

# PRACTICAL LATTICE-BASED CRYPTOGRAPHY IN PALISADE

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*INCLUDES NEW RESULTS OF JOINT WORK WITH  
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*SIMONS INSTITUTE WORKSHOP  
LATTICES: FROM THEORY TO PRACTICE  
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The NJIT logo features the letters "NJIT" in a red, serif font, with a red swoosh underneath. The logo is set against a dark blue rounded rectangular background.

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The Duality logo consists of a stylized blue and white geometric icon above the word "Duality" in a white, sans-serif font. The logo is set against a dark blue rounded rectangular background.

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# HIGH-LEVEL OVERVIEW

- Lattices and lattice schemes implemented in PALISADE
- FHE Algorithms (most of this talk)
  - A quick introduction to double-CRT/RNS
  - **BFV RNS variants (includes new algorithms)**
  - **CKKS RNS variants (includes new algorithms)**
  - **Comparison of BGV vs BFV**
  - **Comparison of FHEW vs TFHE**
  - Extension of FHE to multi-party scenarios: PRE and Threshold FHE
- Lattice gadget toolkit (a quick review)
  - Efficient trapdoor sampling in RNS
  - Subgaussian sampling in RNS
  - Protocols based on lattice trapdoor sampling

# BASIC FACTS ON LATTICES IMPLEMENTED IN PALISADE

- Both  $q$ -ary integer lattices and cyclotomic rings are supported.
- All ring implementations are based on power-of-two cyclotomic rings:

$$R_q = \mathbb{Z}_q / \langle x^n + 1 \rangle.$$

- General cyclotomic rings are also supported but are not currently used for any special functionality.
- All protocols are based on LWE and/or Ring LWE as the hardness assumption.
- For ring instantiations, modulus  $q$  is either a prime or a product of primes, with all primes congruent to  $1 \pmod{m}$ , where  $m$  is the cyclotomic order. This constraint is set to support efficient NTT.



# LATTICE SCHEMES IMPLEMENTED IN PALISADE

- Homomorphic Encryption
  - Brakerski/Fan-Vercauteren (BFV) scheme [Bra12, FV12]: 2 RNS variants
  - Brakerski-Gentry-Vaikuntanathan (BGV) scheme [BGV14]
  - Cheon-Kim-Kim-Song (CKKS) scheme [CKKS17]: 2 RNS variants
  - Ducas-Micciancio (FHEW) [DM15] and Chillotti-Gama-Georgieva-Izabachene (TFHE) [CGGI16] schemes
  - Stehle-Steinfeld scheme (based on NTRU) [SS11]
- Multi-Party HE Extensions
  - Proxy Re-Encryption [PRSV17]
  - Threshold FHE [AJLTVW12, LTV11]

# LATTICE SCHEMES IMPLEMENTED IN PALISADE (CONT'D)

- Schemes based on lattice trapdoors
  - Gentry-Peikert-Vaikuntanathan (GPV) Digital Signature and Identity-based Encryption [GPV08]
  - Ciphertext-policy Attribute-based Encryption (ABE) [ZZG12]
  - Key-policy ABE [BGG+14]
  - Conjunction obfuscation [BVWW16]
  - Token-based conjunction obfuscation [CC17]
  - Token-based branching program obfuscation [CC17, CVW18]

# INTRODUCTION TO RNS/DOUBLE-CRT: WHY IS IT IMPORTANT?

- The main HE schemes, such as BFV, BGV, and CKKS, work with large integers (polynomial coefficients), which are much larger than native word size. Many other non-HE primitives implemented in PALISADE also work with large integers.
  - Multiprecision arithmetic (implemented in software) is typically highly inefficient for these schemes.
- Instead we use Residue Number System (RNS) or Chinese Remainder Theorem (CRT) representation of polynomial coefficients for the following reasons:
  - RNS works with native (machine-word size) integers: faster (as high as 10x) than multi-precision integer arithmetic.
  - Runtime scales (quasi)linearly with integer size.
  - RNS dramatically improves memory locality.
  - Computations are easily parallelizable.
  - RNS supports efficient GPU/FPGA hardware implementations.

# INTRODUCTION TO RNS/DOUBLE-CRT: WHAT IS IT?

- In ring constructions, we use NTT to perform efficient polynomial multiplications (it is one of the “CRTs” in the Double-CRT concept).
- Large modulus  $q$  is also represented as a smooth integer

$$q = \prod_{i=1}^k q_i,$$

where  $q_i$  are pair-wise coprime, native integers (typically of size between 20 and 60 bits).

