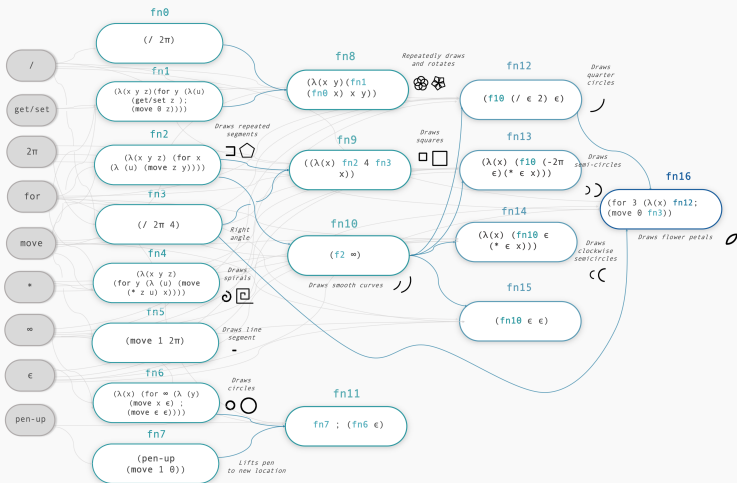
$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r}|^2} \hat{r}$$

# LOGO Turtle Graphics

30 out of 160 tasks



# LOGO Turtle Graphics – learning an interpretable library



`(fn8 5 (fn4 (* ε 2) ∞ ε))`



`(for 7 (λ(x) (fn9 x)))`

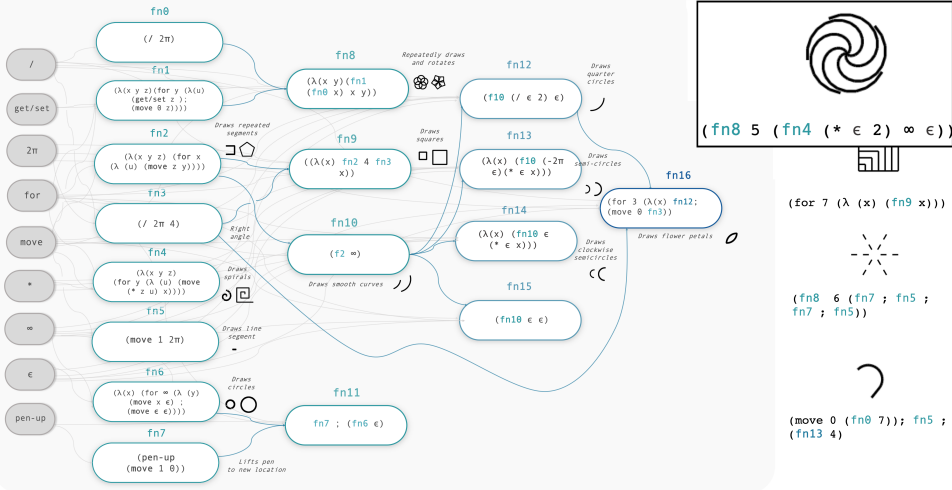


`(fn8 6 (fn7 ; fn5 ; fn7 ; fn5))`

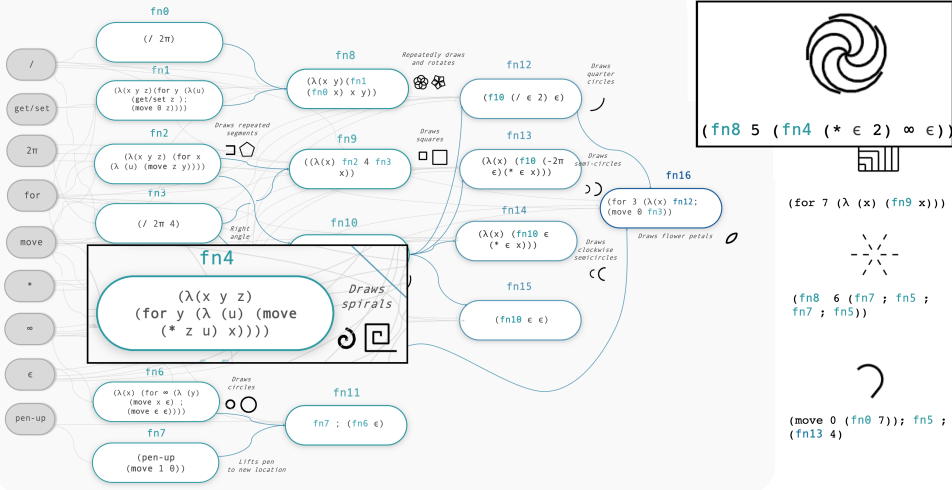


`(move 0 (fn0 7)); fn5 ; (fn13 4)`

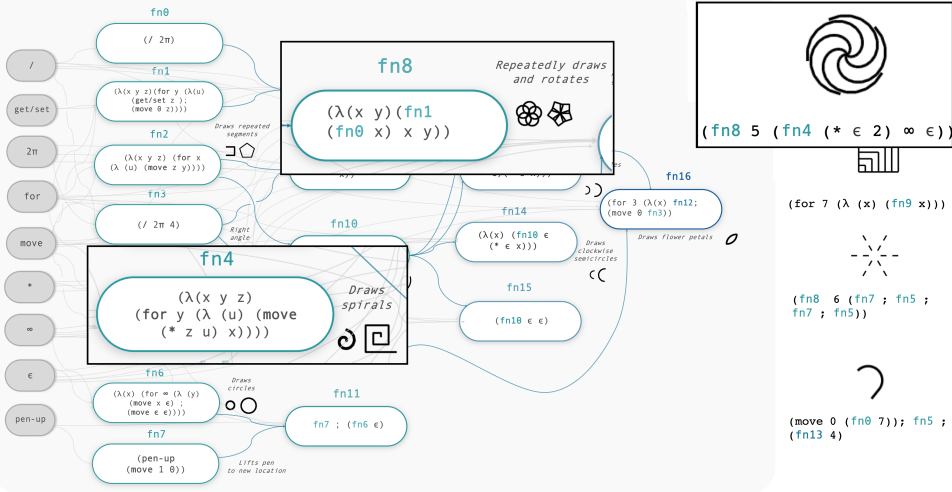
# LOGO Turtle Graphics – learning an interpretable library



