New tools for analysis of Markov chains via high-dimensional expansion

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U. of Washington

Based on joint works with

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Stanford U.

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Georgia Tech

Shayan Oveis Gharan
U. of Washington

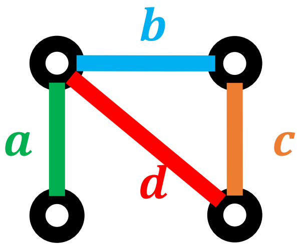
Eric Vigoda
Georgia Tech

Cynthia Vinzant
Institute for Advanced Study

Polynomials in Combinatorics and Probability

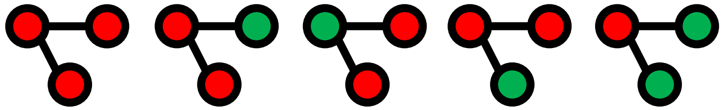
Study a distribution μ on $2^{[n]}$ through some associated "generating" polynomial

Spanning Trees:



$$x_a x_b x_c + x_a x_b x_d + x_a x_c x_d$$

Hardcore Model:



$1 \quad \lambda \quad \lambda \quad \lambda \quad \lambda^2$

$$1 + 3\lambda + \lambda^2$$

Ising Configurations:

$$\sum_{\sigma:V\to\{\pm 1\}} \beta^{\#\{uv\in E:\sigma(u)\neq\sigma(v)\}}\, x^{\#\{u:\sigma(u)=+1\}}$$

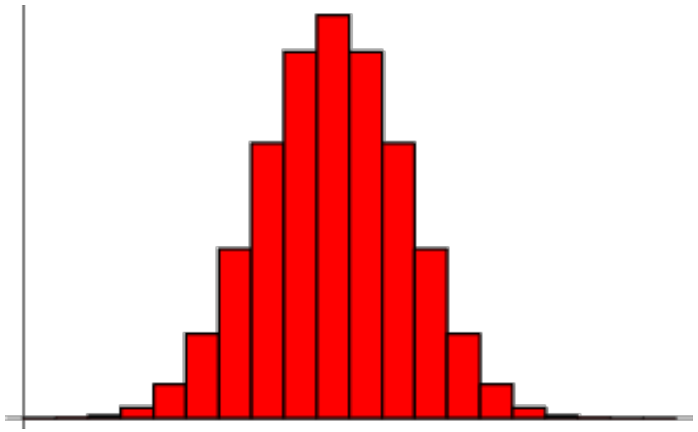
Potts model/q-Colorings:

$$\sum_{\sigma:V\to[q]} z^{\#\{\text{monochromatic edges of } \sigma\}}$$

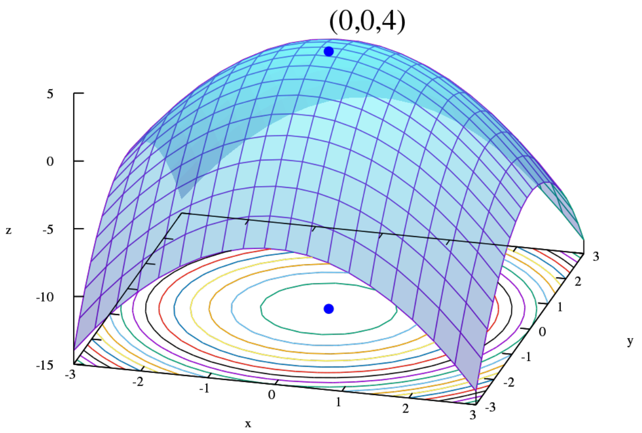
Polynomials in Combinatorics and Probability

Study a distribution μ on $2^{[n]}$ through some associated "generating” polynomial

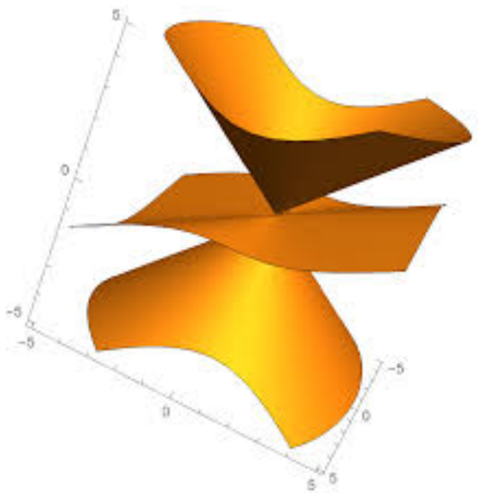
What properties of the polynomial enable efficient sampling of μ or efficient counting?



Coefficients



Function



Roots/Zeroes

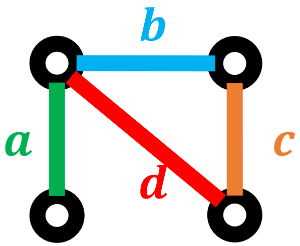
Zeroes and Algorithms

Root-free region

For instance, a la Barvinok

Computational Tractability for Counting and Sampling

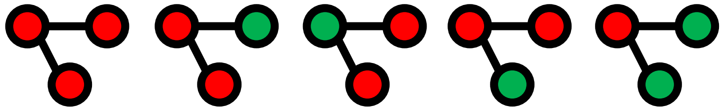
Spanning Trees:



$x_a x_b x_c + x_a x_b x_d + x_a x_c x_d$

Real Stable (via Kirchhoff Matrix Tree Thm)

Hardcore Model:



1 λ λ λ λ^2

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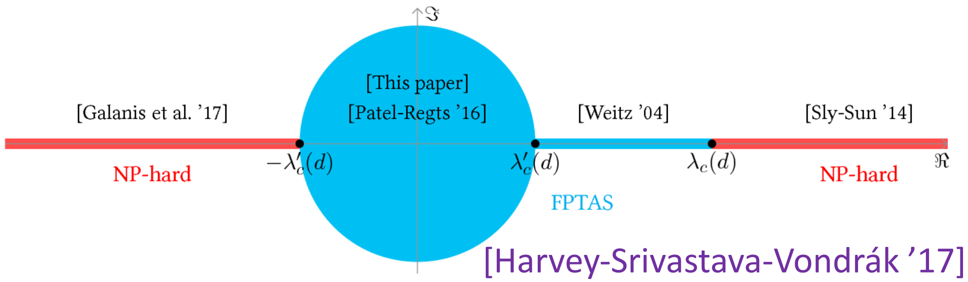

[Harvey-Srivastava-Vondrák '17]

Ising Configurations:

$$\sum_{\sigma:V\to\{\pm 1\}} \beta^{\#\{uv\in E:\sigma(u)\neq\sigma(v)\}} x^{\#\{u:\sigma(u)=+1\}}$$

Lee-Yang Thm [LY'52], Fisher '65

Potts model/q-Colorings:

$$\sum_{\sigma:V\to[q]} z^{\#\{\text{monochromatic edges of } \sigma\}}$$

[Liu-Sinclair-Srivastava '19]

Zeroes and Algorithms

Root-free region → For instance, a la Barvinok → Computational Tractability for Counting and Sampling

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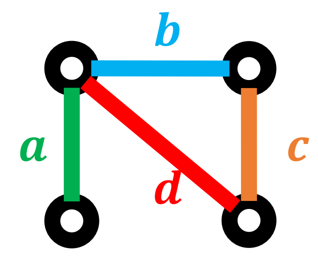

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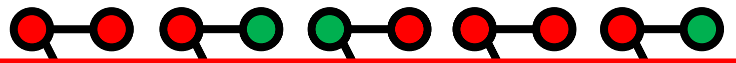
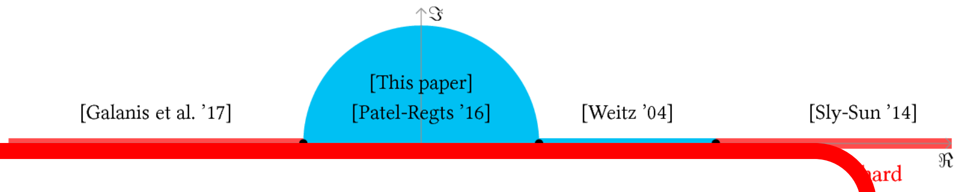


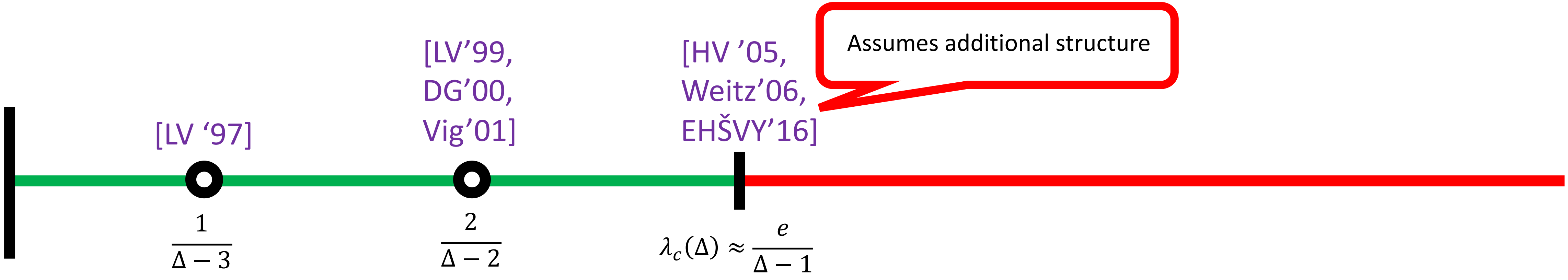


Main Drawback of Interpolation/Correlation Decay: Typically only polynomial-time assuming bounded-degreeness. This seems inherent to the algorithm, and not just the analysis.