- The large numbers (polynomial coefficients) are represented using CRT as residues **mod**  $q_i$ . This CRT decomposition, which is often called the RNS representation, corresponds to the second “CRT” in the Double-CRT notion.

# CHALLENGES OF BFV SCHEME

While addition, multiplication, and automorphism in Double-CRT is trivial (done independently for each small residue), several procedures in BFV require special RNS algorithms to avoid the use of multiprecision arithmetic. These include:

- Scaling in decryption

$$m := \left[ \left[ \frac{t}{q} \cdot [\langle \mathbf{sk}, \mathbf{ct} \rangle]_q \right]_t \right].$$

- Scaling in homomorphic multiplication (tensor product w/o modular reduction)

$$\mathbf{ct}^* := \left[ \left[ \frac{t}{q} \cdot \mathbf{ct}_1 \otimes \mathbf{ct}_2 \right]_q \right].$$

- Ciphertext digit decomposition in key switching (e.g., in relinearization)

$$\mathbf{G}^{-1}(\mathbf{ct}[last]).$$



# BFV SCHEME IN RNS

- Two RNS variants of BFV are known (both are implemented in PALISADE)
  - The Bajard-Eynard-Hasan-Zucca (BEHZ) variant [BEHZ16] that uses only integer arithmetic (based on small Montgomery reduction and auxiliary moduli).
  - The Halevi-Polyakov-Shoup (HPS) variant [HPS19] that uses both integer and floating-point arithmetic.
  - The main differences are in the RNS procedures used for decryption and homomorphic multiplication. RNS digit decomposition is done using the same technique.
- Until very recently, it was believed that
  - The HPS variant requires extended floating-point precision (long doubles and quadfloat) to support moduli  $> 45$  bits.
  - The BEHZ variant has a higher noise growth than HPS [BPAVR19].

# HPS RNS VARIANT: MAIN DEFINITIONS

- We work with the following CRT reconstructions of a large integer  $x$

$$x = \left( \sum_{i=1}^k [x_i \cdot \tilde{q}_i]_{q_i} \cdot q_i^* \right) - v \cdot q, \text{ where } v \in \mathbb{Z}$$

$$x = \left( \sum_{i=1}^k x_i \cdot \tilde{q}_i \cdot q_i^* \right) - v' \cdot q, \text{ where } v' \in \mathbb{Z}$$

$$q_i^* = q/q_i \in \mathbb{Z}_q, \quad \tilde{q}_i = (q_i^*)^{-1} \pmod{q_i} \in \mathbb{Z}_{q_i}$$

# HPS RNS VARIANT: SCALING IN DECRYPTION

$$y := \left[ \left[ \frac{t}{q} \cdot x \right] \right]_t = \left[ \left[ \left( \sum_{i=1}^k x_i \cdot \tilde{q}_i \cdot q_i^* \right) \cdot \frac{t}{q} - v' \cdot q \cdot \frac{t}{q} \right] \right]_t = \left[ \left[ \left( \sum_{i=1}^k x_i \cdot \left( \tilde{q}_i \cdot \frac{t}{q_i} \right) \right) \right] \right]_t$$

We separate integer and fractional parts (both are pre-computed):

$$t \frac{\tilde{q}_i}{q_i} = \omega_i + \theta_i, \text{ where } \omega_i \in \mathbb{Z}_t \text{ and } \theta_i = \left[ -\frac{1}{2}, \frac{1}{2} \right)$$

Main challenge is in the approximation error accumulating from rounding  $\sum_{i=1}^k x_i \cdot \theta_i$

The goal is to choose such  $q_i$  that

$$\sum_{i=1}^k x_i \cdot \varepsilon_i < \frac{1}{4}, \text{ where } |\varepsilon_i| \leq 2^{-52}$$

If we limit  $k$  to 32, then the highest modulus bit size supported with doubles is 45.