# LOGO Turtle Graphics – learning an interpretable library

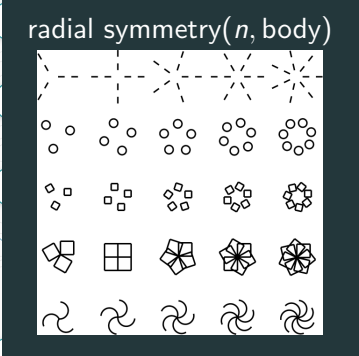
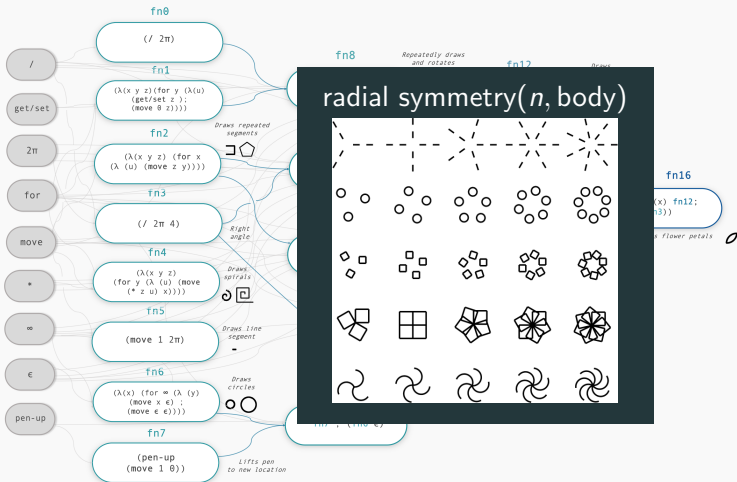


# LOGO Turtle Graphics – learning an interpretable library





# LOGO Turtle Graphics – learning an interpretable library



fn16  
(x) fn12;  
fn3))  
flower petals

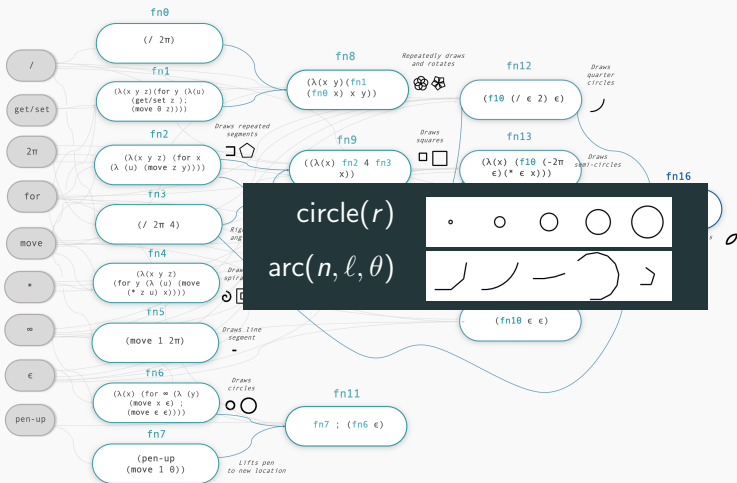
(fn8 5 (fn4 (\* € 2) ∞ €))

(for 7 (λ (x) (fn9 x)))

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(move 0 (fn0 7)); fn5 ; (fn13 4)

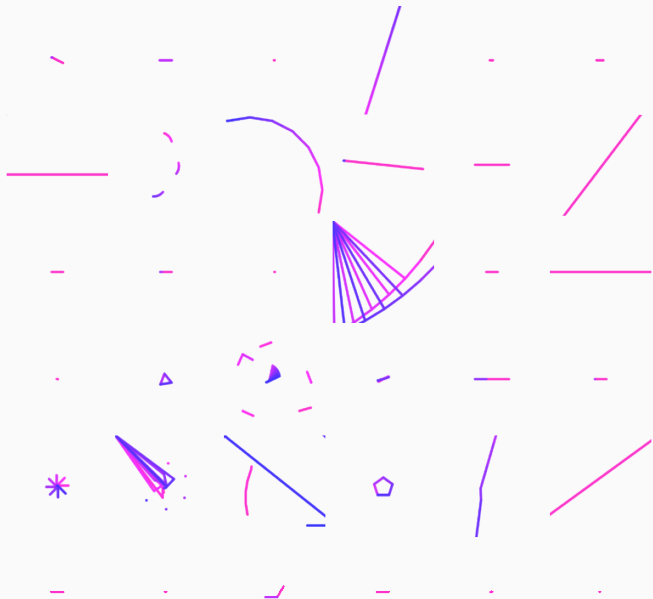
# LOGO Turtle Graphics – learning an interpretable library



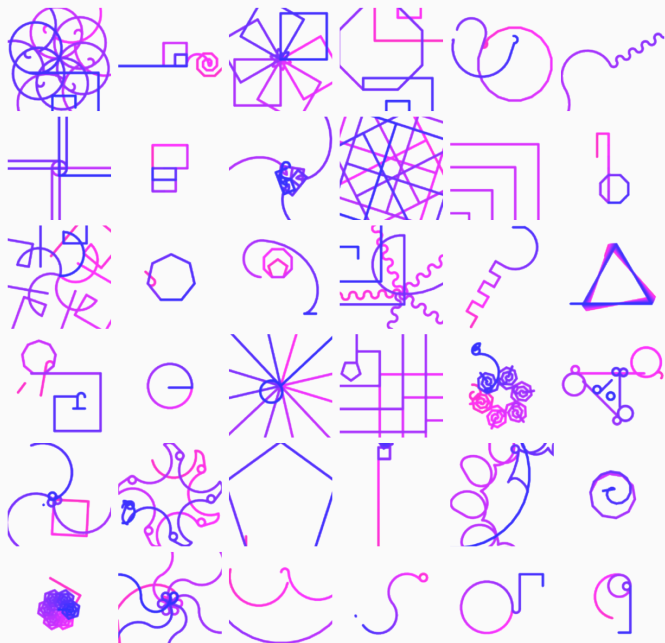
Graphical outputs and their corresponding function calls:

- $(\text{fn8 } 5 (\text{fn4 } (* \epsilon 2) \infty \epsilon))$
- $(\text{for } 7 (\lambda(x) (\text{fn9 } x)))$
- $(\text{fn8 } 6 (\text{fn7 } ; \text{fn5 } ; \text{fn7 } ; \text{fn5}))$
- $(\text{move } 0 (\text{fn0 } 7)); \text{fn5 } ; (\text{fn13 } 4)$