Potts model/q-Colorings: $\sum_{\sigma: V \to [q]} z^{\#\{\text{monochromatic edges of } \sigma\}}$

[Liu-Sinclair-Srivastava '19]

MCMC vs. Interpolation/Correlation Decay (for Hardcore Model)



Exists FPTAS

- via Correlation Decay [Weitz '06]
- via interpolation [Peters-Regts'17, Patel-Regts '17]

Hardness:

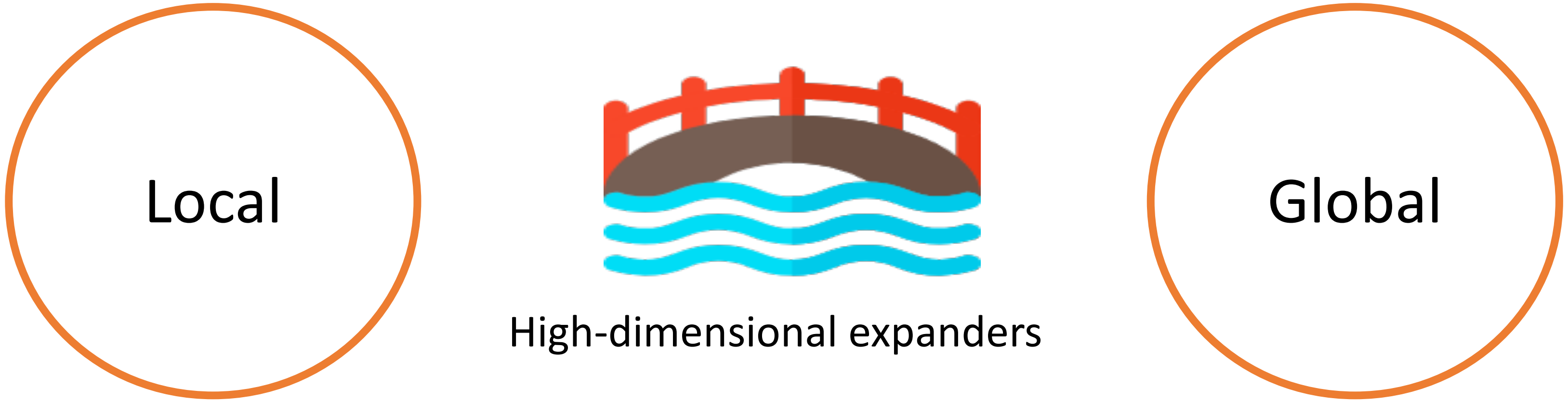
- Slow mixing [LV'97,'99, DFJ'02, MWW'07]
- NP-Hardness [Sly'10, SS'14, GGŠVY'14, GŠV'15'16]

Some Recent Results

Thm [Anari-L.-Oveis Gharan ‘20, Chen-L.-Vigoda ‘20]: The Glauber dynamics mixes rapidly for 2-spin systems in the correlation decay regime.

Thm [Feng-Guo-Yin-Zhang ‘20, Chen-Galanis-Štefankovič-Vigoda ‘20]: The Glauber dynamics mixes rapidly for q-colorings on triangle-free graphs with max-degree-Δ when $q > 1.763\Delta$

Gives $n^{1/\delta}$ algorithm as opposed to $n^{(\log \Delta)/\delta}$


Local
High-dimensional expanders
Global

High-Dimensional Expanders in TCS

Random Walks/Markov chains

[Parzanchevski ’13, Evra-Golubev-Lubotzky ’14, Kaufman-Mass ’16, Parzanchevski-Rosenthal ’16, Lubetzky-Lubotzky-Parzanchevski ‘17, **Oppenheim ’18, Kaufman-Oppenheim ’18**, Chapman-Parzanchevski ’19, **Anari-L.-Oveis Gharan-Vinzant ‘19, Cryan-Guo-Mousa ‘20, Alev-Lau ’20, Anari-L.-Oveis Gharan ‘20, Chen-L.-Vigoda ’20, Feng-Guo-Yin-Zhang ‘20, Chen-Galanis-Štefankovič-Vigoda ‘20**]

PCPs & Property Testing

[Kaufman-Lubotzky ’14, Dinur-Kaufman ’17, Dinur-Harsha-Kaufman-Ron-Zewi’19, Dikstein-Dinur ‘19, Dinur-Meshulam ’19, Gotlib-Kaufman ‘19, Kaufman-Mass ‘20]

High-dimensional expansion

[Boros-Füredi ’84, Pach ‘98, Lubotzky-Samuels-Vishne ’05, Linial-Meshulam ‘06, Kahle ‘07’09’13’14, Gromov ’10, Fox-Gromov-Lafforgue-Naor-Pach ’11, Kaufman-Kazhdan-Lubotzky ’14, Lubotzky-Meshulam-Mozes ‘15, Lubotzky-Luria-Rosenthal ’16 ’18, Lubotzky ‘17, Dotterrer-Kaufman-Wagner ’18]

Boolean Functions & CSPs

[Dikstein-Dinur-Filmus-Harsha ’18, Alev-Jeronimo-Tulsiani ‘19]

Error-Correcting Codes

[Kaufman-Mass ’18, Dinur-Harsha-Kaufman-Navon-Shma ’18, Alev-Jeronimo-Quintana-Srivastava-Tulsiani ‘20]

Outline

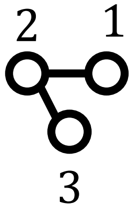
The High-Order Walk

High-Dimensional Expansion: Beyond Log-Concavity

Correlation Decay and Expansion

Future Directions

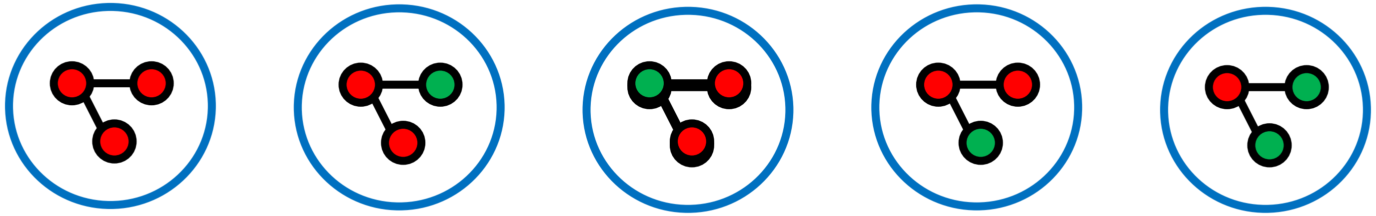
The High-Order/Down-Up Walk [Kaufman-Mass '16]

Stable Sets of 

"in"

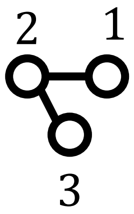
"out"

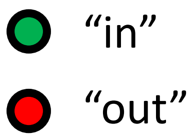
$$g_\mu(x_1, x_2, x_3, x_{\bar{1}}, x_{\bar{2}}, x_{\bar{3}}) = x_{\bar{1}}x_{\bar{2}}x_{\bar{3}} + \lambda x_1 x_{\bar{2}} x_{\bar{3}} + \lambda x_{\bar{1}} x_2 x_{\bar{3}} + \lambda x_{\bar{1}} x_{\bar{2}} x_3 + \lambda^2 x_1 x_{\bar{2}} x_3$$



The High-Order/Down-Up Walk [Kaufman-Mass ‘16]

Stable Sets of





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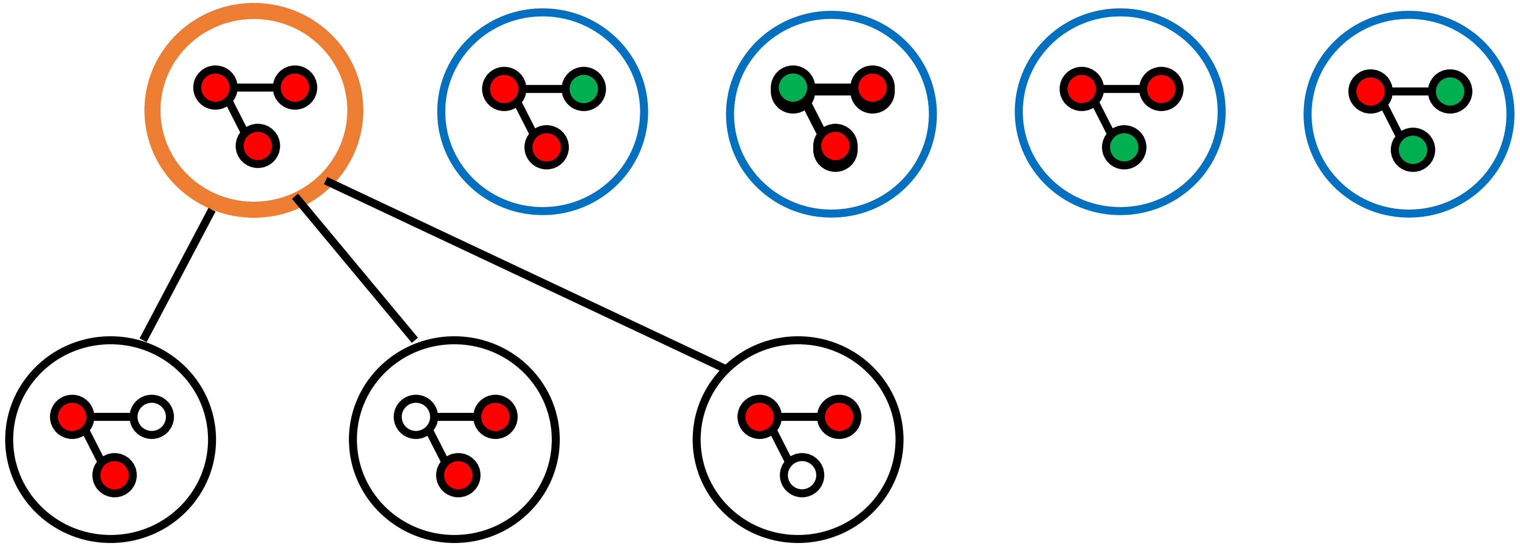
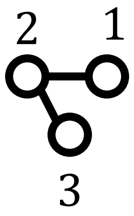


