The robust polynomial method and a subvolume law for locally gapped frustration-free 2D spin systems

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Upcoming section

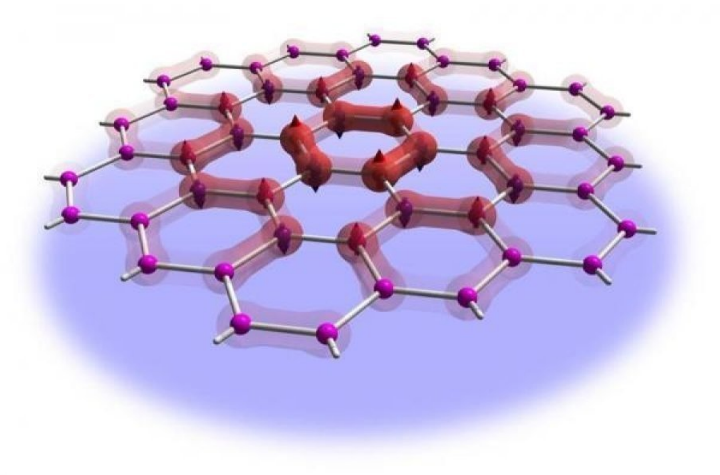
Introduction

Bounding entanglement entropy

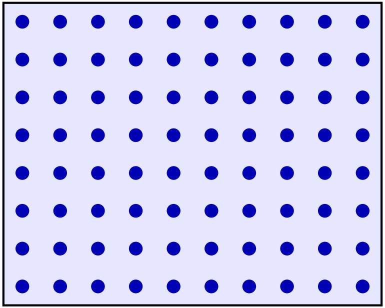
Polynomials

Sub-volume law

Discussion



Square lattice



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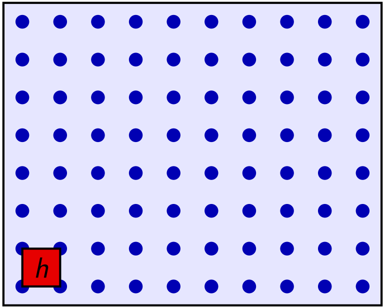

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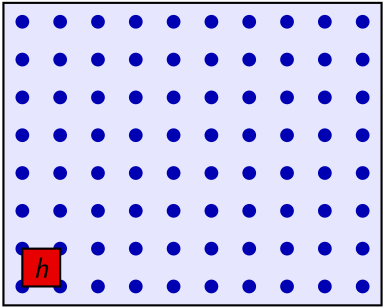

- Suppose $0 \preceq h \preceq \mathrm{I}$.

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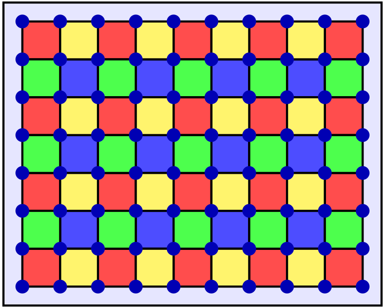

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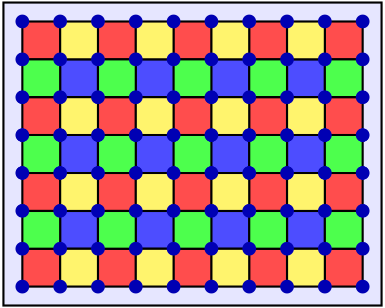

- $H = \sum_{i=1}^{n-1} \sum_{j=1}^{L-1} (\mathrm{I} \otimes \mathrm{I} \otimes \ldots h_{i,j} \otimes \ldots \mathrm{I})$.

Ground state and frustration-free assumption

- Ground state $|\Omega\rangle$.
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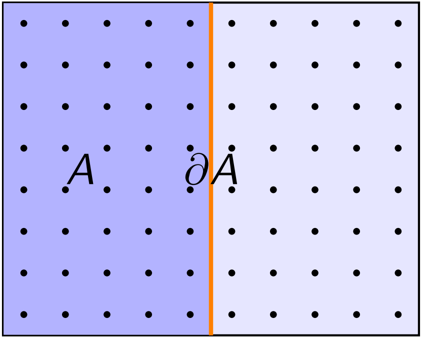
- Frustation-free (FF).
  - $h_{i,j} |\Omega\rangle = 0, \quad \forall$ i,j.
- FF allows us to choose $h_{i,j}^2 = h_{i,j}$.
  - Mapping $h_{i,j} \to \text{span}(h_{i,j})$ does not change $|\Omega\rangle$ and changes γ by a constant.