# What does DreamCoder dream of? (before learning)

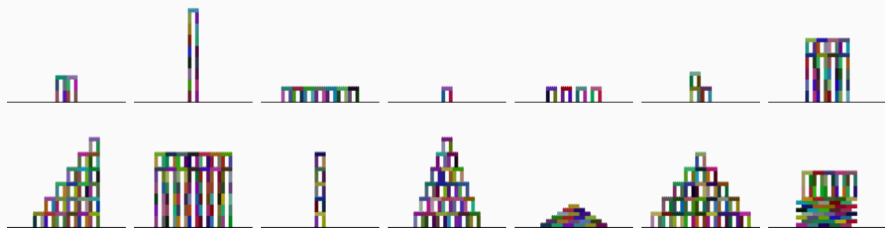


## What does DreamCoder dream of? (after learning)



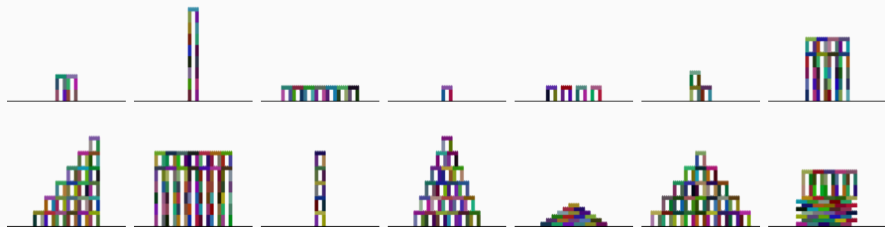
# Planning to build towers

example tasks (112 total)

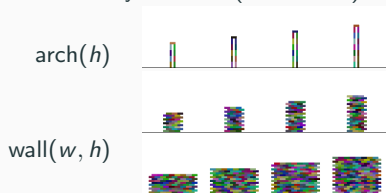


# Planning to build towers

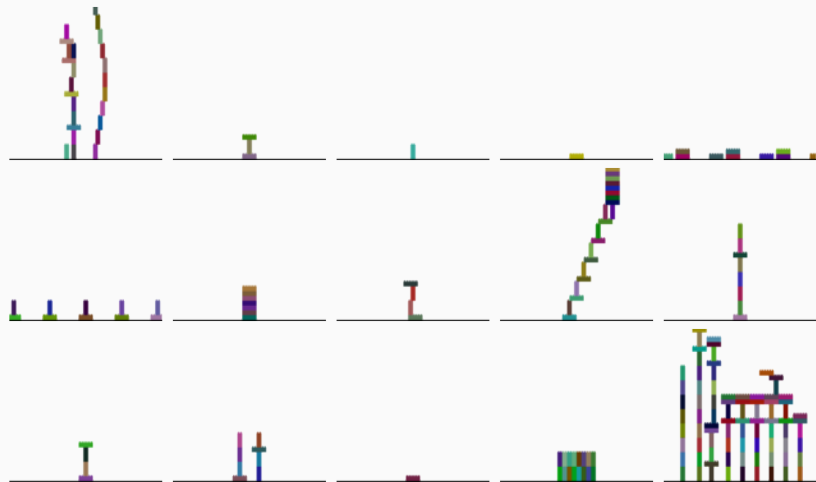
example tasks (112 total)



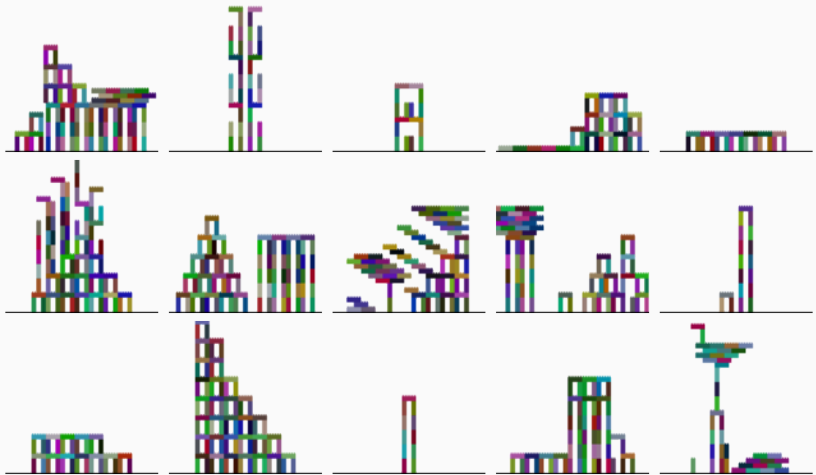
learned library routines ( $\approx 20$  total)



# Dreams before learning

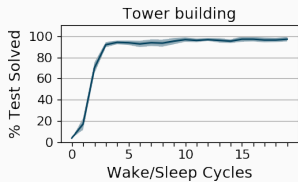


# Dreams after learning

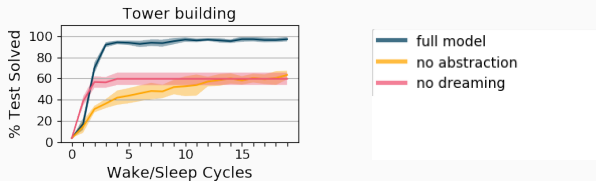




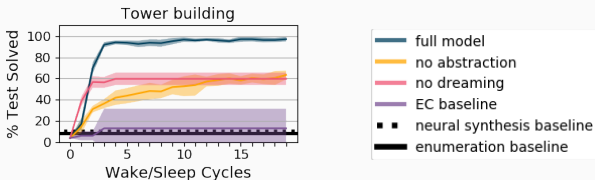
# Learning dynamics



# Learning dynamics

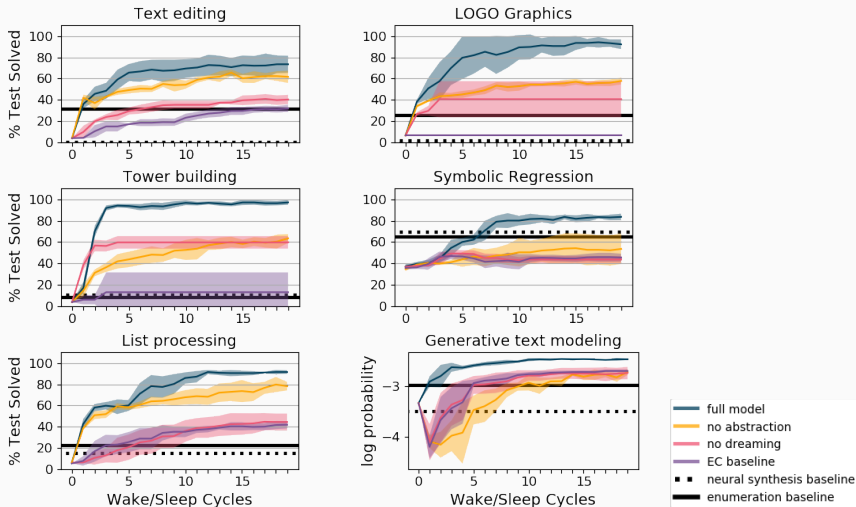


# Learning dynamics



baselines: Exploration-Compression, EC [Dechter et al. 2013]  
neural program synthesis, RobustFill [Devlin et al. 2017]  
24 hours of brute-force enumeration

