Secure Multi-Party Quantum Computation with a Dishonest Majority

Yfke Dulek, Alex Grilo, Stacey Jeffery,
Christian Majenz, Christian Schaffner

arXiv: 1909.13770 , @EuroCrypt 2020

seminar talk @Simons, Tuesday, March 17, 2020

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CWI

QuSoft
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Thanks!

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Multi-party computation (MPC)

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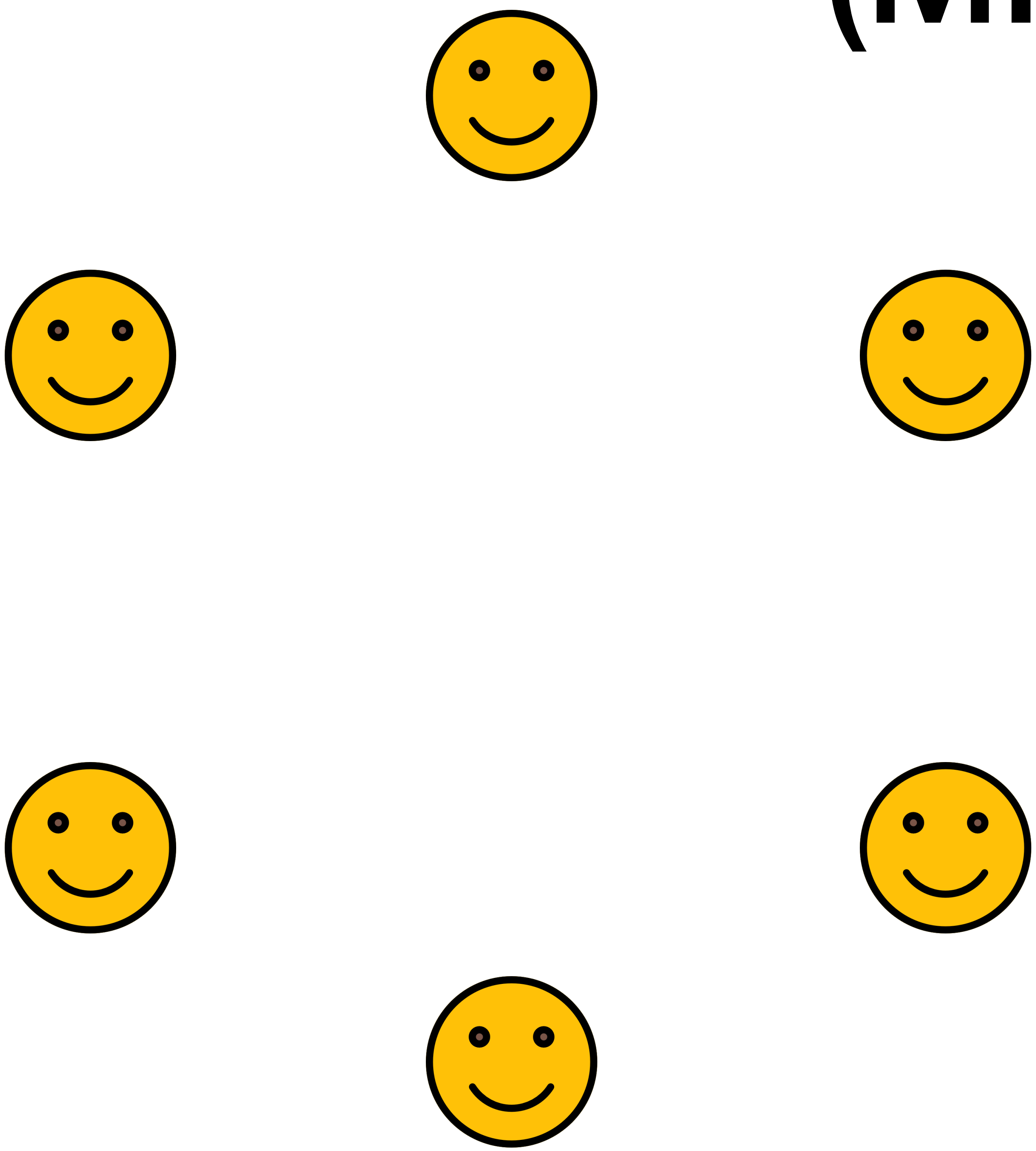

Icons by Pixel perfect.

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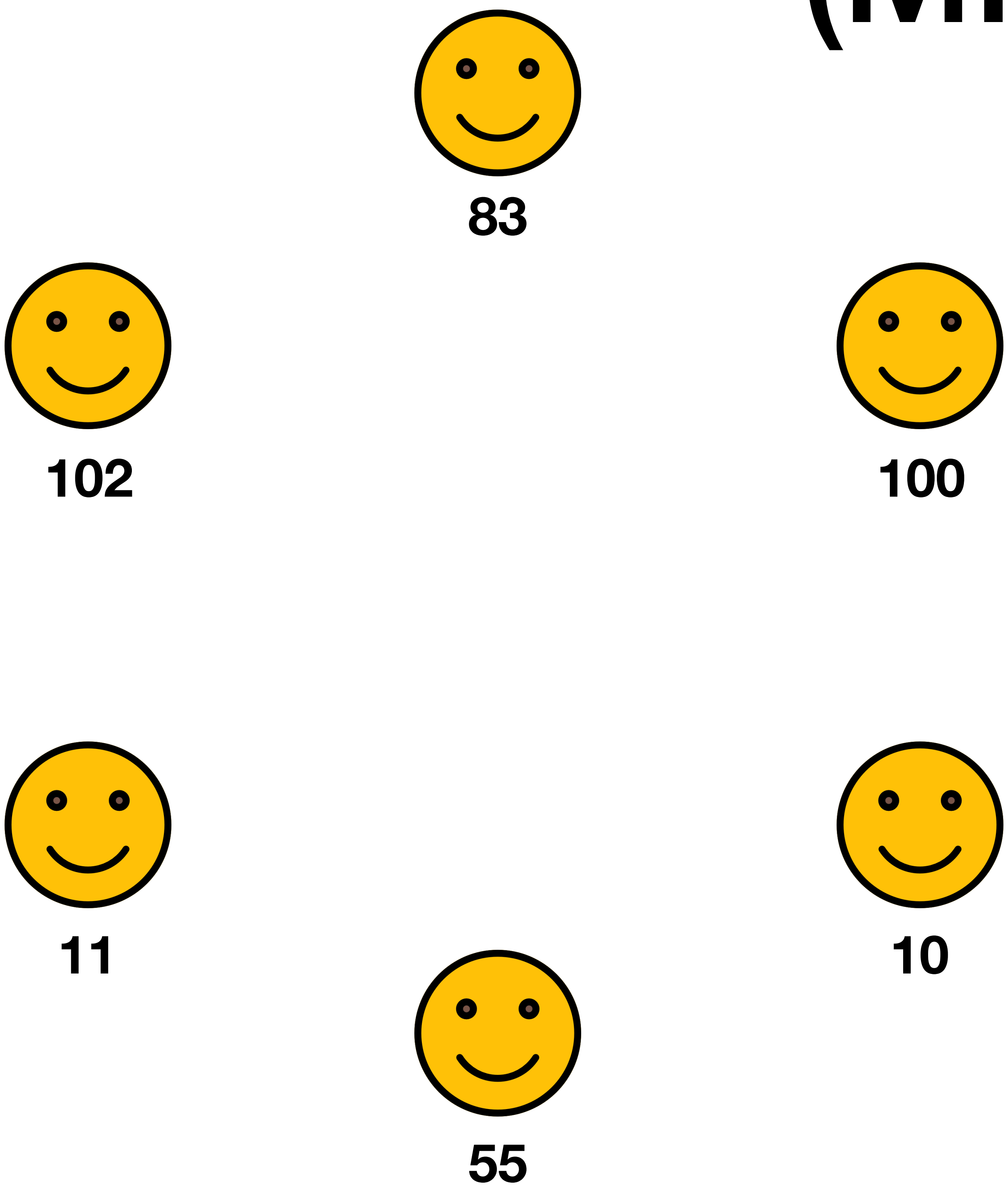

Input (player i): x_i

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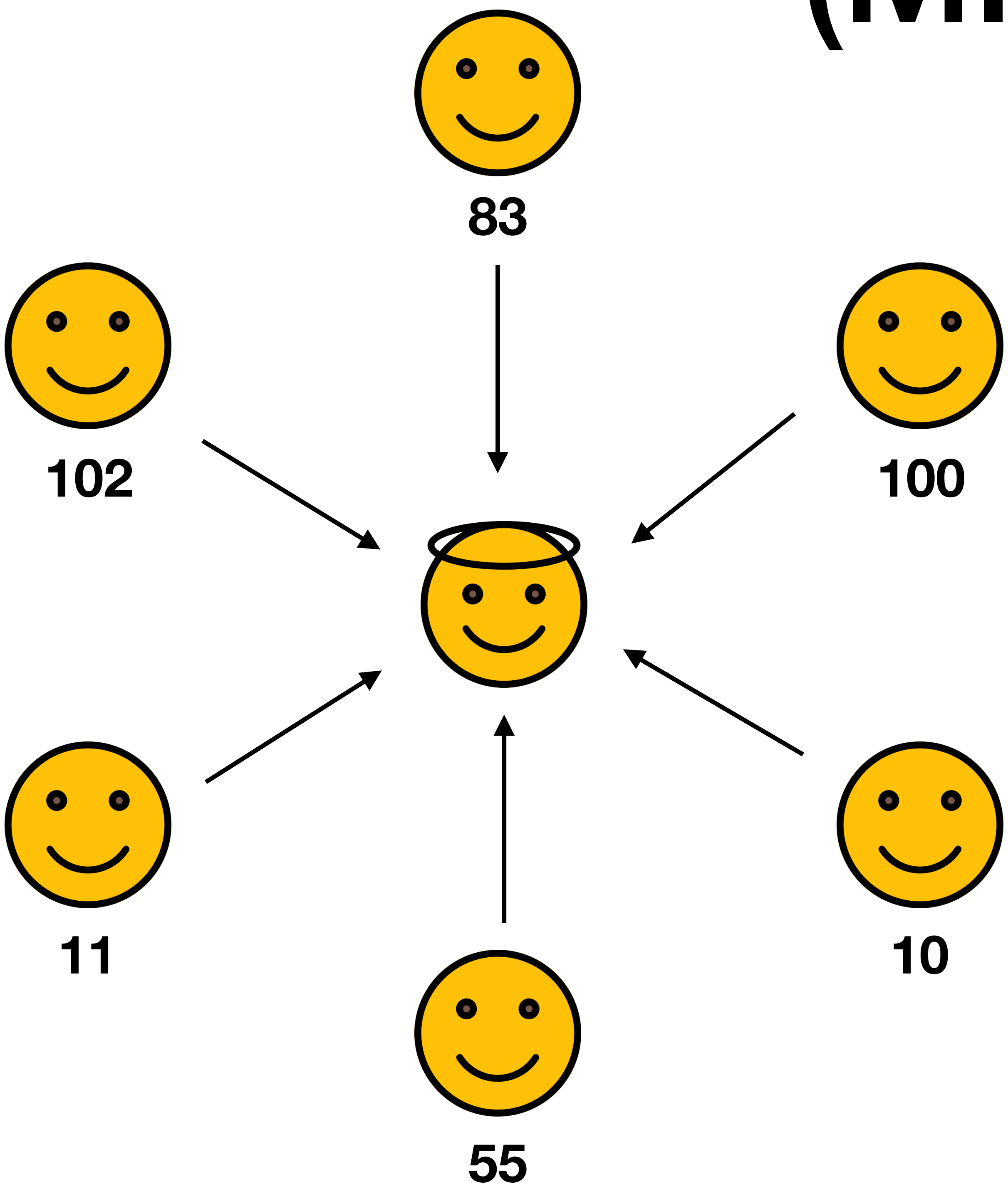

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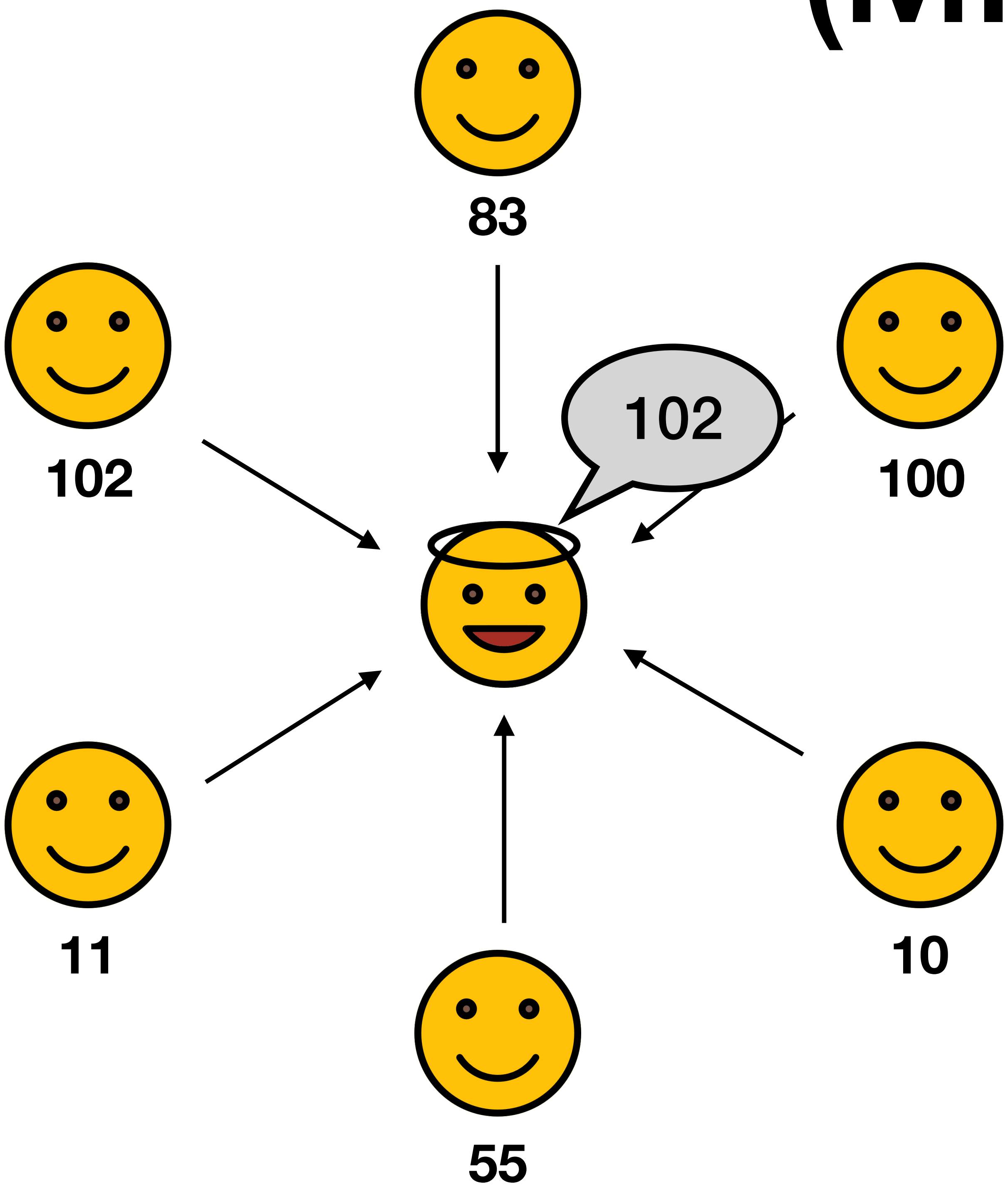

Input (player i): x_i

Output: $f(x_1, \ldots, x_k)$

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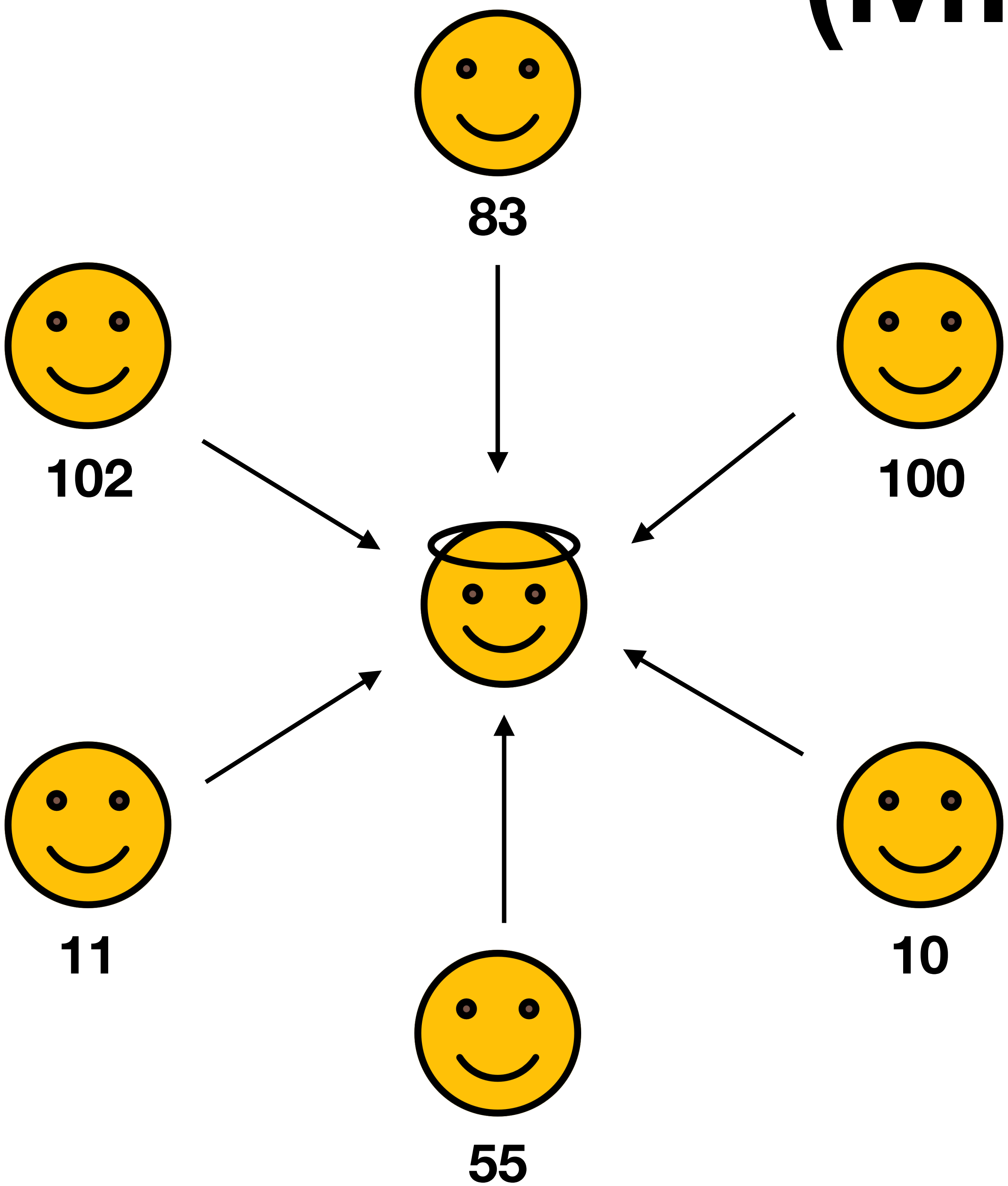

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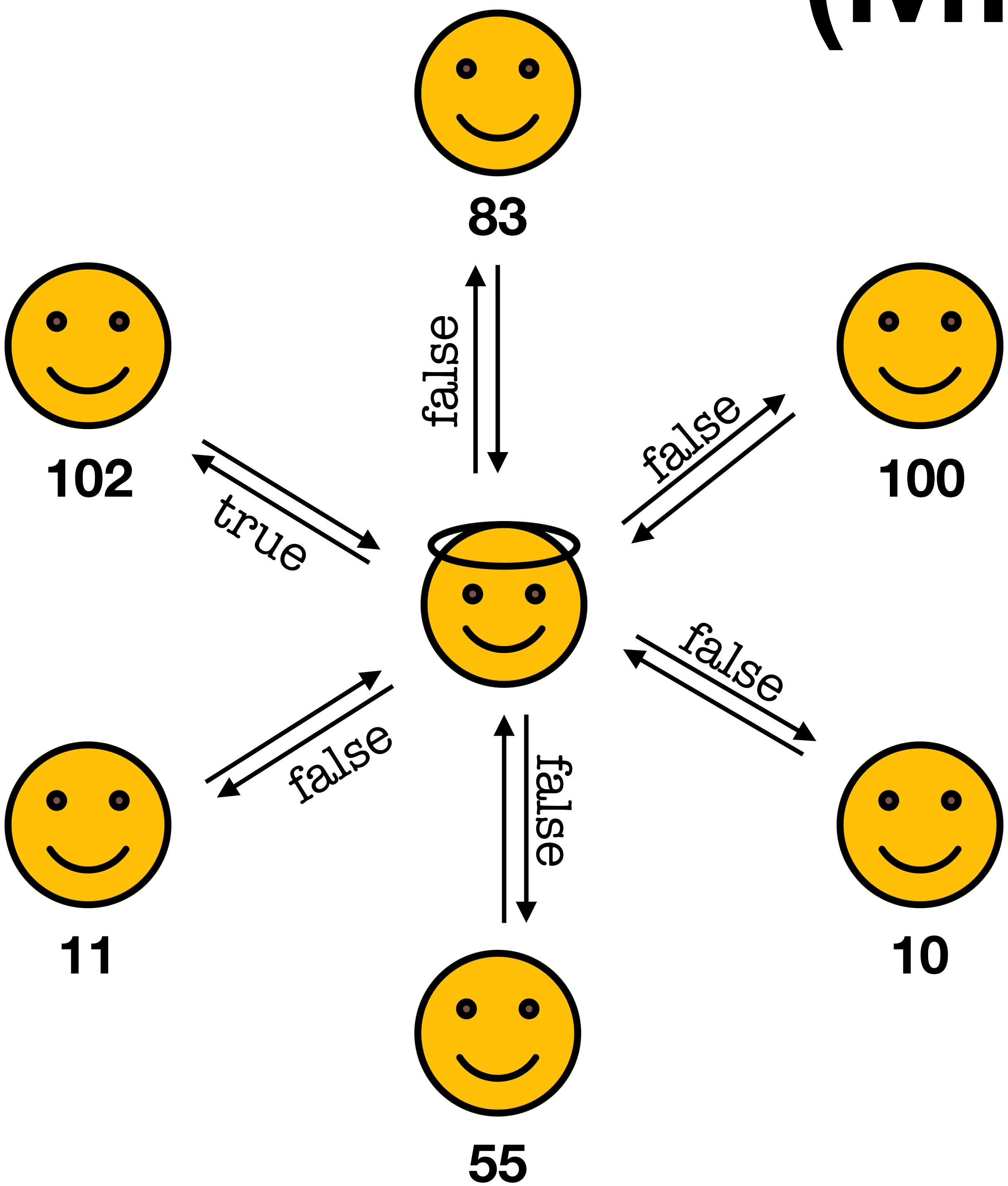

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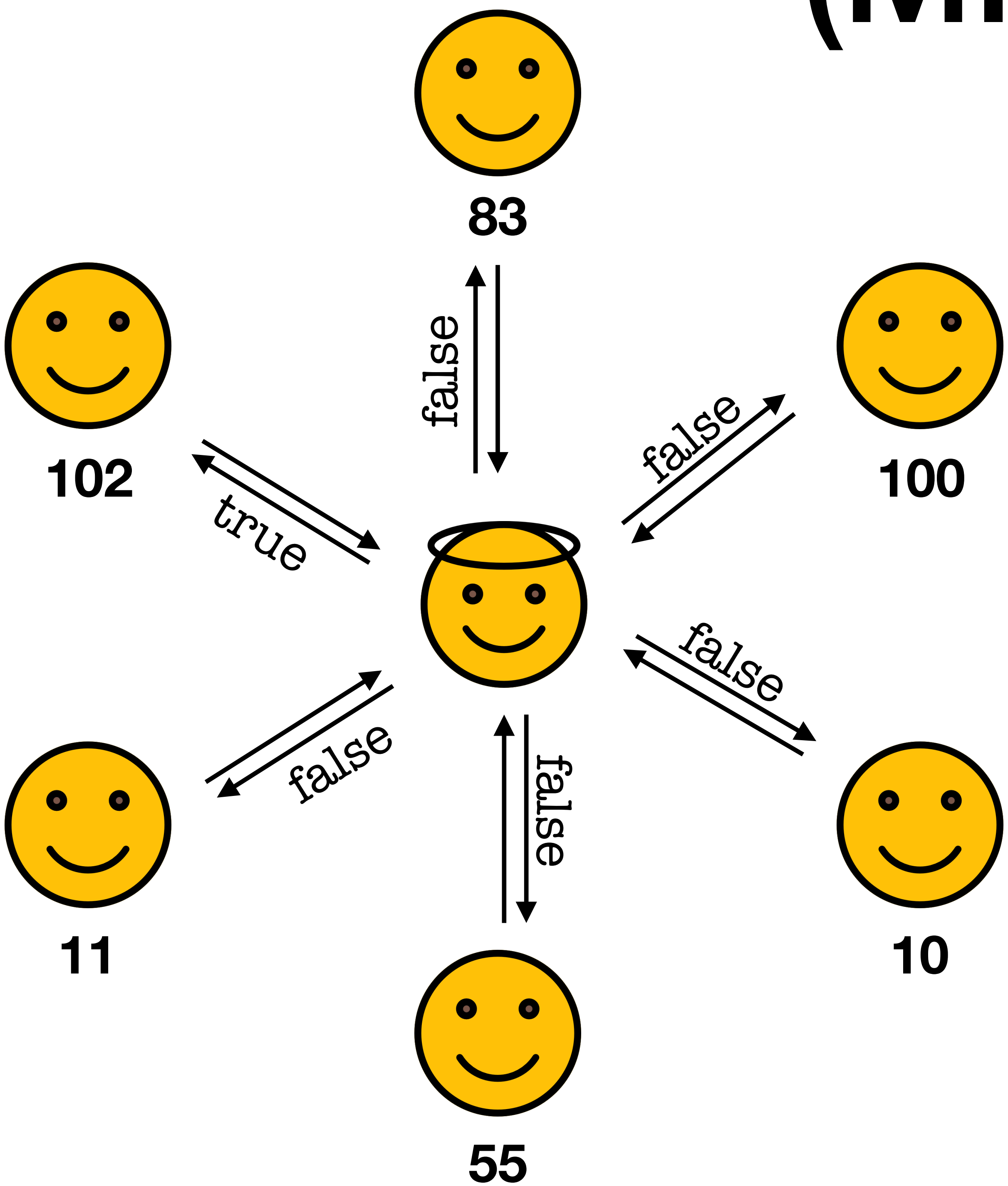

Input (player i): x_i

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Output (player i): $f_i(x_1, \ldots, x_k)$

This is the **ideal** situation.

What if there is no  ?

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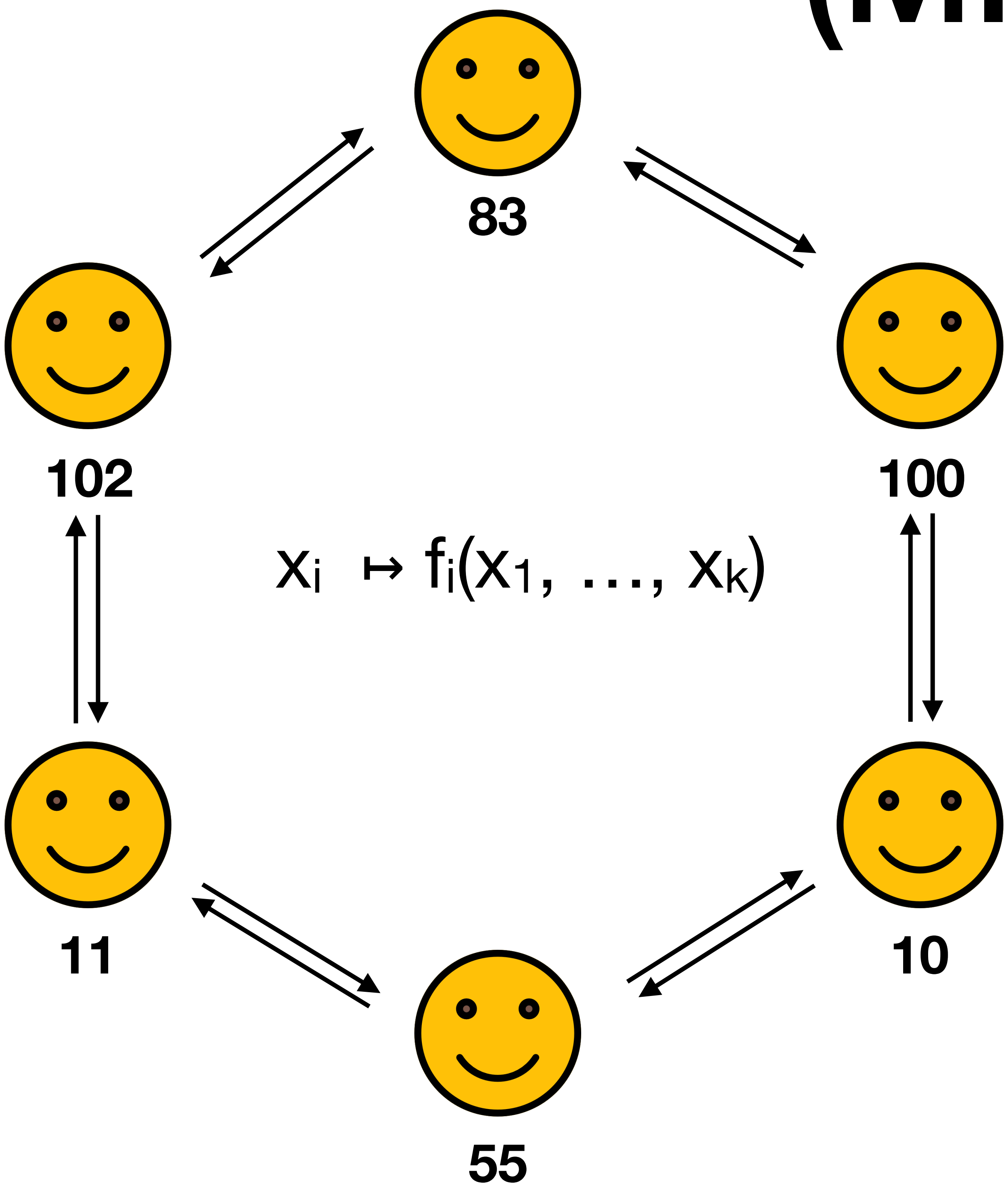

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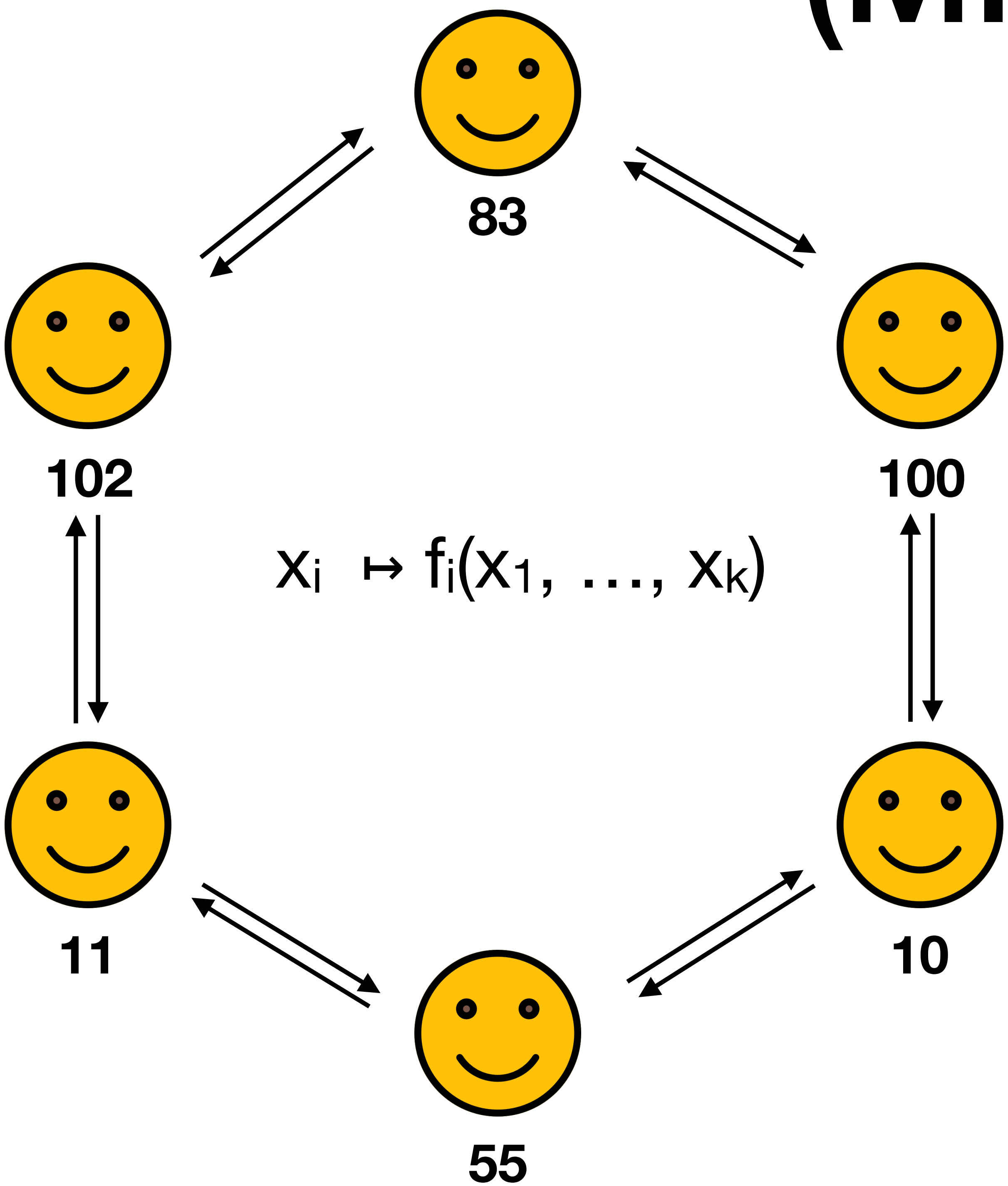
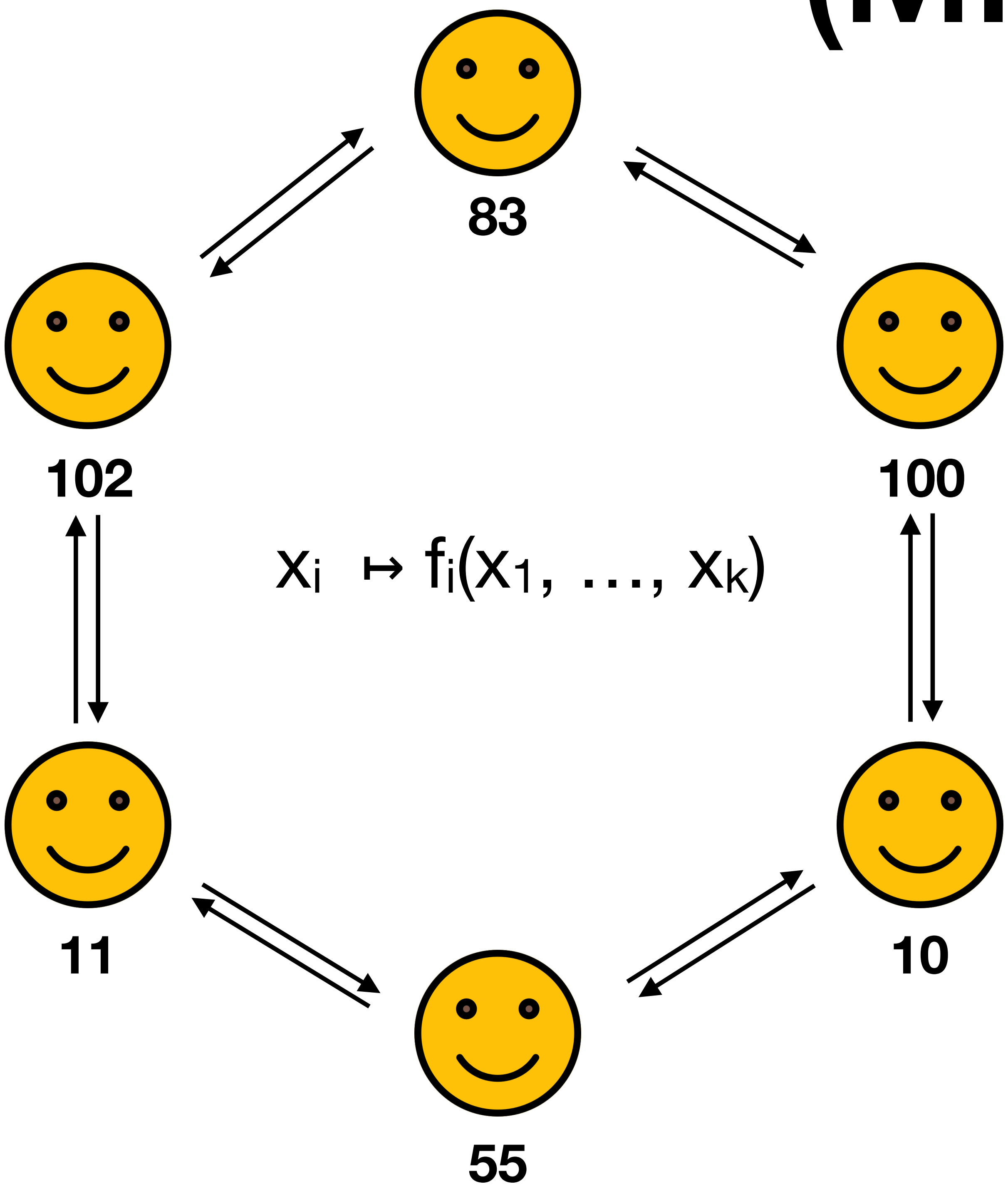