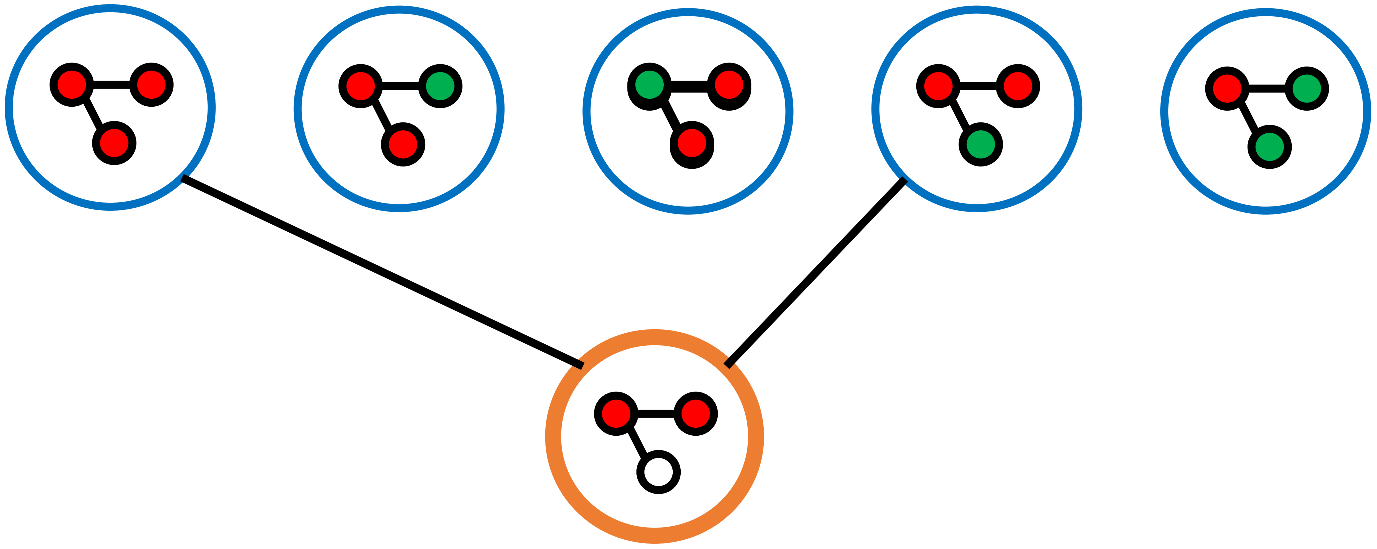
$x_{\bar{2}}x_{\bar{3}}$ $\quad$ $x_{\bar{1}}x_{\bar{3}}$ $\quad$ $x_{\bar{1}}x_{\bar{2}}$

The High-Order/Down-Up Walk [Kaufman-Mass ‘16]

Stable Sets of 

“in” (green)
“out” (red)

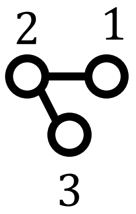
$$g_\mu(x_1, x_2, x_3, x_{\bar{1}}, x_{\bar{2}}, x_{\bar{3}}) = x_{\bar{1}}x_{\bar{2}}x_{\bar{3}} + \lambda x_1 x_{\bar{2}} x_{\bar{3}} + \lambda x_{\bar{1}} x_2 x_{\bar{3}} + \lambda x_{\bar{1}} x_{\bar{2}} x_3 + \lambda^2 x_1 x_{\bar{2}} x_3$$

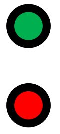


$x_{\bar{1}}x_{\bar{2}}$

The High-Order/Down-Up Walk [Kaufman-Mass ‘16]

Stable Sets of





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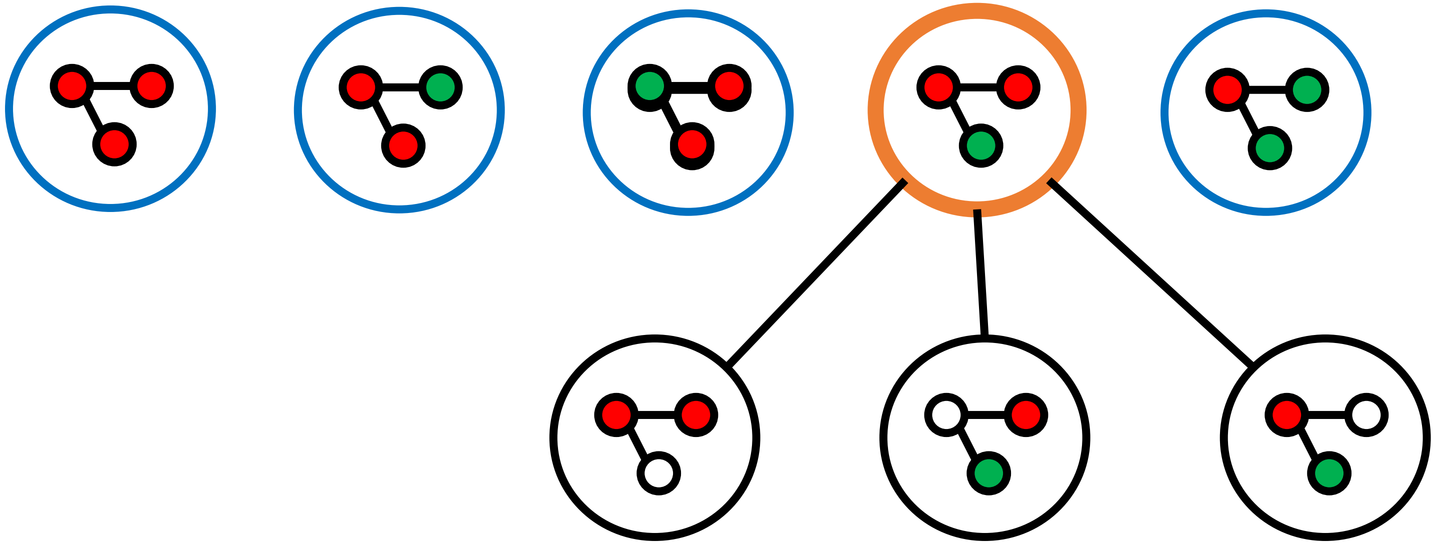

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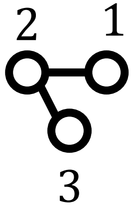
Stable Sets of 

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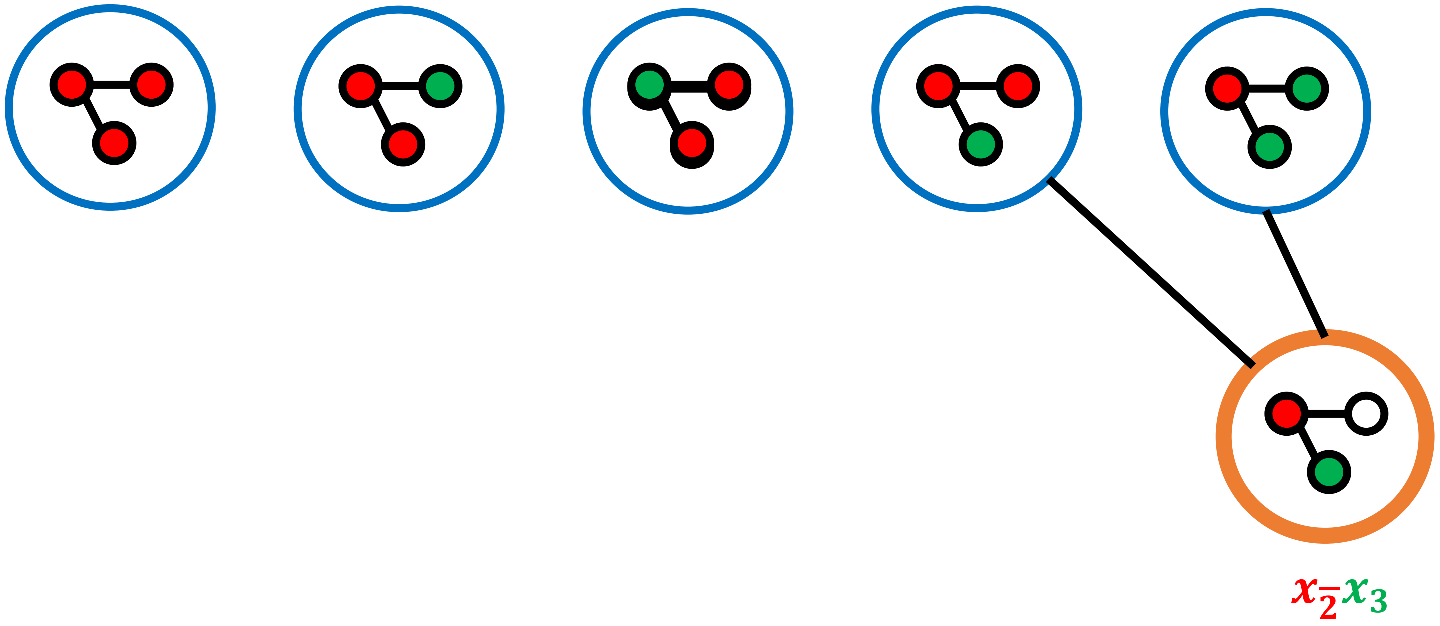