# Learning dynamics



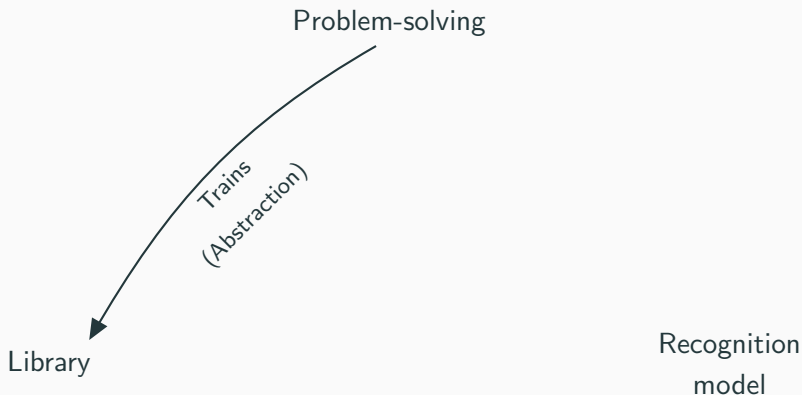
# Synergy between recognition model and library learning

Problem-solving

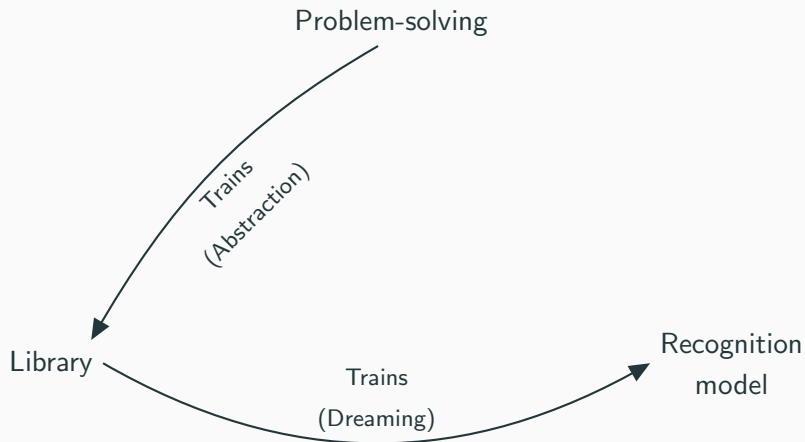
Library

Recognition  
model

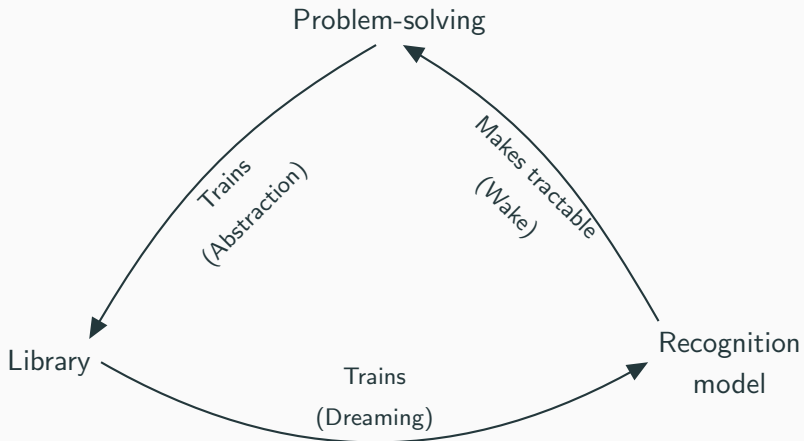
# Synergy between recognition model and library learning



# Synergy between recognition model and library learning

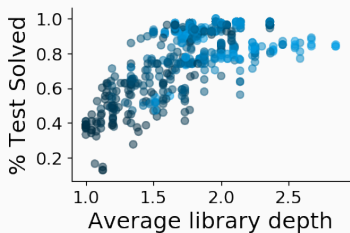


# Synergy between recognition model and library learning





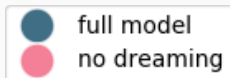
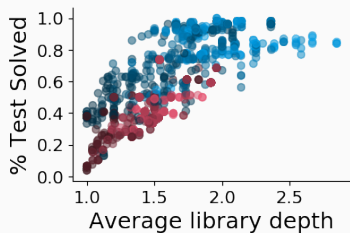
# Evidence for dreaming bootstrapping better libraries



Darker: Early in learning

Brighter: Later in learning

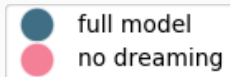
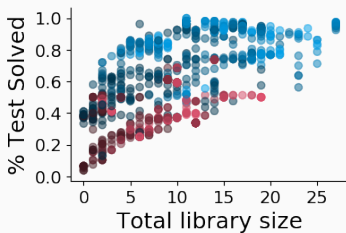
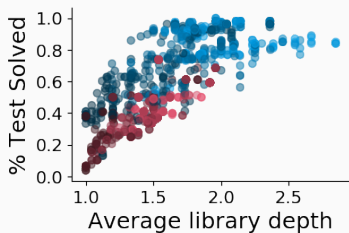
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From learning libraries,  
to learning languages

# From learning libraries, to learning languages

modern functional programming → physics

# From learning libraries, to learning languages

1950's Lisp → modern functional programming → physics

# Physics Formula Sheet

## Mechanics

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$$

$$a_c = \frac{v^2}{r}$$

$$|\vec{F}_{\text{spring}}| = k|\vec{x}|$$

$$v = v_0 + at$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$PE_{\text{spring}} = \frac{1}{2}kx^2$$

$$v_x^2 - v_{x0}^2 = 2a(x - x_0)$$

$$\omega = \omega_0 + \alpha t$$

$$T_{\text{spring}} = 2\pi\sqrt{\frac{m}{k}}$$

$$\vec{a} = \frac{\Sigma \vec{F}}{m} = \frac{\vec{F}_{\text{net}}}{m}$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$T_{\text{pendulum}} = 2\pi\sqrt{\frac{\ell}{g}}$$

$$|\vec{F}_{\text{friction}}| \leq \mu |\vec{F}_{\text{Normal}}|$$

$$v = f\lambda$$

$$\vec{p} = m\vec{v}$$

$$x = A\cos(2\pi ft)$$

$$|\vec{F}_{\text{gravity}}| = G \frac{m_1 m_2}{r^2}$$

$$\Delta \vec{p} = \vec{F}\Delta t$$

$$\vec{\alpha} = \frac{\Sigma \vec{\tau}}{I} = \frac{\vec{\tau}_{\text{net}}}{I}$$

$$|\vec{F}_{\text{gravity}}| = mg$$

$$KE = \frac{1}{2}mv^2$$

$$\vec{\tau} = r \times F$$

$$PE_{\text{gravity}} = -G \frac{m_1 m_2}{r}$$

$$\Delta PE = mg\Delta y$$

$$L = I\omega$$

$$\rho = \frac{m}{V}$$

$$\Delta E = W = Fd\cos\theta$$

$$\Delta L = \tau\Delta t$$

$$KE = \frac{1}{2}I\omega^2$$

## Electricity

$$|\vec{F}_e| = k \left| \frac{q_1 q_2}{r^2} \right|$$

$$\Delta V = IR$$

$$R = \frac{\rho \ell}{A}$$

$$I = \frac{\Delta q}{\Delta t}$$

$$P = I\Delta V$$

$$R_{\text{series}} = R_1 + R_2 + \dots + R_n$$

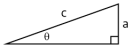
$$\frac{1}{R_{\text{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$$