# HPS VARIANT: NEW ALGORITHM (JOINT WORK WITH ANDREY KIM)

- Inspired by the Brakerski-Vaikuntanathan digit decomposition technique, originally proposed for key switching in the BV scheme [BV11].
- We decompose  $x_i$  into high and low digits:

$$x_i = x_i^{(h)} \cdot 2^r + x_i^{(l)}$$

- Precompute at parameter generation

$$t \frac{\tilde{q}_i}{q_i} = \omega_i^{(l)} + \theta_i^{(l)}, \text{ where } \omega_i^{(l)} \in \mathbb{Z}_t \text{ and } \theta_i^{(l)} = \left[-\frac{1}{2}, \frac{1}{2}\right)$$
$$t \frac{[2^r \tilde{q}_i]_{q_i}}{q_i} = \omega_i^{(h)} + \theta_i^{(h)}, \text{ where } \omega_i^{(h)} \in \mathbb{Z}_t \text{ and } \theta_i^{(h)} = \left[-\frac{1}{2}, \frac{1}{2}\right)$$

- During decryption, we compute

$$\sum_{i=1}^k x_i \cdot \left(t \frac{\tilde{q}_i}{q_i}\right) = \sum_{i=1}^k x_i^{(l)} \cdot \left(\omega_i^{(l)} + \theta_i^{(l)}\right) + x_i^{(h)} \cdot \left(\omega_i^{(h)} + \theta_i^{(h)}\right)$$

# HPS VARIANT: NEW ALGORITHM (CONT'D)

- The approximation error is now

$$\sum_{i=1}^k x_i^{(l)} \cdot \varepsilon_i^{(l)} + x_i^{(h)} \cdot \varepsilon_i^{(h)}, \text{ where } |\varepsilon_i^{(l)}|, |\varepsilon_i^{(h)}| \leq 2^{-52}$$

- We can easily support 60-bit (or higher) moduli using *doubles* by setting  $r$  to 30 bits as the sum will always be much smaller than  $1/4$ .
- The technique is also efficient as we can perform only regular integer multiplications (no need for intermediate modular reductions in all practical cases).
- The technique is readily extendible to any size/number of digits and can be used to reduce the floating-point approximation error in any scenario where intermediate floating-point computations are used for modular integer computations.

# HPS VARIANT: CRT BASIS EXTENSION

- Extend to modulus  $p$

$$[x]_p = \left[ \left( \sum_{i=1}^k [x_i \cdot \tilde{q}_i]_{q_i} \cdot q_i^* \right) - v \cdot q \right]_p.$$

- Estimate  $v$  using floating-point arithmetic

$$v = \left\lfloor \left( \sum_{i=1}^k [x_i \cdot \tilde{q}_i]_{q_i} \cdot q_i^* \right) / q \right\rfloor = \left\lfloor \sum_{i=1}^k \frac{[x_i \cdot \tilde{q}_i]_{q_i}}{q_i} \right\rfloor.$$

- Compute

$$[x]_p = \left[ \left( \sum_{i=1}^k [x_i \cdot \tilde{q}_i]_{q_i} \cdot [q_i^*]_p \right) - v \cdot [q]_p \right]_p$$

- The approximation error introduced by floating-point computations does not practically affect the noise in the homomorphic multiplication (less than 1 bit)



# HPS BFV VARIANT: CURRENT SINGLE-THREADED RESULTS

<i>L</i>	<i>n</i>	$\log_2 q$	<i>k</i>	<i>Encryption</i> [ms]	<i>Addition</i> [ms]	<i>Multiplication</i> [ms]	<i>Rotation</i> [ms]	<i>Decryption</i> [ms]
1	$2^{11}$	50	1	0.42	0.008	1.84	0.31	0.09
3	$2^{12}$	100	2	1.18	0.026	5.90	0.66	0.31
5	$2^{13}$	180	3	3.10	0.077	17.7	2.82	0.88
10	$2^{13}$	240	4	7.77	0.218	48.6	9.80	2.21
20	$2^{14}$	420	7	12.2	0.425	98.8	28.4	3.76
30	$2^{15}$	660	11	38.4	1.53	383	144	12.3
50	$2^{16}$	1020	17	118	5.72	1,556	717	39.0
100	$2^{17}$	2100	35	510	24.9	11,252	6,435	171

From the practical perspective, the BFV implementation in PALISADE is now used in a commercial product of Duality (for an encrypted SQL-like query).

# NOTES ON COMPARISON OF HPS AND BEHZ RNS VARIANTS

- Thanks to Vincent Zucca and Ahmad Q. A. Al Badawi, PALISADE has a highly optimized implementation of the BEHZ variant, and we can perform a fair comparison of both variants.
- Based on the current implementation of both in PALISADE and complexity analysis
  - The runtimes are approximately the same (with differences only within 10%).
  - The noise growth is the same in both variants (as recently suggested in [MENSZ19]).
  - The only significant (usability) difference is that HPS is simpler, and easier to implement.
- It should be noted that both variants can co-exist and their RNS procedures can be mixed in the same implementation of BFV.