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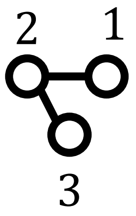
Stable Sets of 

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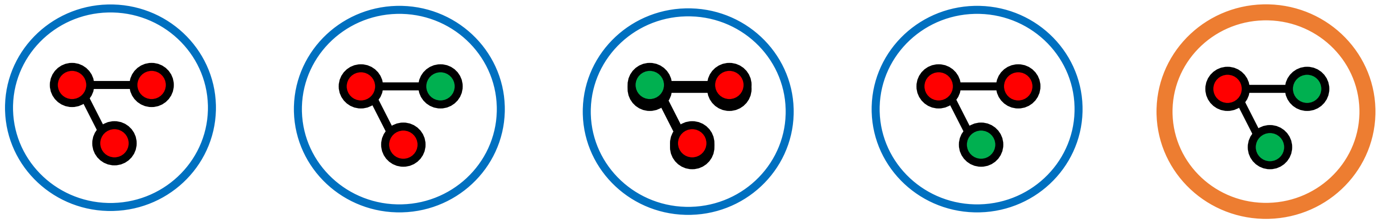
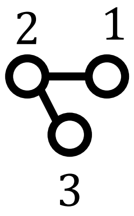


Current stable set S

1. Pick $v \in V$ uniformly at random (and “uncolor” it)
2. Move to $S - v$ with probability p and $S \cup v$ o.w. (i.e. recolor v)

This is the **Glauber dynamics**

The High-Order/Down-Up Walk [Kaufman-Mass ‘16]

Stable Sets of 

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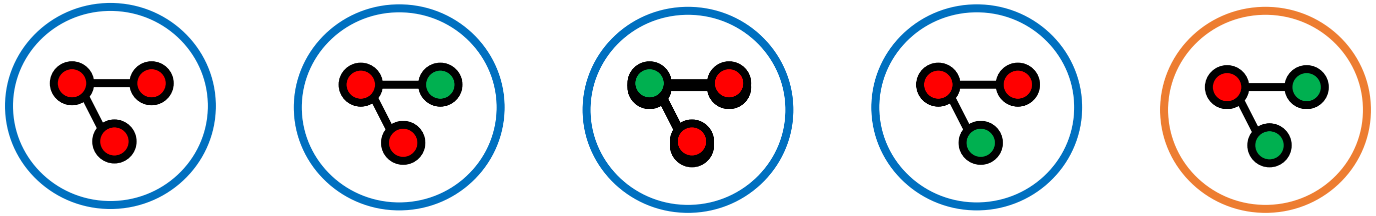

P_{downup} = transition probability matrix

Goal: Upper bound $\lambda_2(P_{downup})$ away from 1

Under what conditions on g_μ?

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Log-Concavity

g_μ is log-concave if: $\lambda_2(\nabla^2 g_\mu) \leq 0$

Log-Concavity

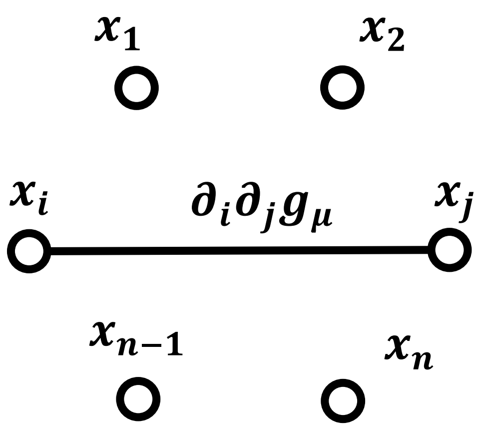
g_μ is strongly log-concave if: $\lambda_2\left(\nabla^2 \partial_{i_1} \ldots \partial_{i_k} g_\mu\right) \le 0$ for all $i_1, \ldots, i_k$
[Gurvits ‘06, Anari-Oveis Gharan-Vinzant ‘19, Brändén-Huh ‘19]

Thm [Anari-L.-Oveis Gharan-Vinzant ‘19]: If g_μ is strongly log-concave, then $\lambda_2\left(P_{downup}\right) \ge 1 - \frac{1}{r}$. This implies $O(r^2 \log n)$ mixing.

Mixing time now down to $O(r \log r)$ [Cryan-Guo-Mousa ‘19, Anari-L.-Oveis Gharan-Vinzant ‘20]

Log-Concavity as “Expansion”

g_μ is log-concave if: $\lambda_2(\nabla^2 g_\mu) \leq 0$



$$\widetilde{\nabla}^2 g_\mu = \frac{1}{d-1} diag(\nabla g_\mu)^{-1} \nabla^2 g_\mu$$

Transition matrix $\widetilde{\nabla}^2 g_\mu$ has spectral gap ≥ 1.
It is an unbelievably good expander.

Such strong expansion **only** holds for matroids

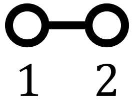
“Approximate” Log-Concavity

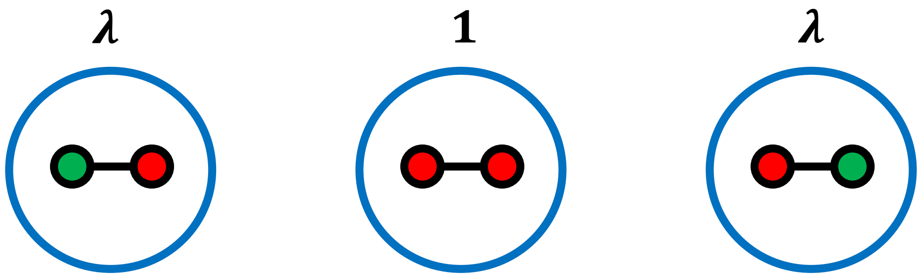
g_μ is “approximately” log-concave if: $\lambda_2\left(\nabla^2 g_\mu\right) \leq \text{"}small\text{"}$

“Approximate” Log-Concavity

Equivalent to log-concavity of $g_\mu(x_1^\alpha, \dots, x_n^\alpha)$

g_μ is “approximately” log-concave if: $\lambda_2\left(\widetilde{\nabla}^2 g_\mu\right) \le \alpha$

Stable Sets of 



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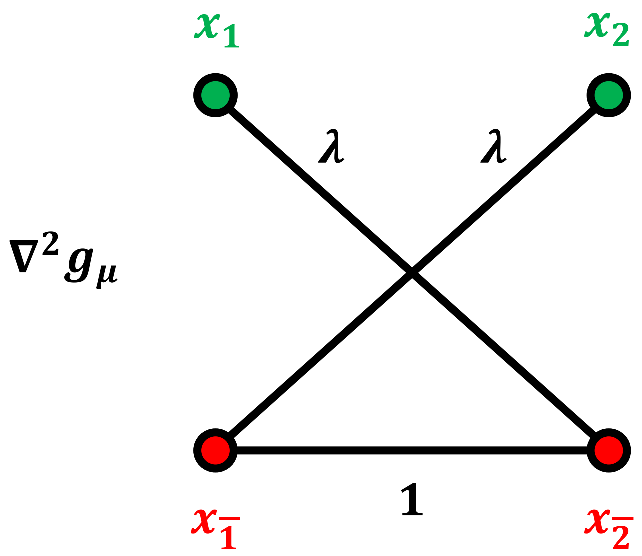

$$\lambda_2\left(\widetilde{\nabla}^2 g_\mu\right) = \lambda > 0$$

Mixing from High-Dimensional Expansion

g_μ is $(\alpha_0, \ldots, \alpha_{n-2})$-local spectral expander: $\lambda_2\left(\widetilde{\nabla}^2 \partial_{i_1} \ldots \partial_{i_k} g_\mu\right) \leq \alpha_k$ for all $i_1, \ldots, i_k$
[Dinur-Kaufman '17, Oppenheim '18, Kaufman-Oppenheim '18]

Thm [Anari-L.-Oveis Gharan '20, Chen-L.-Vigoda '20]: $\alpha_k \leq O(\frac{1}{n-k})$ for 2-spin systems in "correlation decay" regime

Thm [Alev-Lau '20]: If g_μ is $(\alpha_0, \ldots, \alpha_{n-2})$-local spectral expander, then $\lambda_2(P_{downup}) \leq 1 - \frac{1}{n}\prod_{k=0}^{n-2}(1-\alpha_k)$

Thm [Anari-L.-Oveis Gharan '20, Chen-L.-Vigoda '20]: The Glauber dynamics mixes rapidly for 2-spin systems in the correlation decay regime.