## Geometry

Rectangle  $A = bh$     Rectangular Solid  $V = \ell wh$     Triangle  $A = \frac{1}{2}bh$

Circle  $A = \pi r^2$     Cylinder  $V = \pi r^2 \ell$     Sphere  $V = \frac{4}{3}\pi r^3$   
 $C = 2\pi r$      $S = 2\pi r\ell + 2\pi r^2$      $S = 4\pi r^2$

## Trigonometry



$$c^2 = a^2 + b^2$$

$$\sin\theta = \frac{a}{c} \quad \cos\theta = \frac{b}{c} \quad \tan\theta = \frac{a}{b}$$

## Variables

a = acceleration

A = amplitude

A = Area

b = base length

C = circumference

d = distance

E = energy

f = frequency

F = force

h = height

I = current

I = rotational inertia

KE = kinetic energy

k = spring constant

L = angular momentum

$\ell$  = length

m = mass

P = power

p = momentum

q = charge

r = radius

R = resistance

S = surface area

T = period

t = time

PE = potential energy

V = electric potential

V = volume

v = velocity

w = width

W = work

x = position

y = height

$\alpha$  = angular acceleration

$\lambda$  = wavelength

$\mu$  = coefficient of friction

# Growing languages for vector algebra and physics

## Initial Primitives

map

zip

cons

empty

cdr

power

fold

car

+

-

\*

/

0

1

$\pi$

## Physics Equations

### Newton's Second Law

$$\vec{a} = \frac{1}{m} \sum_i \vec{F}_i$$

### Parallel Resistors

$$R_{total} = \left( \sum_i \frac{1}{R_i} \right)^{-1}$$

### Work

$$U = \vec{F} \cdot \vec{d}$$

### Force in a Magnetic Field

$$|\vec{F}| = q|\vec{v} \times \vec{B}|$$

### Kinetic Energy

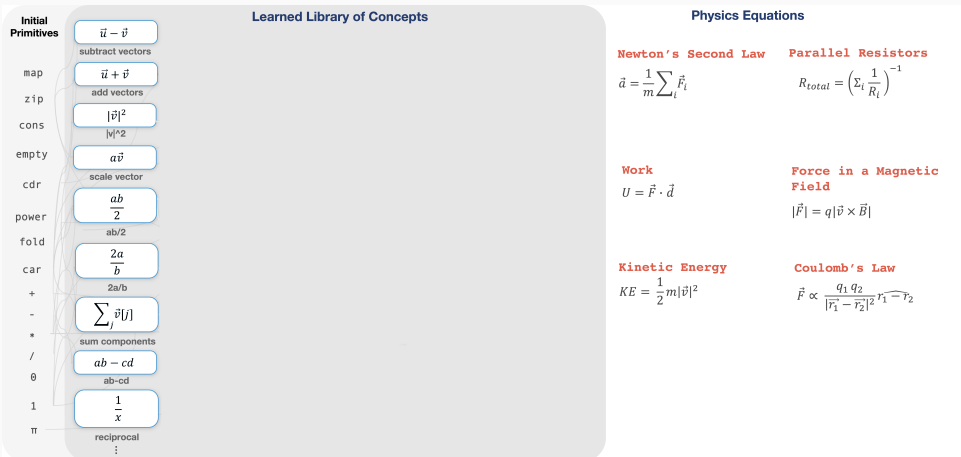
$$KE = \frac{1}{2} m |\vec{v}|^2$$

### Coulomb's Law

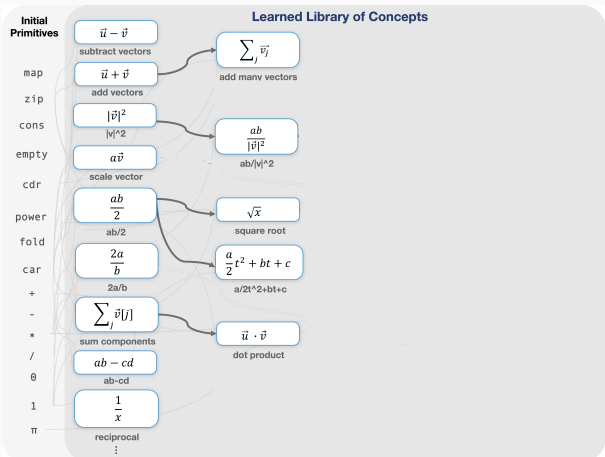
$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^2} \widehat{r_1 - r_2}$$



# Growing languages for vector algebra and physics



# Growing languages for vector algebra and physics



## Physics Equations

### Newton's Second Law

$$\vec{a} = \frac{1}{m} \sum_i \vec{F}_i$$

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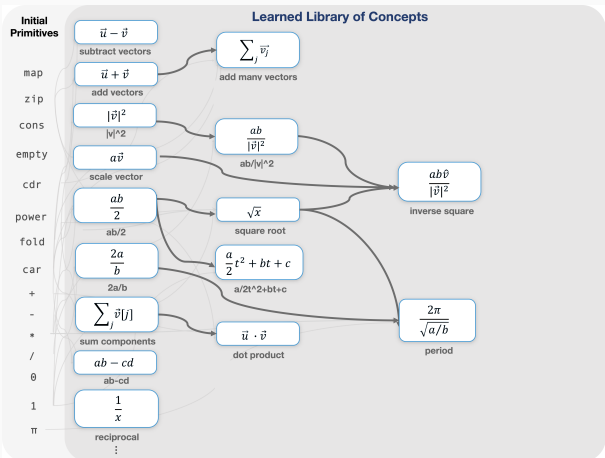
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# Growing languages for vector algebra and physics