We want:

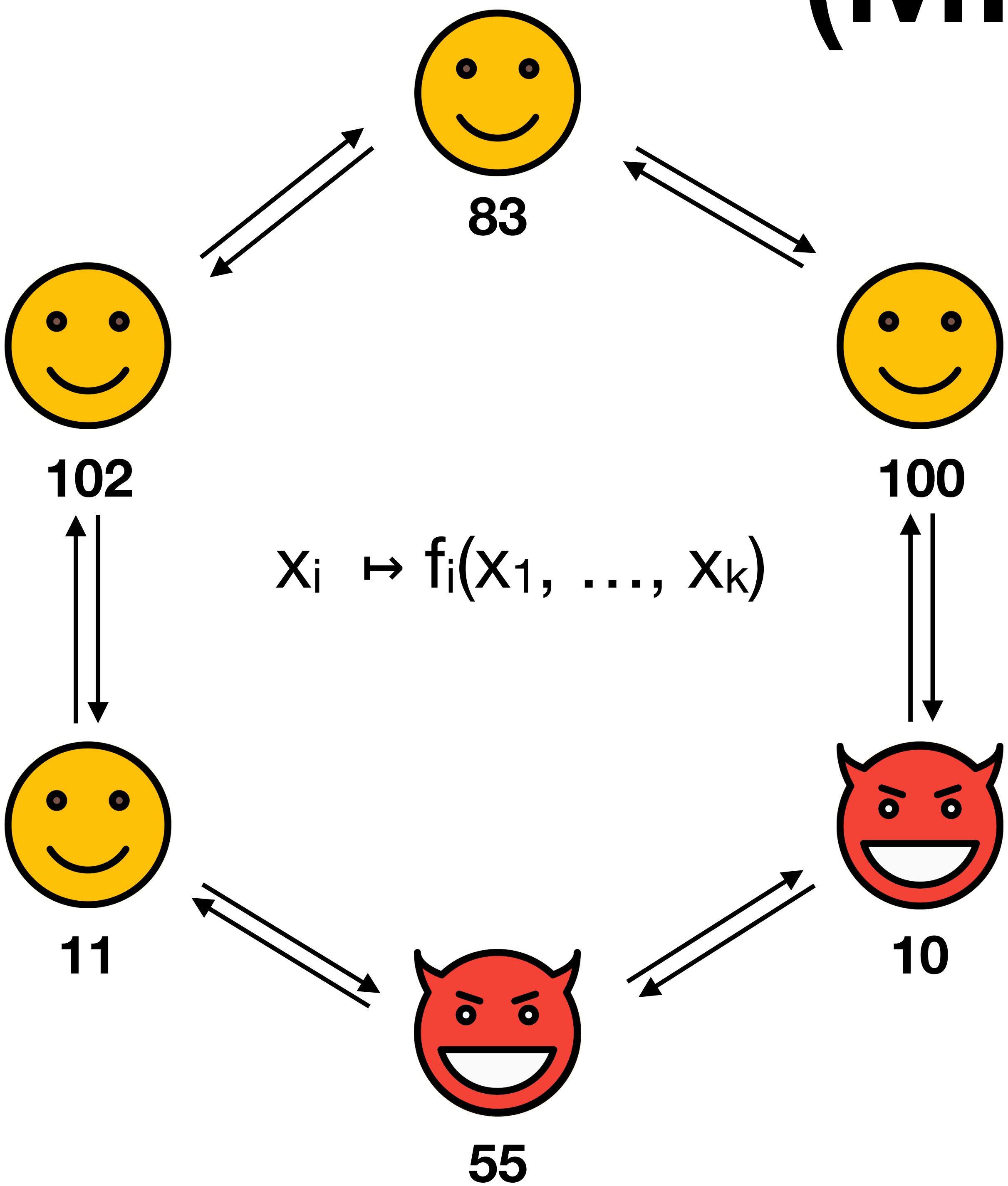
Multi-party computation (MPC)



We want:

- input privacy

Multi-party computation (MPC)



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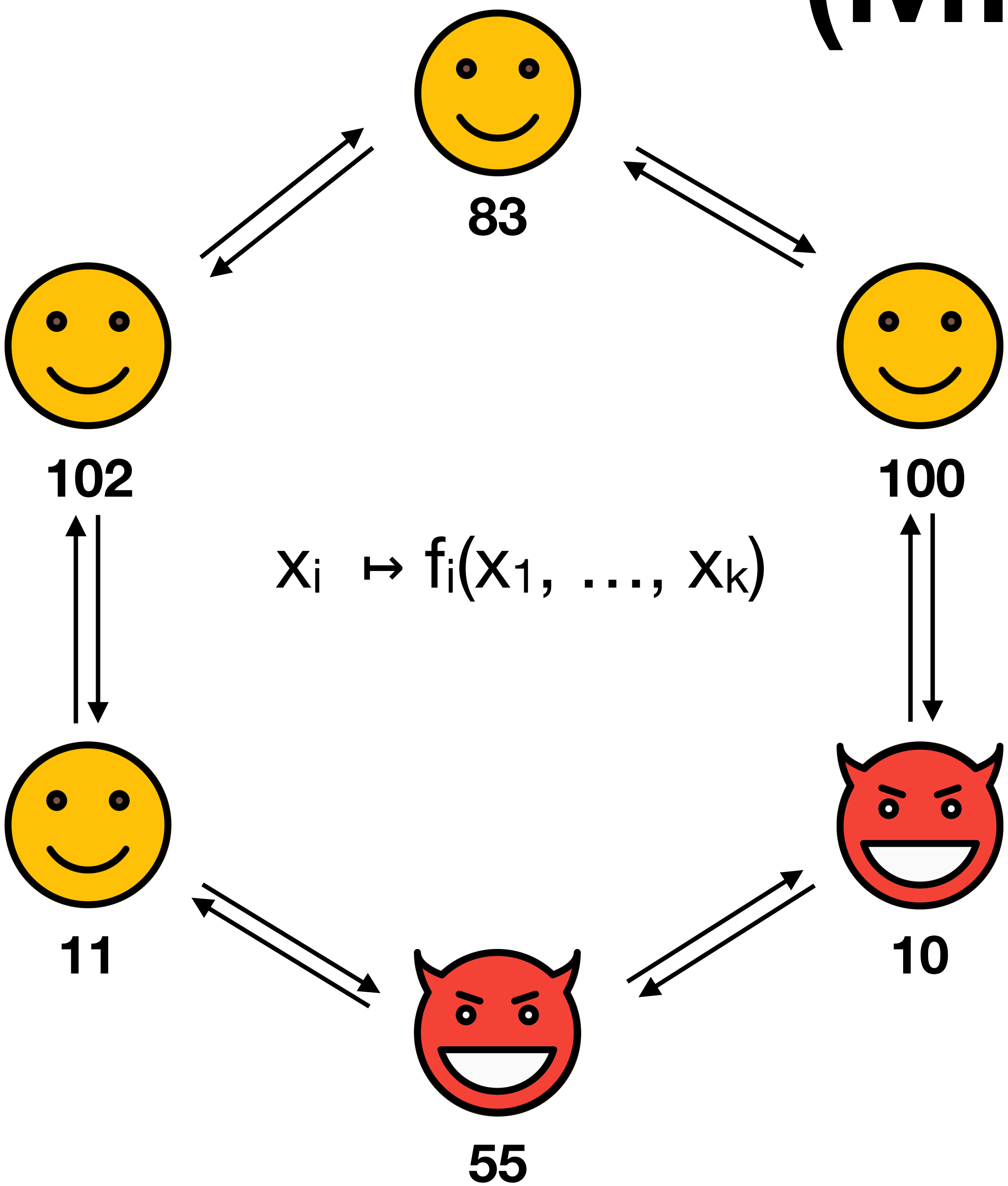
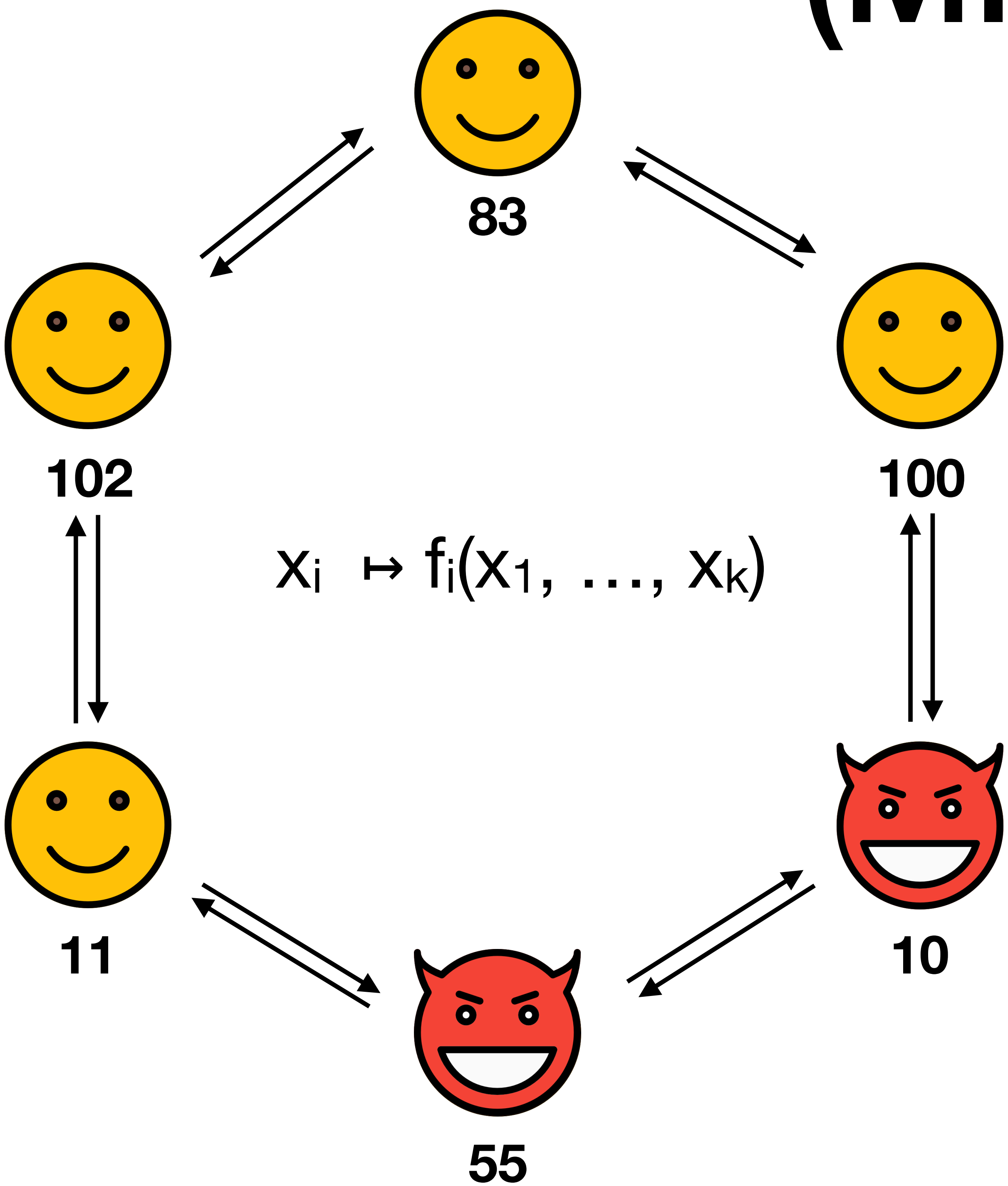


We want:

- input privacy
- correctness

Multi-party computation (MPC)



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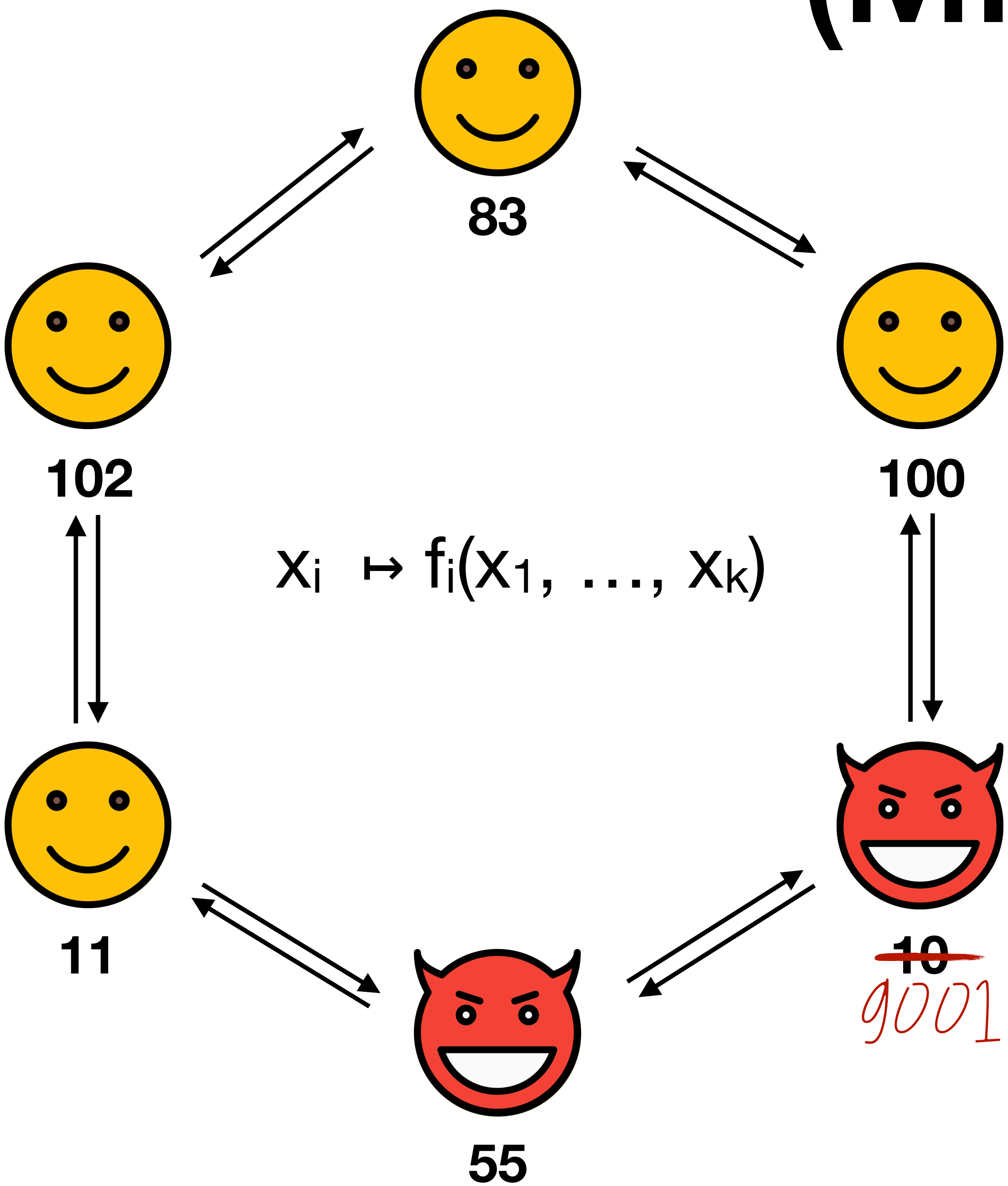
We cannot prevent:

Multi-party computation (MPC)



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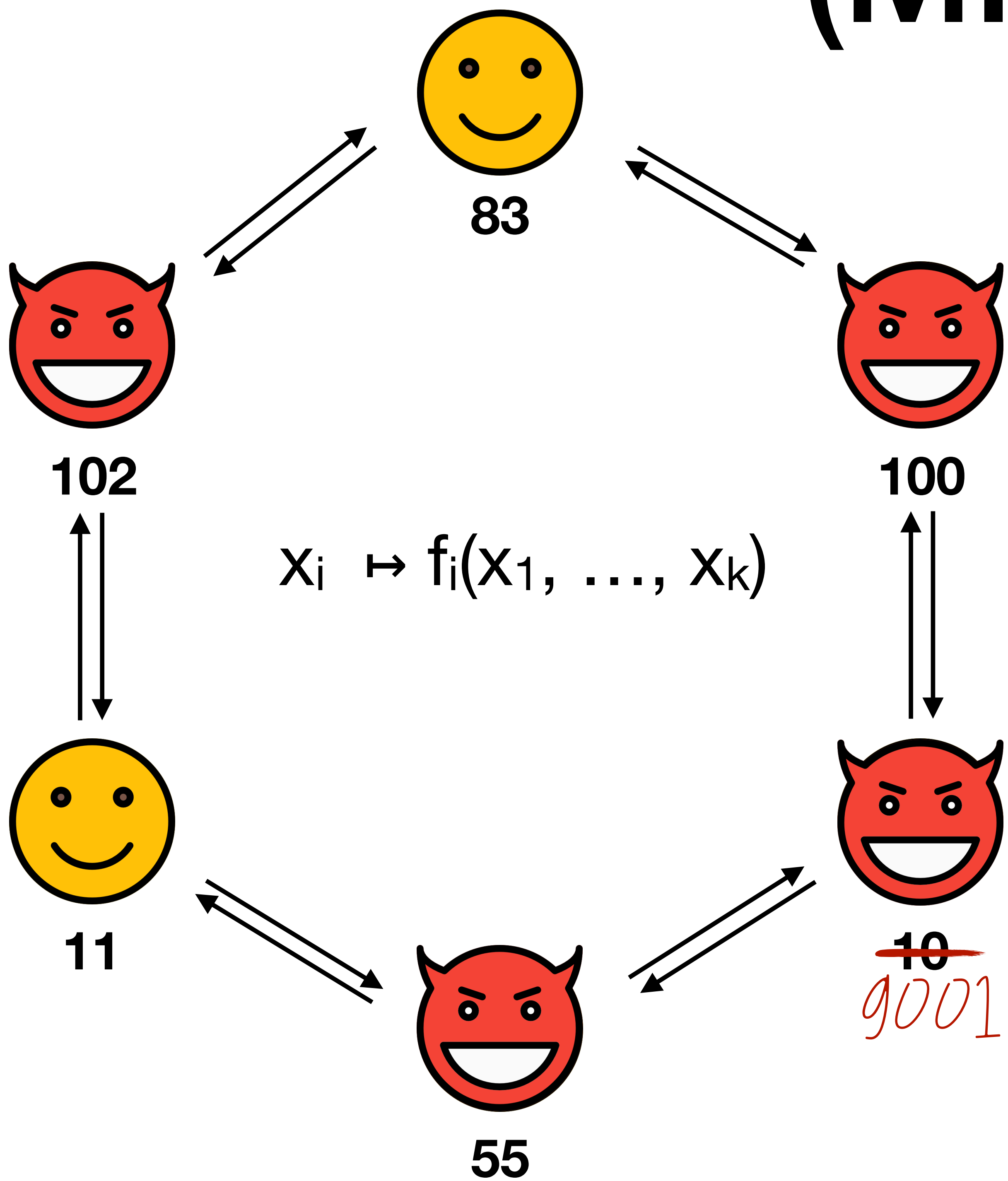
Multi-party computation (MPC)



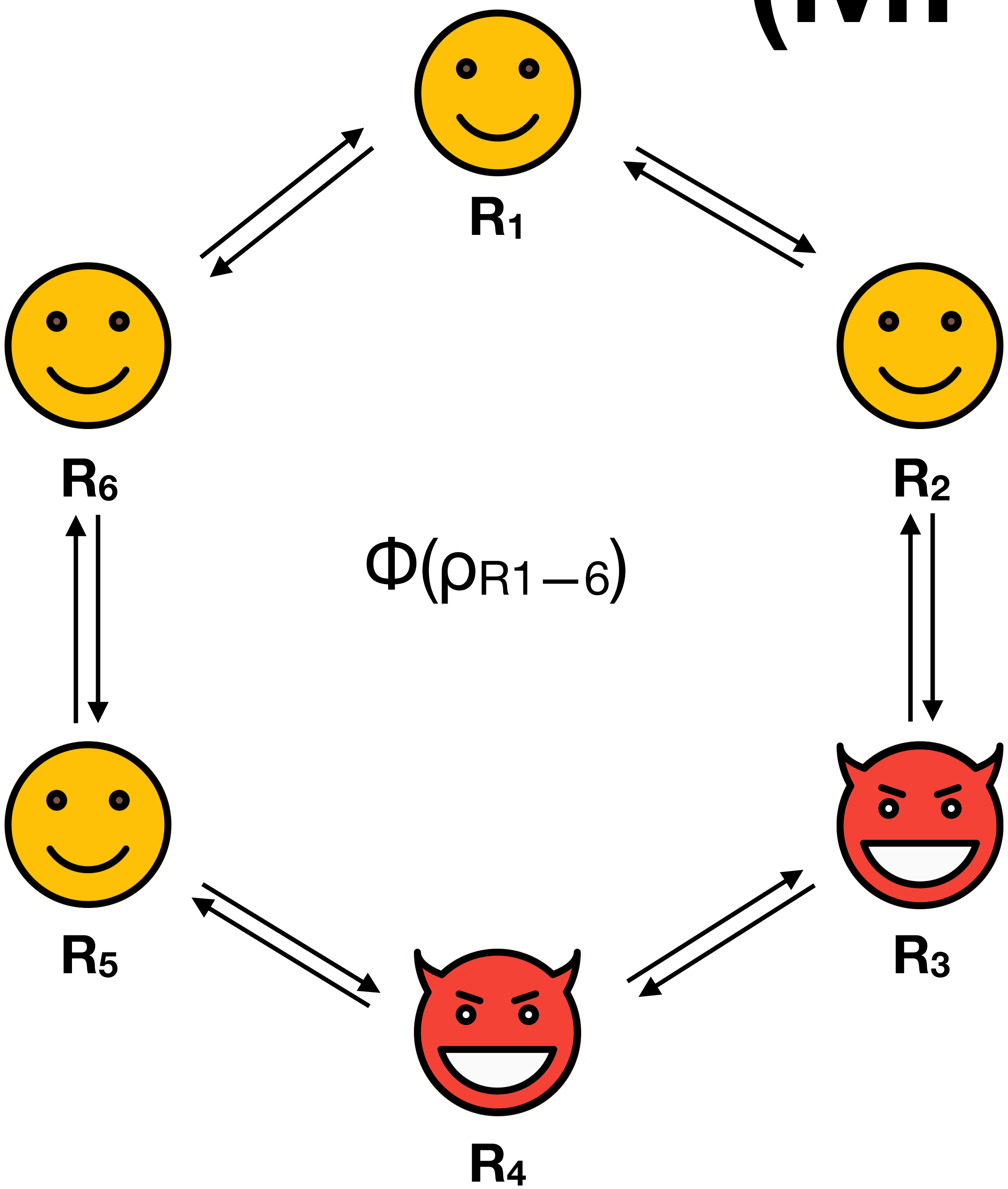
We want:

- input privacy
- correctness

We cannot prevent:

- lying about inputs
- unfairness

Goal: Quantum MPC (MPQC)



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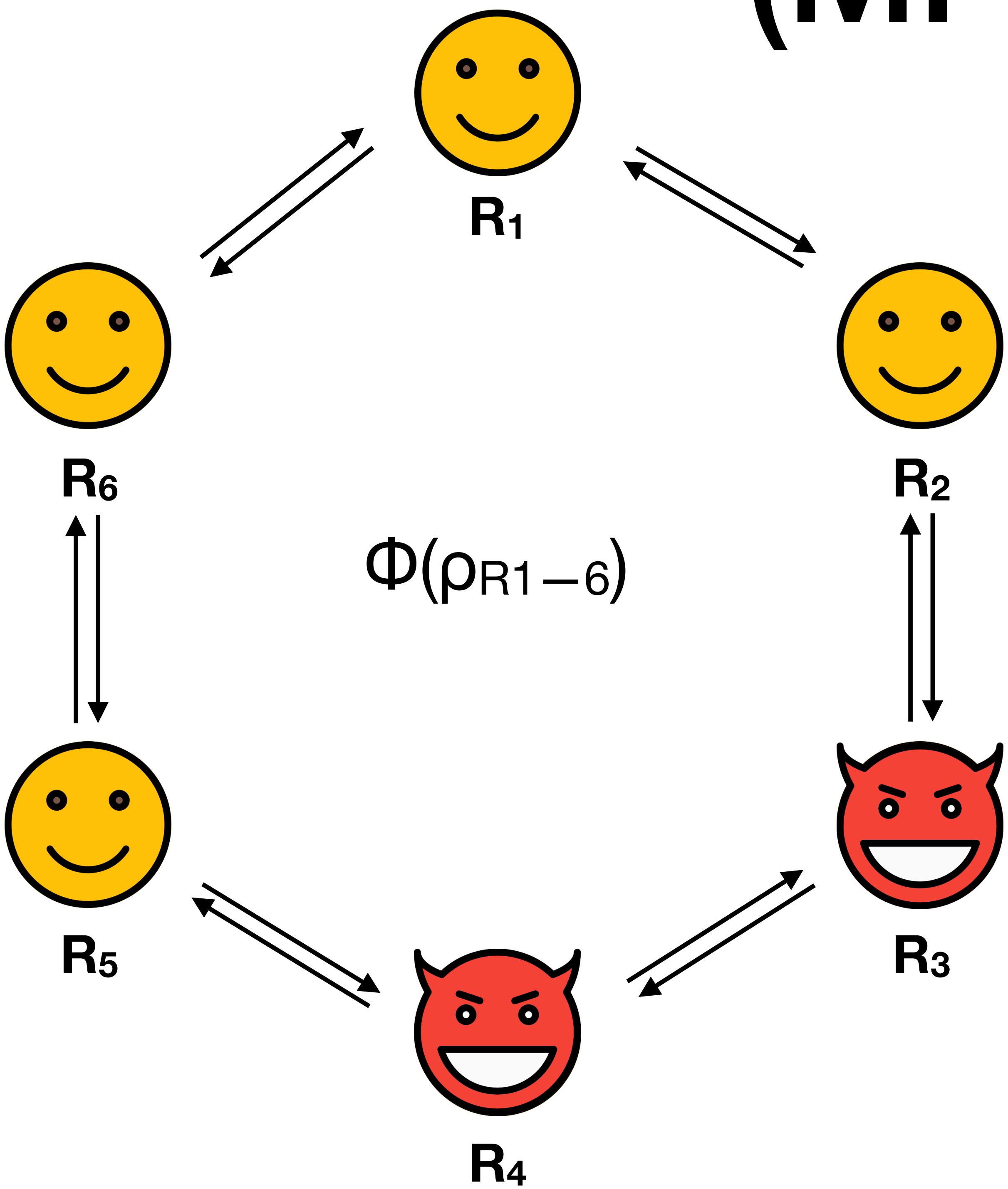

This talk: protocol for MPQC

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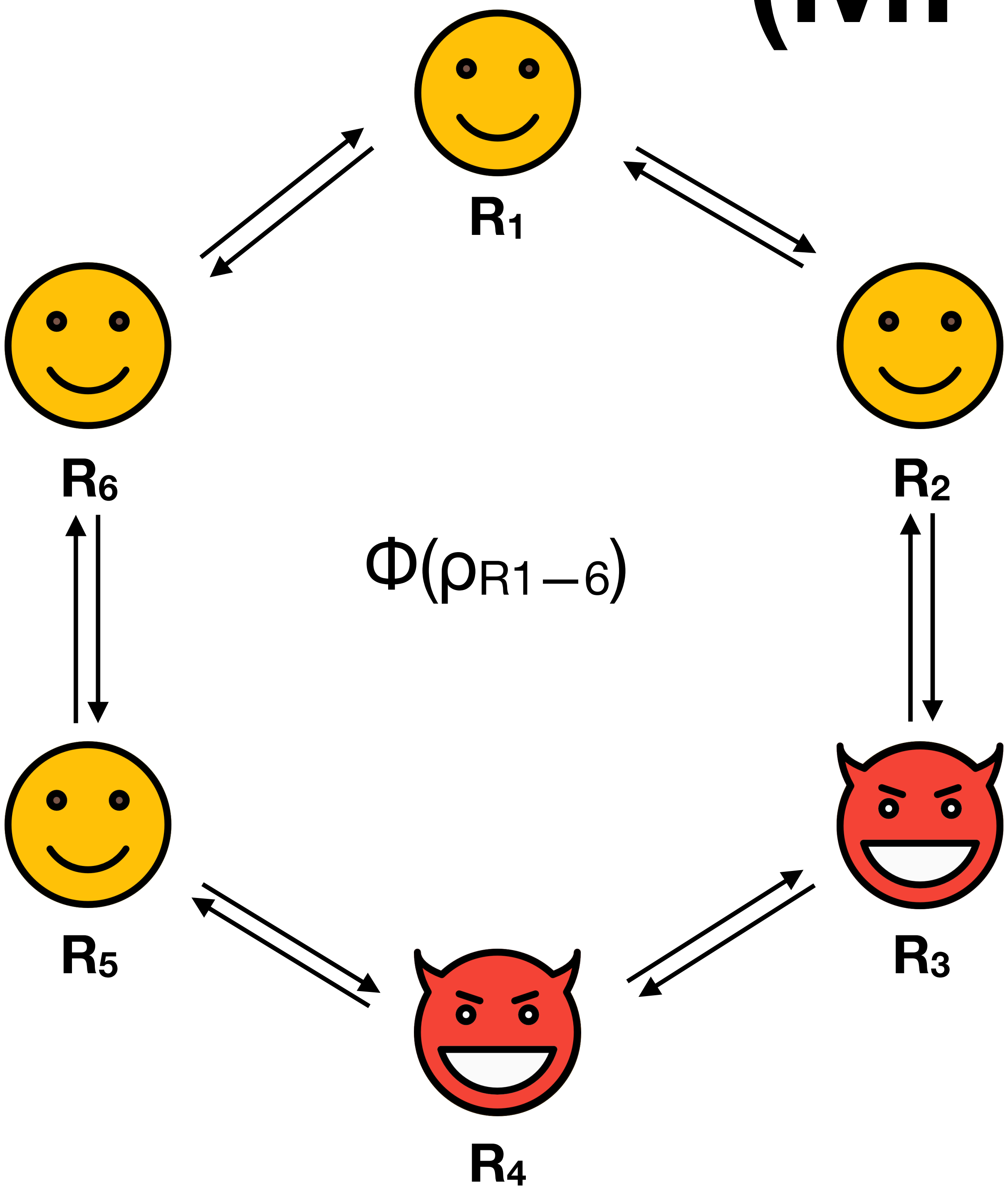

This talk: protocol for MPQC

- Up to k-1 😈

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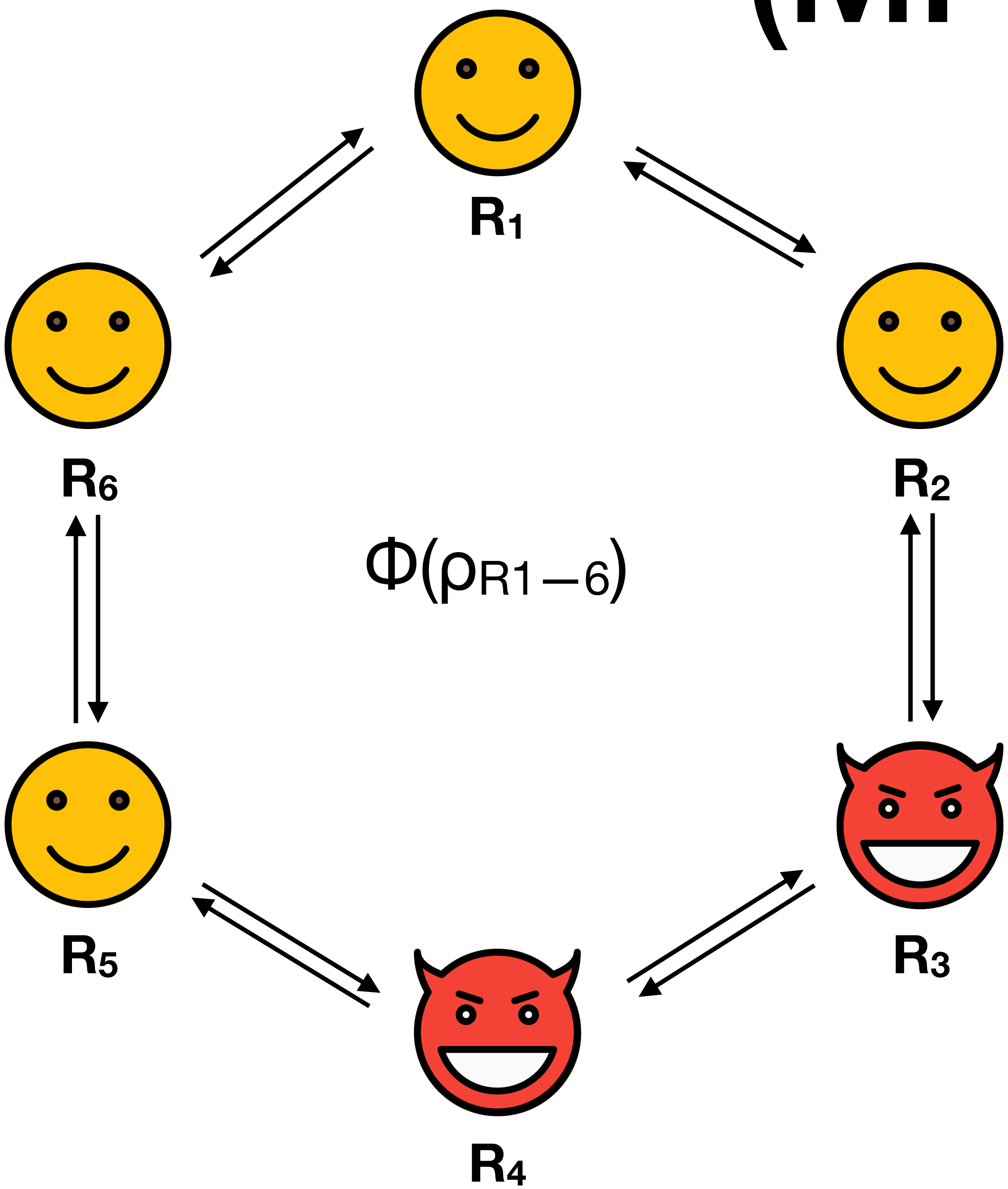