Mixing from High-Dimensional Expansion

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Thm [Feng-Guo-Yin-Zhang '20, Chen-Galanis-Štefankovič-Vigoda '20]: $\alpha_k \leq O(\frac{1}{n-k})$ for q-colorings on triangle-free graphs with max-degree-Δ when $q > 1.763\Delta$

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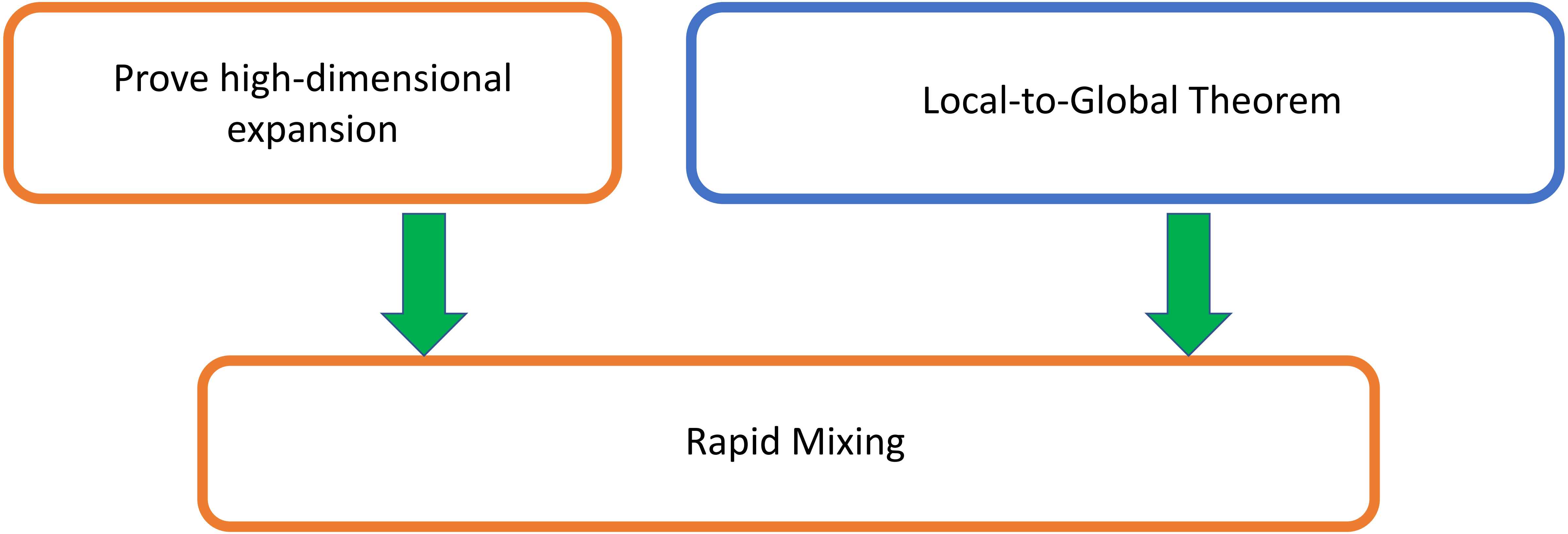
The Strategy



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Trickle Down for Polynomials

Thm [Oppenheim '18]: If $\nabla^2 g_\mu$ is connected and $\lambda_2(\widetilde{\nabla}^2 \partial_i g_\mu) \leq \alpha, \forall i$, then $\lambda_2(\widetilde{\nabla}^2 g_\mu) \leq \frac{\alpha}{1-\alpha}$

Trickle Down for Polynomials

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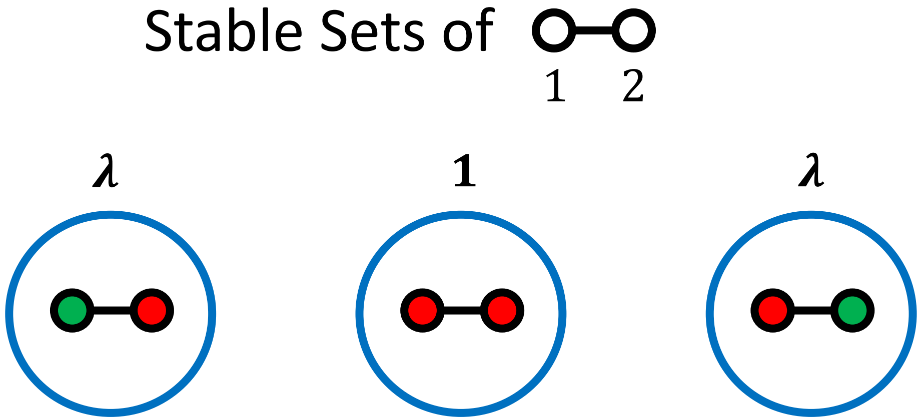
Works beautifully for matroids

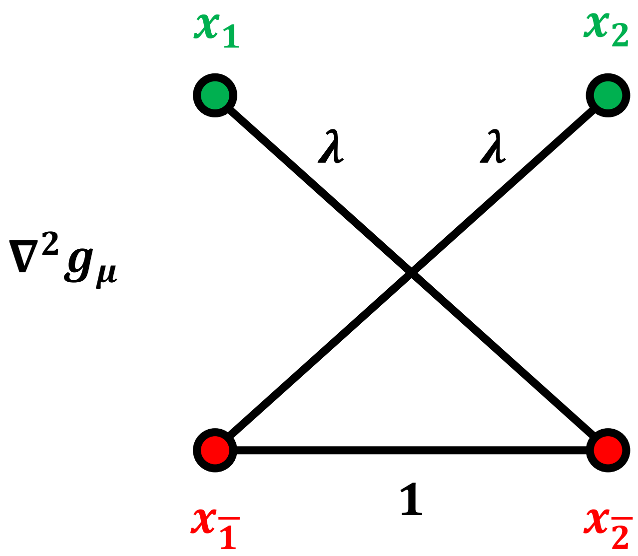
Such strong expansion **only** holds for matroids

Where Trickling Down Fails

Thm [Oppenheim '18]: If $\nabla^2 g_\mu$ is connected and $\lambda_2(\widetilde{\nabla}^2 \partial_i g_\mu) \leq \alpha, \forall i$, then $\lambda_2(\widetilde{\nabla}^2 g_\mu) \leq \frac{\alpha}{1-\alpha}$



$$g_\mu(x_1, x_2, x_{\bar{1}}, x_{\bar{2}}) = x_{\bar{1}} x_{\bar{2}} + \lambda x_1 x_{\bar{2}} + \lambda x_{\bar{1}} x_2$$



$\lambda_2(\widetilde{\nabla}^2 g_\mu) = \lambda \geq \Omega(1)$ so trickling down is useless.

Where Trickling Down Fails

Thm [Oppenheim '18]: If $\nabla^2 g_\mu$ is connected and $\lambda_2\left(\widetilde{\nabla}^2 \partial_i g_\mu\right) \leq \alpha, \forall i$, then $\lambda_2\left(\widetilde{\nabla}^2 g_\mu\right) \leq \frac{\alpha}{1-\alpha}$

g_μ is high-dimensional expander if: $\lambda_2\left(\widetilde{\nabla}^2 \partial_{i_1} \dots \partial_{i_k} g_\mu\right) \leq \alpha_k$ for all $i_1, \dots, i_k$

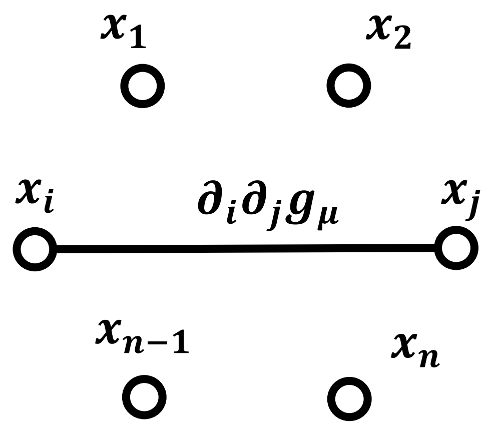
Trickling down needs: $\alpha_{d-2} \leq \frac{1}{d}$ so that $\alpha_{d-3} \leq \frac{1}{d-1}, \dots, \alpha_1 \leq \frac{1}{3}, \alpha_0 \leq \frac{1}{2}$

What typically happens: $\alpha_{d-2} \leq \frac{1}{2}, \alpha_{d-3} \leq \frac{1}{3}, \dots, \alpha_1 \leq \frac{1}{d-1}, \alpha_0 \leq \frac{1}{d}$

Influences and Eigenvalues

Claim: $\widetilde{\nabla}^2 g_\mu(i,j) = \frac{1}{d-1}\Pr[j \mid i]$

Intuition: $\frac{\partial_i \partial_j g_\mu}{g_\mu} = \Pr[i,j]$ and $\frac{\partial_i g_\mu}{g_\mu} = \Pr[i]$