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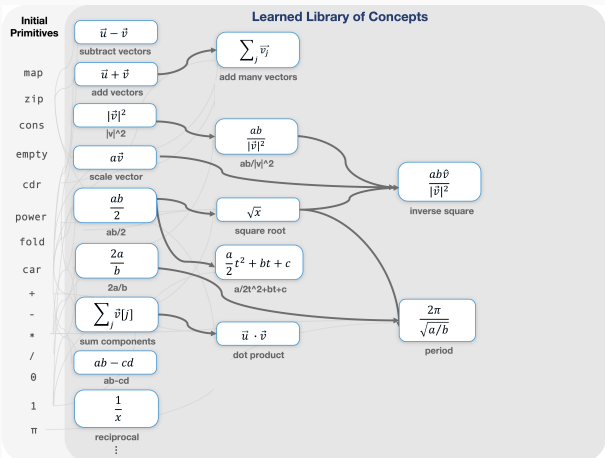
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# Growing languages for vector algebra and physics



## Physics Equations

### Newton's Second Law

$$\vec{a} = \frac{1}{m} \sum_i \vec{F}_i$$

(scale-vector(reciprocal m)  
(add-many-vectors Fs))

### Parallel Resistors

$$R_{total} = \left( \sum_i \frac{1}{R_i} \right)^{-1}$$

(reciprocal (sum-components  
(map  $\lambda(r)$  (reciprocal r))  
Rs))

### Work

$$U = \vec{F} \cdot \vec{d}$$

(dot-product F d)

### Force in a Magnetic Field

$$|\vec{F}| = q|\vec{v} \times \vec{B}|$$

(\* q (ab-cd v\_x b\_y v\_y b\_x))

### Kinetic Energy

$$KE = \frac{1}{2} m |\vec{v}|^2$$

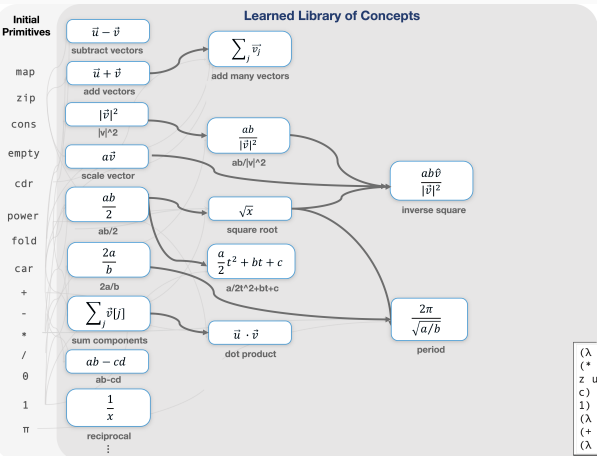
(ab/2 m (|\vec{v}|^2 v))

### Coulomb's Law

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^2} \vec{r}_1 - \vec{r}_2$$

(inverse-square q\_1 q\_2  
(subtract-vectors r\_1 r\_2))

# Growing languages for vector algebra and physics



## Physics Equations

### Newton's Second Law

$$\vec{a} = \frac{1}{m} \sum_i \vec{F}_i$$

(scale-vector(reciprocal m)  
(add-many-vectors Fs))

### Work

$$U = \vec{F} \cdot \vec{d}$$

(dot-product F d)

### Kinetic Energy

$$KE = \frac{1}{2} m |\vec{v}|^2$$

(ab/2 m (|\vec{v}|^2 v))

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$$R_{total} = \left( \sum_i \frac{1}{R_i} \right)^{-1}$$

(reciprocal (sum-components  
(map (λ(r) (reciprocal r))  
Rs)))

### Force in a Magnetic Field

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(\* q (ab-cd v\_x b\_y v\_y b\_x))

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(inverse-square q\_1 q\_2  
(subtract-vectors r\_1 r\_2))

```
(λ (x y z u) (map (λ (v) (* (/
(* (power (/ (* x x) (fold (zip
z u (λ (w a) (- w a)) 0) (λ (b
c) (+ (* b b) c)))) (/ (* 1
1) (+ 1 1))) y) (fold (zip z u
(λ (d e) (- d e))) 0) (λ (f g)
(+ (* f f) g)))) v)) (zip z u
(λ (h i) (- h i))))
```

Solution to Coulomb's Law if expressed in initial primitives

# Growing a language for recursive programming

## Initial Primitives

Y combinator

cons

car

cdr

nil

if

nil?

+

-

0

1

=

## Recursive Programming Algorithms

### Stutter

[■ ■] → [■ ■ ■ ■]  
[■ ■ ■] → [■ ■ ■ ■ ■ ■]

### Take every other

[■ ■ ■ ■] → [■ ■]  
[■ ■ ■ ■ ■ ■] → [■ ■ ■]

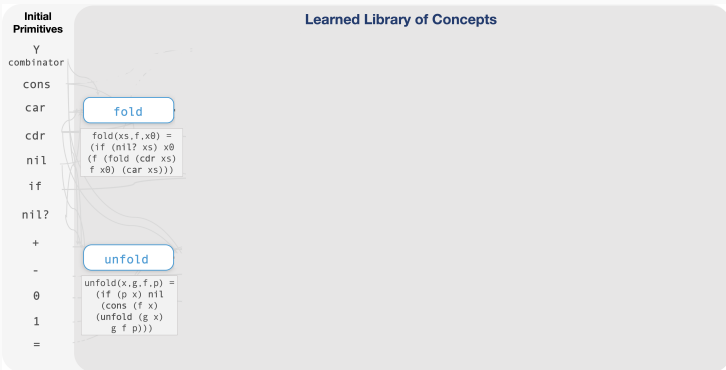
### List lengths

[[■ ■ ■], [■]] → [3 1]  
[[■ ■], [], [■]] → [2 0 1]

### List differences

[1 8 2], [0 5 1] → [1 3 1]  
[2 3 6], [1 2 4] → [1 1 2]

# Growing a language for recursive programming



## Recursive Programming Algorithms

### Stutter

```
[■ ■] → [■ ■ ■ ■]  
[■ ■ ■] → [■ ■ ■ ■ ■ ■]  
(fold A (λ (u v) (cons  
v (cons v u))) nil)
```

### Take every other

```
[■ ■ ■ ■] → [■ ■]  
[■ ■ ■ ■ ■ ■] → [■ ■ ■]
```

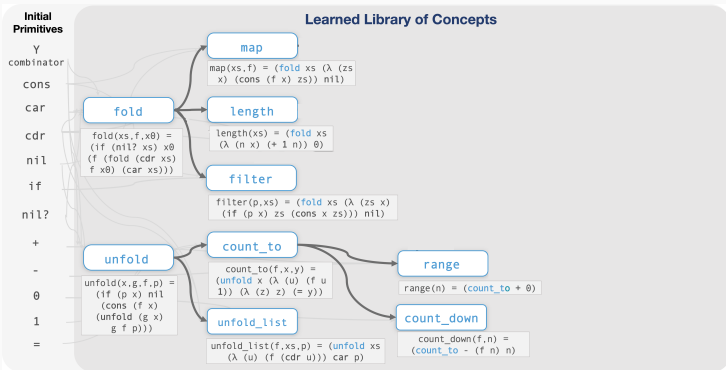
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# Growing a language for recursive programming