Entanglement entropy



Bound on $\mathrm{S}(\Omega_A)$?

Entanglement entropy

- Area law: $\mathrm{S}(\Omega_A) = \mathcal{O}(|\partial A|)$.
- Trivial volume law: $\mathrm{S}(\Omega_A) = \mathcal{O}(|\partial A|^2)$.
- Sub-volume law: $\mathrm{S}(\Omega_A) = \mathcal{O}(|\partial A|^c)$ for some $1 < c < 2$.

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Area law conjecture: Unique ground state of a gapped hamiltonian ($\gamma =$ some constant) satisfies an area law across every bi-partition ∂A.

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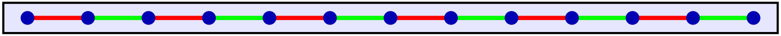
Most quantum states satisfy volume law. Thus area/sub-volume laws show that ground states are 'simpler' than most quantum states.

Results in 1D

Hastings [2007]	$exp\left(\mathcal{O}(1/\gamma)\right)$
Aharonov, Arad, Landau, Vazirani [2011] (FF)	$exp\left(\mathcal{O}(1/\gamma)\right)$
Arad, Landau, Vazirani [2012] (FF)	$\mathcal{O}(1/\gamma^3)$
Arad, Kitaev, Landau, Vazirani [2013]	$\mathcal{O}(1/\gamma)$



Conjecture of Gosset, Huang [2016]: Scaling for FF is $\mathcal{O}\left(\frac{1}{\sqrt{\gamma}}\right)$.

Implications of 1D area law

- 1D area law implies gapped ground state can be approximated by a Matrix-Product State of 'small' bond dimension.
- Supports the success of Density Matrix Renormalization Group algorithm (White [1992]).
- Polynomial time algorithm for ground states (Landau, Vidick, Vazirani [2013]; Arad, Landau, Vidick, Vazirani [2016]).

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- Adiabatic assumption. Cho [2014].
- Assumptions on specific heat. Brandão, Cramer [2015].

For commuting hamiltonian: $[h_{i,j}, h_{i',j'}] = 0$, area law holds in all dimensions.

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Approximation to ground space

- Schmidt rank of an operator K: smallest D such that

$$K = \sum_{i=1}^{D} K_A^i \otimes K_{A^c}^i.$$

- We denote it by $\mathrm{SR}(K)$.

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  - $\| K_{imaginary} - |\Omega\rangle\langle\Omega| \|_1 \leq \varepsilon$.
  - $\mathrm{SR}(K_{imaginary}) =$ small.

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  - $\mathrm{S}(\Omega_A) \leq \log \text{small} + \underbrace{\varepsilon |A|}_{\text{(Alicki-Fannes)}}$.

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  - $\mathrm{S}(\Omega_A) \leq \log \text{small} + \underbrace{\varepsilon |A|}_{\text{(Alicki-Fannes)}}$.
- But imaginary $\neq$ real.
- True situation: $\| K - |\Omega\rangle\langle\Omega| \|_\infty \leq \varepsilon$.

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- $\| K - |\Omega\rangle\langle\Omega| \|_\infty \leq \Delta$ and $\mathrm{SR}(K) = D$.
- For FF systems, we also have $K|\Omega\rangle = |\Omega\rangle$.

Approximation to ground space

Theorem (Hastings 2007; Arad, Landau, Vazirani 2012)

$$\underbrace{\mathrm{S}_{min}\left(\Omega_A\right) \leq}_{obvious} \mathrm{S}\left(\Omega_A\right) \leq \frac{\log D}{\log \frac{1}{\Delta}} \mathrm{S}_{min}\left(\Omega_A\right) + \log D.$$

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- What happens if $\frac{\log D}{\log \frac{1}{\Delta}} < 1$?