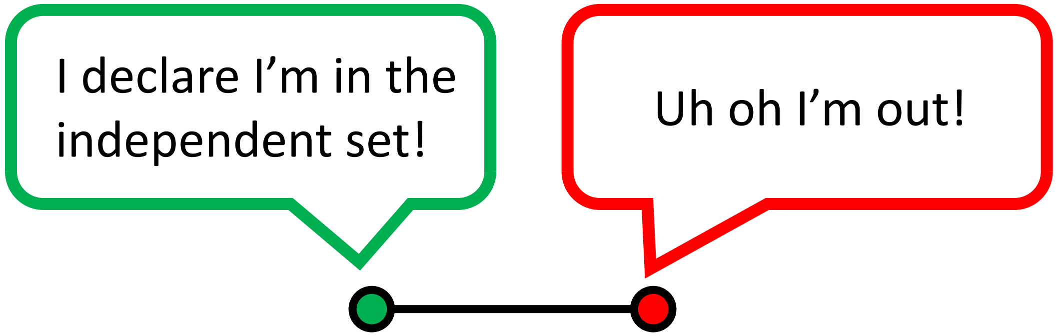


Thm [Anari-L.-Oveis Gharan '20]: $\lambda_2\left(\widetilde{\nabla}^2 g_\mu\right) \leq \frac{1}{n-1} \max_r \sum_v \left|\Pr[\boldsymbol{v} \mid \boldsymbol{r}] - \Pr[\boldsymbol{v} \mid \overline{\boldsymbol{r}}]\right|$

Spatial Mixing/Correlation Decay

Goal: Bound $\sum_v |\Pr[\boldsymbol{v} \mid \boldsymbol{r}] - \Pr[\boldsymbol{v} \mid \overline{\boldsymbol{r}}]|$ for all $r \in G$

Spatial Mixing: $|\Pr[\boldsymbol{v} \mid \boldsymbol{r}] - \Pr[\boldsymbol{v} \mid \overline{\boldsymbol{r}}]| \leq \exp(-d(v,r))$



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(for amenable graphs)

Spatial Mixing → $O(1)$ bound

Bounding Influences: High-Level Strategy

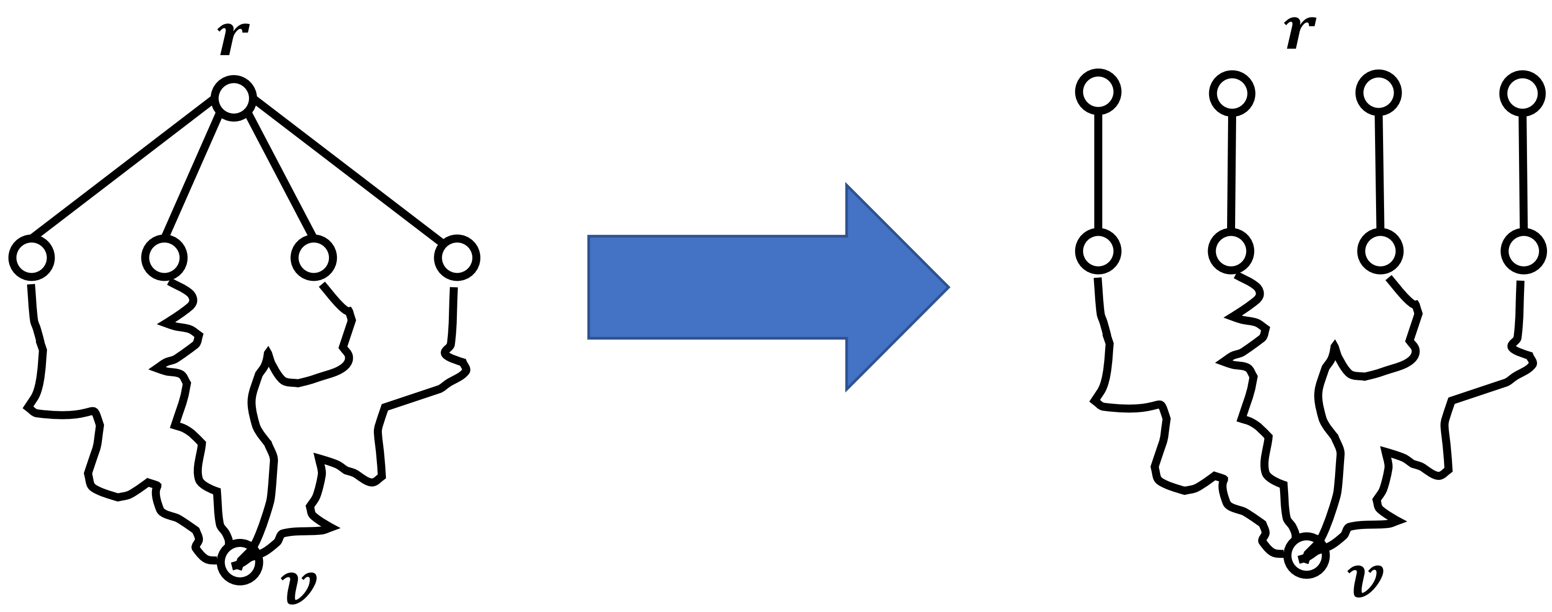
Goal: Bound $\sum_v |\Pr[\boldsymbol{v} \mid \boldsymbol{r}] - \Pr[\boldsymbol{v} \mid \overline{\boldsymbol{r}}]|$ for all $r \in G$

1. Reduction to trees
2. Apply known correlation decay analysis

Bounding Influences: Reduction from Graphs to Trees

Goal: Bound $\sum_v |\Pr[\, v \mid r \,] - \Pr[\, v \mid \overline{r} \,]|$ for all $r \in G$

Thm [Chen-L.-Vigoda '20]: $\sum_{v \in G} |\Pr[v|r] - \Pr[v|\overline{r}]| \le \sum_{u \in T} |\Pr[u|r] - \Pr[u|\overline{r}]|$ where $T = T_{SAW}(G, r)$



Bounding Influences: Reduction from Graphs to Trees

Goal: Bound $\sum_v | \Pr[\, v \mid r \,] - \Pr[v \mid \bar{r}] \,|$ for all $r \in G$

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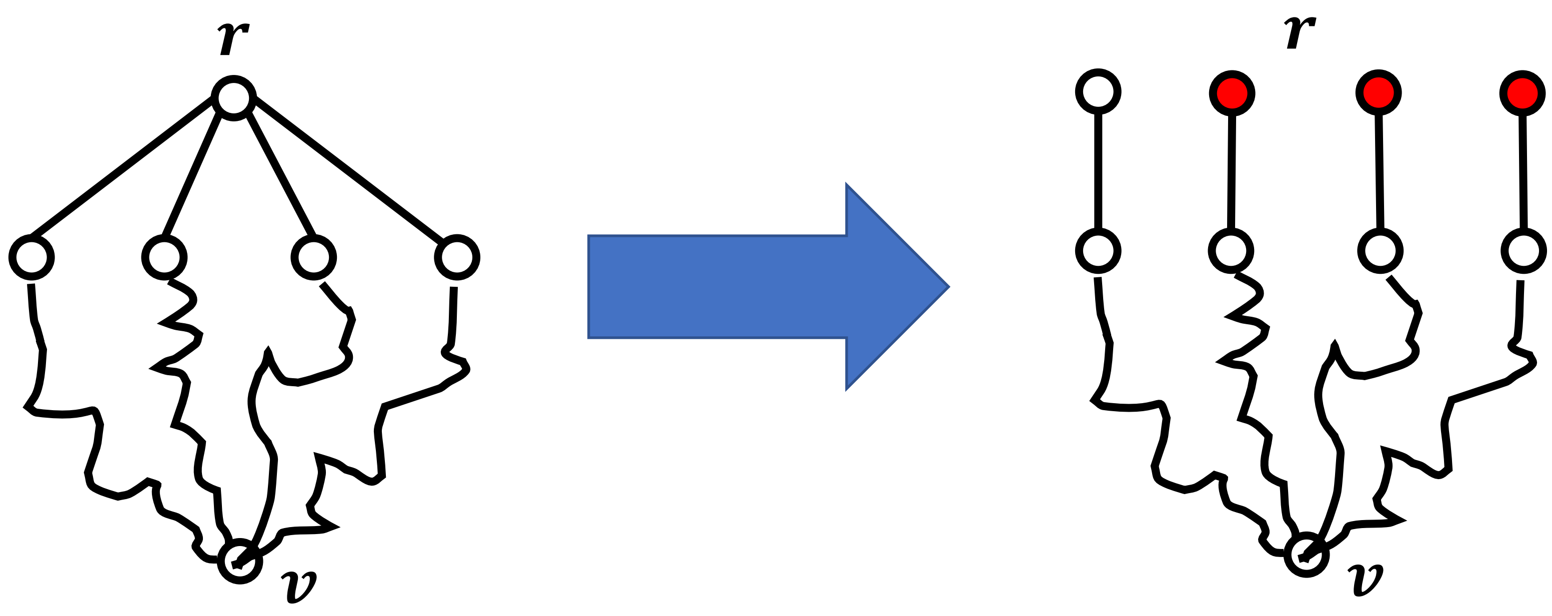