This talk: protocol for MPQC

- Up to k-1 
- Computationally secure

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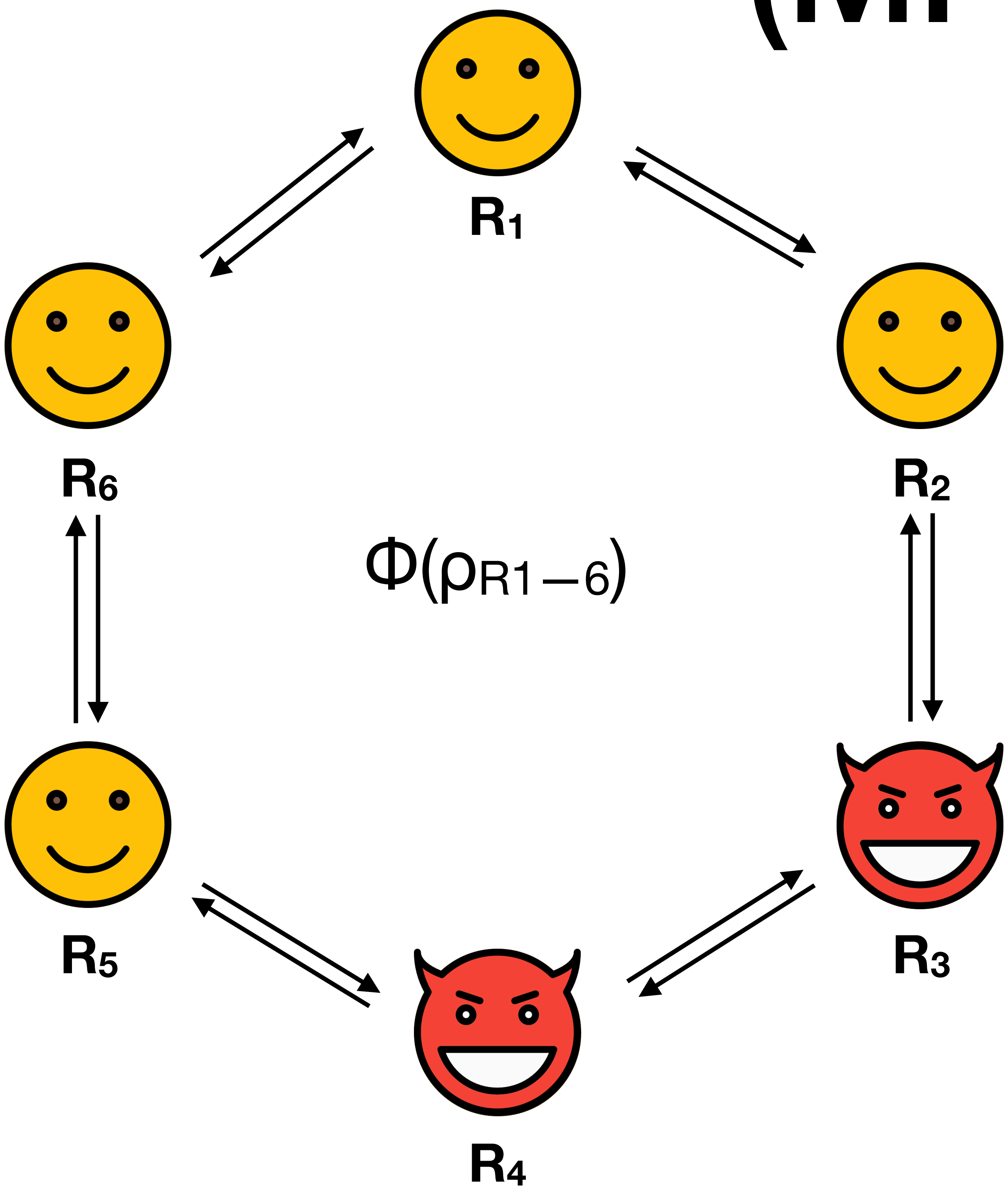

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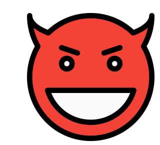
- Up to k-1 
- Computationally secure
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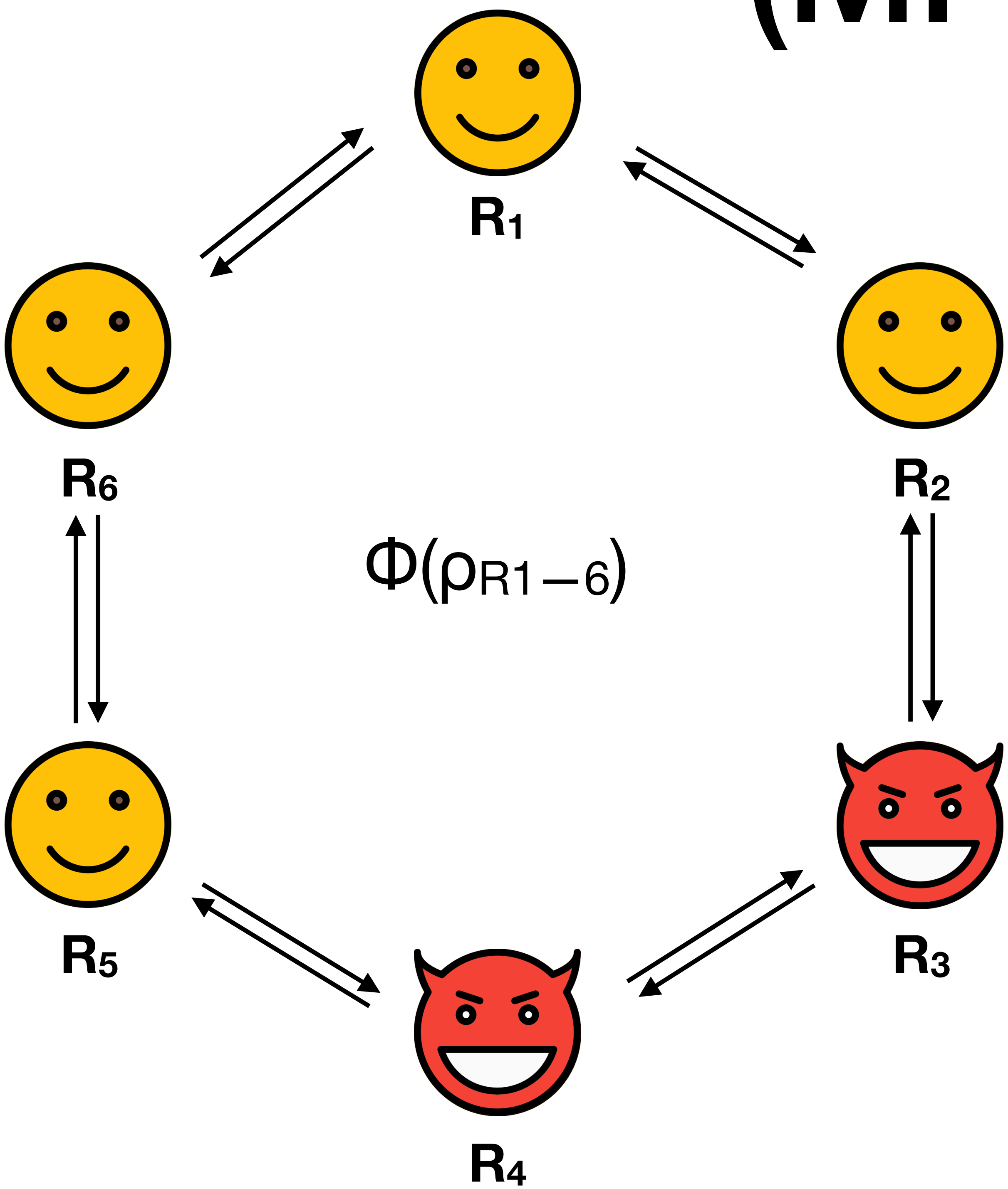

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- Up to k-1 
- Computationally secure
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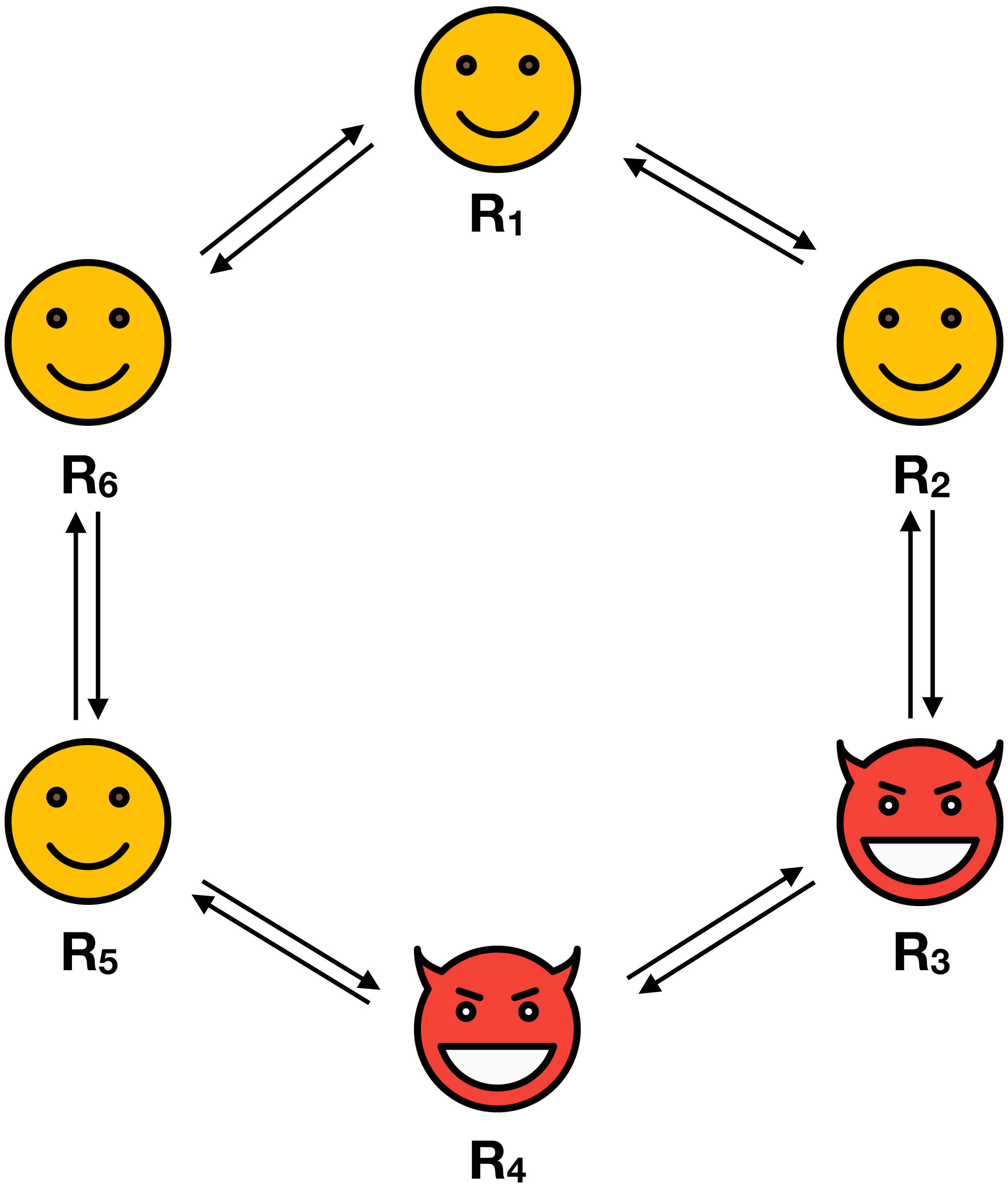
- subroutine: classical MPC 

Previous Approaches

MPQC: two approaches



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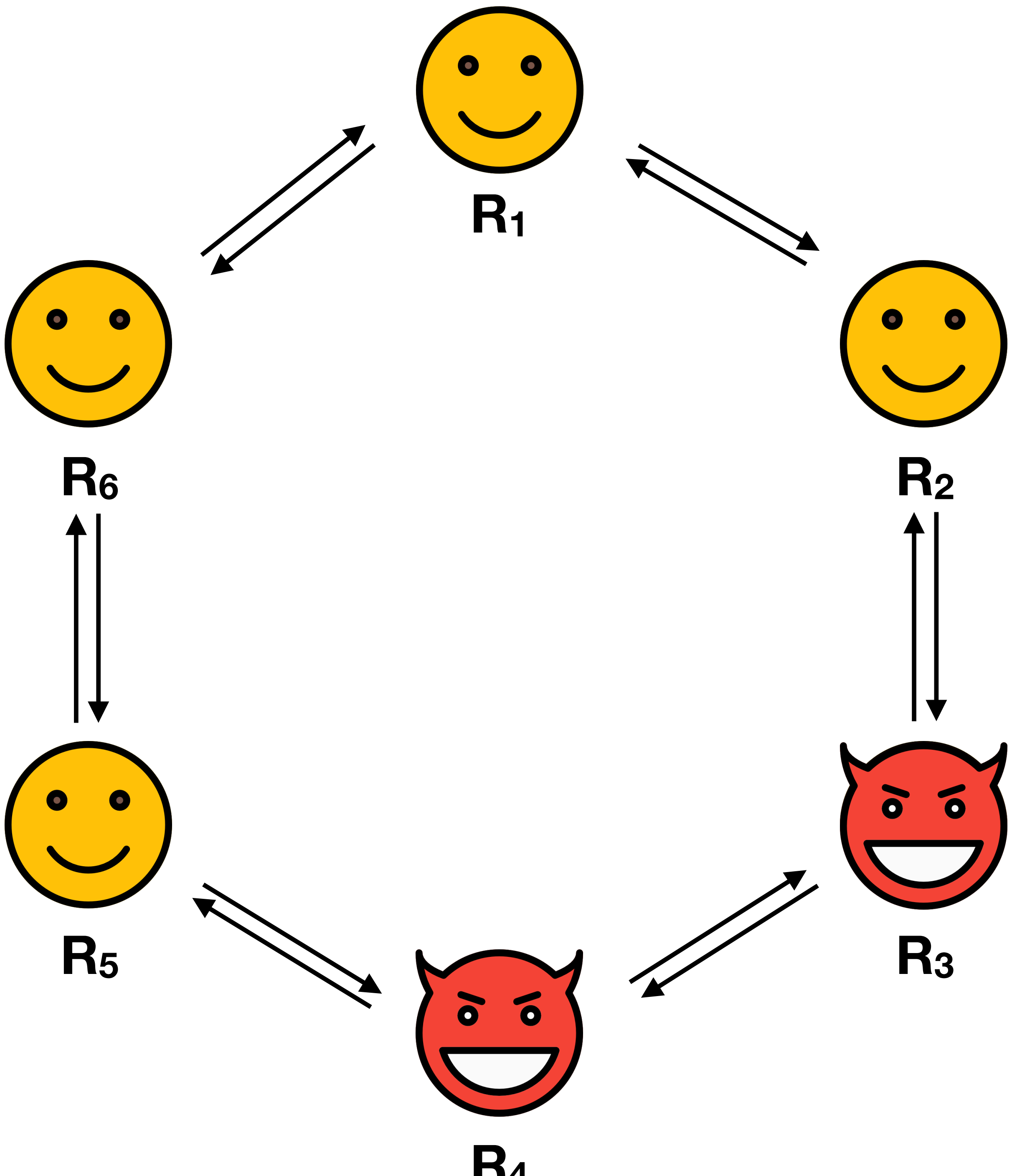

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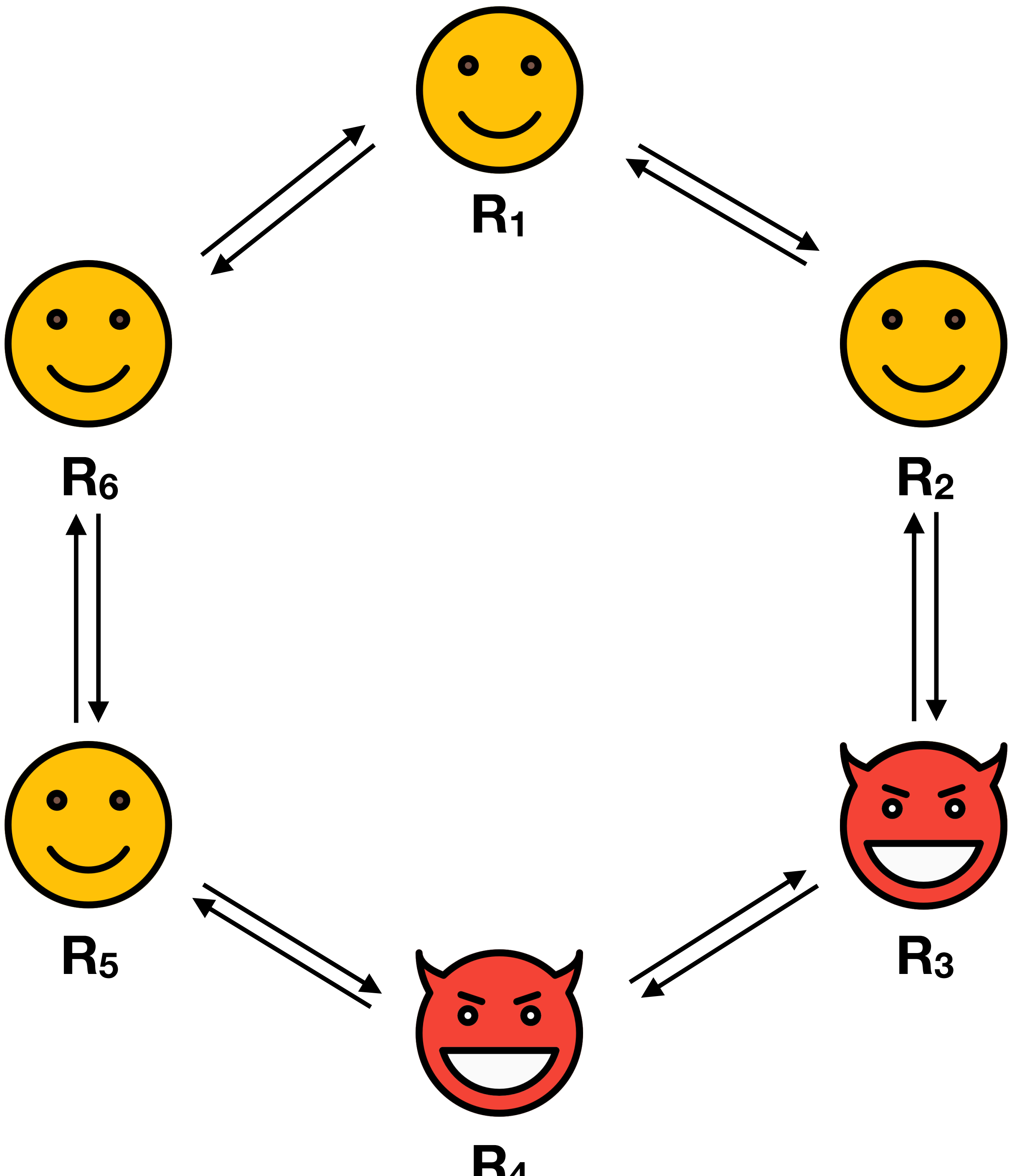

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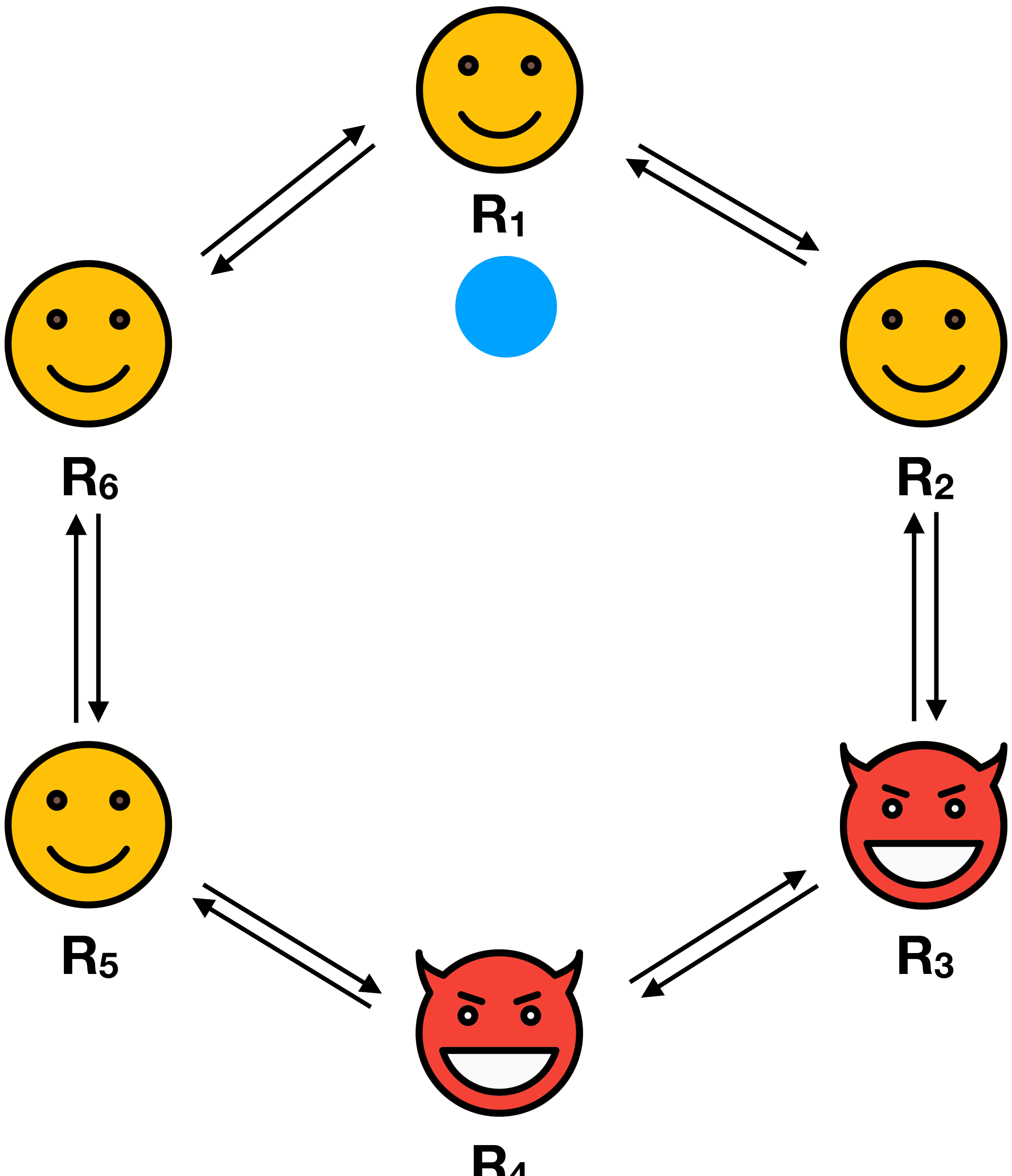

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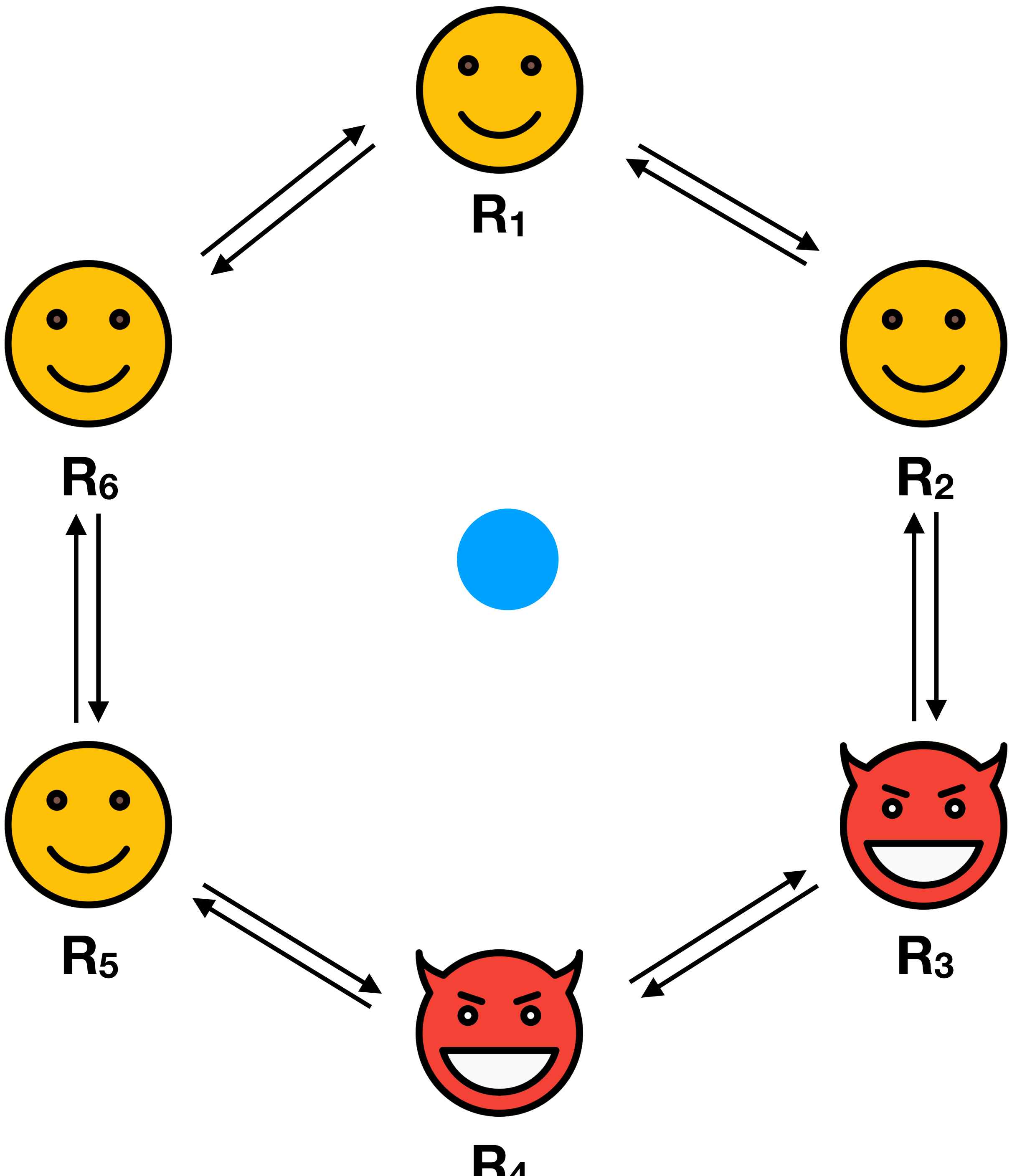

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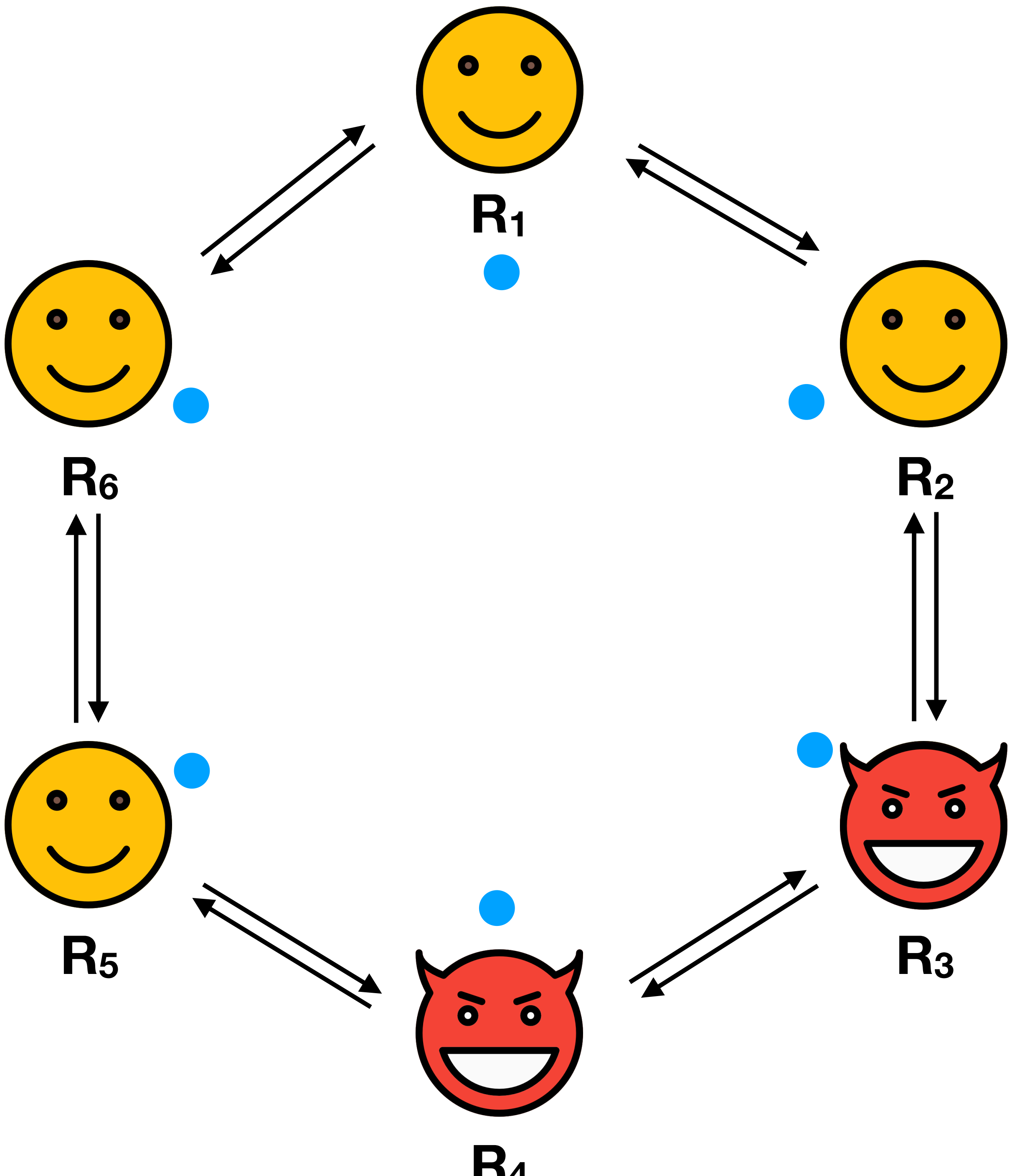

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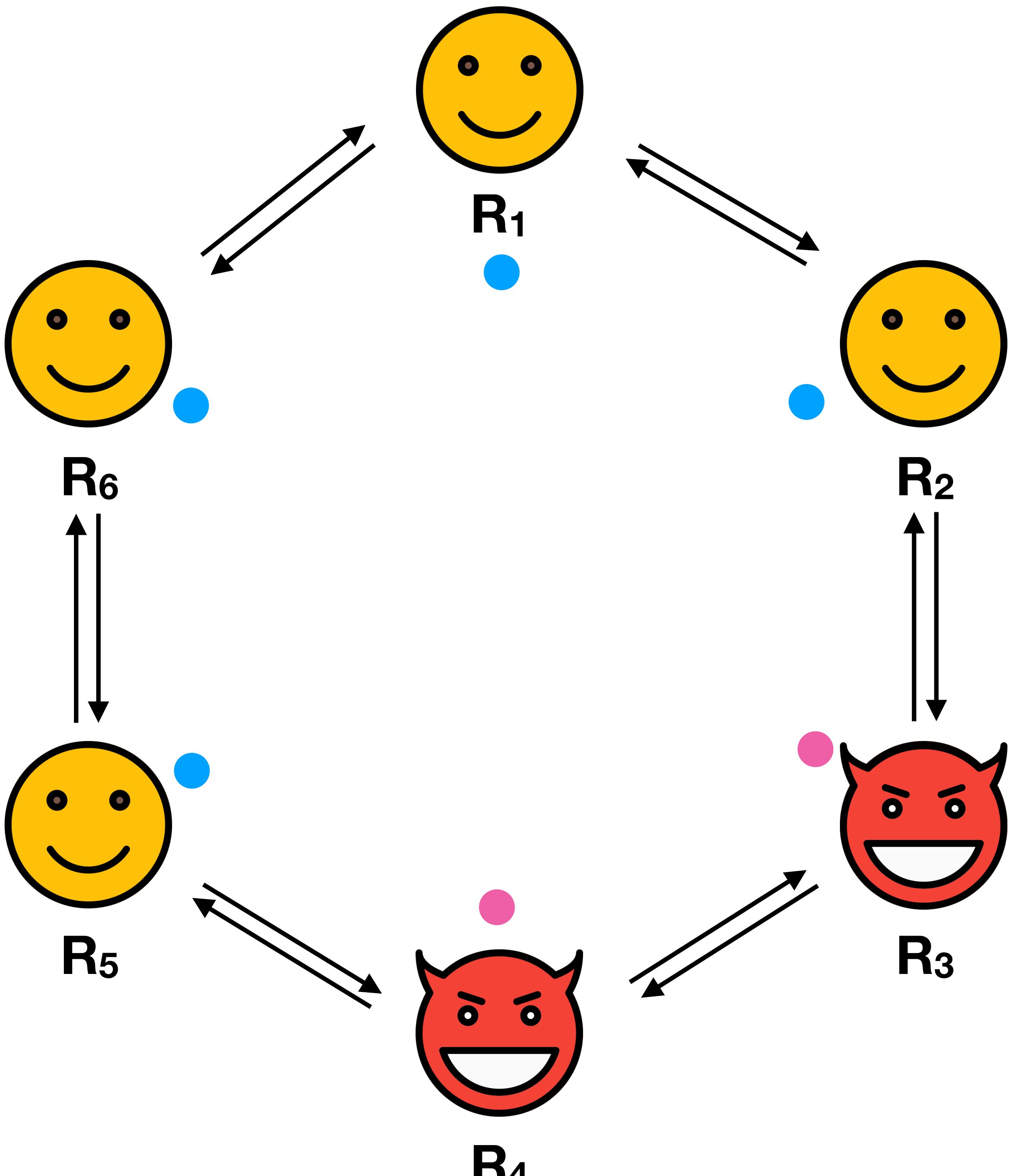