Theorem (Arad, Landau, Vazirani 2012)

If $D\Delta < \frac{1}{2}$ (the AGSP condition), then

$$\mathrm{S}\left(\Omega_A\right) \leq 2 \log D.$$

Polynomial approximation to ground space

- Arad, Landau, Vazirani [2012] and Arad, Kitaev, Landau, Vazirani [2013] viewed K as polynomials of H.
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    - Then $f_{ground}(H) = |\Omega\rangle\langle\Omega|$.
    - Approximate f_{ground} using tools from approximation theory.

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  - Think of a multinomial $h_{7,1} h_{7,3} \underbrace{\ldots}_{d \text{ times}} h_{7,21}$.
- If $d < s$, then AGSP condition is satisfied.
- Unfortunately, a stringent condition in practise.

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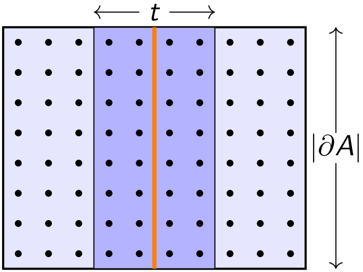

- A family of AGSPs: K_t with degree d_t (in dark blue region) and $\Delta = e^{-s_t}$.

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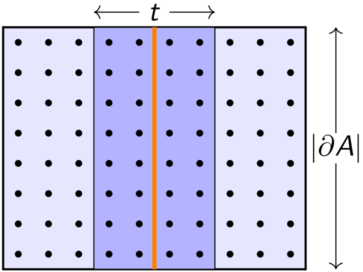

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Theorem (Arad, Landau, Vazirani (2012); Arad, Kitaev, Landau, Vazirani (2013))

If $d_t \leq t \cdot s_t$, then AGSP condition is satisfied and $\mathrm{S}\left(\Omega_A\right) \leq t|\partial A|$.

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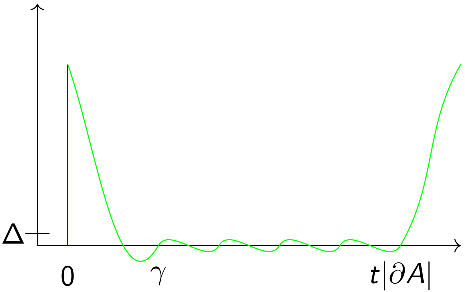

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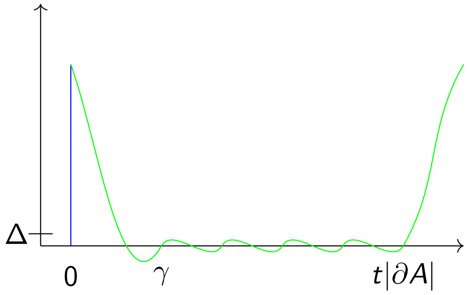

- Chebyshev polynomials, for a given degree, achieve the smallest approximation.
- Let us assume $\gamma =$ constant, till penultimate slide.

Chebyshev approximation

- Chebyshev polynomial achieves $d_t = \sqrt{t|\partial A|}$ and $\Delta = e^{-s_t} = \frac{1}{3}$.
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- Evaluating $d_t \leq ts_t$, we get $t > |\partial A|$.
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- Still a volume law. Note: any improvement would give subvolume law.
- But what about commuting case?

Improved Chebyshev approximations for integer points

Inclusion-exclusion: Exact and approximate

Jeff Kahn, Nathan Linial & Alex Samorodnitsky

Combinatorica **16**, 465–477(1996) | Cite this article

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- This polynomial was discovered independently in (Buhrman, Cleve, de Wolf, Zalka [1999]), who showed that quantum query complexity for AND_n with error ε requires $\sqrt{n \log \frac{1}{\varepsilon}}$ queries, instead of $\sqrt{n} \log \frac{1}{\varepsilon}$ queries.
  - $\log \frac{1}{\varepsilon} \equiv s_t$ and queries $\equiv d_t$.

Improved Chebyshev approximations for integer points

- Analysis: setting $d_t = t|\partial A|$, we get $s_t = t|\partial A|$. Thus, $d_t \leq ts_t$ can be satisfied with $t = 1$ (constant). Hence, $\mathrm{S}\left(\Omega_A\right) \leq t|\partial A| = |\partial A|$.