Bounding Influences: Reduction from Graphs to Trees

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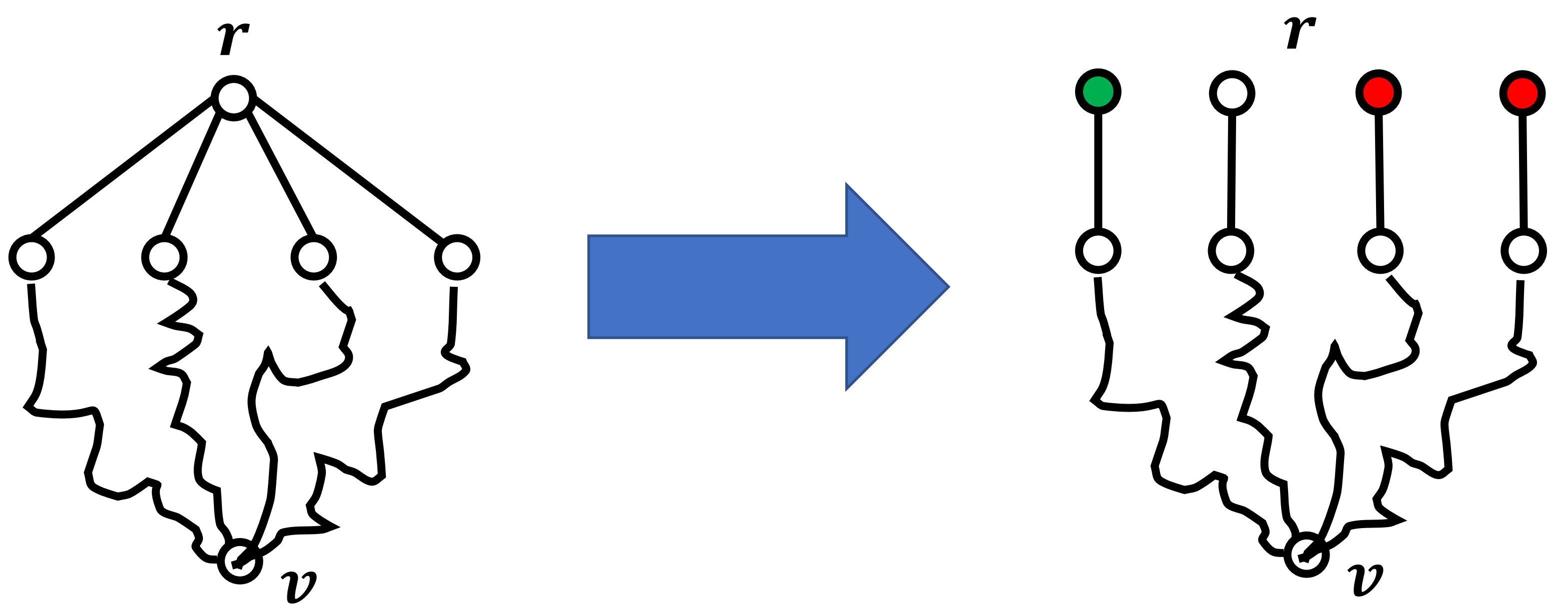

Bounding Influences: Reduction from Graphs to Trees

Goal: Bound $\sum_v |\Pr[\, v \mid r \,] - \Pr[v \mid \bar{r}]|$ for all $r \in G$

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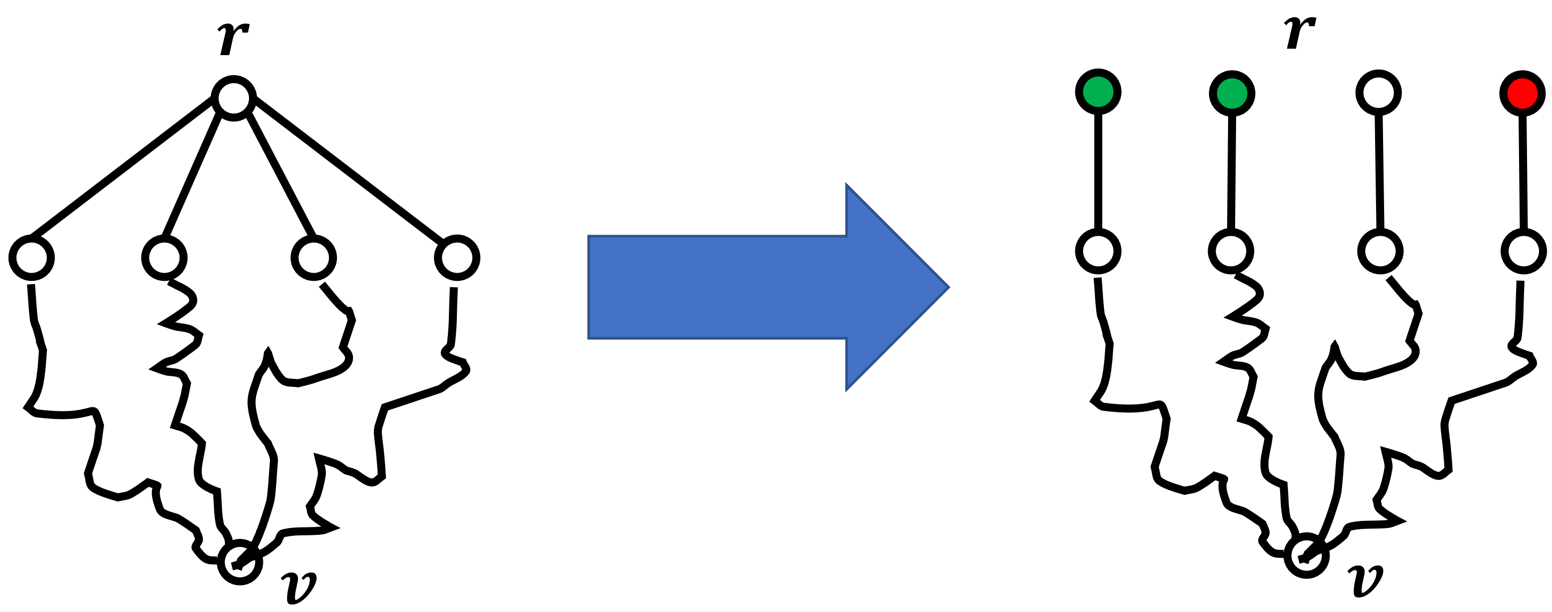
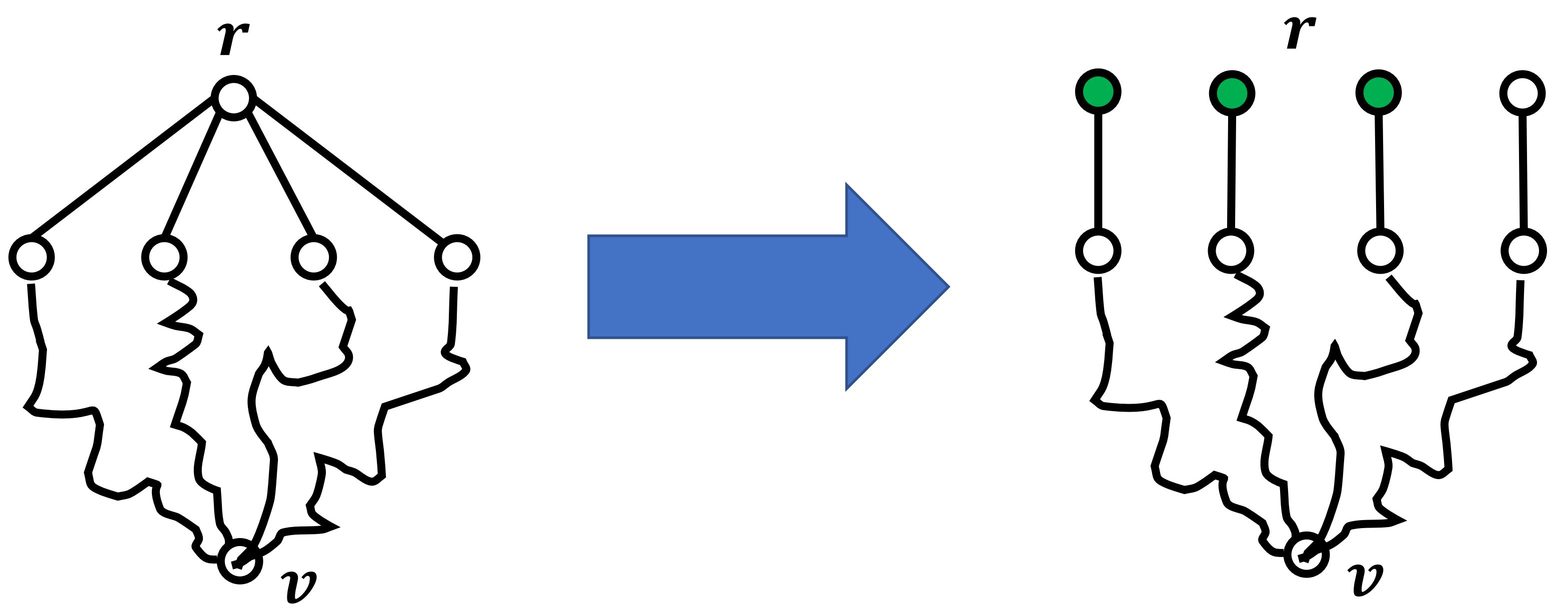


Bounding Influences: Reduction from Graphs to Trees

Goal: Bound $\sum_v |\Pr[v \mid r] - \Pr[v \mid \bar{r}]|$ for all $r \in G$

Thm [Chen-L.-Vigoda '20]: $\sum_{v \in G} |\Pr[v|r] - \Pr[v|\bar{r}]| \leq \sum_{u \in T} |\Pr[u|r] - \Pr[u|\bar{r}]|$ where $T = T_{SAW}(G, r)$