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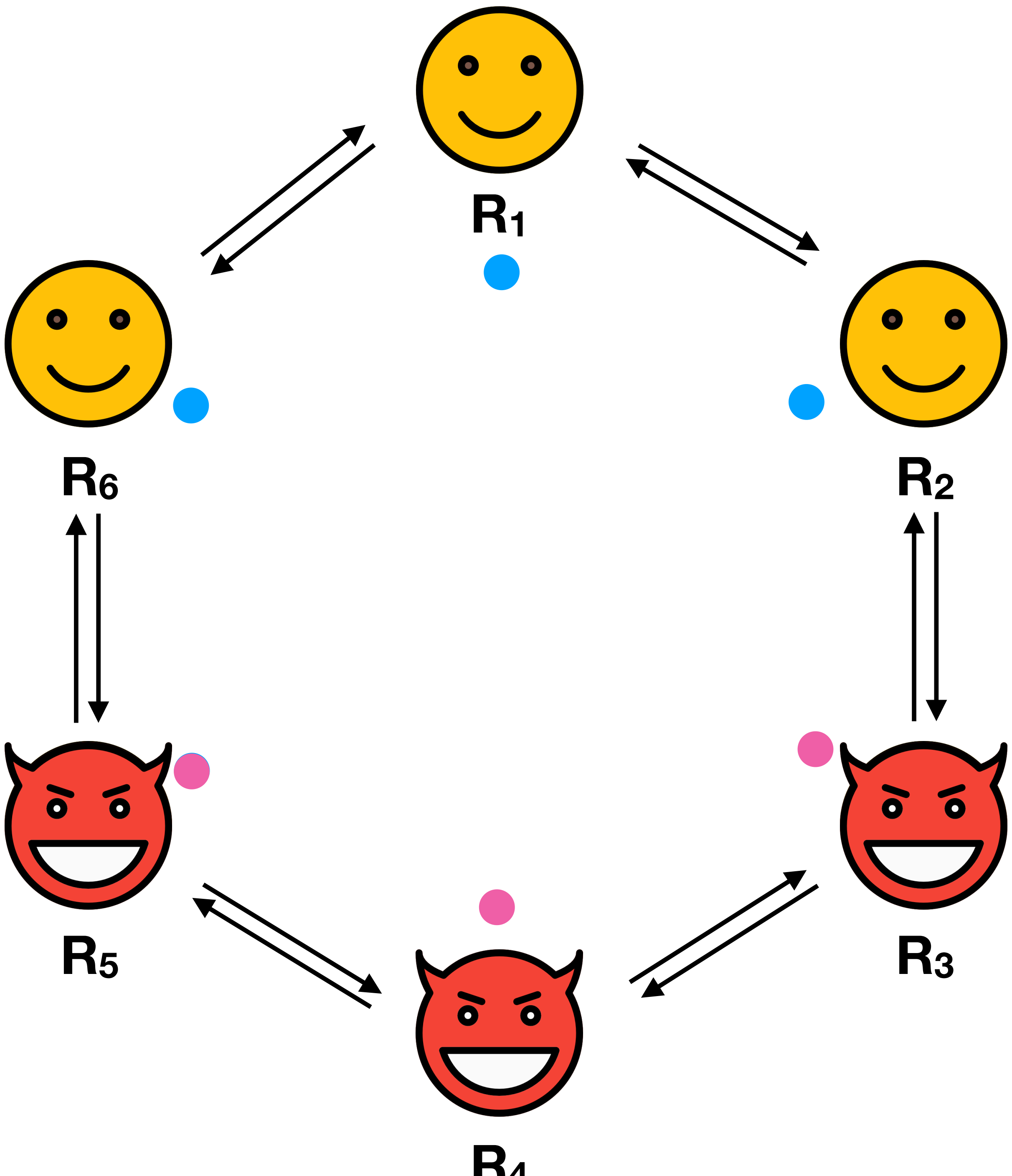

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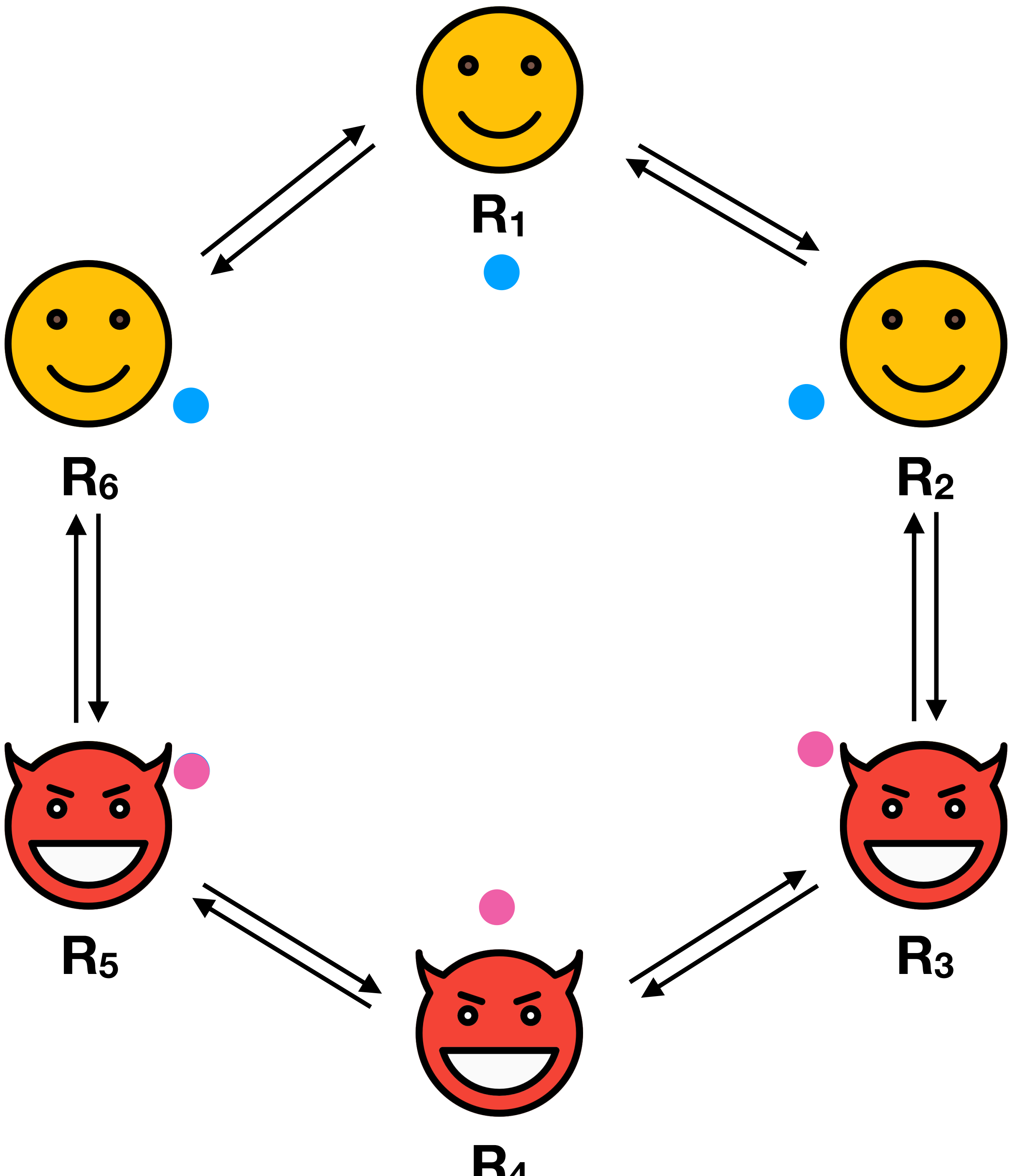

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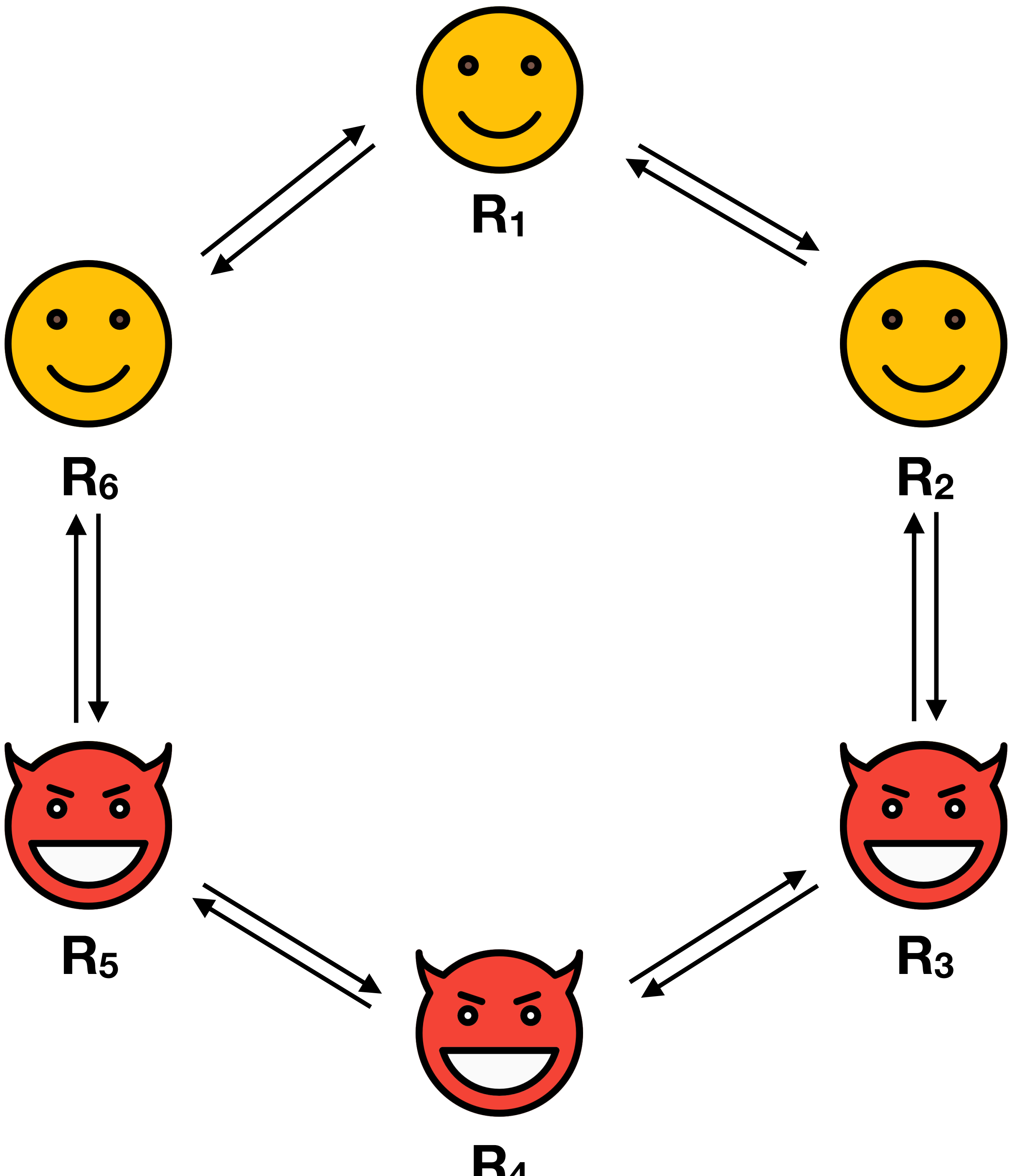

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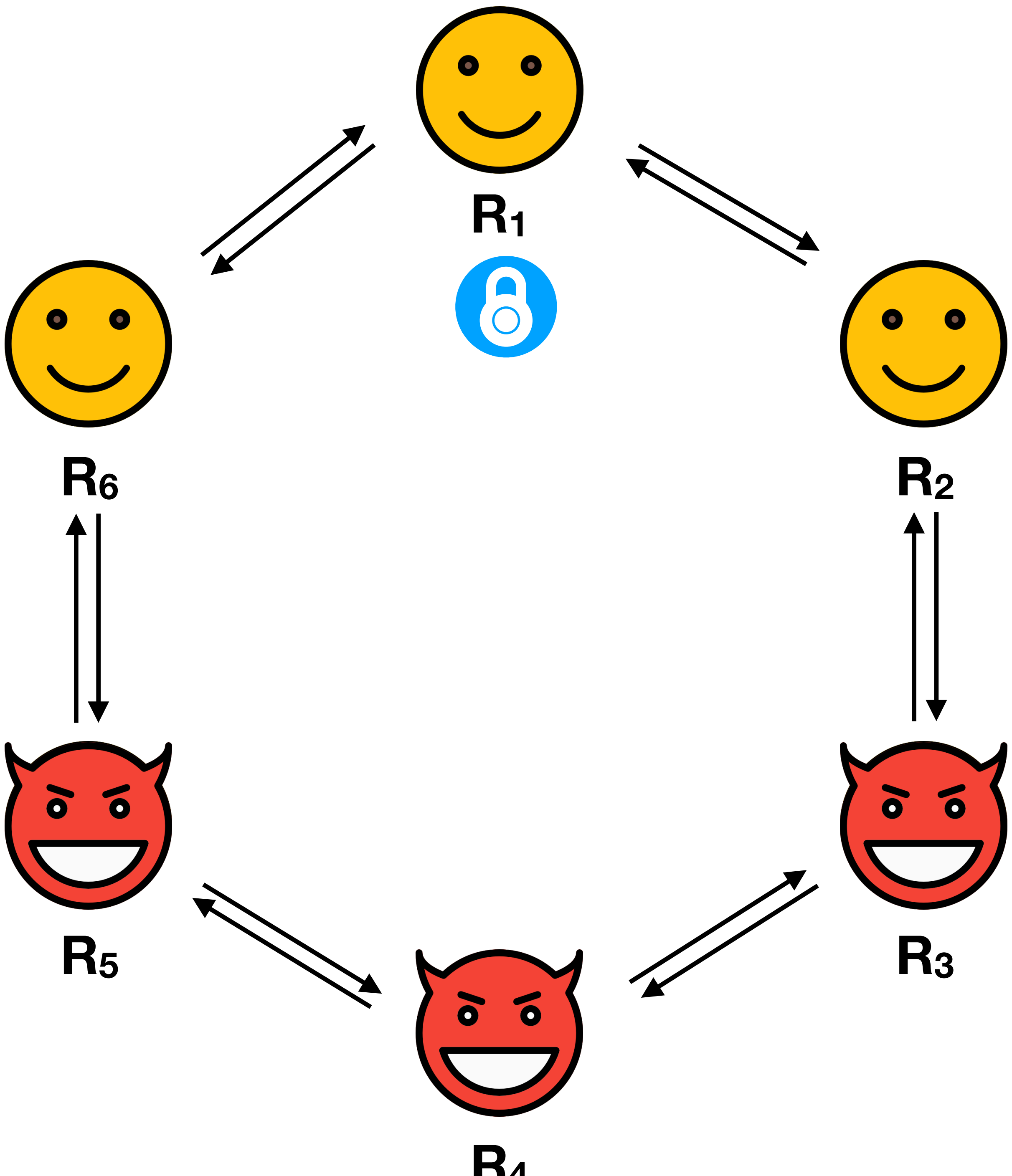

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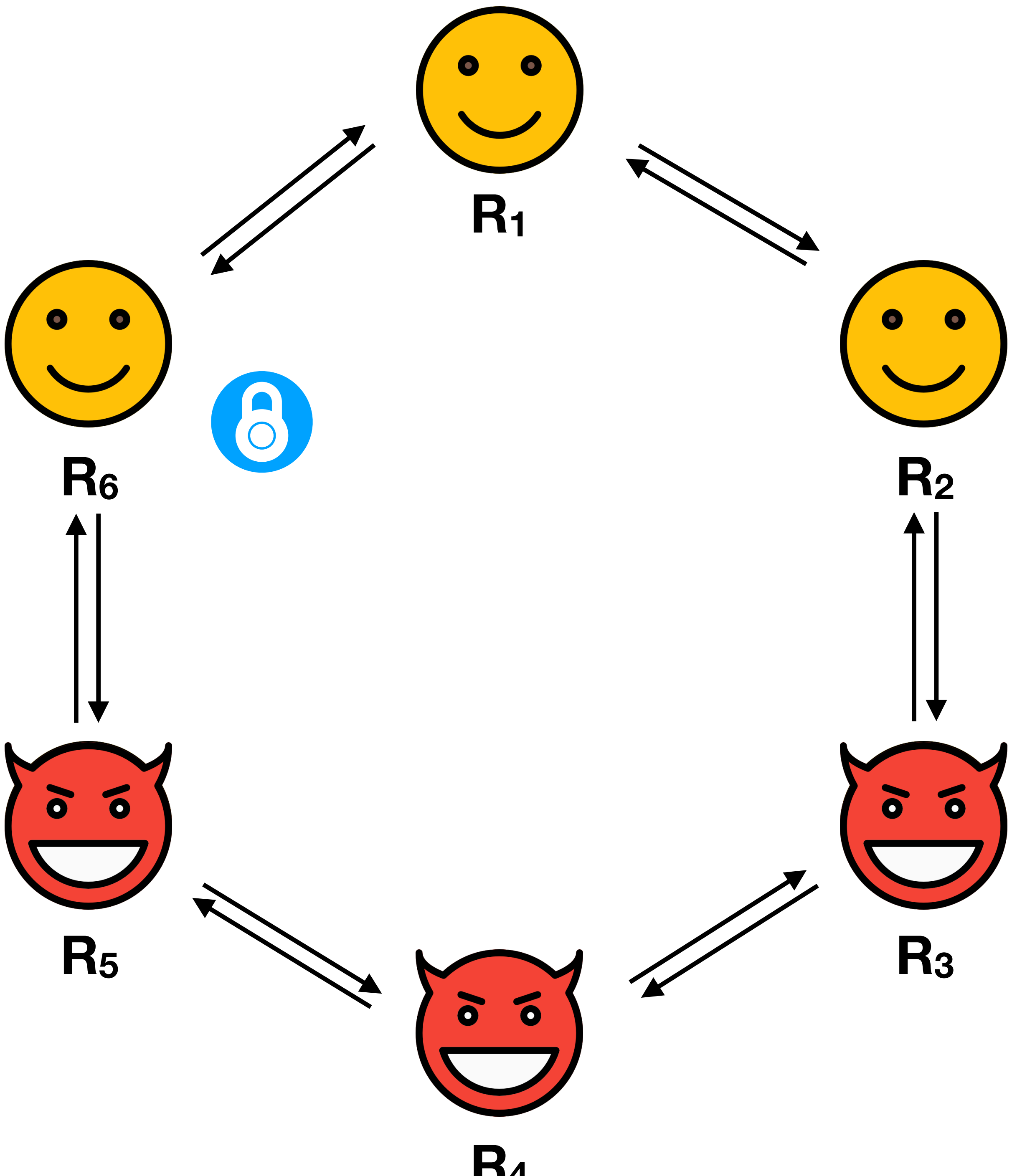

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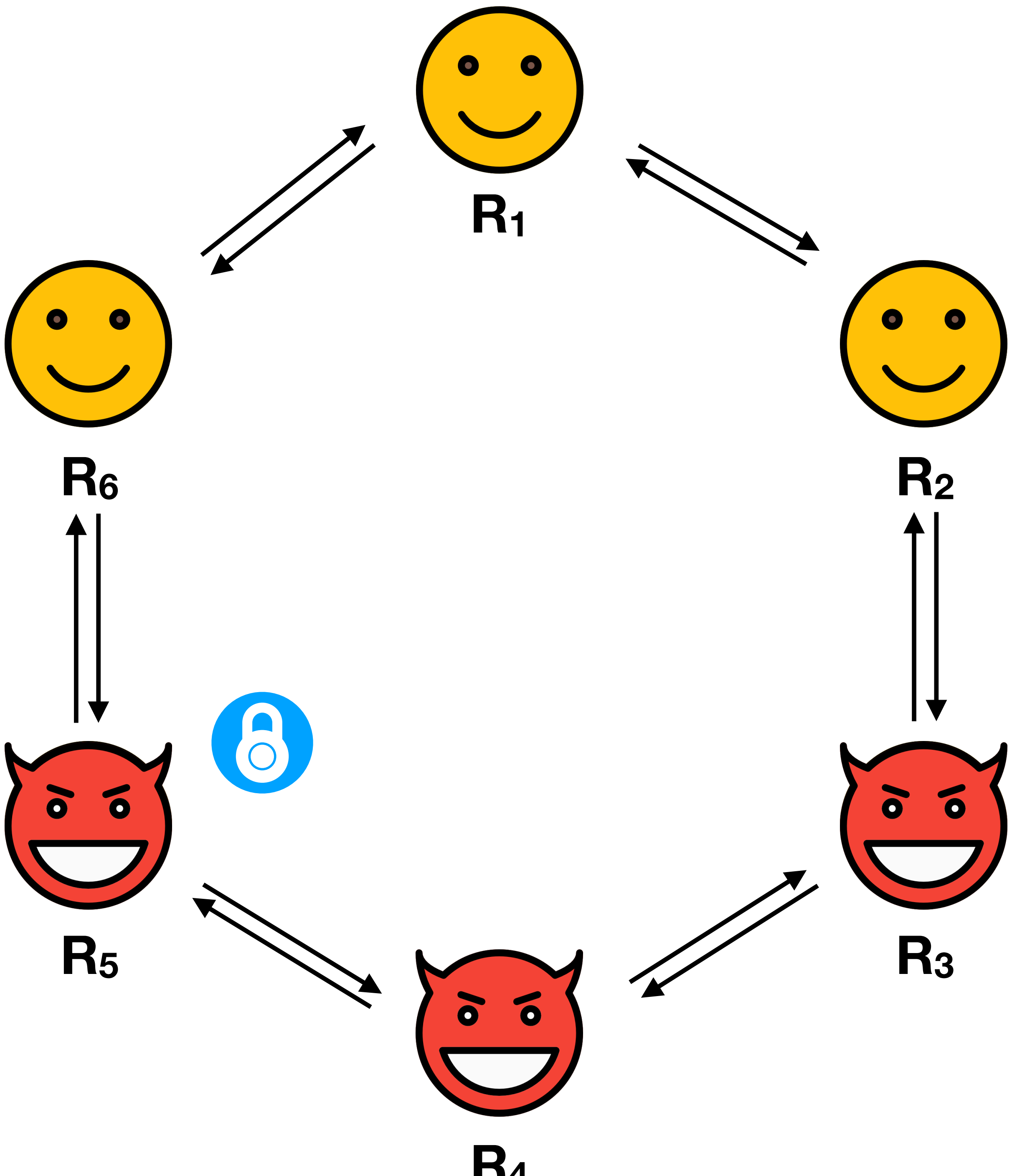

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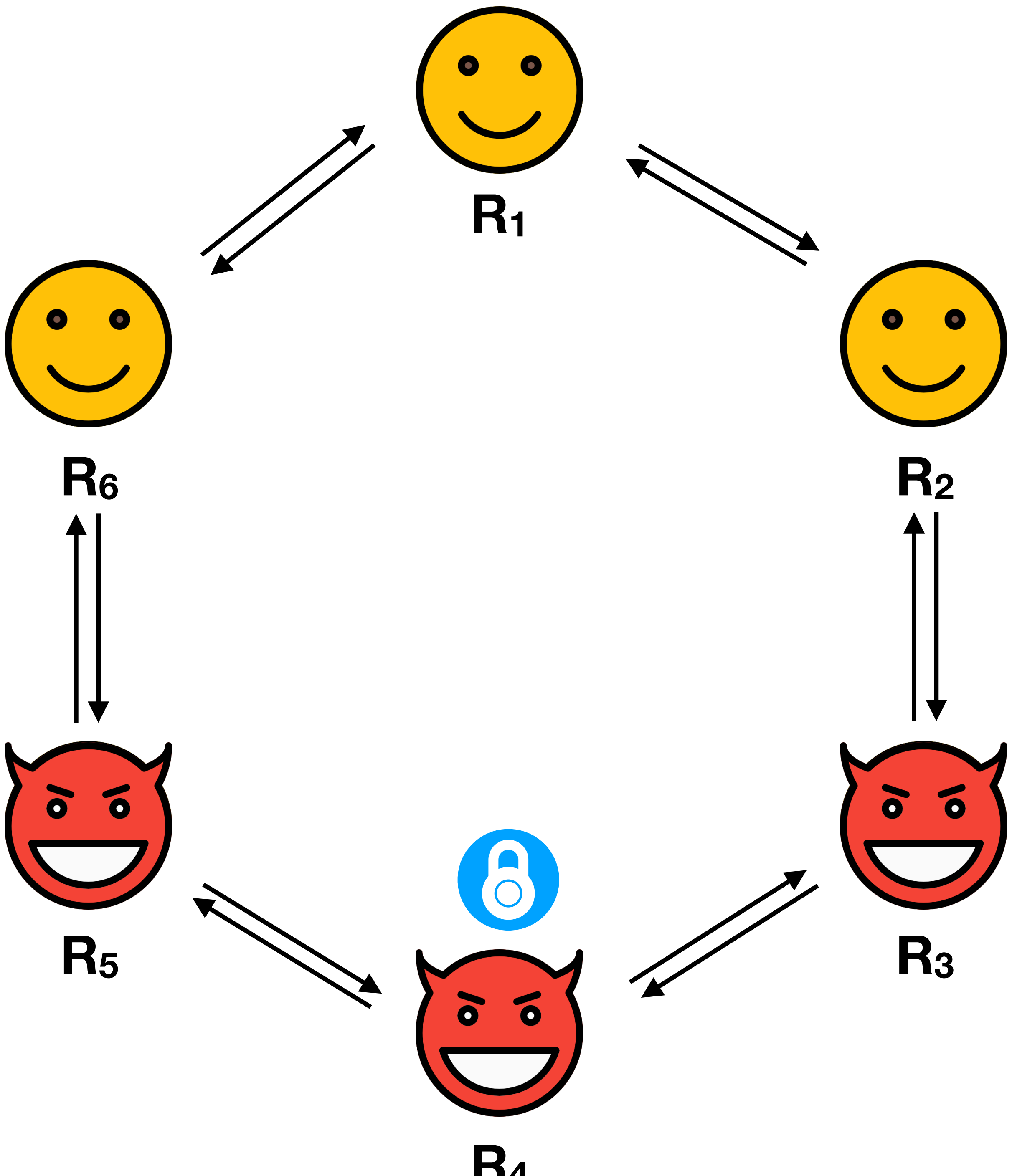

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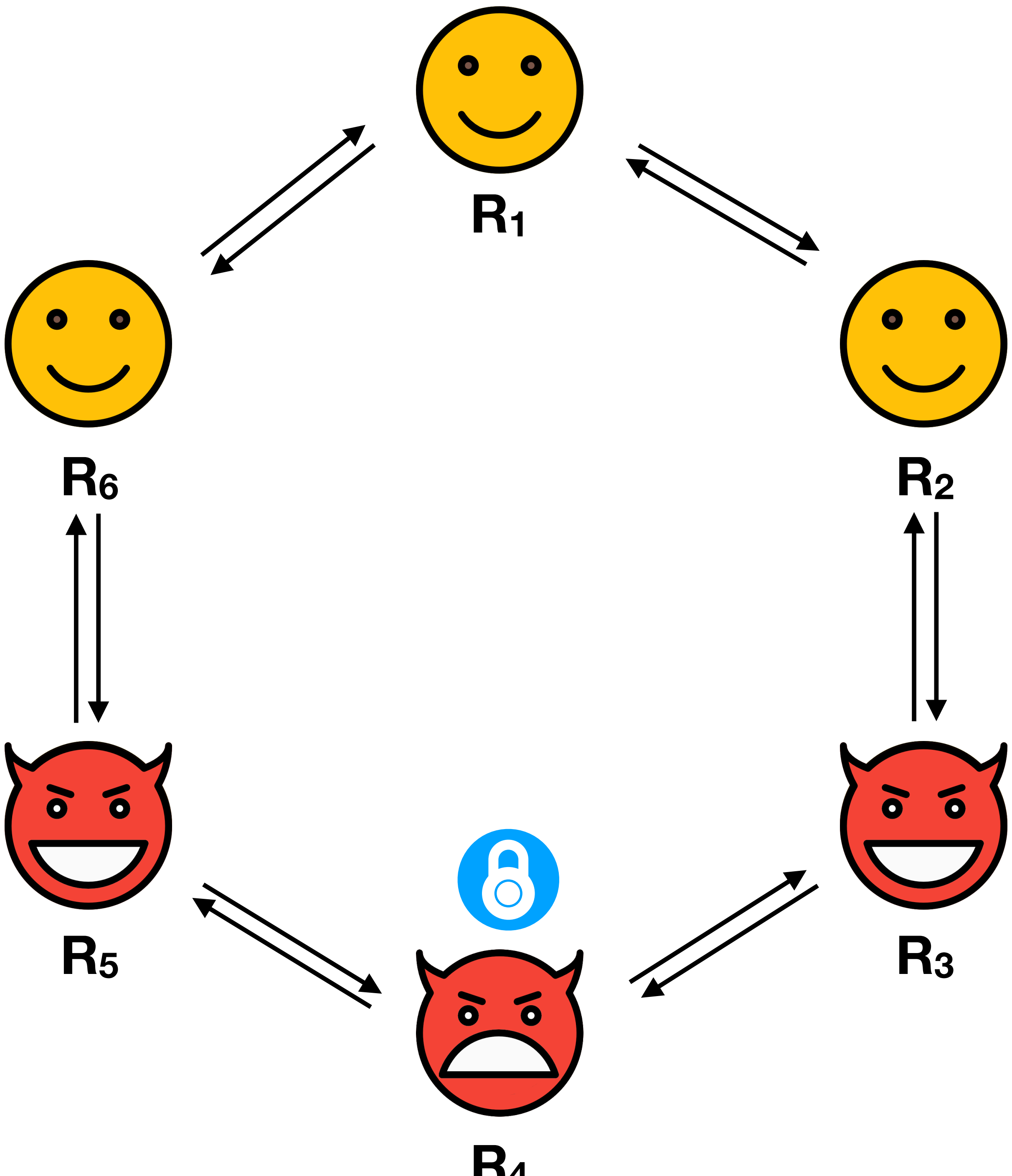

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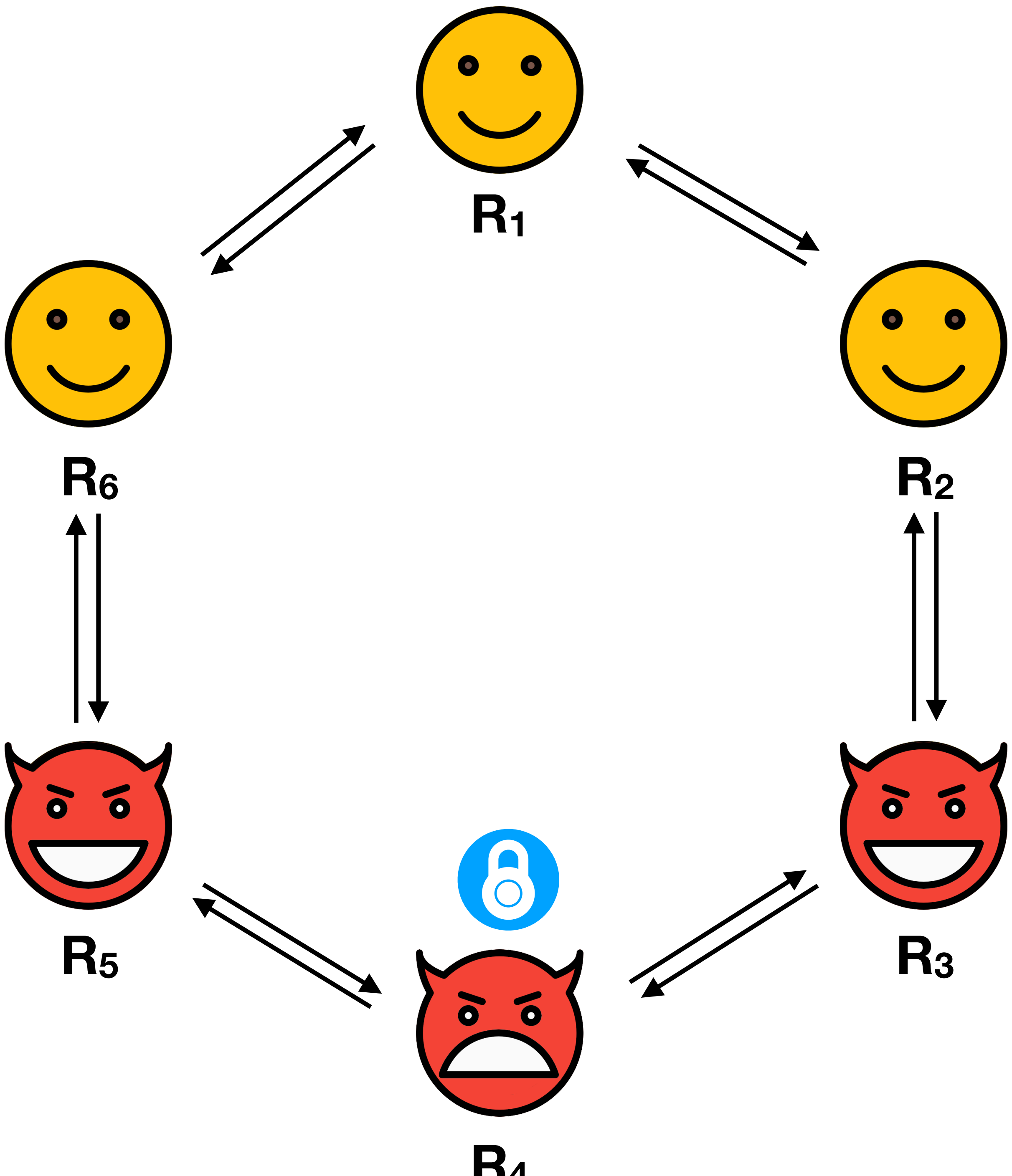

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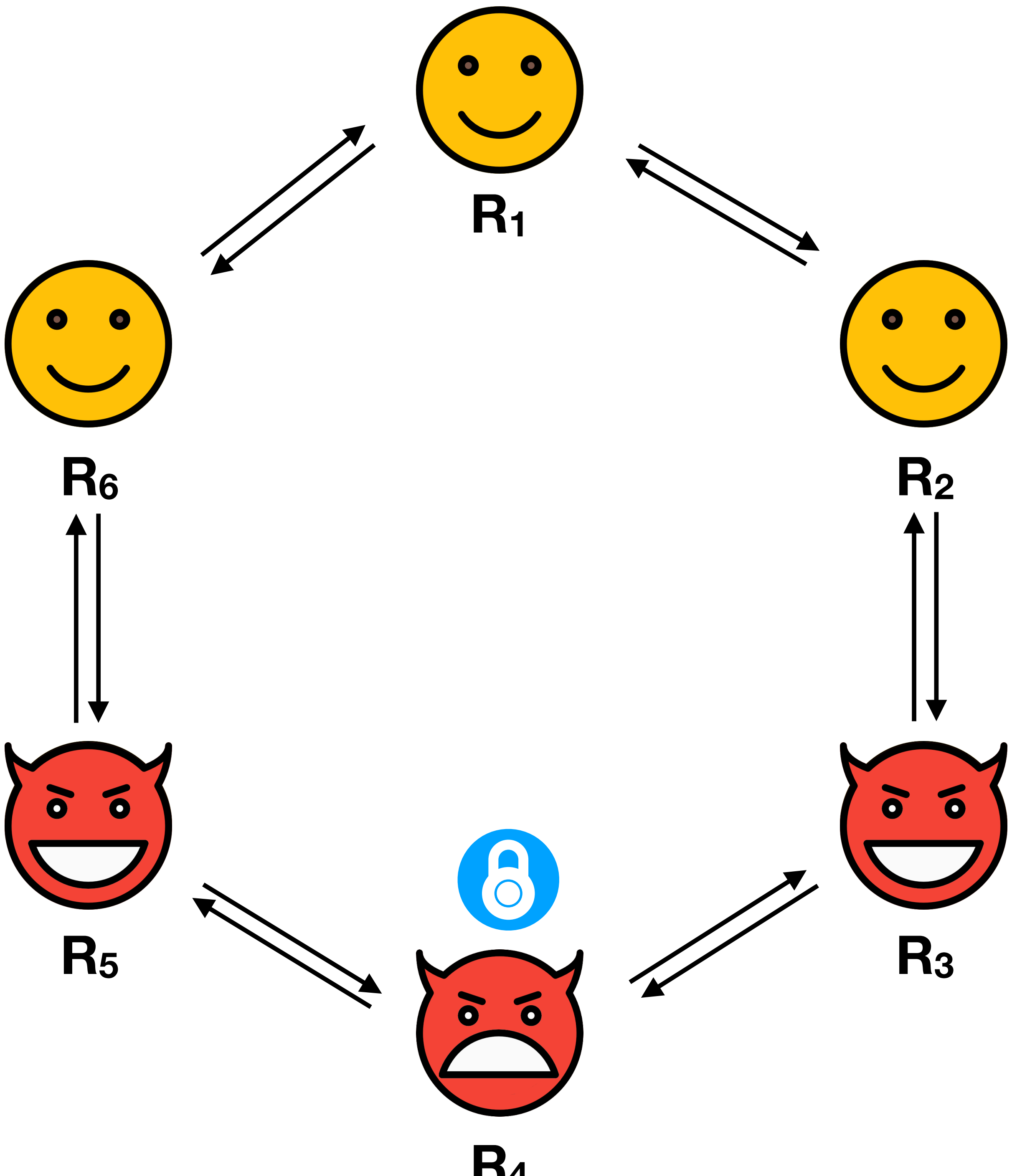

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Introduction

Authentication

Computation

Magic-state generation

Summary

Clifford code

Remember Yfke's tutorial: https://www.youtube.com/watch?v=yEjh8qJQqsM

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Theorem (informal): for any A on $n+1$ qubits, the probability that A changes $|\psi\rangle$, but is not detected at decoding is very small (2^{-n}).