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- Unfortunately, the construction heavily uses the integer spectrum and can't be generalized to continuous spectrum of a non-commuting Hamiltonian.

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Approach

- Make the improved Chebyshev approximation more friendly for non-commuting case, using robust polynomials.
- Use it, battling non-commutativity, to get improved approximation in 2D (requires local gap assumption).

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  - Take a binary string $(x_1, x_2, \ldots x_m)$ and corrupt its values to $(x_1 + e_1, x_2 + e_2, \ldots x_m + e_m) \in \mathbb{R}^m$. Here $e_i \in (-\frac{1}{10}, \frac{1}{10})$.

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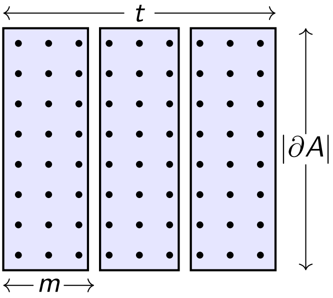
  - Feed in corrupted input to Rob_p. It holds that

$$\mathrm{Rob}_p(x_1 + e_1, x_2 + e_2, \ldots x_m + e_m) = p(x_1, x_2, \ldots x_m) \pm 2^{-d}.$$

Improved approximation for AND



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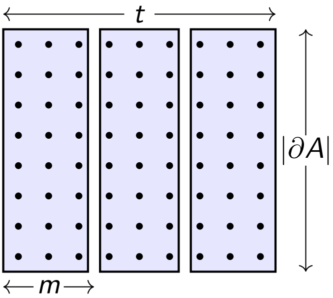

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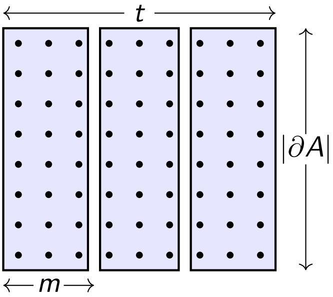

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- Approximate $\mathrm{AND}_{t|\partial A|}$ by $\mathrm{Rob}_{\mathrm{AND}_{\frac{t}{m}}} \circ (q)^{\times \frac{t}{m}}$.

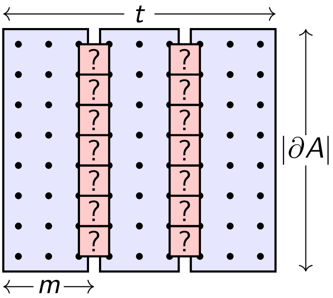
Improved approximation for AND

- Degree is $d_t = \frac{2t}{m} \times \sqrt{m|\partial A|} = \frac{2t\sqrt{|\partial A|}}{\sqrt{m}}$.
- Error is $e^{-s_t} = 2^{-\frac{t}{m}}$.
- Since $m = \frac{4t^2|\partial A|}{d_t^2}$, we recover

$$s_t = \frac{d_t^2}{4t|\partial A|}.$$

- Since $1 \leq m \leq t$, we also recover the constraint $2\sqrt{t|\partial A|} \leq d_t \leq 2t|\partial A|$.

Lifting to local hamiltonian setting



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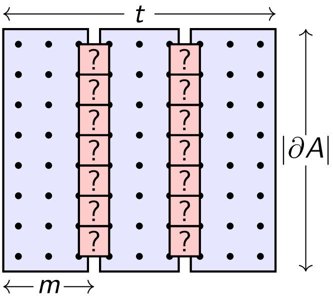

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- Quantum friendly: Approximate $\mathrm{AND}_{m|\partial A|}$ by Chebyshev polynomial q with error $\frac{1}{10}$ and degree $\approx \sqrt{m|\partial A|}$.
- Robust polynomial is also quantum friendly.