## Recursive Programming Algorithms

### Stutter

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[■ ■] → [■ ■ ■ ■]
[■ ■ ■] → [■ ■ ■ ■ ■ ■]
(fold A (λ (u v) (cons v (cons v u))) nil)
```

### Take every other

```
[■ ■ ■ ■] → [■ ■]
[■ ■ ■ ■ ■ ■] → [■ ■ ■]
(unfold_list cdr A nil?)
```

### List lengths

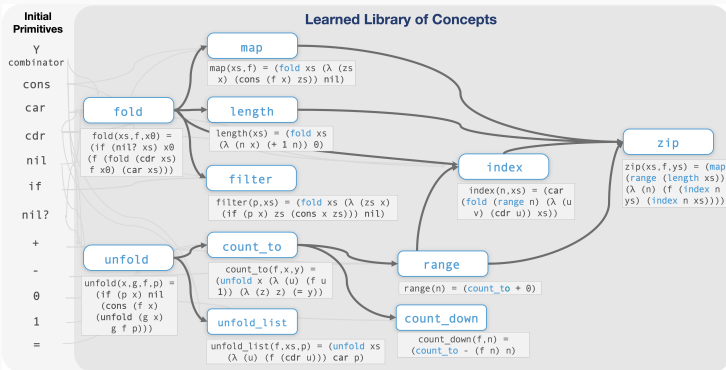
```
[[■ ■ ■], [■]] → [3 1]
[[■ ■], [], [■]] → [2 0 1]
(map A length)
```

### List differences

```
[1 8 2], [0 5 1] → [1 3 1]
[2 3 6], [1 2 4] → [1 1 2]
```



# Growing a language for recursive programming



## Recursive Programming Algorithms

### Stutter

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[■ ■] → [■ ■ ■]
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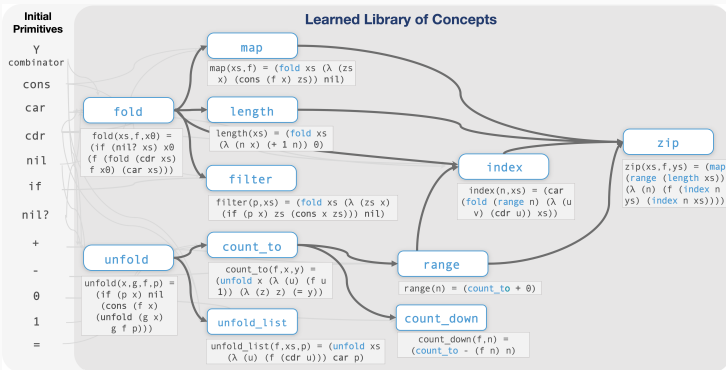
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(zip A - B)
    
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# Growing a language for recursive programming



## Recursive Programming Algorithms

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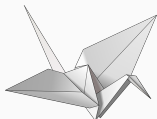
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(map A length)
    
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### List differences

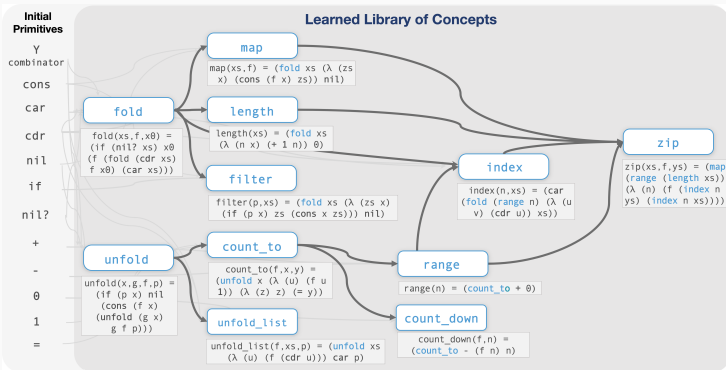
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Origami Programming: Jeremy Gibbons, 2003

# Growing a language for recursive programming



## Recursive Programming Algorithms

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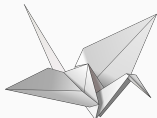
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(zip A - B)
```

1 year of compute. 5 days on 64 CPUs.



Origami Programming: Jeremy Gibbons, 2003

Library learning interacts synergistically with neural synthesis:  
bootstrapping, more than sum of parts

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Symbols aren't necessarily interpretable. Grow the language based on experience to make it more powerful *and* more human understandable

Library learning interacts synergistically with neural synthesis:  
bootstrapping, more than sum of parts

Symbols aren't necessarily interpretable. Grow the language based on experience to make it more powerful *and* more human understandable

Learning-from-scratch is possible in principle. Don't do it. But program induction makes it convenient to build in what we know how to build in, and then learn on top of that

the end.

# Collaborators

Josh  
Tenenbaum



Armando  
Solar-Lezama



Max Nye



Cathy Wong



Mathias Sable-Meyer



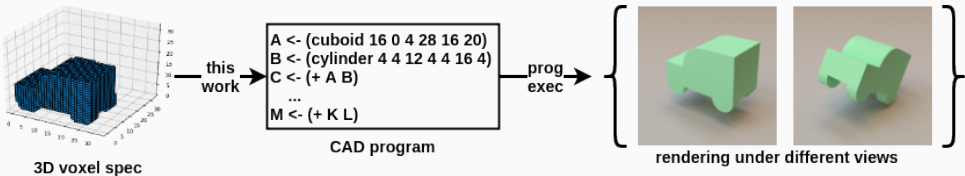
Lucas Morales



**thank  
you**



# 3D program induction



Challenge: combinatorial search!

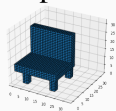
Branching factor:  $> 1.3$  million per line of code,  $\approx 20$  lines of code  
search space size:  $(1.3 \text{ million})^{20} \approx 10^{122}$  programs

Ellis\*, Nye\*, Pu\*, Sosa\*, Tenenbaum, Solar-Lezama. NeurIPS 2019.