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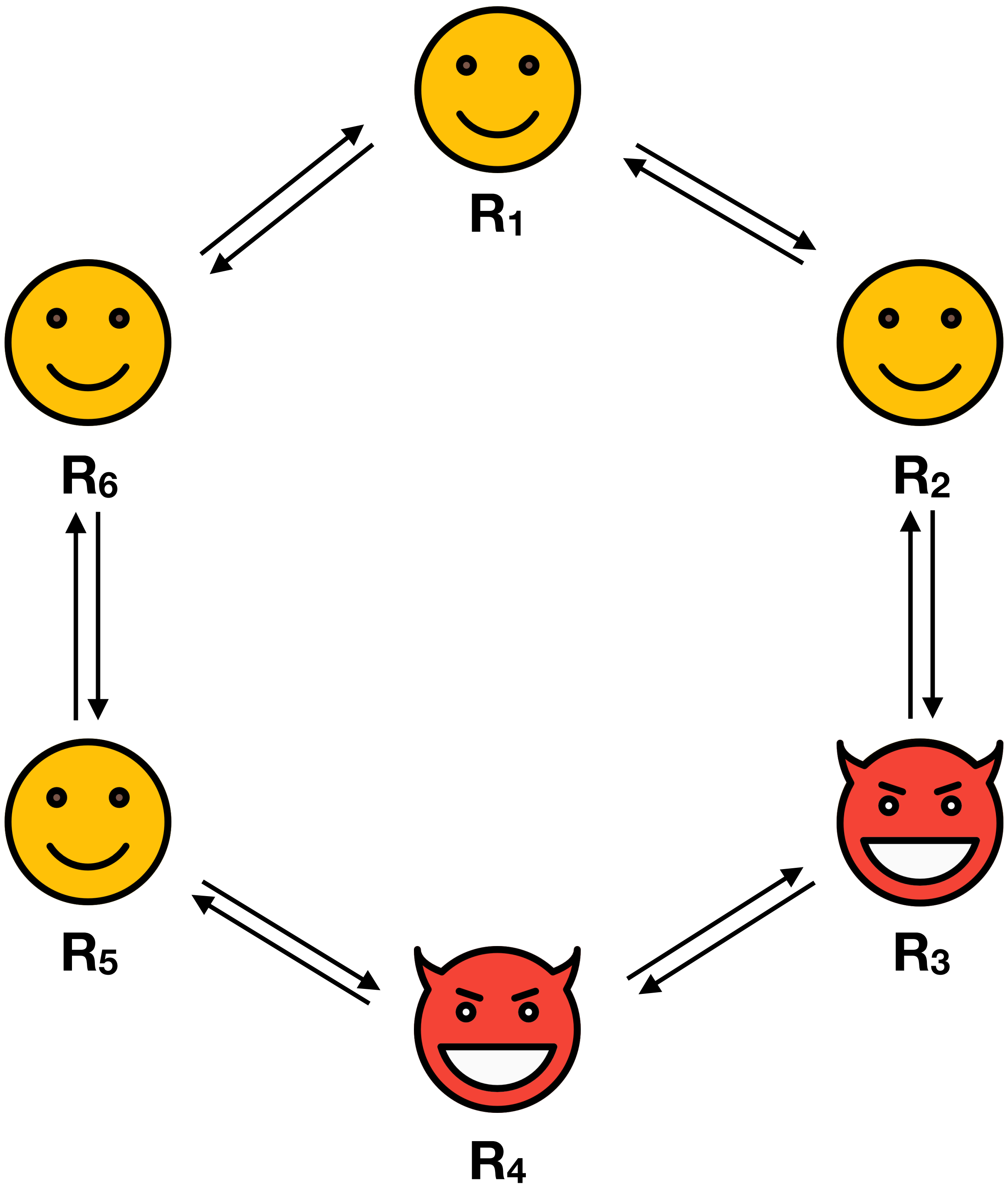
Bonus: the Clifford code also provides privacy.

Clifford code in MPQC



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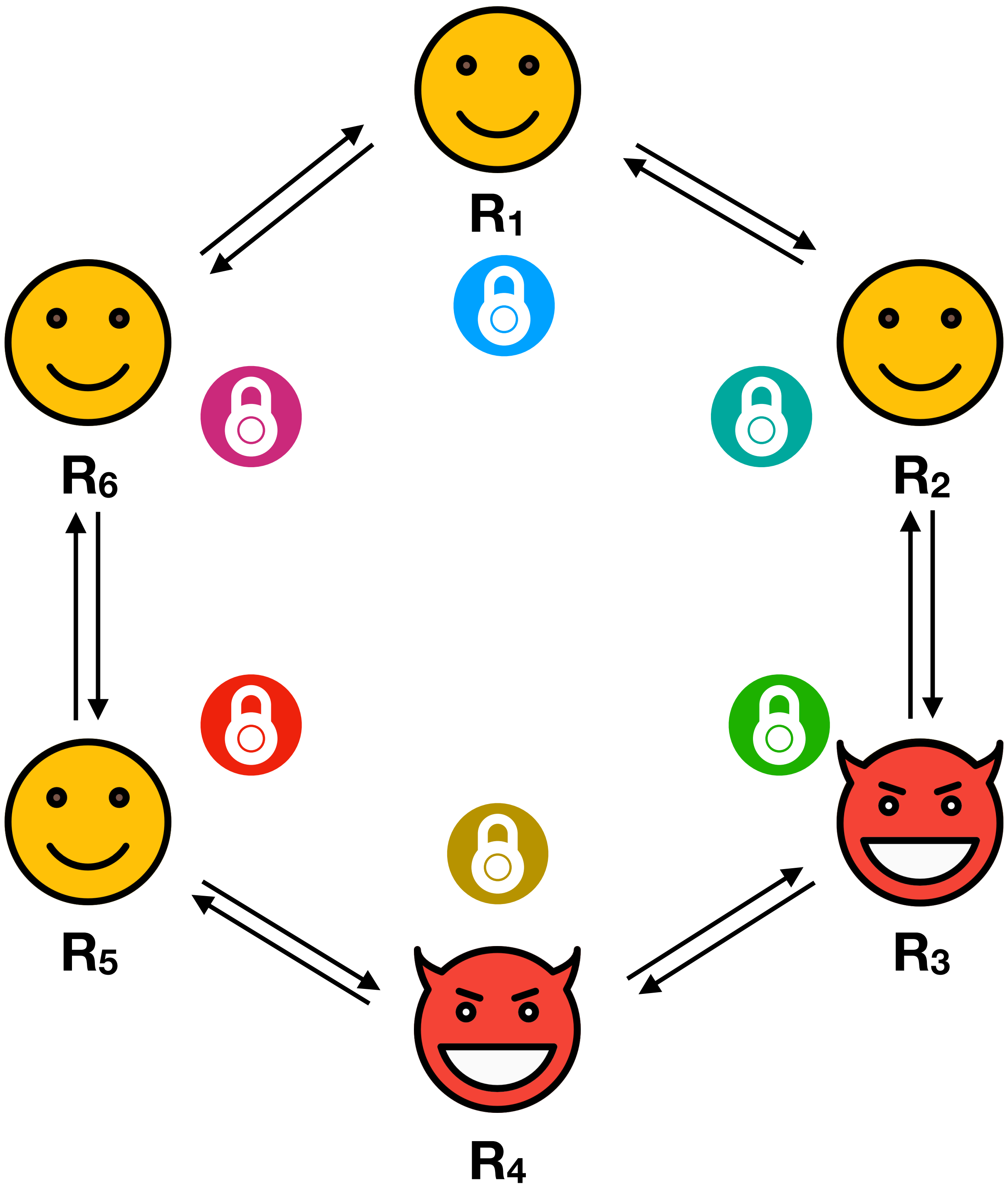

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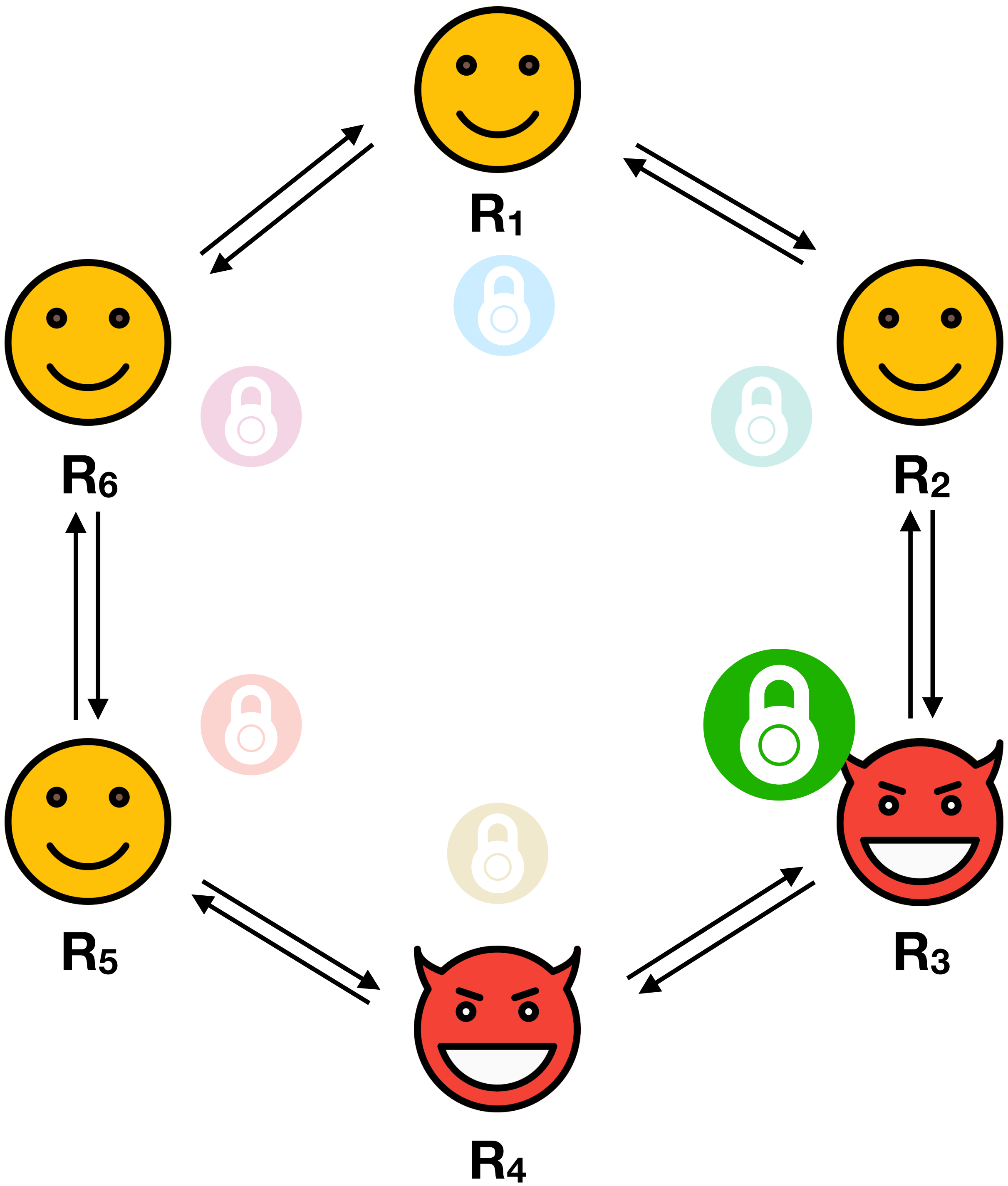

- What if the encoding player is dishonest?

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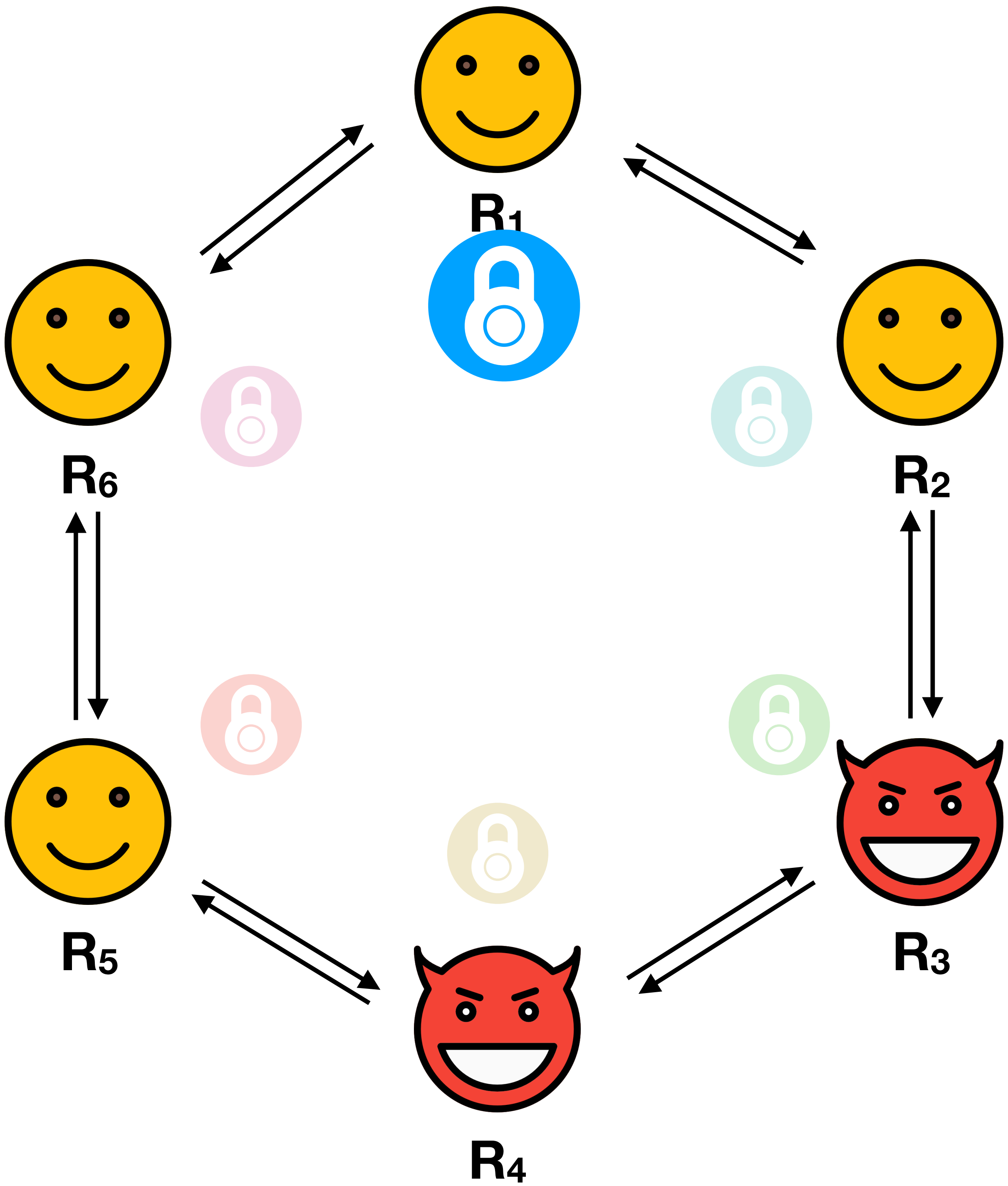

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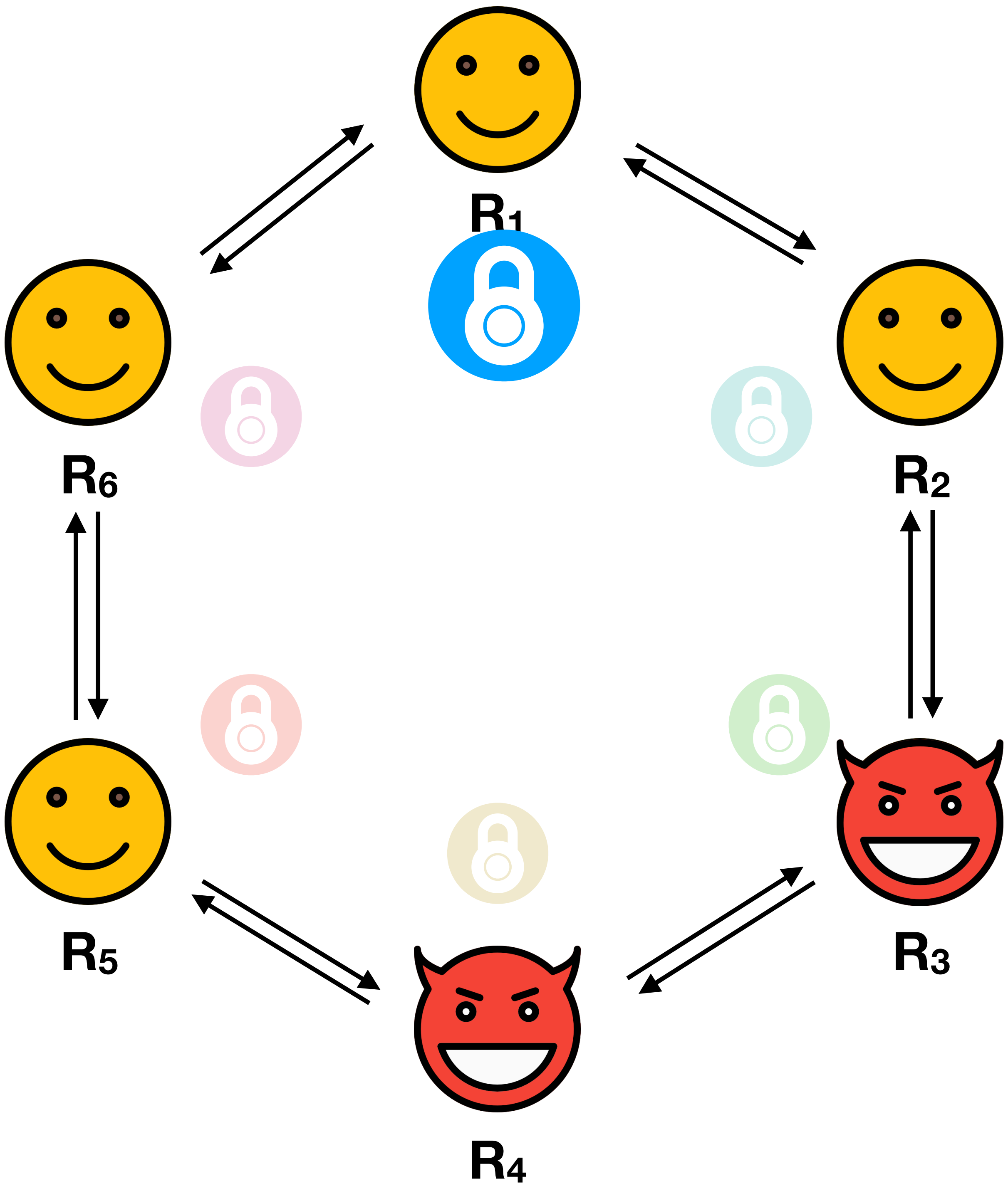
Clifford code in MPQC



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Answers: use classical multi-party computation! 

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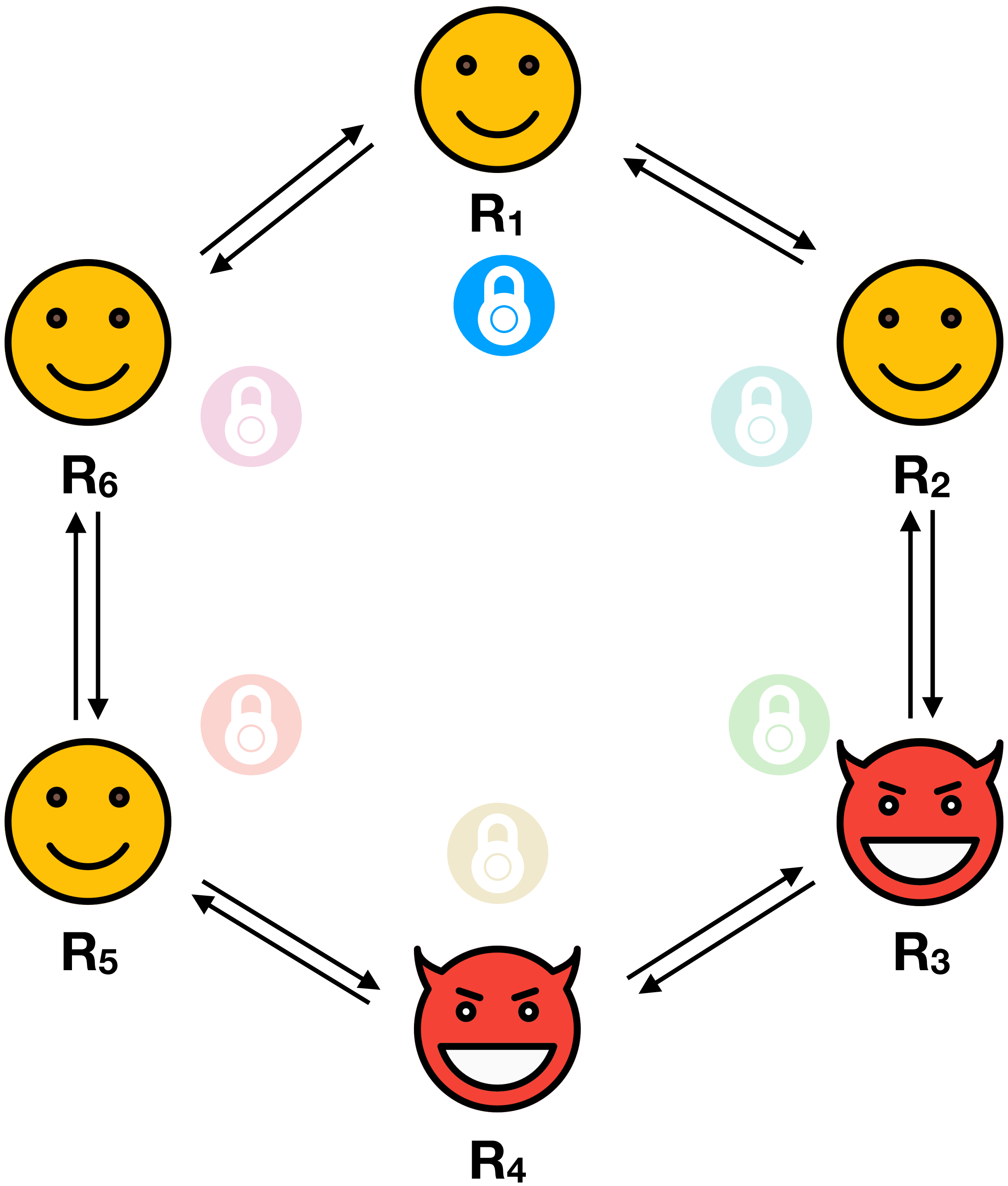

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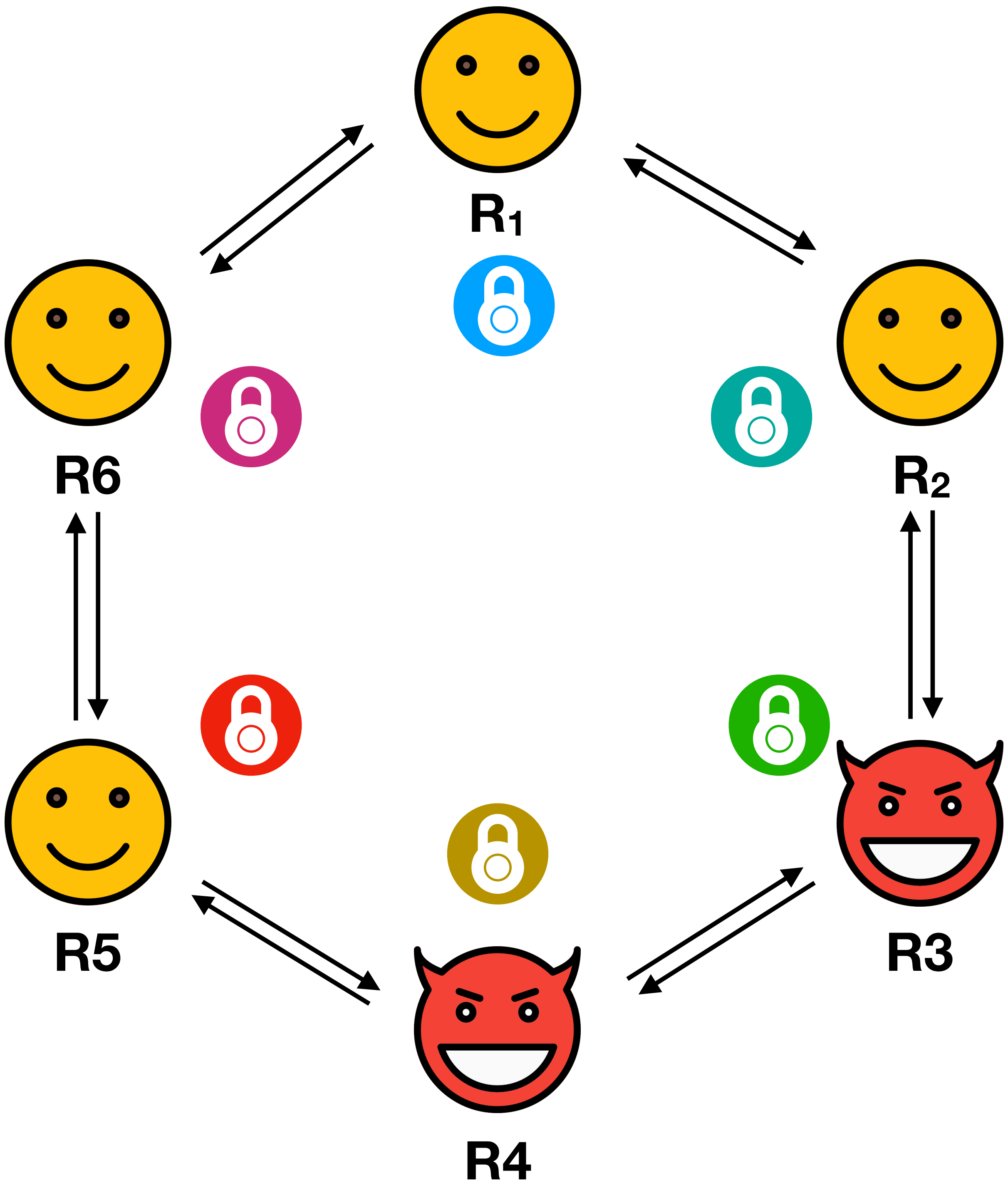

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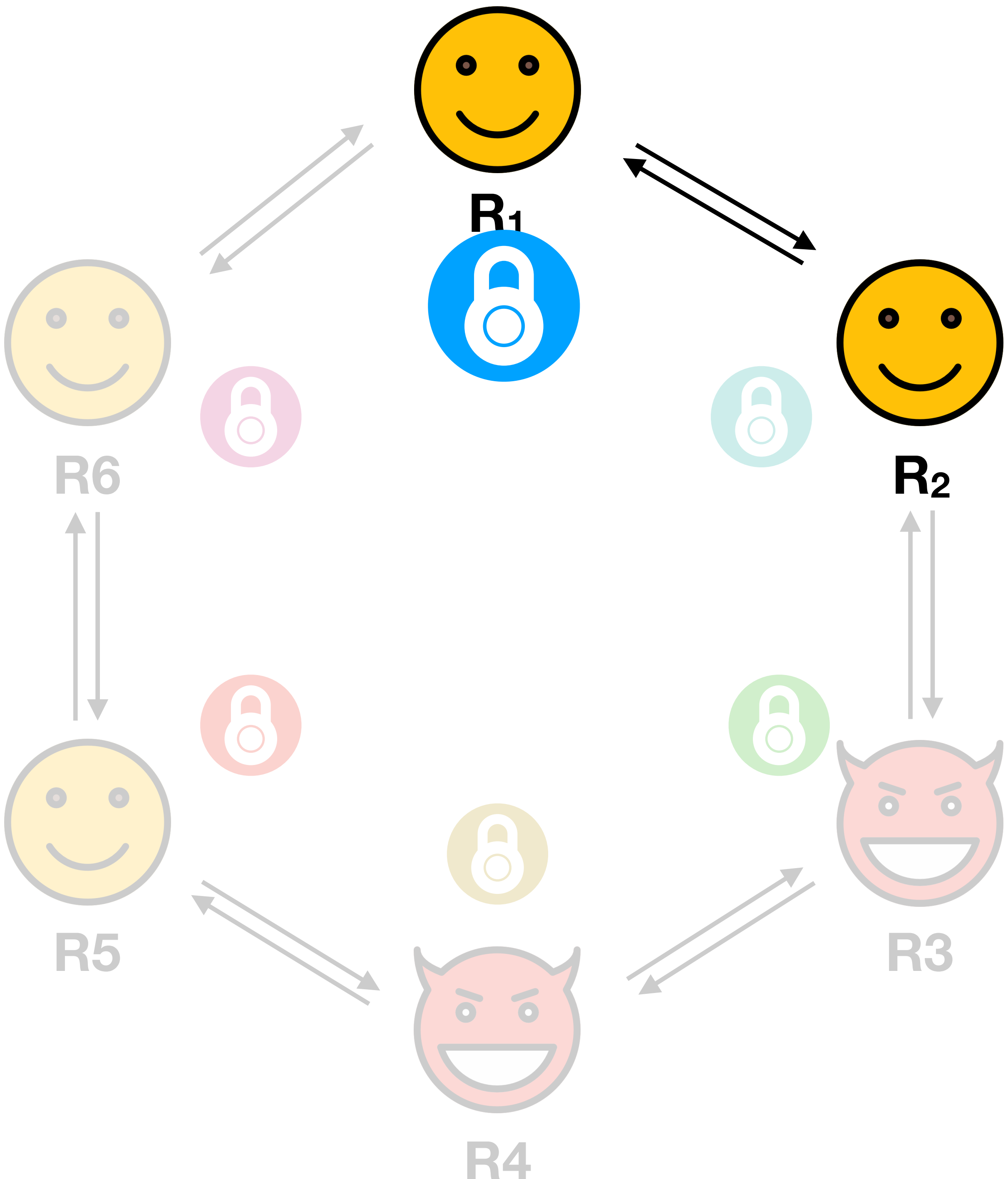

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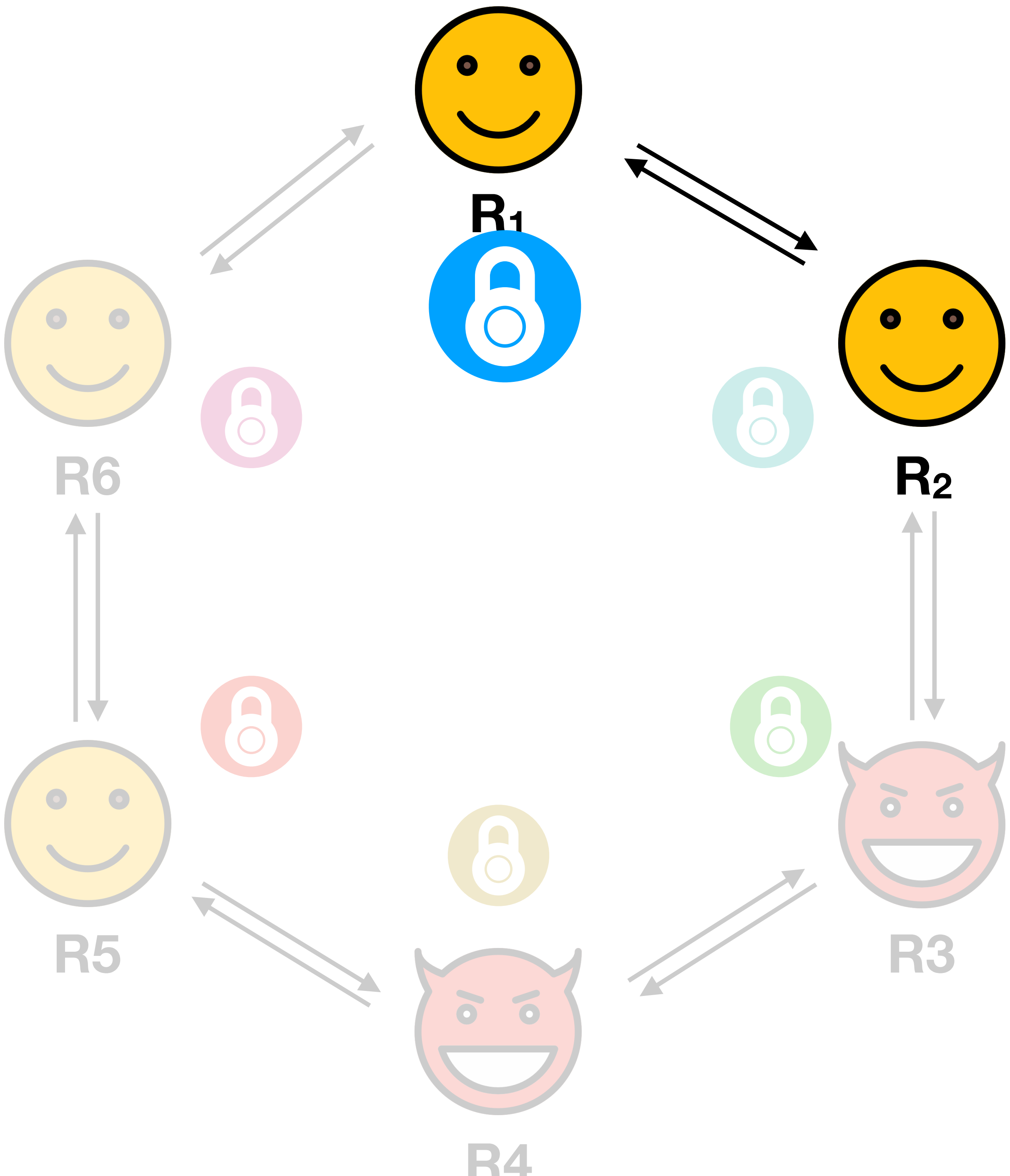