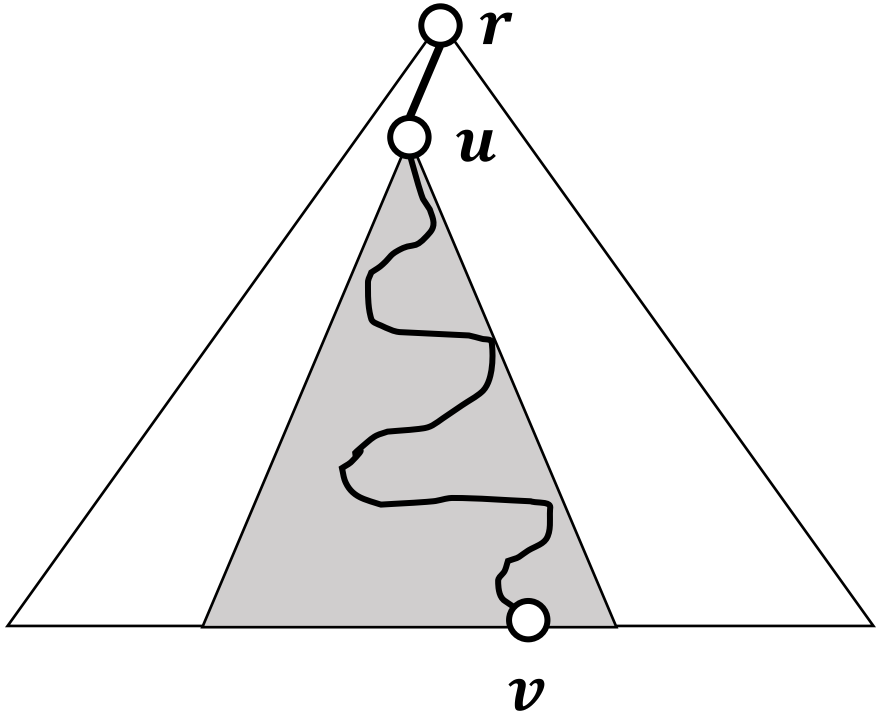


Bounding Influences: for Trees

Goal: Bound $\sum_v |\Pr[v \mid r] - \Pr[v \mid \bar{r}]|$ for all $r \in T$

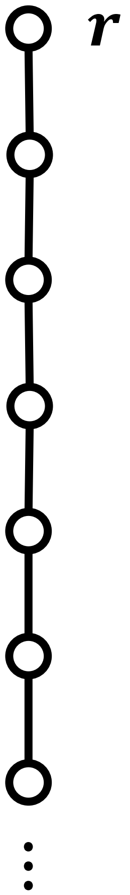
Obs: $(\Pr[u|r] - \Pr[u|\bar{r}]) \cdot (\Pr[v|u] - \Pr[v|\bar{u}]) = \Pr[v|r] - \Pr[v|\bar{r}]$

Cor: $|\Pr[v \mid r] - \Pr[v \mid \bar{r}]| \leq \left(\frac{\lambda}{1+\lambda}\right)^{d(r,v)}$



Bounding Each Vertex Separately

Cor: $|\Pr[v \mid r] - \Pr[v \mid \bar{r}]| \leq \left(\frac{\lambda}{1+\lambda}\right)^{d(r,v)}$

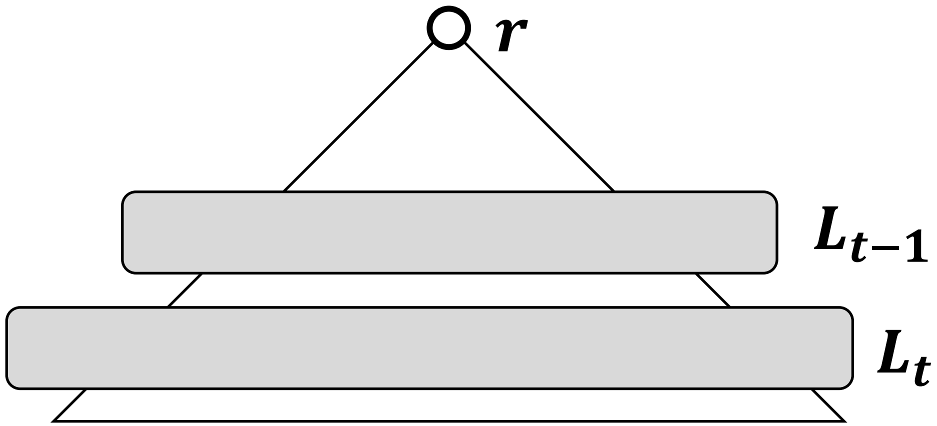


There can be $\approx (\Delta - 1)^t$ vertices at distance t

$$\sum_{v \in T} |\Pr[v \mid r] - \Pr[v \mid \bar{r}]| \lesssim \sum_{t=1}^{\infty} \left(\frac{\lambda}{1+\lambda}\right)^t (\Delta - 1)^t$$

Works only for $\lambda < \frac{1}{\Delta - 1}$

Amortize Over Levels

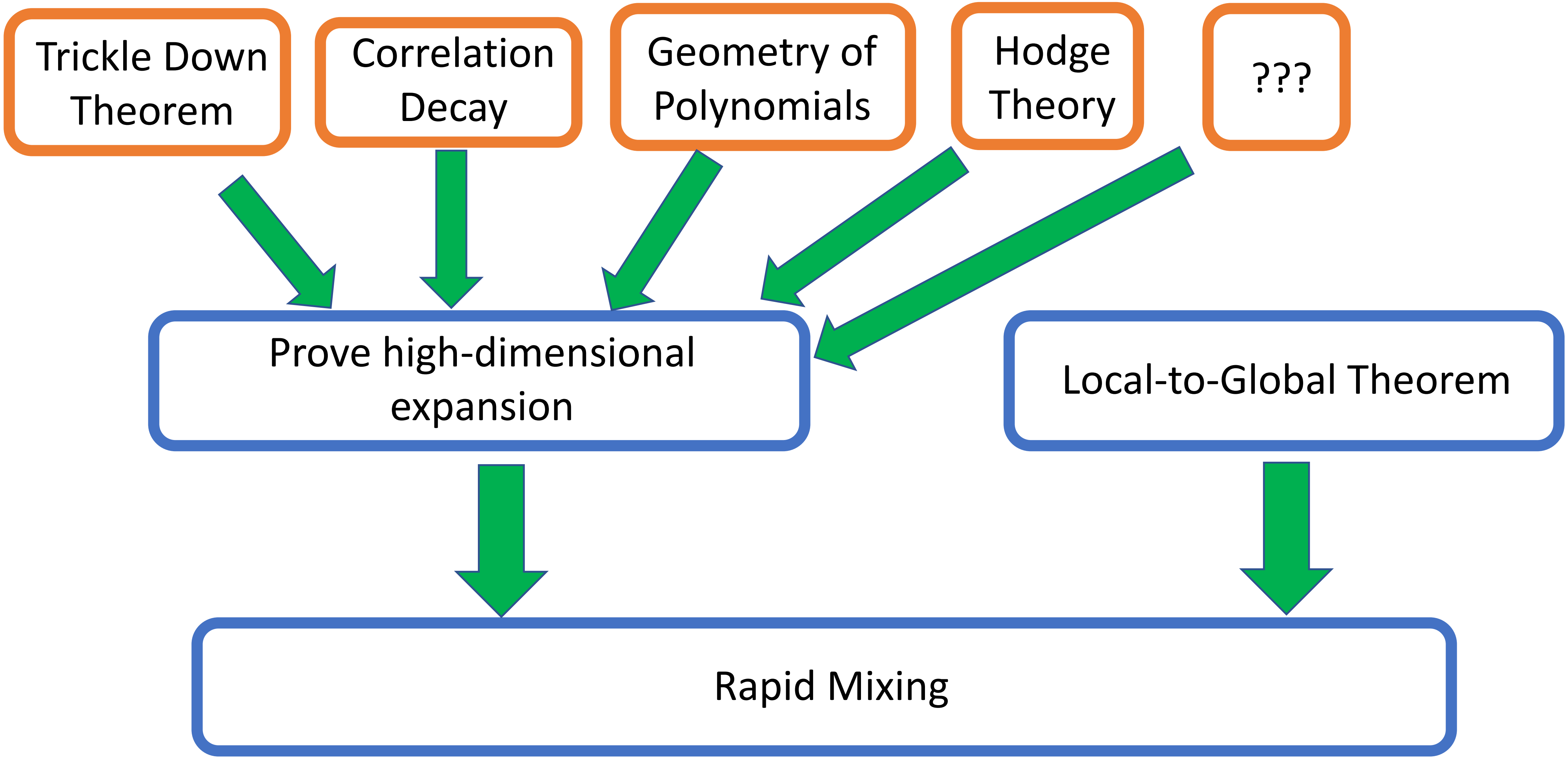


In correlation decay regime,

$$\sum_{v\in L_t} |\Pr[v \mid r] - \Pr[v \mid \bar{r}]| \lesssim (1-\delta) \sum_{v\in L_{t-1}} |\Pr[v \mid r] - \Pr[v \mid \bar{r}]|$$

$$\sum_{v\in T} |\Pr[v \mid r] - \Pr[v \mid \bar{r}]| \lesssim \sum_{t=1}^{\infty} (1-\delta)^t \lesssim \frac{1}{\delta}$$

The Strategy



Open Problems

New sampling applications?

New methods to certify expansion?

Fast algorithms?

Refinements of known local-to-global results?

Modified logarithmic Sobolev Inequalities?
[Cryan-Guo-Mousa ‘20]

Open Problems

New sampling applications?

New methods to certify expansion?

Fast algorithms?

Analysis of other chains besides Glauber dynamics?

Applications beyond sampling, such as optimization?

Thanks!