# MAIN CHALLENGE OF CKKS SCHEME IN RNS: RESCALING

- Modulus switching, which is used for rescaling in CKKS, is defined as

$$[\mathbf{c}']_q = \left\lfloor \frac{q}{Q} \cdot [\mathbf{c}]_Q \right\rfloor.$$

- In CKKS rescaling,  $\Delta = Q/q$  corresponds to the scaling factor for encoding the floating-point data.
- In original (multiprecision) CKKS,  $\Delta$  is typically  $2^p$ , and both  $Q$  and  $q$  are a power of 2.
- In RNS variants of HE schemes,  $Q$  and  $q$  are decomposed into products of small primes to support fast NTT, and  $q$  typically divides  $Q$ .
- Rescaling requires scaling down by  $2^p$  but we cannot choose such  $q$  and  $Q$  that will give us  $2^p$  almost exactly. Our best approximation is a small prime  $Q/q$  that is close to  $2^p$ .

# “APPROXIMATE” RESCALING RNS VARIANT OF CKKS

- Initial CKKS RNS variants, such as [CHKKS18] and [BGPRV19], treat the rescaling approximation error as extra “noise” in the CKKS scheme.
- The RNS moduli  $q_i$  are chosen such that  $2^p/q_i$  stays in the range  $(1 - 2^{-\varepsilon}, 1 + 2^{-\varepsilon})$ , where  $2^{-\varepsilon}$  is kept as small as possible.
- When we perform the first multiplication and rescale operation, the effective scaling factor changes from  $2^p$  to  $2^{2p}/q_L$ .
  - We introduce an approximation error of roughly  $p - \varepsilon$  bits.
- If we alternate the moduli around  $2^p$ , i.e.,  $q_{i-1} < 2^p$  and  $q_i > 2^p$ , we can reduce the approximation error after multiple levels.
- Still, the error keeps growing as we increase the ring dimension and depth of computation, and the rescaling approximation error dominates over other CKKS noise (typically exceeding it by several bits). Can we avoid this approximation error?

# “EXACT” RESCALING RNS VARIANT OF CKKS (JOINT WORK WITH ANTONIS PAPADIMITRIOU)

- Yes, we can eliminate the approximation error but the cost can be too high.
  - The rescaling error is deterministic and depends only on  $2^p$  and  $q_i$ 's.
  - Hence we can always adjust the scaling factor by performing an extra scalar multiplication.
- A naïve solution is to multiply by a scaling factor after every multiplication.
  - While preserving the original scaling factor  $2^p$ , this seems to double the number of levels.
- Can we do better than this?

# “EXACT” RESCALING RNS VARIANT OF CKKS (JOINT WORK WITH ANTONIS PAPADIMITRIOU) – CONT’D

## Challenge 1

How to efficiently adjust the scaling factors, minimizing the performance cost?

## Our Solution

Assign a different scaling factor to each level:

- Use the “natural order” of CKKS

Level	Scaling factor
$L$	$q_L$
$L-1$	$q_L$
$L-2$	$q_L^2 / q_{L-1}$
$L-3$	$(q_L^2 / q_{L-1})^2 / q_{L-2}$
...	...

- Avoids any scalar multiplication for ciphertexts following the natural order.
- We need to do scalar multiplication only for ciphertexts at different levels.



# “EXACT” RESCALING RNS VARIANT OF CKKS (JOINT WORK WITH ANTONIS PAPADIMITRIOU) – CONT’D

## Challenge 2

How to hide scaling factor adjustments from the user, i.e., do them automatically?

## Our Solution

- Keep track of the scaling factors for each level
- Automatically adjust the scaling factor when needed
- Automatically rescale to enforce the natural order of scaling factors

# “EXACT” RESCALING RNS VARIANT OF CKKS (JOINT WORK WITH ANTONIS PAPADIMITRIOU) – CONT’D

## Challenge 3

What logic should we use for automated rescaling?

## Our Solution

Rescale right before the next multiplication.