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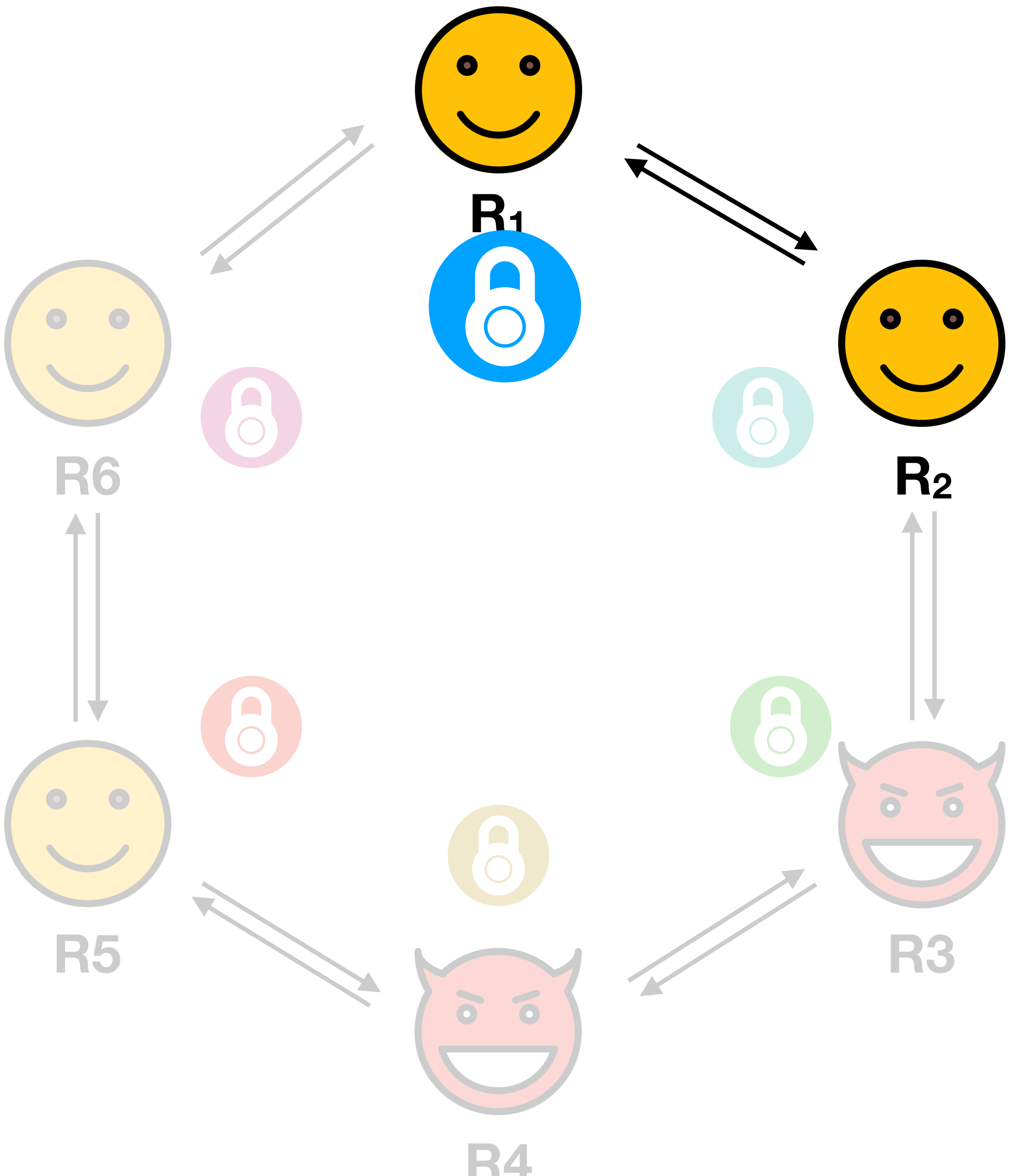

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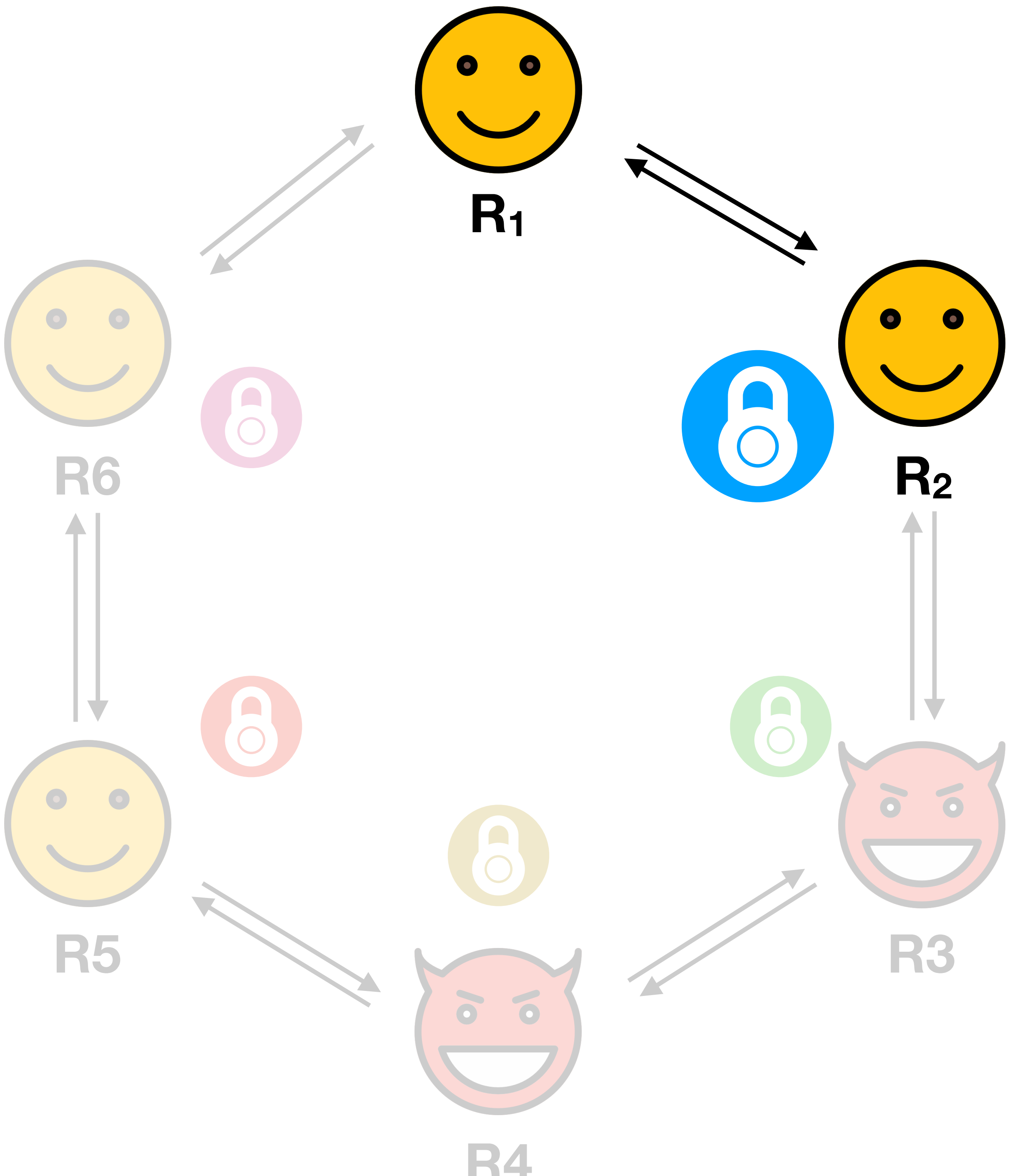

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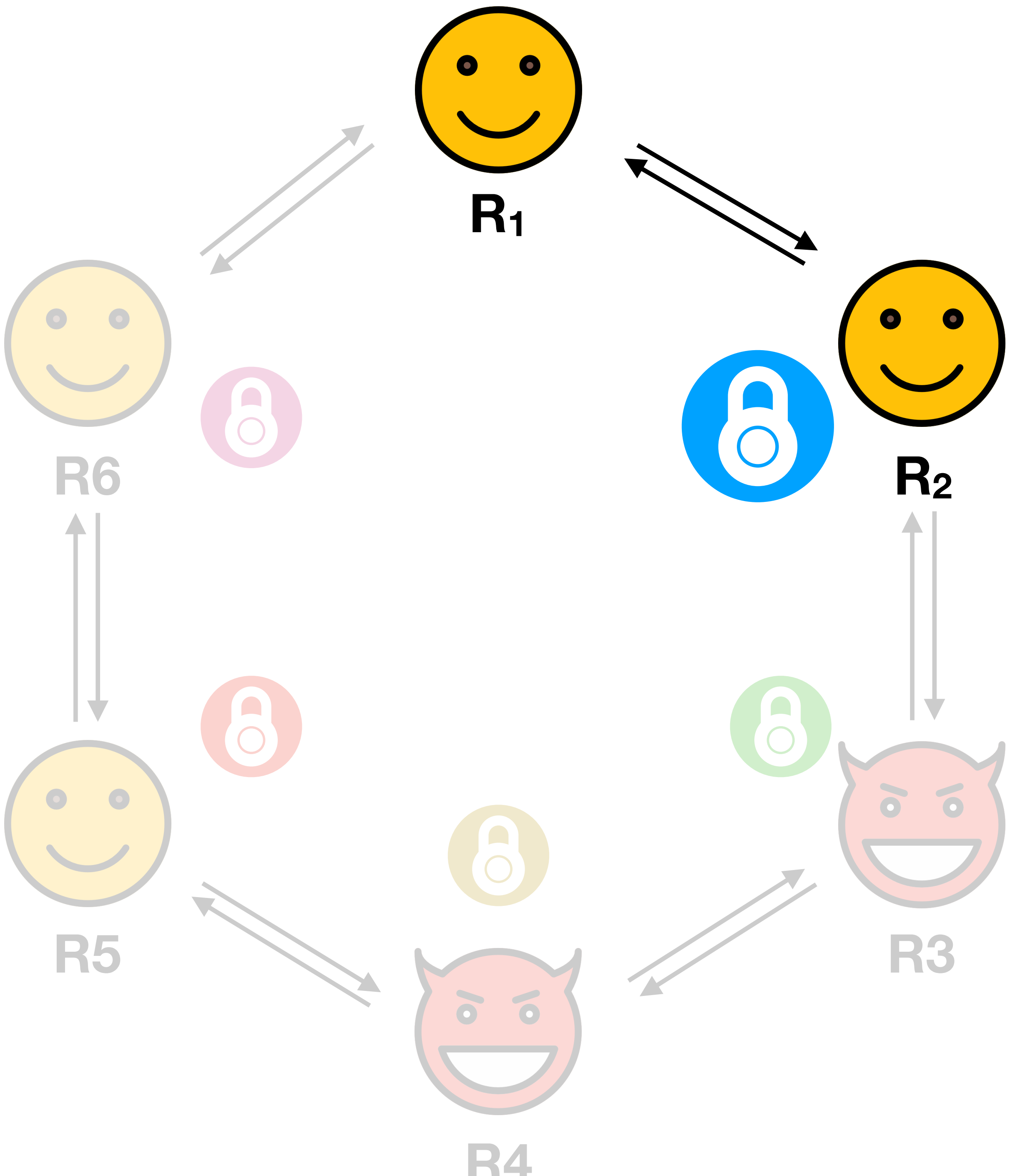

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PLAYER 2'S TRAPS ACCESSIBLE

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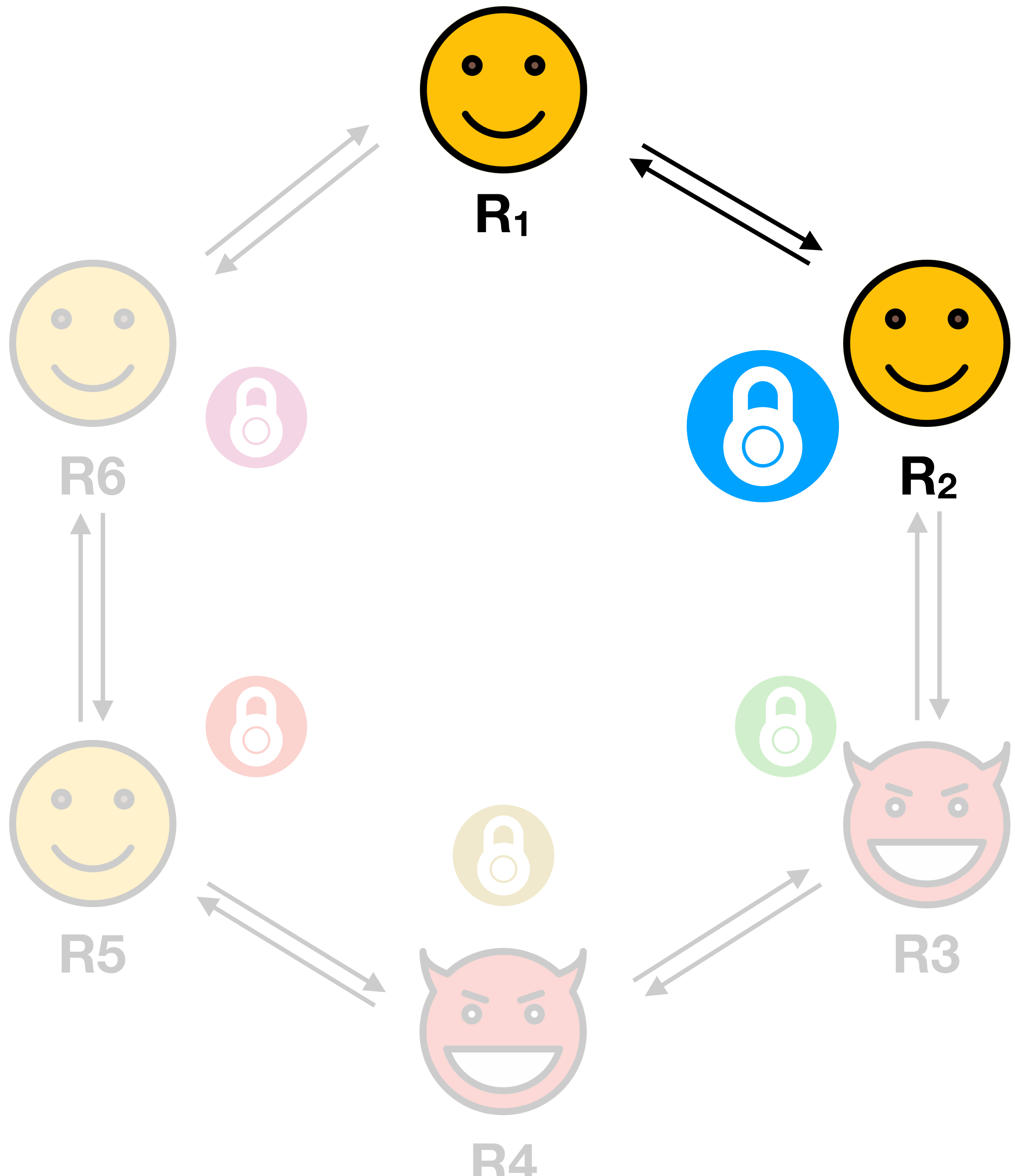

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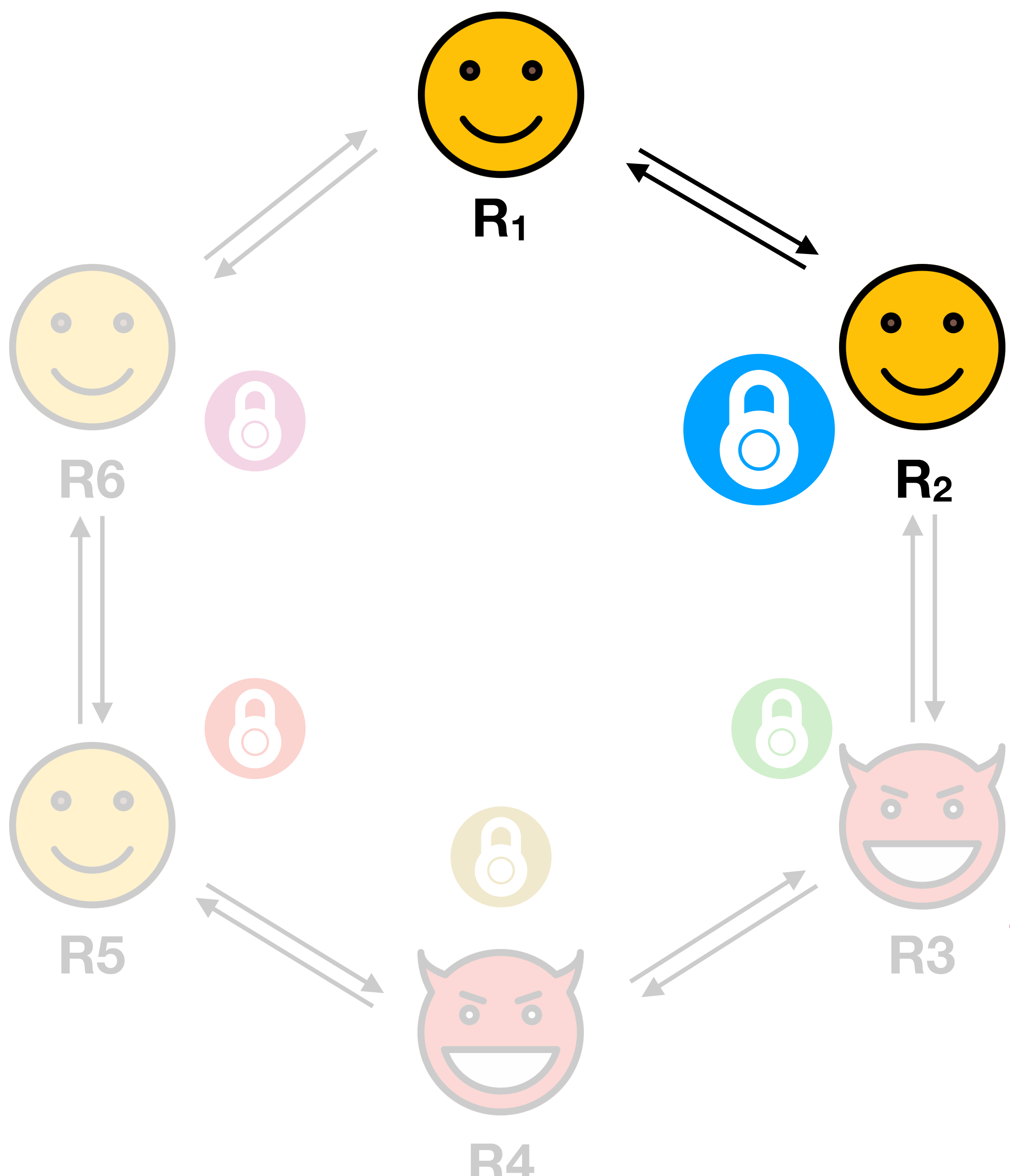

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PLAYER 1'S TRAPS INACCESSIBLE

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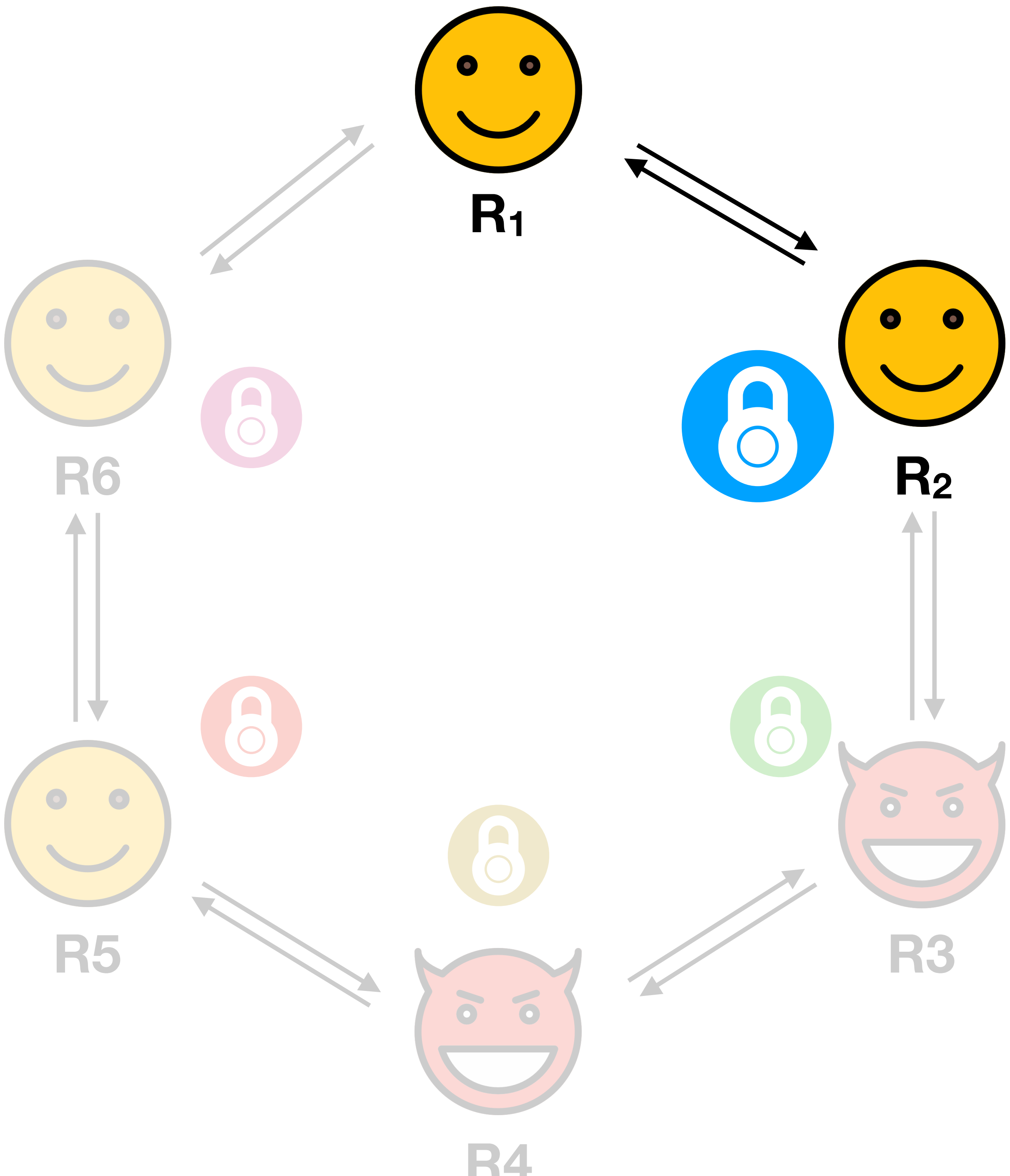
MAKE ACCESSIBLE WITH CLASSICAL MPC:

$$f(C_1, C_2) = (C_2' \otimes I^{\otimes n})(C_1^\dagger \otimes I^{\otimes n})C_2^\dagger$$

First idea: stack encodings



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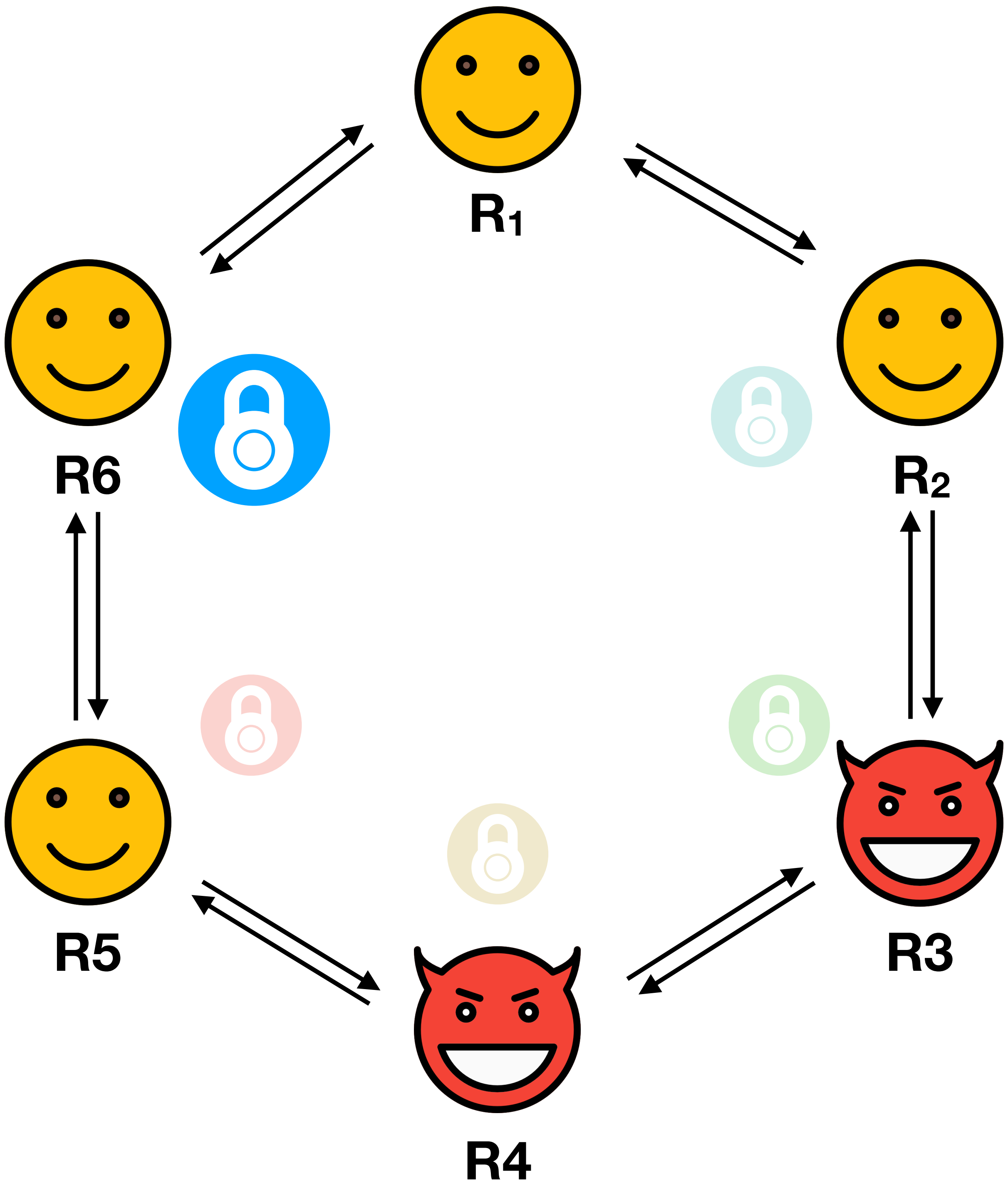

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$$\downarrow$$

$$C_4(C_3(C_2(C_1(|\psi\rangle \otimes |0^n\rangle) \otimes |0^n\rangle) \otimes |0^n\rangle) \otimes |0^n\rangle)$$

$$\downarrow$$

$$C_5(C_4(C_3(C_2(C_1(|\psi\rangle \otimes |0^n\rangle) \otimes |0^n\rangle) \otimes |0^n\rangle) \otimes |0^n\rangle) \otimes |0^n\rangle)$$

$$\downarrow$$

$$C_6(C_5(C_4(C_3(C_2(C_1(|\psi\rangle \otimes |0^n\rangle) \otimes |0^n\rangle) \otimes |0^n\rangle) \otimes |0^n\rangle) \otimes |0^n\rangle) \otimes |0^n\rangle)$$

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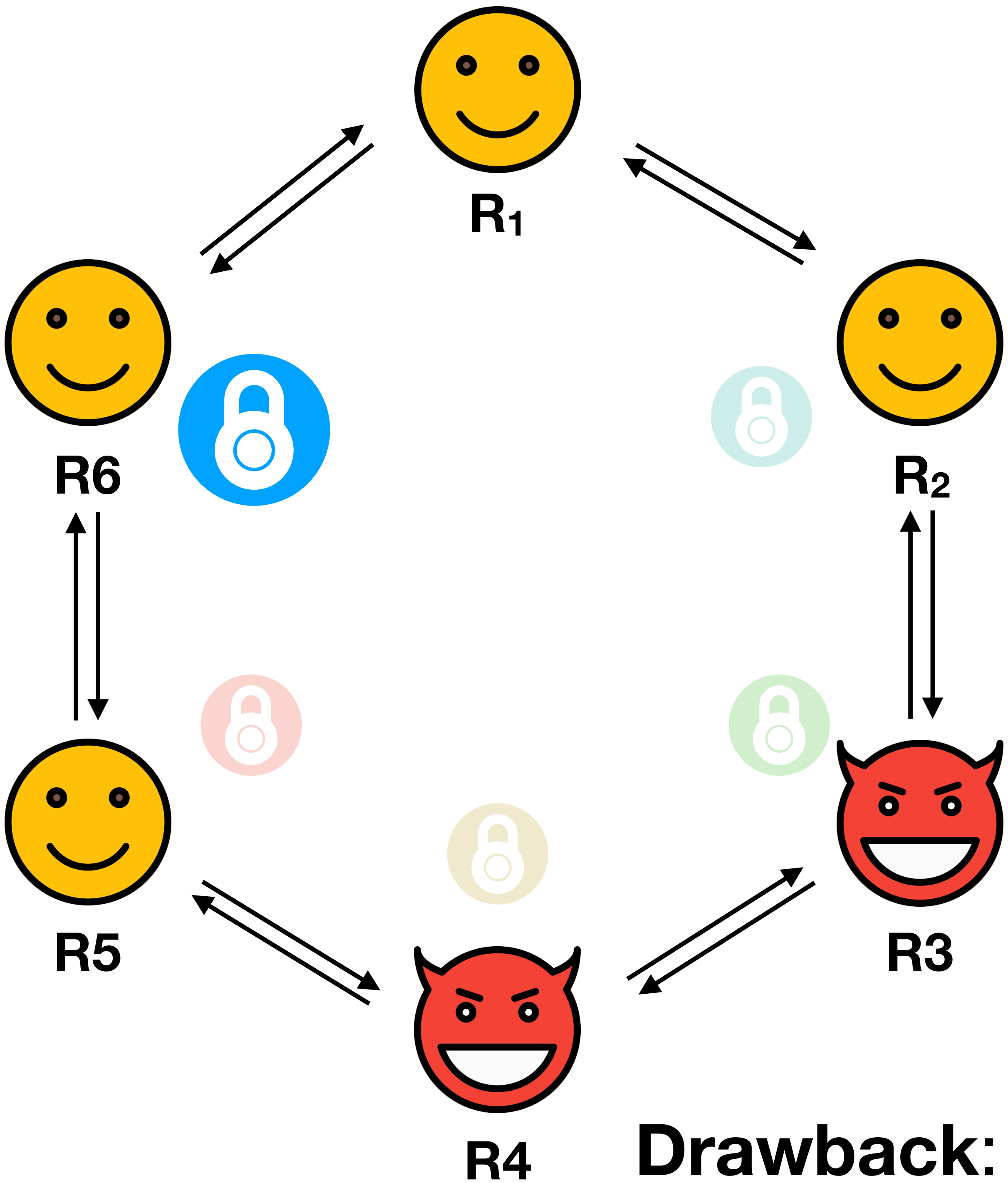

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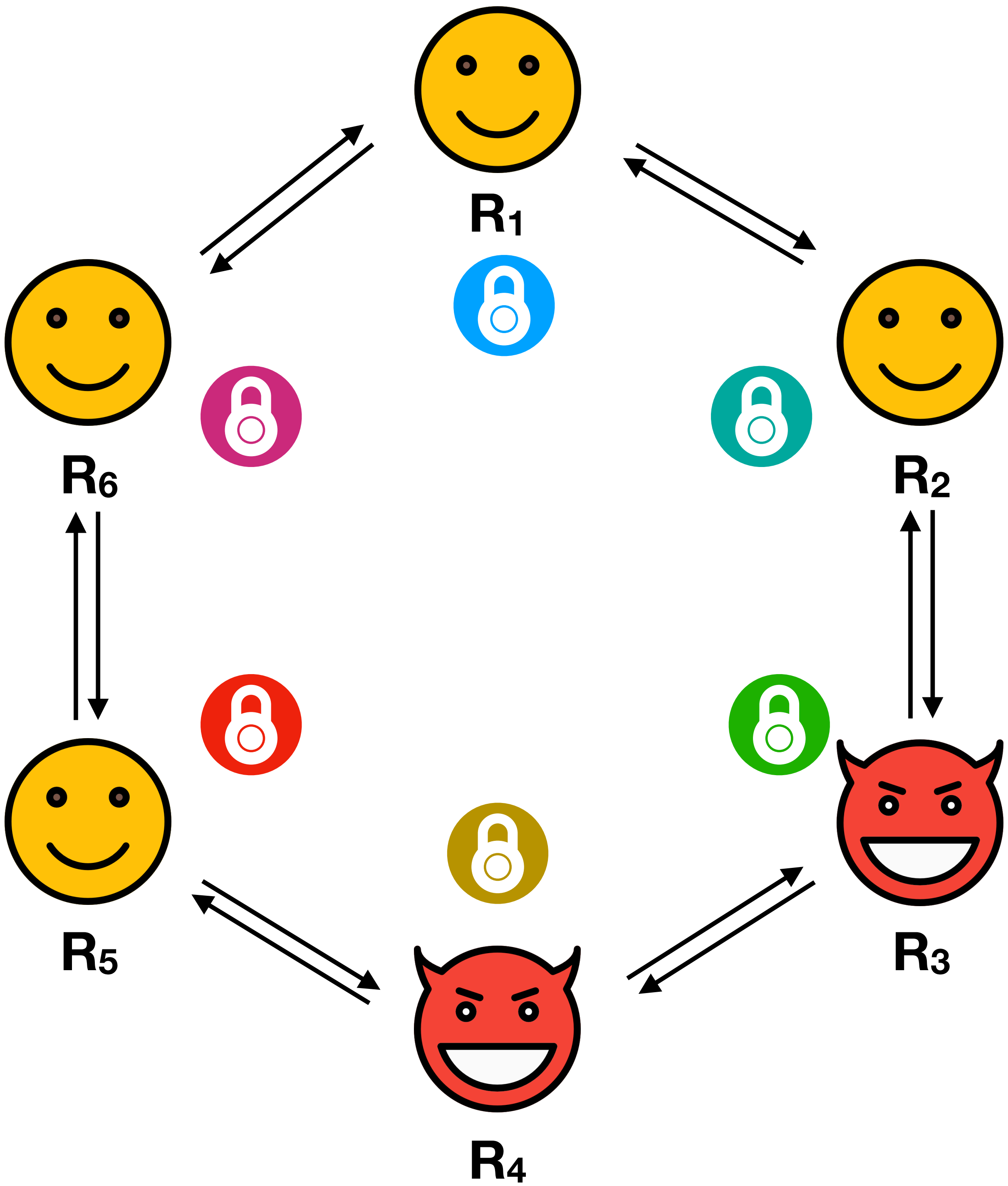
Drawback: very large ciphertexts ($nk + 1$)