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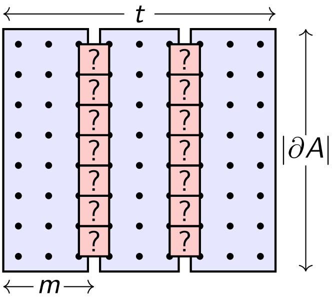
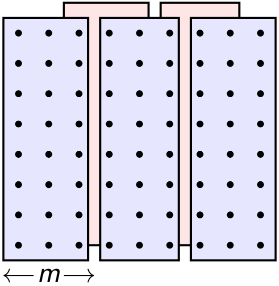


- Assume that the hamiltonian on the blue blocks is also gapped: local gap assumption.
- Quantum friendly: Approximate $\mathrm{AND}_{m|\partial A|}$ by Chebyshev polynomial q with error $\frac{1}{10}$ and degree $\approx \sqrt{m|\partial A|}$.
- Robust polynomial is also quantum friendly.
- But we are missing out the ground space of H.

Coarse-grained detectability lemma



- Coarse-grained detectability lemma (A., Arad, Vidick [2016]; Aharonov, Arad, Landau, Vazirani [2011]): The 'AND' of blue and red projectors, that is,

$$\mathrm{Blue}_1 \cdot \mathrm{Blue}_2 \cdot \mathrm{Blue}_3 \cdot \mathrm{Red}_1 \cdot \mathrm{Red}_2$$

is e^{-m} close to the ground space on $t|\partial A|$ qudits.

Subvolume law of 5/3

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Theorem (A., Arad, Gosset, 2019)

For locally gapped FF spin systems (local gap constant), we have

$$\mathrm{S}\left(\Omega_A\right) = \tilde{\mathcal{O}}\left(|\partial A|^{5/3}\right).$$

How far can this go?

- Due to non-commutativity, a degree d_t polynomial can only be expected to achieve

$$e^{-s_t} = \underbrace{e^{-\frac{d_t^2}{t|\partial A|}}}_{\textit{Improved Chebyshev}} + \underbrace{e^{-t}}_{\text{Detectability lemma}} .$$

- If this were the correct behaviour, we would get $\mathrm{S}\left(\Omega_A\right) \approx |\partial A|^{3/2}$ (Work in progress).

Upcoming section

Introduction

Bounding entanglement entropy

Polynomials

Sub-volume law

Discussion

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- Helpful FF example from Michalakis, Zwolak (2011):

$$H = \sum_{i=1}^{N-1} \left(|00\rangle\langle 00|_{i,i+1} + |11\rangle\langle 11|_{i,i+1} + \frac{\delta_{i=even}}{3N} |01\rangle\langle 01|_{i,i+1} \right),$$

- Ground state is $|010101\ldots\rangle$, spectral gap is $\frac{2}{3}$, but local gap is $\frac{1}{3N}$.

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- Ground state is $|010101\ldots\rangle$, spectral gap is $\frac{2}{3}$, but local gap is $\frac{1}{3N}$.
- But mapping $h_{i,j} \to \text{span}(h_{i,j})$, new H has local gap 1.

Local gap assumption

- For every FF hamiltonian H, is there a transformation to H' that is a sum of projectors, such that $\gamma_{loc} \geq \gamma^c$, for some constant c?

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- Are constructions from Cubitt, Perez-Garcia-Wolf (2015); Bausch, Cubitt, Lucia, Perez-Garcia (2018) counter examples?
- Note that γ can be lower bounded in terms of γ_{loc} due to Knabe's theorem (1988) and its generalizations: Gosset, Mozgunov [2015]; Kastoryano, Lucia [2017]; Lemm, Mozgunov [2018]; etc.

Representation of 2D ground state

- Recent work of Abrahamsen [2020] shows subexponential algorithms for preparing locally gapped FF ground states.
- It is possible to show that there exist PEPS representations with better scaling for such ground states?
- Can AGSPs circumvent the information theoretic limitation of "area law doesn't imply PEPS with polynomial bond dimension" (Ge, Eisert 204)?

Scaling with gap, and a coincidence (?)

- Prior works: $\frac{|\partial A|^2}{\gamma}$.
- Our result: $\frac{|\partial A|^{5/3}}{\gamma^{5/6}}$.
- Hopeful conjecture: $\frac{|\partial A|^{3/2}}{\gamma^{3/4}}$.
- Gosset-Huang conjecture: 1D scaling of FF systems is $\frac{1}{\sqrt{\gamma}}$ (the correlation length).

Thank you for your attention!