- Perform all computations between two multiplications using the “noise budget” of the first multiplication. So these intermediate operations are almost noise-free.
- There is a performance penalty as most operations work with an effective scaling factor of  $\sim 2^{2p}$  (and we keep one extra modulus in the CRT basis of the current ciphertext), but it is relatively small for most cases.

# “EXACT” RESCALING RNS VARIANT OF CKKS (JOINT WORK WITH ANTONIS PAPADIMITRIOU) – CONT’D

## Challenge 4

We noticed that the natural-order scaling factors start diverging either towards 0 or infinity after a certain number of levels (like 20), hence preventing the use of this RNS variant for deeper computations.

## Our Solution

Choose  $q_i$ 's so that the scaling factor changes as little as possible from one level to the next.

```
SelectModuli( $n, L, p$ ):  
   $q_L := \text{FirstPrime}(p, n)$   
   $q_{next} := q_L$   
   $q_{prev} := q_L$   
   $sf_L := q_L; sf_{L-1} := q_L$   
   $ctr := 0$   
  for  $i = L - 2, \dots, 1$   
     $sf_i := \frac{(sf_{i+1})^2}{q_{i+1}}$   
    if  $ctr \bmod 2 = 0$   
       $q_{prev} := \lfloor sf_i \rfloor - 2n - \lfloor \lfloor sf_i \rfloor \rfloor_{2n} + 1$   
       $q_i := \text{PreviousPrime}(q_{prev}, n)$   
    else  
       $q_{next} := \lfloor sf_i \rfloor + 2n - \lfloor \lfloor sf_i \rfloor \rfloor_{2n} + 1$   
       $q_i := \text{NextPrime}(q_{next}, n)$   
     $ctr := ctr + 1$   
   $q_0 := \text{PreviousPrime}(60, n)$   
  return  $q$ 
```

# “EXACT” RESCALING RNS VARIANT OF CKKS (JOINT WORK WITH ANTONIS PAPADIMITRIOU) – SUMMARY

## Exact vs Approximate RNS Rescaling

- The rescaling error in the exact RNS variant is similar to the error in the “textbook” CKKS. In the approximate RNS variant, the rescaling error is often higher than CKKS noise.
  - Net effect: higher output precision (same as in “textbook” CKKS) can be supported.
- Rescaling and “modulus switching” (level reduction without changing the scale) are done automatically in the exact RNS variant. Rescaling is performed manually in the approximate RNS variant.
- The exact RNS variant is typically 1.1x to 1.5x slower (depending on the computations) than the approximate variant due to extra scalar multiplications and “delayed” rescaling. But it is much simpler for the user and provides a higher precision.

# KEY SWITCHING IN CKKS AND BGV

Key switching plays a crucial role in CKKS and BGV (and to some extent in BFV). It is often the main performance bottleneck of many computations. We consider the following two techniques as most practical options for RNS variants of CKKS and BGV:

- The RNS decomposition technique proposed in [BEHZ16] (similar in concept to Brakerski-Vaikuntanathan (BV) digit decomposition [BV11])
  - Often faster for a small number of levels.
  - May introduce significant noise when performing rotations.
- Hybrid key switching (a hybrid of Gentry-Halevi-Smart key switching [GHS12] and RNS decomposition) [HK20]
  - Often faster for deeper computations.
  - Introduces auxiliary moduli (thus increasing the effective ciphertext modulus used in estimating the LWE work factor).
  - Generally has lower noise growth.

# COMPARISON OF BGV VS BFV

Comparison of the single-threaded runtime for BGV and BFV to compute a chain of  $2^{depth} - 1$  homomorphic multiplications (using the binary tree approach). The security level was set to 128 bits according to the HomomorphicEncryption.org security standard.

t	depth	BGV [ms]	BFV [ms]	BGV Speedup
2	2	4.8	35.2	7.3x
2	3	34.8	82.1	2.4x
2	4	112	261	2.3x
2	5	318	554	1.7x
2	6	872	1,136	1.3x
65537	2	9.7	51.6	5.3x
65537	3	35.1	121	3.4x
65537	4	256	730	2.9x
65537	5	740	1,508	2.0x
65537	6	1,999	4,060	2.0x