Public authentication test



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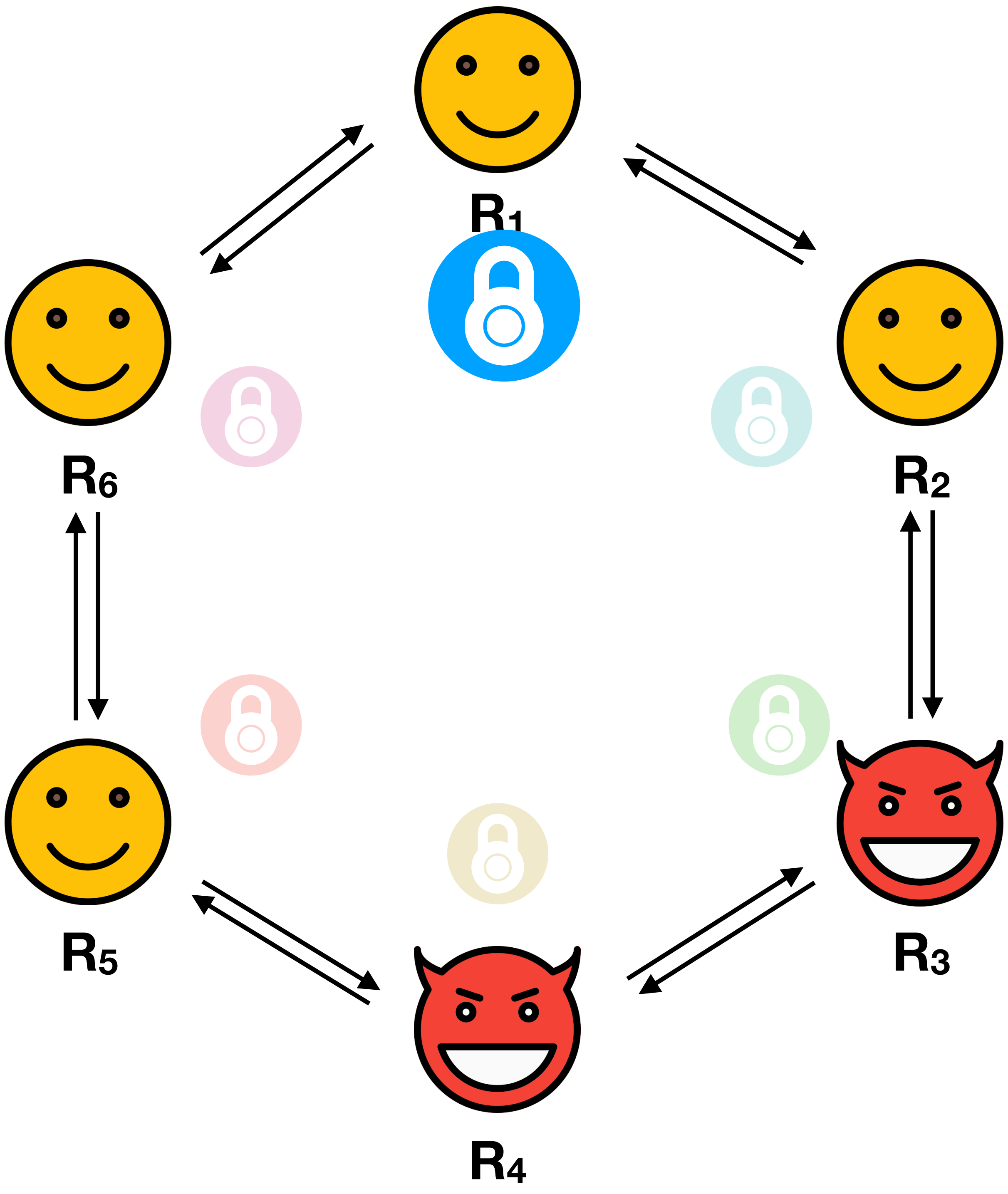

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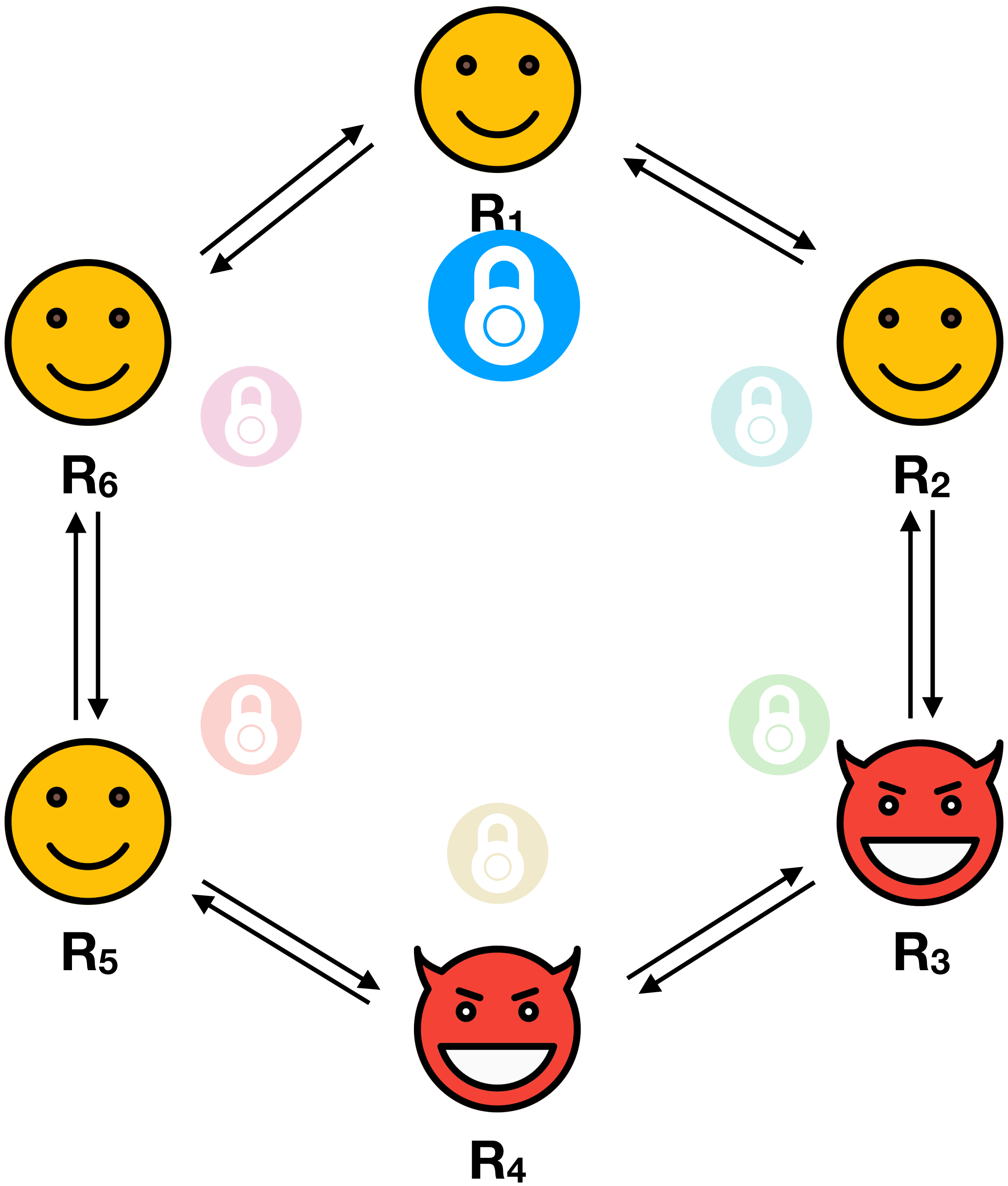
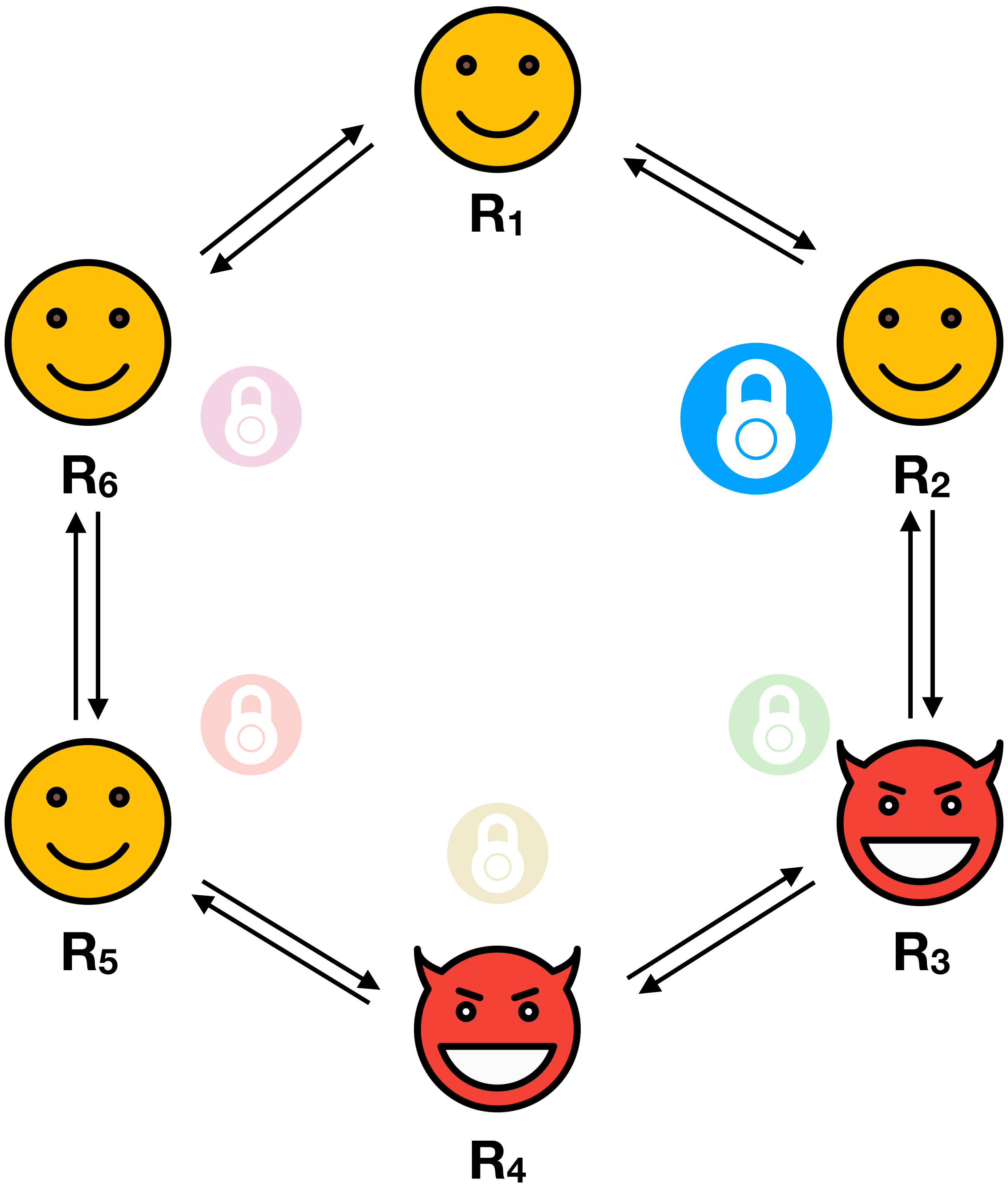


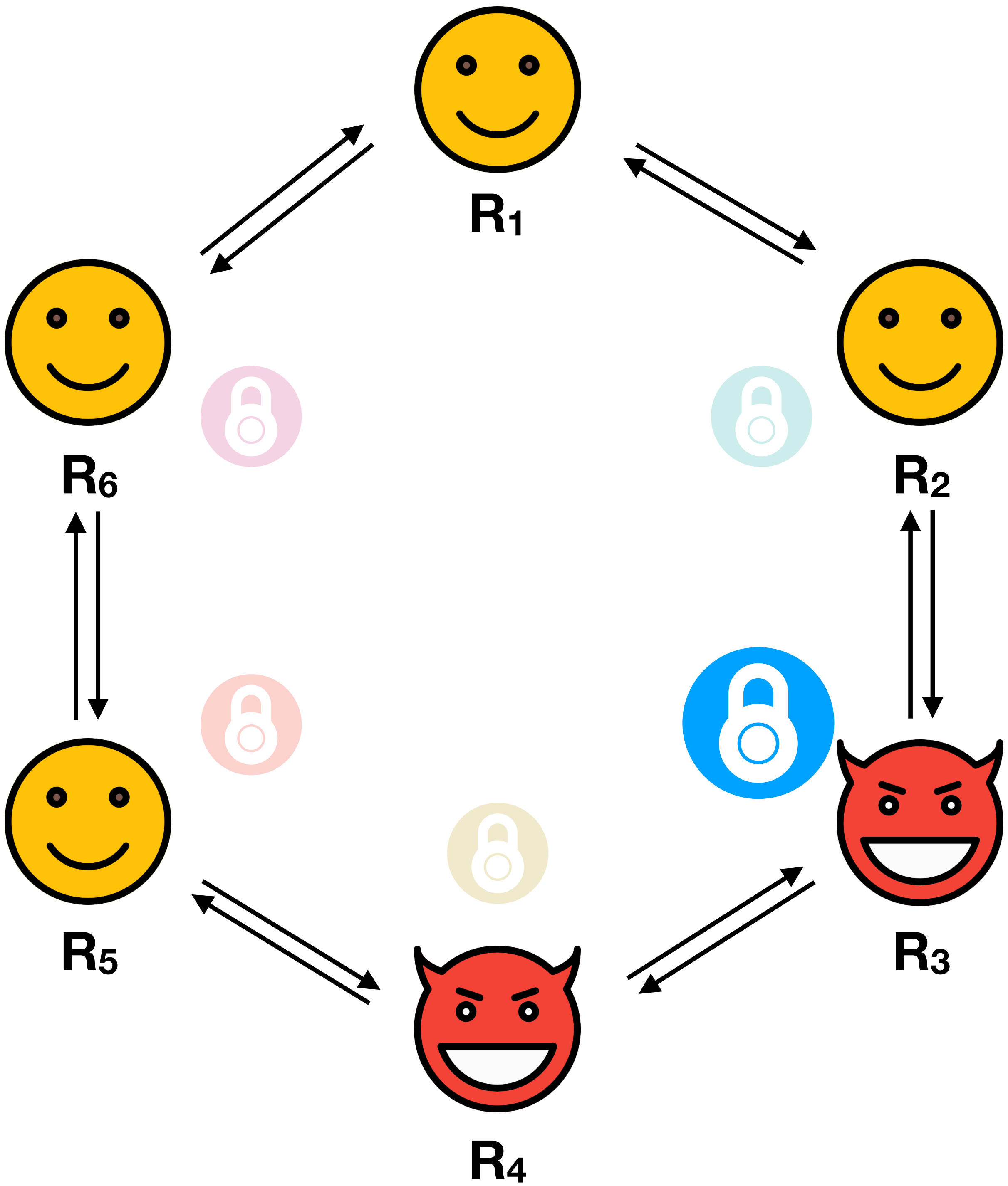
$$C_1(|\psi\rangle \otimes |0^{2n}\rangle)$$

Public authentication test



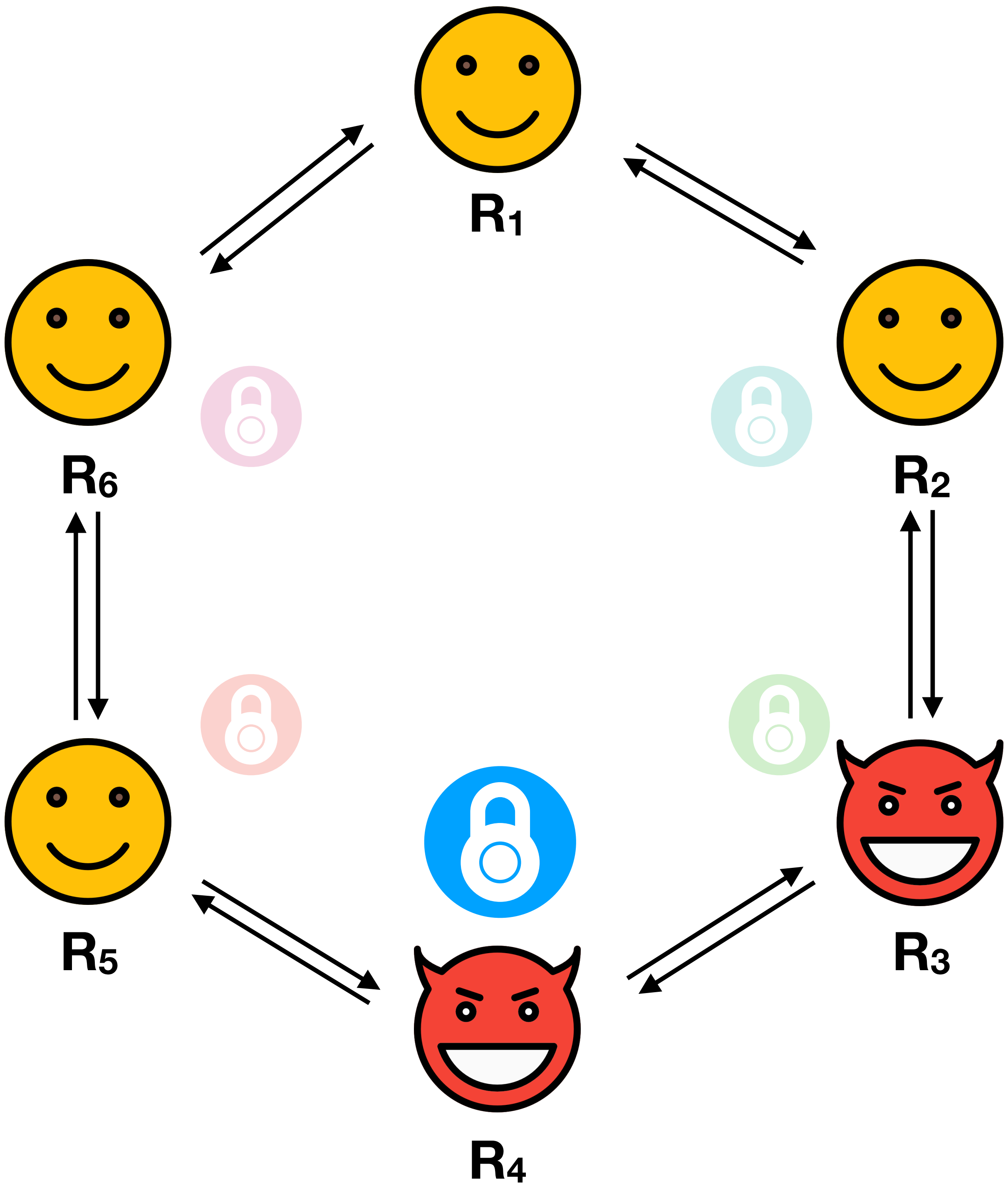
$$C_2 C_1 (|\psi\rangle \otimes |0^{2n}\rangle)$$

Public authentication test



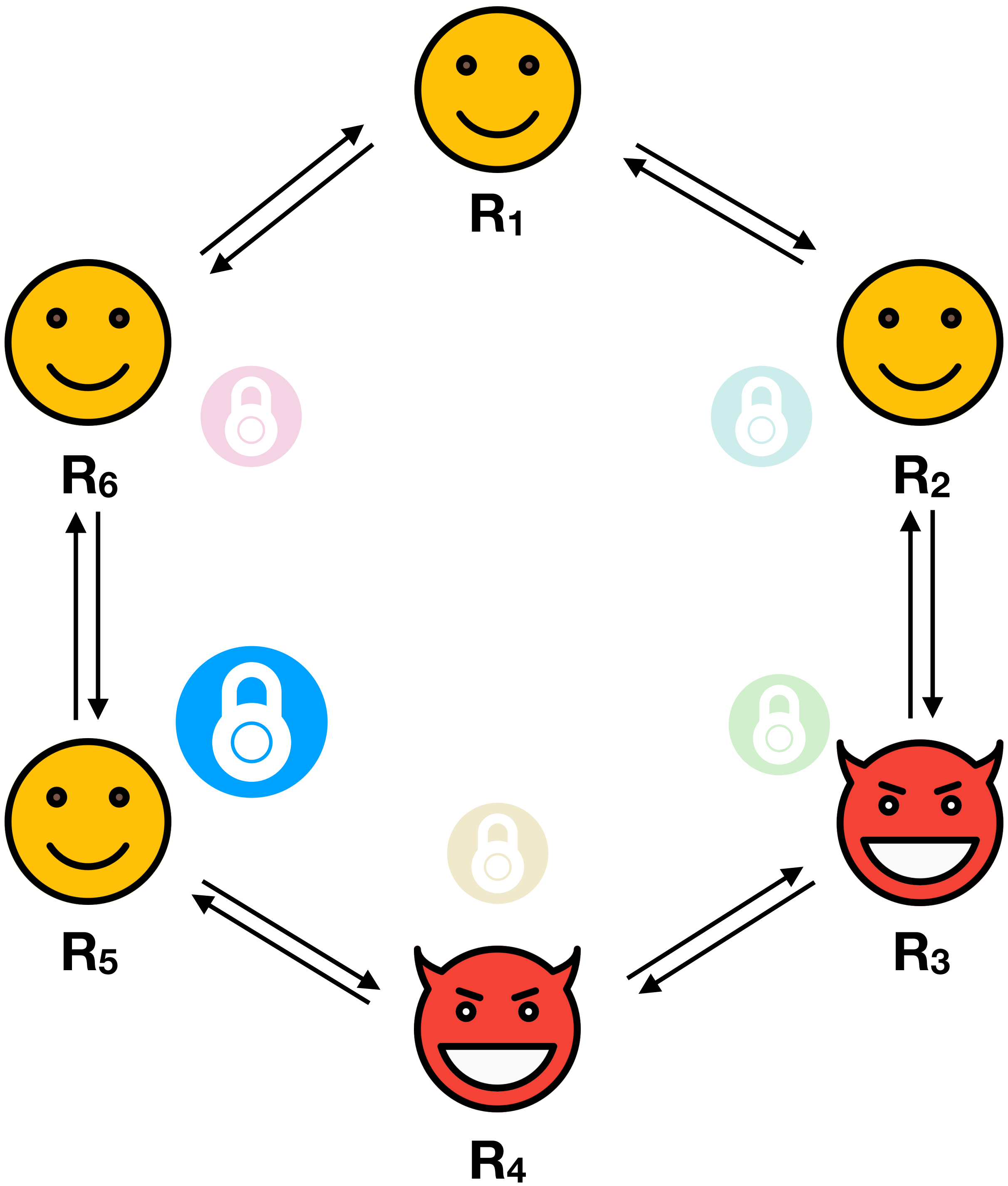
$$C_3 C_2 C_1 (|\psi\rangle \otimes |0^{2n}\rangle)$$

Public authentication test



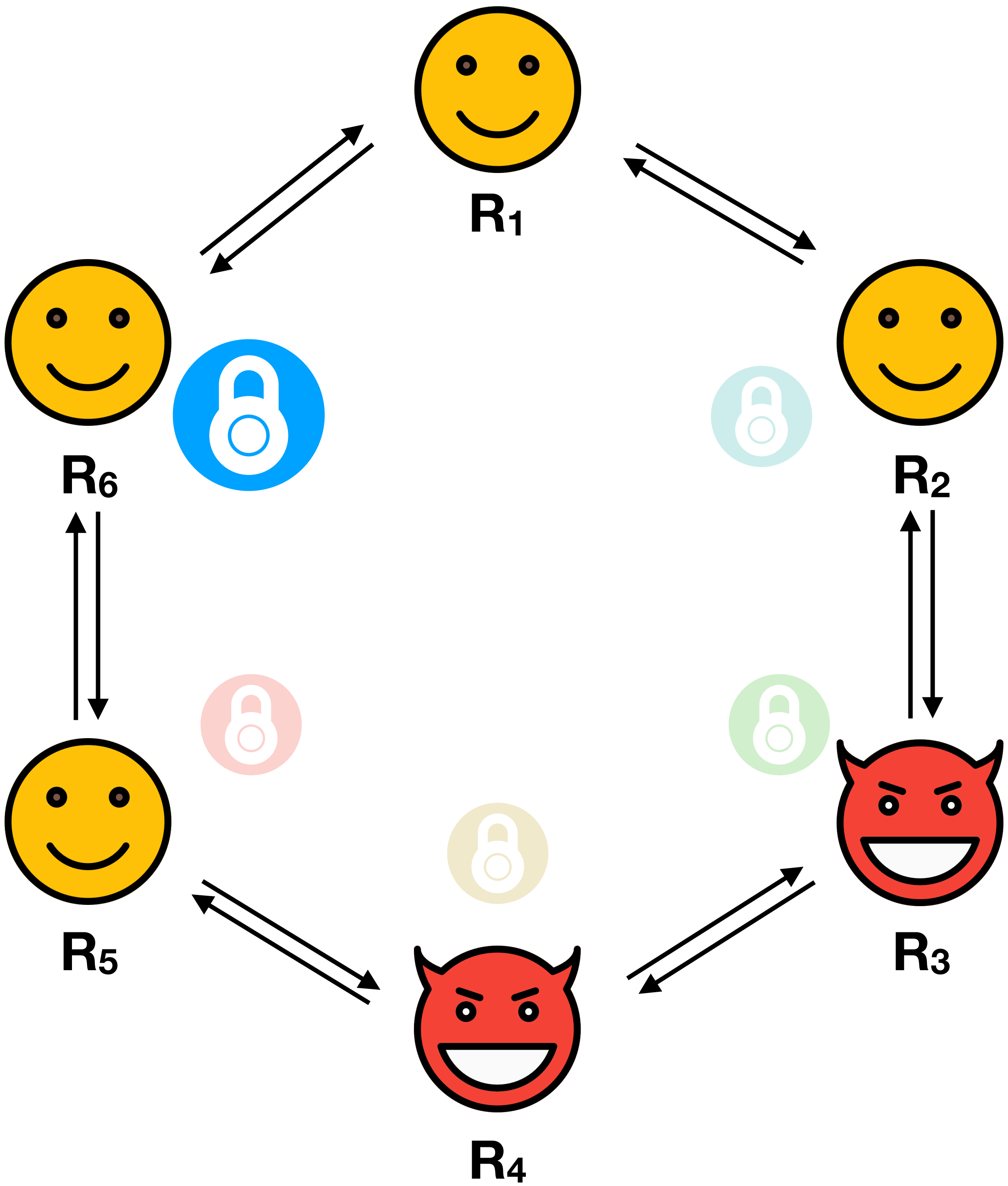
$$C_4C_3C_2C_1(|\psi\rangle \otimes |0^{2n}\rangle)$$

Public authentication test



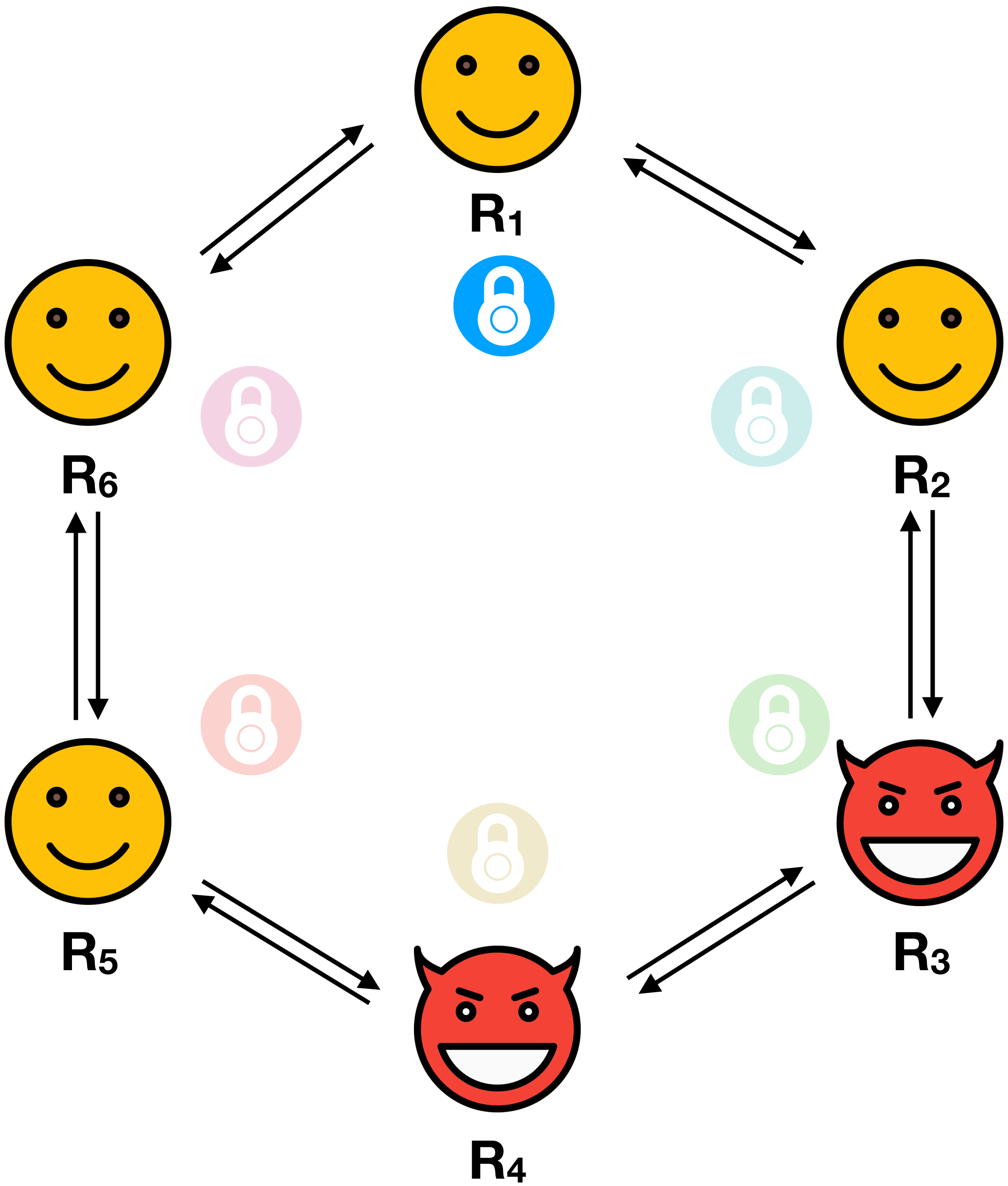
$$C_5C_4C_3C_2C_1(|\psi\rangle \otimes |0^{2n}\rangle)$$

Public authentication test



$$C_6C_5C_4C_3C_2C_1(|\psi\rangle \otimes |0^{2n}\rangle)$$

Public authentication test



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Public authentication test

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Public authentication test

$$\underbrace{C_6C_5C_4C_3C_2C_1}_{C}(|\psi\rangle \otimes |0^{2n}\rangle)$$

Public authentication test

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UNKNOWN TO ALL

Public authentication test

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PLAYER I CREATED THESE

UNKNOWN TO ALL

Public authentication test

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PLAYER I CREATED THESE

UNKNOWN TO ALL

Using classical MPC: 

Public authentication test

$$\underbrace{C_6 C_5 C_4 C_3 C_2 C_1}_{C}(|\psi\rangle \otimes |0^{2n}\rangle)$$

PLAYER I CREATED THESE

UNKNOWN TO ALL

Using classical MPC: 

- Select $g \in_R GL(2n, \mathbb{F}_2)$. Note: $g(y) = 0^{2n}$ iff $y = 0^{2n}$

  Lemma: apply random g and measure n traps

  $\approx$ measure $2n$ traps

Public authentication test

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PLAYER I CREATED THESE

UNKNOWN TO ALL

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Public authentication test

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Public authentication test

$$\underbrace{C_6C_5C_4C_3C_2C_1}_{C}(|\psi\rangle \otimes |0^{2n}\rangle)$$

UNKNOWN TO ALL (C)

PLAYER 1 CREATED THESE ($|0^{2n}\rangle$)

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Result: authenticated state $C'(|\psi\rangle \otimes |0^n\rangle)$

Public authentication test

Public authentication test

One player **performs** the test: applies Clifford, measures, …

Public authentication test

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All players **verify** the test through classical MPC 

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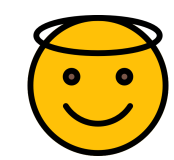
All players **verify** the test through classical MPC 

The test can be used:

- to test encodings (as in previous slide);

Public authentication test

One player **performs** the test: applies Clifford, measures, …

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The test can be used:

- to test encodings (as in previous slide);
- to test whether a computation step was executed honestly