\*equal contribution

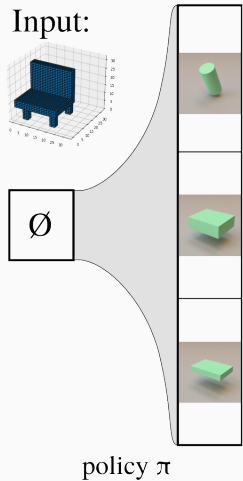
Solution: stochastic **tree search** + learn **policy** that writes code  
+ learn **value** function that assesses execution of program so far;  
analogous to **AlphaGo** [Silver et al. 2016]

Input:

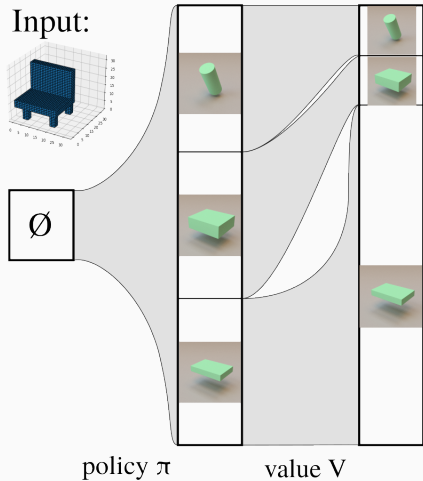


$\emptyset$

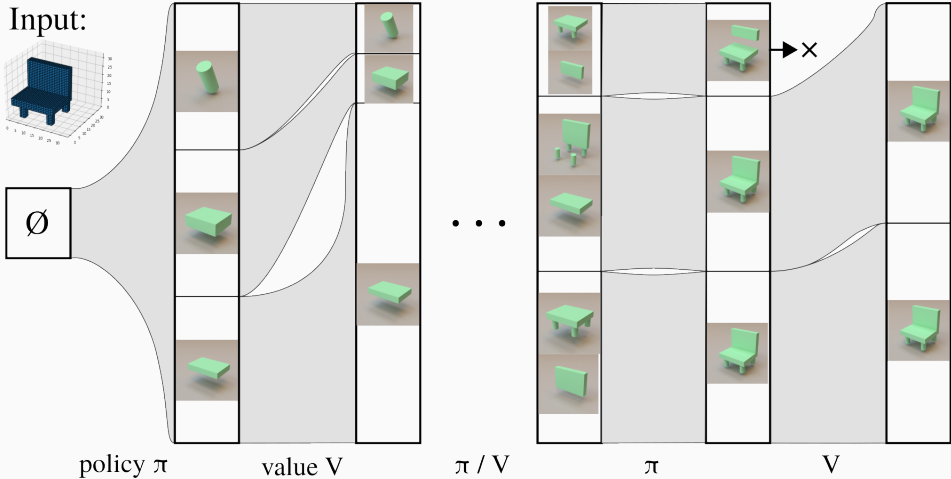
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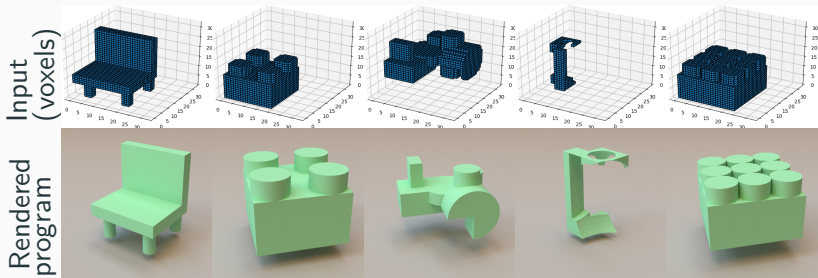
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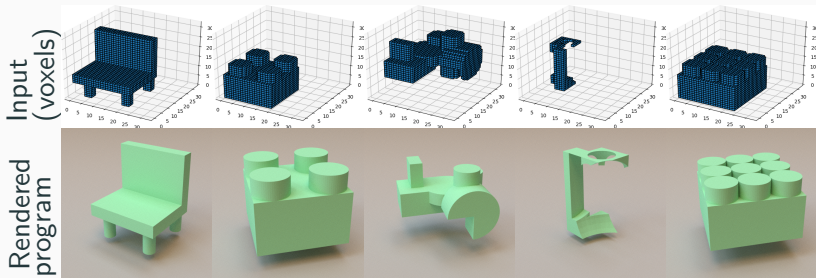
# 3D program induction



Ellis\*, Nye\*, Pu\*, Sosa\*, Tenenbaum, Solar-Lezama. NeurIPS 2019.

\*equal contribution

# 3D program induction



same architecture learns to synthesize text editing programs  
(FlashFill, Gulwani 2012)

Ellis\*, Nye\*, Pu\*, Sosa\*, Tenenbaum, Solar-Lezama. NeurIPS 2019.

\*equal contribution

# Library structure: Text Editing

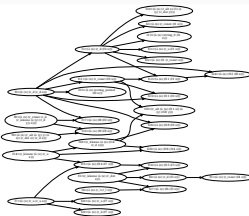
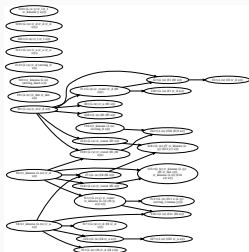
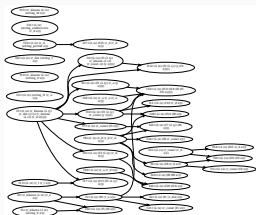
DreamCoder learns libraries for FlashFill-style text editing [Gulwani 2012]



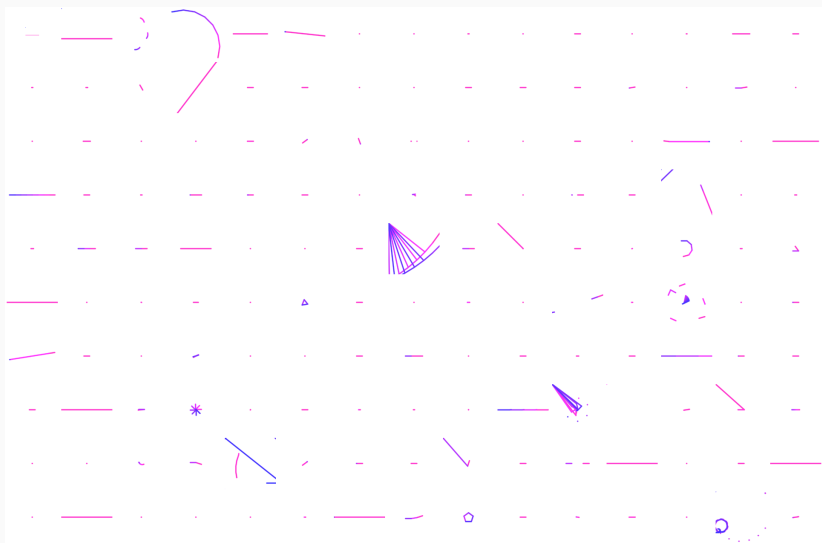


# Library structure: Generating Text

Libraries for probabilistic generative models over text:  
data from crawling web for CSV files



# 150 random dreams before learning



# 150 random dreams after learning