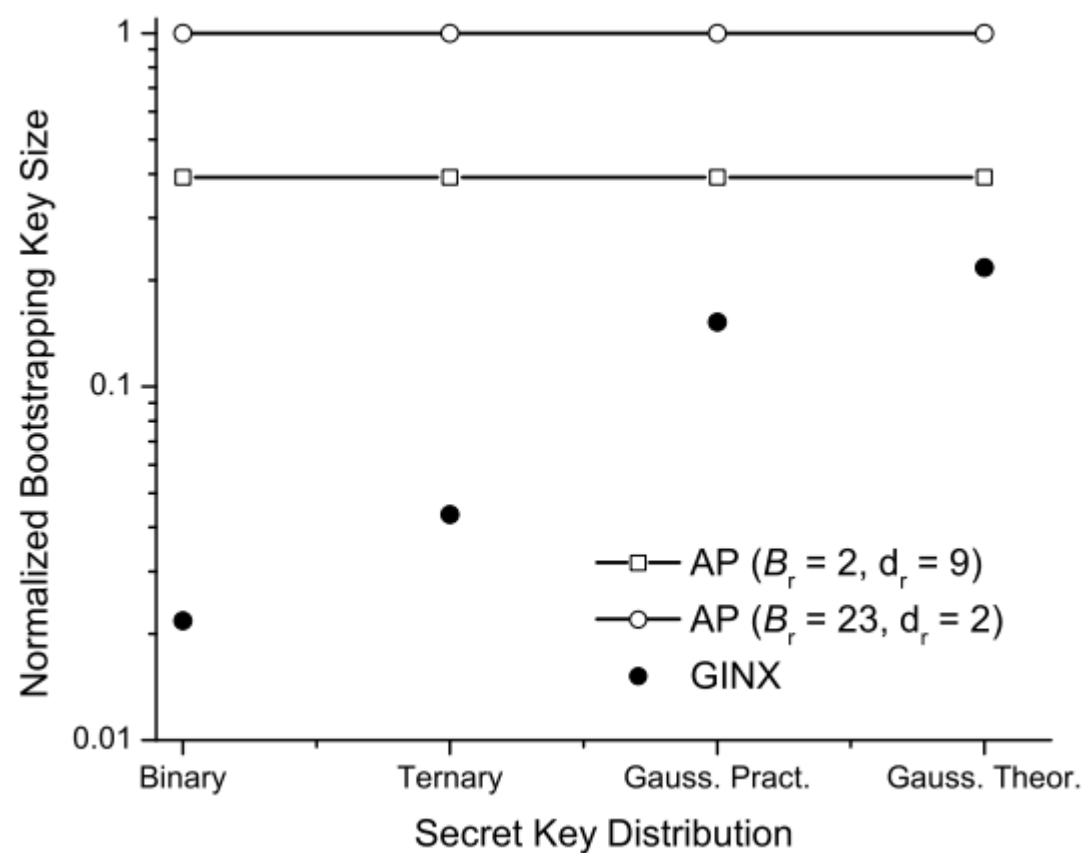
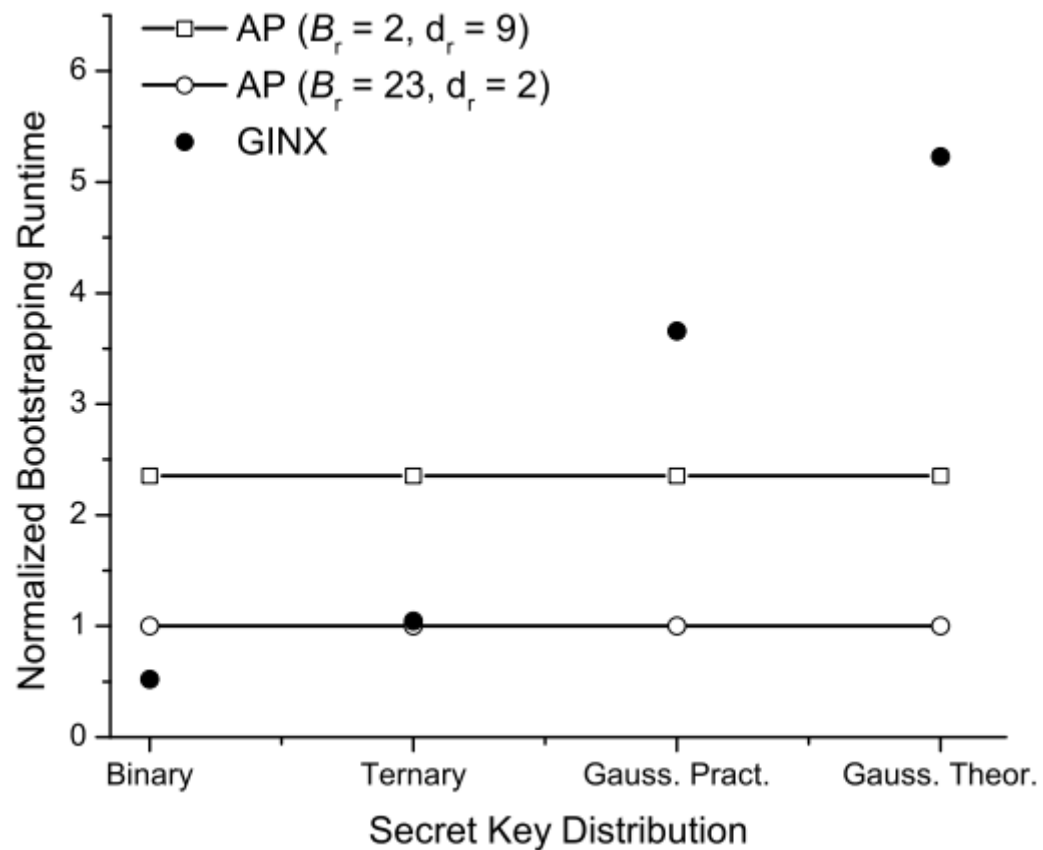


# FHEW VS TFHE

We recently presented a framework that integrates the Ducas-Micciancio (FHEW) and Chillotti-Gama-Georgieva-Izabachene (TFHE) schemes [MP20]. The results can be summarized as follows:

- When instantiated using LWE/Ring LWE, both schemes are the same except for the bootstrapping procedure: Alperin-Sherif—Peikert (AP) for FHEW vs Gamas-Izabachene-Nguyen-Xie (GINX) for TFHE.
- All other optimizations and enhancements, such as circuit bootstrapping, proposed in later TFHE papers equally apply to FHEW.
- Though the original TFHE scheme is formulated for binary secret distributions, it can be extended to ternary and Gaussian secrets.

# FHEW VS TFHE (CONT'D)



# MULTI-PARTY EXTENSIONS OF FHE

In [PRSV17], we showed that PRE can be built using the key switching procedure in BGV.

- We subsequently improved the security (changed from CPA to security under Honest Re-encryption Attacks [Coh19]) and extended it to BFV and CKKS.
- The current runtimes are one order of magnitude better than in [PRSV17] and look favorably compared to non-lattice constructions.
- There is an ongoing work with Aloni Cohen in this area.

Single-key FHE has also been extended in PALISADE to the multi-party threshold FHE setting using the ideas from [AJLTVW12, LTV11].

- Threshold FHE support has been added to BGV, BFV, and CKKS.
- The algorithms are being improved, and the enhanced implementation will soon be publicly available.

# LATTICE GADGET TOOLKIT

PALISADE provides an implementation of a recent lattice gadget toolkit developed in [GM18] and [GMP19]. The toolkit includes new gadget matrices and sampling algorithms:

- Efficient MP12 trapdoor sampling for lattices with arbitrary moduli, particularly, power-of-2 cyclotomic rings.
- Efficient subgaussian sampling for the same lattices.
- CRT (RNS) gadgets.
- RNS algorithms for discrete gaussian and subgaussian sampling.

The focus has been on general-purpose lattice trapdoor sampling algorithms that can support complex protocols, such as key-policy ABE and program obfuscation.

# PROTOCOLS BASED ON LATTICE TRAPDOORS

The following protocols are implemented in PALISADE (please see the references for implementation details):

- GPV digital signature and IBE, ciphertext-policy ABE [GPRRS18, GPRRSS19].
- Key-policy ABE in RNS [DDPRSSS18,GMP19].
- Conjunction obfuscation based on GGH15 in RNS [CDGKPRRS18].
- Token-based obfuscation of conjunctions and branching programs based on GGH15 in RNS [CGMPR18].

# MORE INFORMATION ON PALISADE

- PALISADE website
  - <https://palisade-crypto.org/>
- Stable release
  - <https://gitlab.com/palisade/palisade-release>
  - Current version: 1.9.2
- Development repo
  - <https://gitlab.com/palisade/palisade-development>
  - Includes latest code, preview releases, and additional lattice schemes
- Contact
  - [contact@palisade-crypto.org](mailto:contact@palisade-crypto.org)

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# THANK YOU

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