Introduction
Authentication
Computation
Magic-state generation
Summary

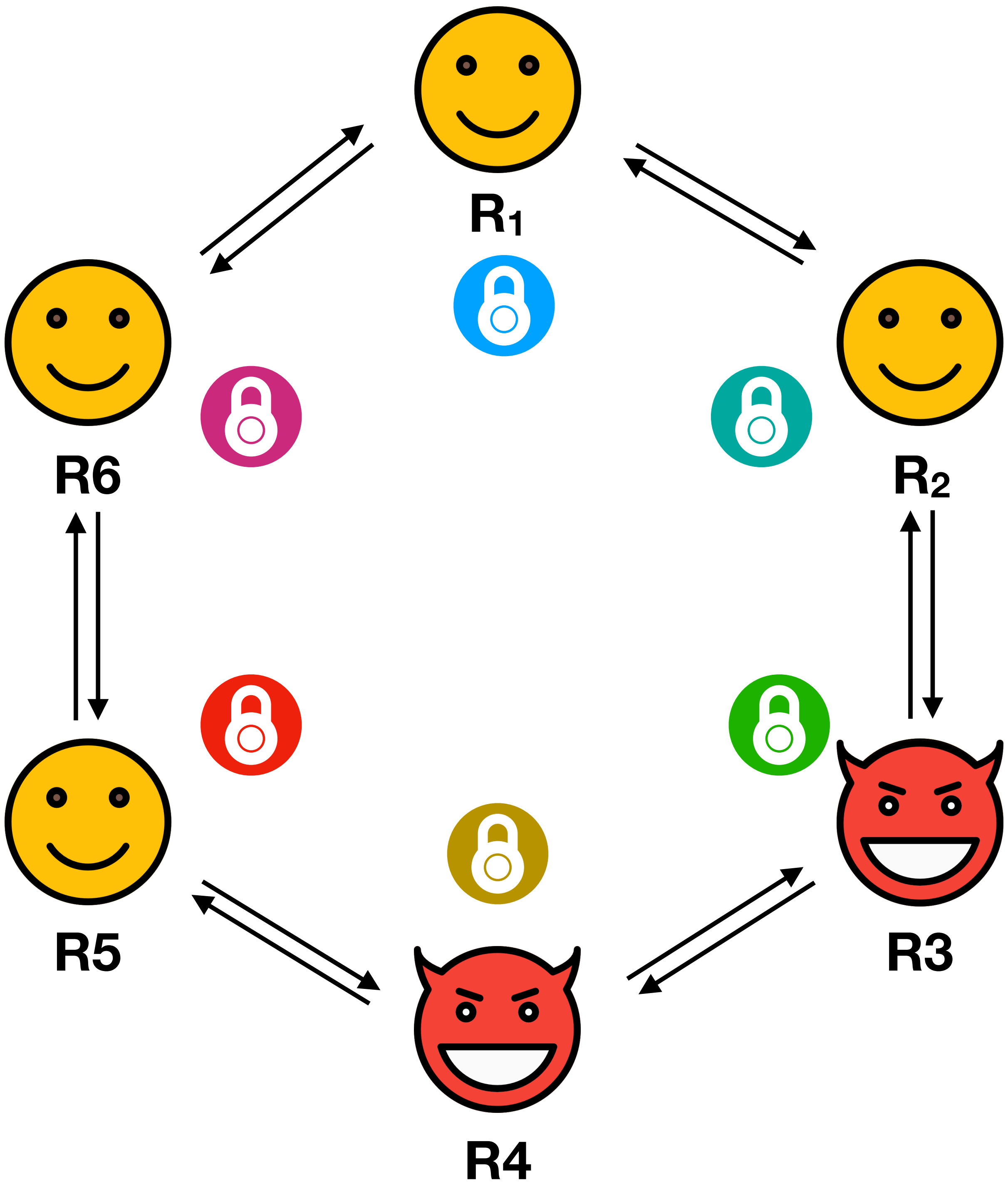
Computation

$|\psi\rangle$ $G|\psi\rangle$

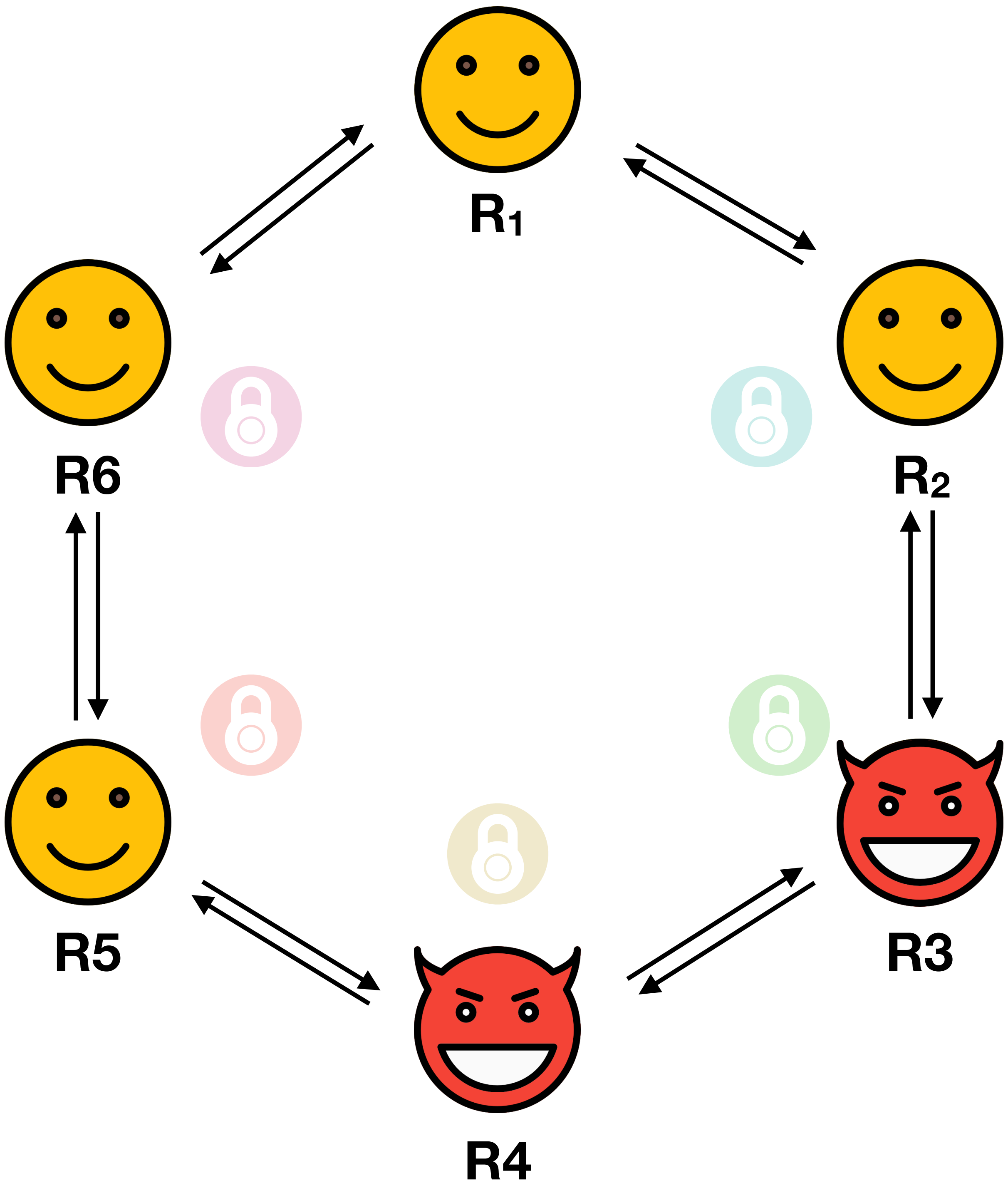
Protocols ($C(|\psi\rangle \otimes |0^n\rangle) \mapsto C'(G|\psi\rangle \otimes |0^n\rangle)$) for these G :

- 1-qubit Cliffords
- CNOT (2-qubit Clifford)
- T (non-Clifford)
- Computational-basis measurement

Single-qubit Cliffords

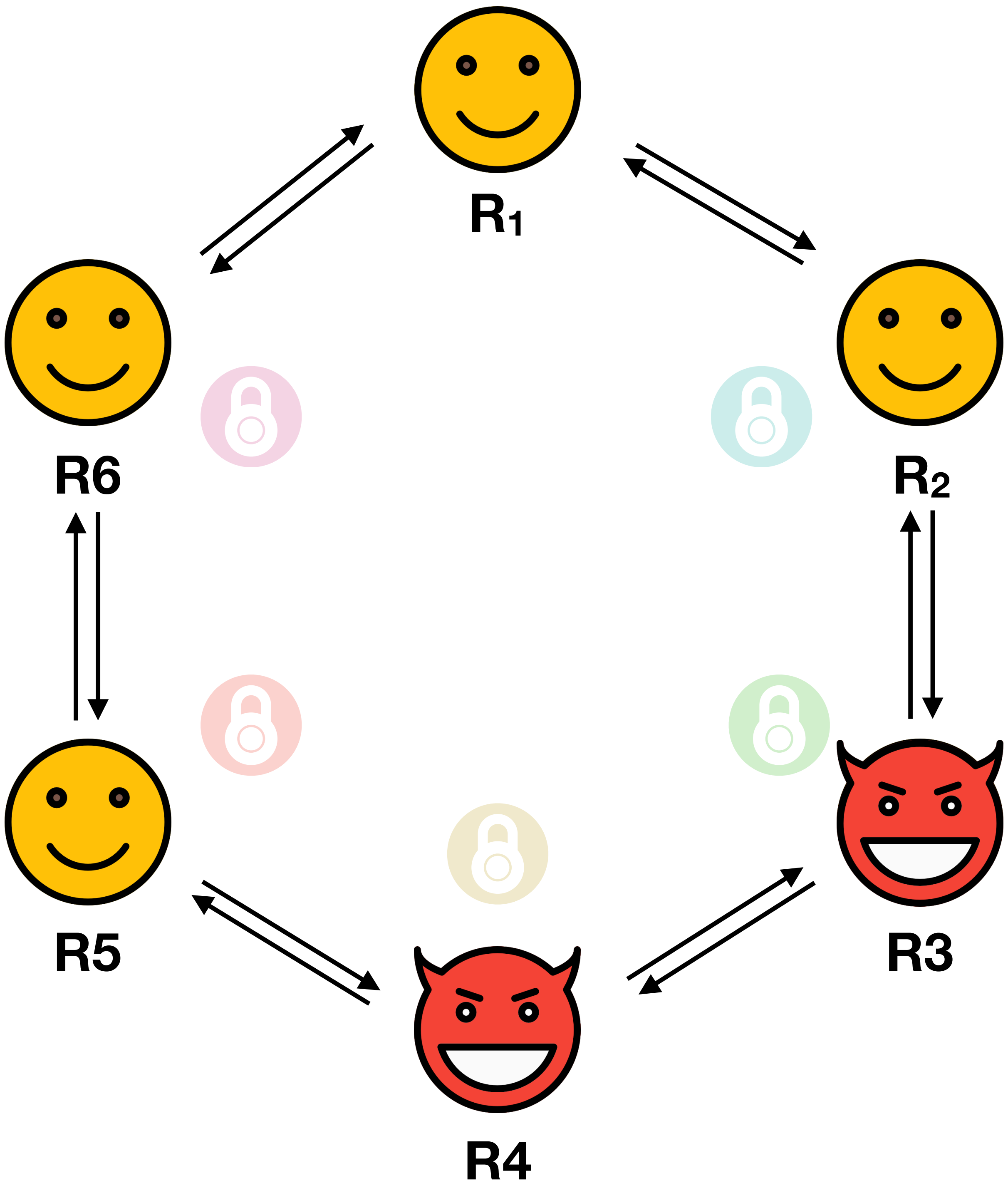


Single-qubit Cliffords



$$= C(|\psi\rangle \otimes |0\rangle^{\otimes n})$$

Single-qubit Cliffords

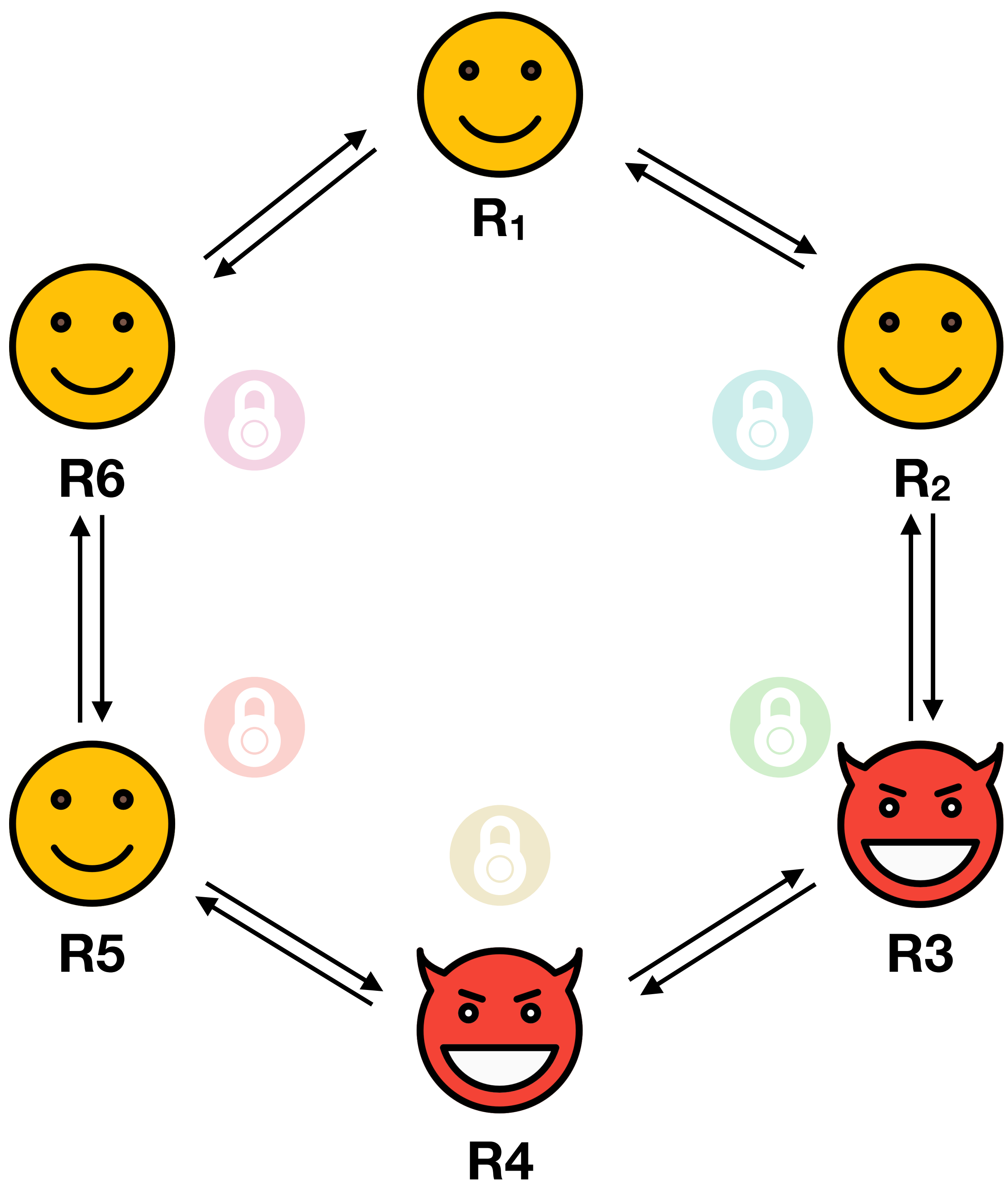


 $= C(|\psi\rangle \otimes |0\rangle^{\otimes n})$

Using classical MPC: update classical key 

$$C \mapsto C' := C(G^\dagger \otimes I^{\otimes n})$$

Single-qubit Cliffords



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Then will decode to

$$(C')^\dagger C(|\psi\rangle \otimes |0\rangle^{\otimes n})$$

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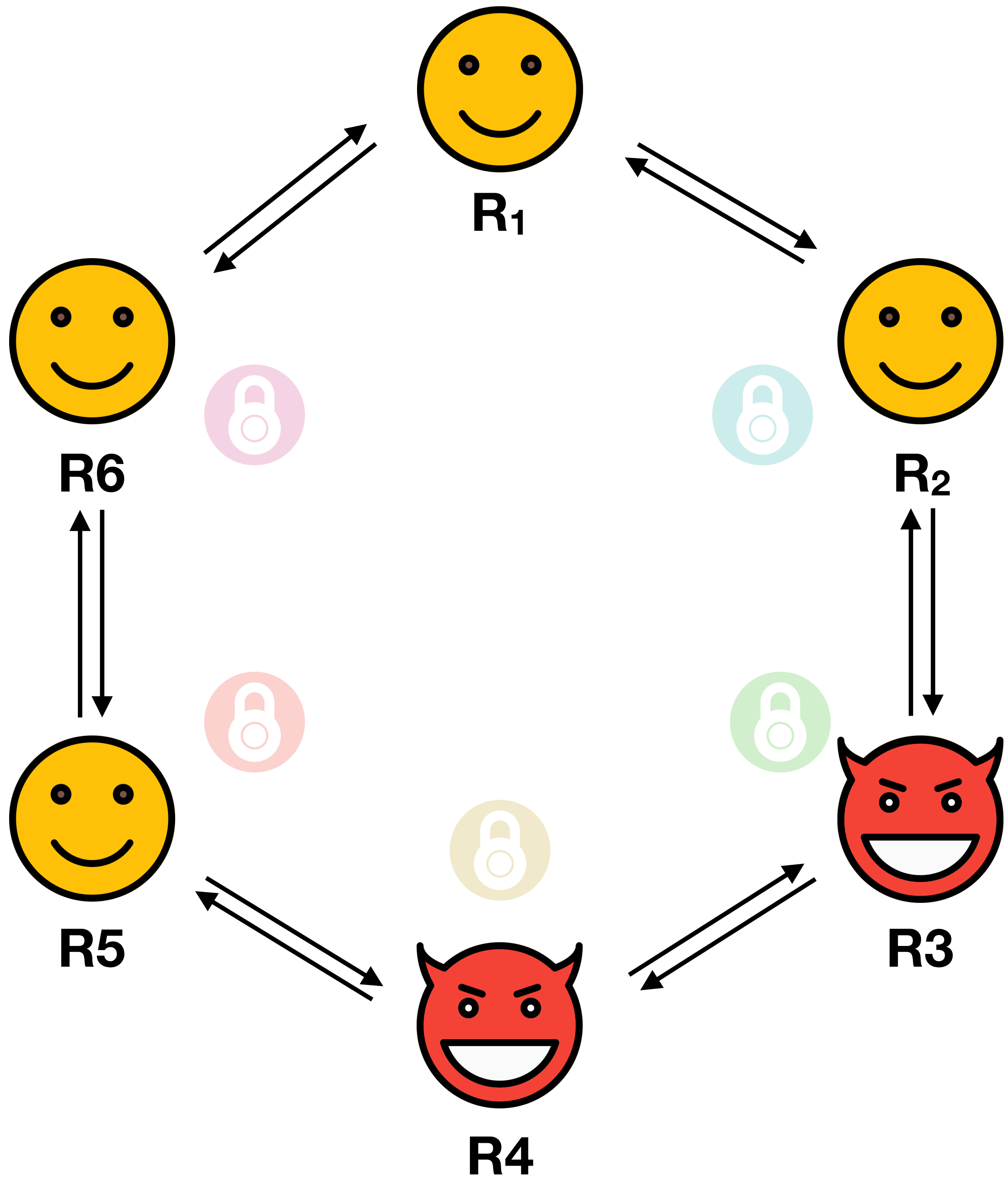

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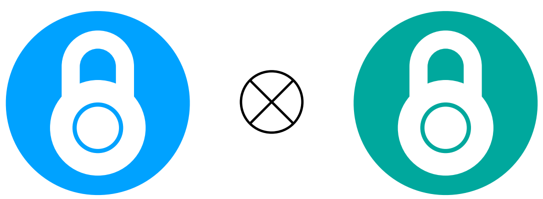
$$= G|\psi\rangle \otimes |0\rangle^{\otimes n}$$

CNOT

CNOT



CNOT

 $\otimes$  $= (C_1 \otimes C_2)(|\psi_1\rangle \otimes |0^n\rangle \otimes |\psi_2\rangle \otimes |0^n\rangle)$

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Same strategy does not work:
$(C_1 \otimes C_2)(CNOT^\dagger \otimes I^{\otimes 2n})$ is not in product form.

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Instead:

CNOT

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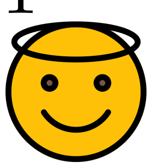
Instead:

- Player 1 applies $(C_1' \otimes C_2')CNOT(C_1^\dagger \otimes C_2^\dagger)$ for freshly random C_1', C_2' .



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Instead:

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- Player 1 executes public authentication test. 😇

Non-Clifford gate $T = \begin{bmatrix} 1 & 0 \\ 0 & e^{\pi i/4} \end{bmatrix}$

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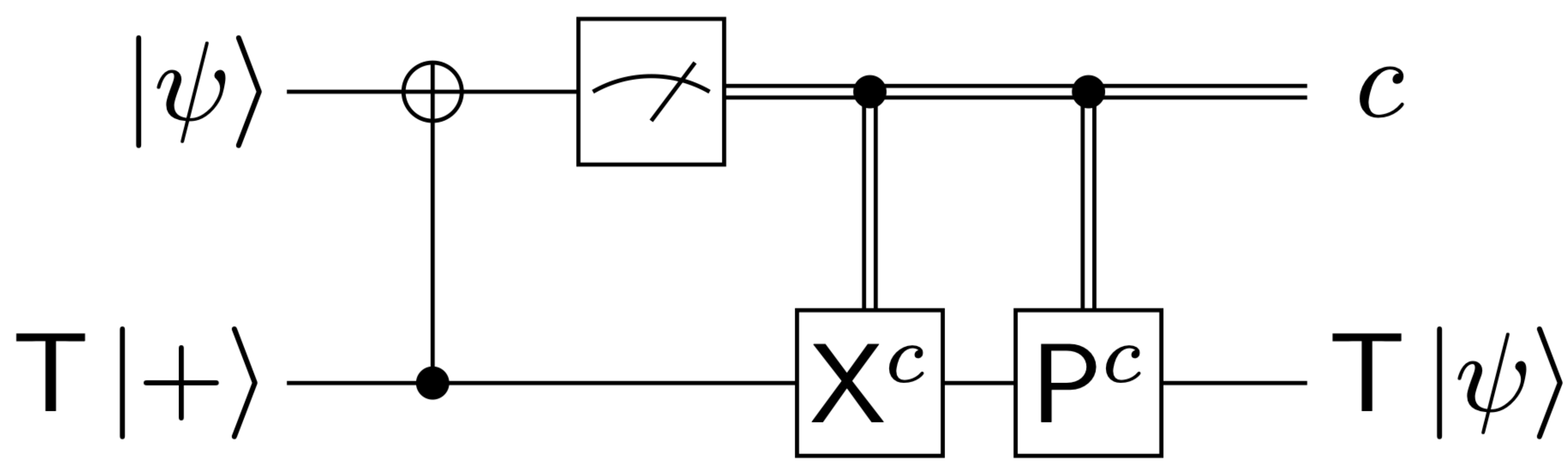
Magic-state computation:



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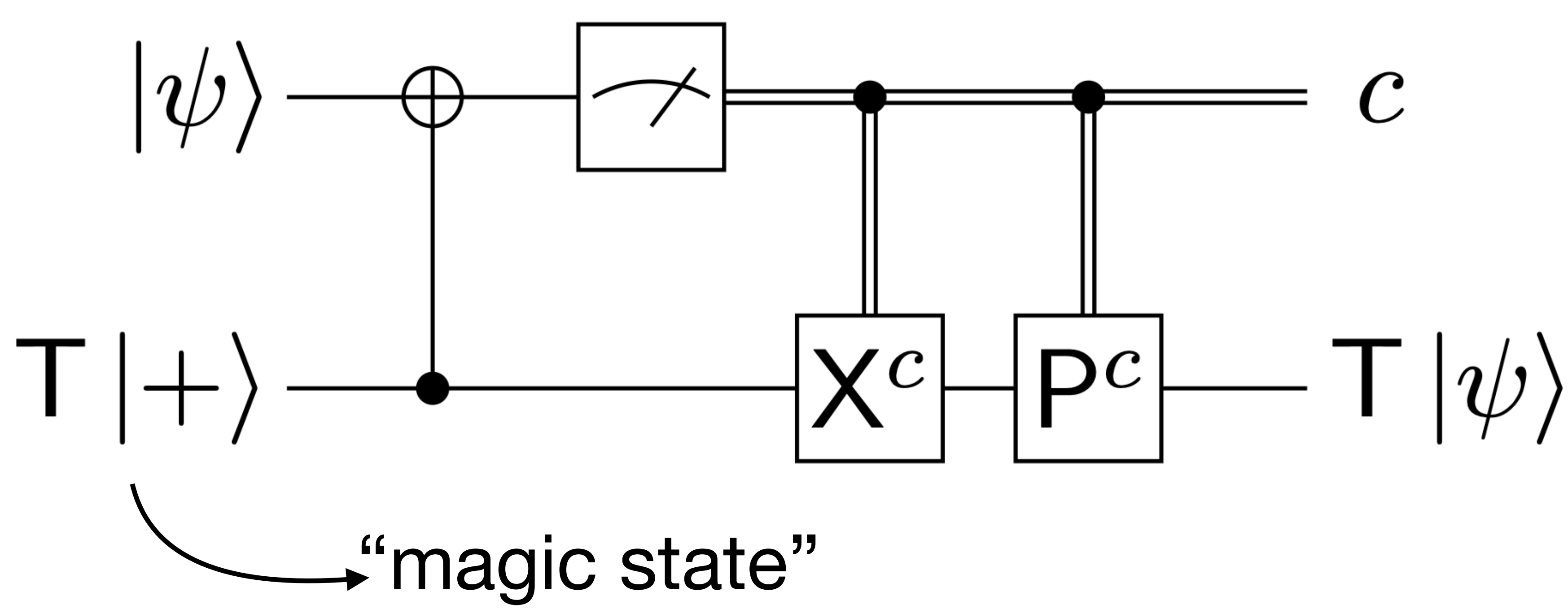

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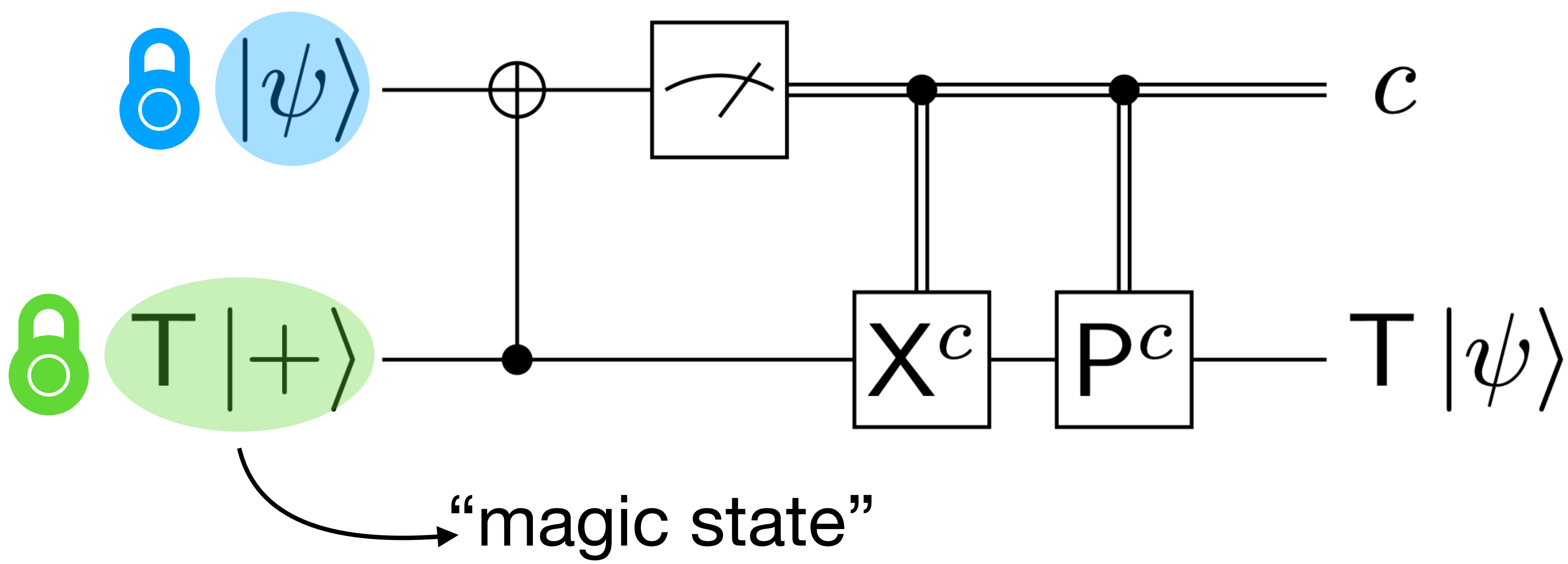

$$C_1(|\psi\rangle \otimes |0^n\rangle) \otimes C_2(T|+\rangle \otimes |0^n\rangle)$$

Non-Clifford gate $T = \begin{bmatrix} 1 & 0 \\ 0 & e^{\pi i/4} \end{bmatrix}$

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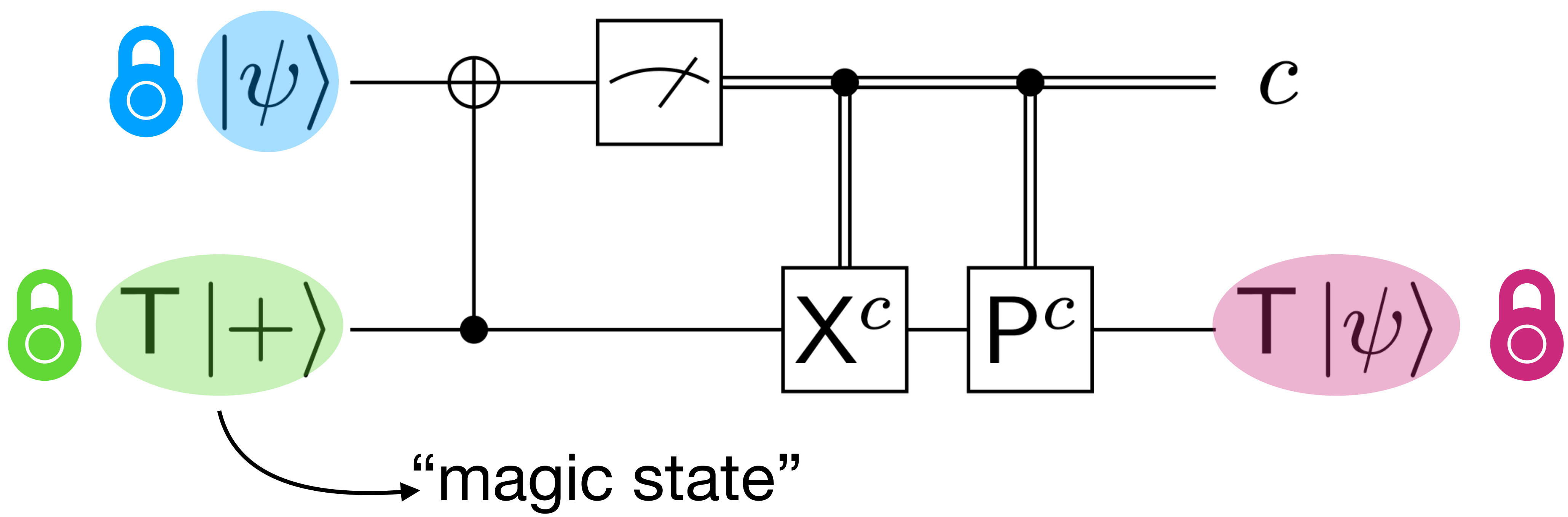
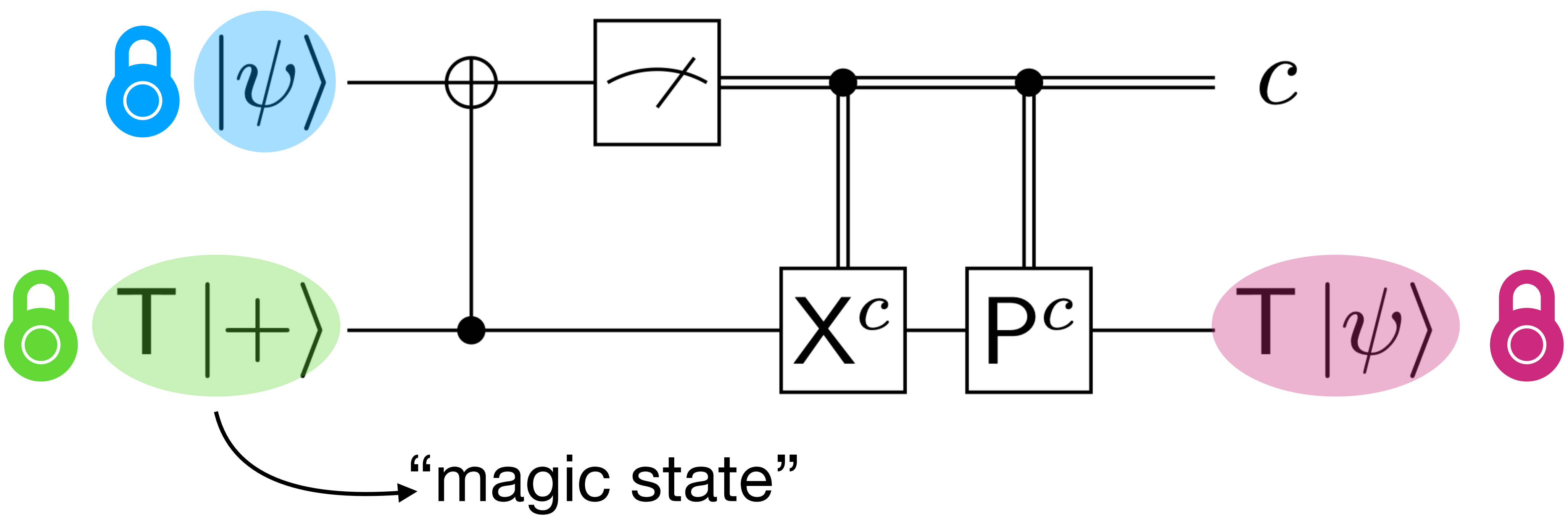


$$C_1(|\psi\rangle \otimes |0^n\rangle) \otimes C_2(T|+\rangle \otimes |0^n\rangle) \mapsto C_3(T|\psi\rangle \otimes |0^n\rangle)$$

Non-Clifford gate $T = \begin{bmatrix} 1 & 0 \\ 0 & e^{\pi i/4} \end{bmatrix}$

Magic-state computation:



$$C_1(|\psi\rangle \otimes |0^n\rangle) \otimes C_2(T|+\rangle \otimes |0^n\rangle) \mapsto C_3(T|\psi\rangle \otimes |0^n\rangle)$$

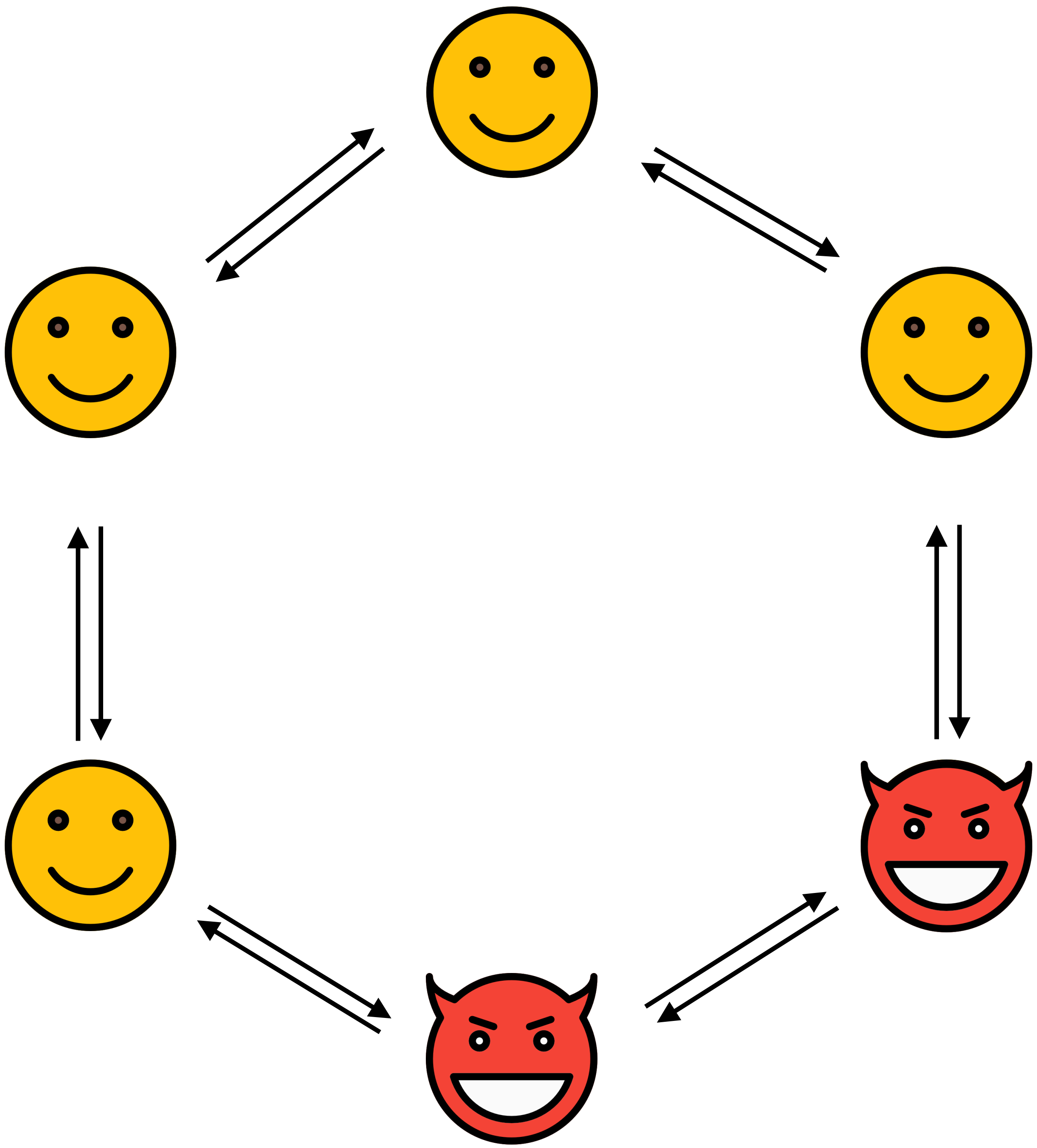
Nobody can be trusted to create encoded magic states!

Introduction
Authentication
Computation

Magic-state generation

Summary

Magic-state generation



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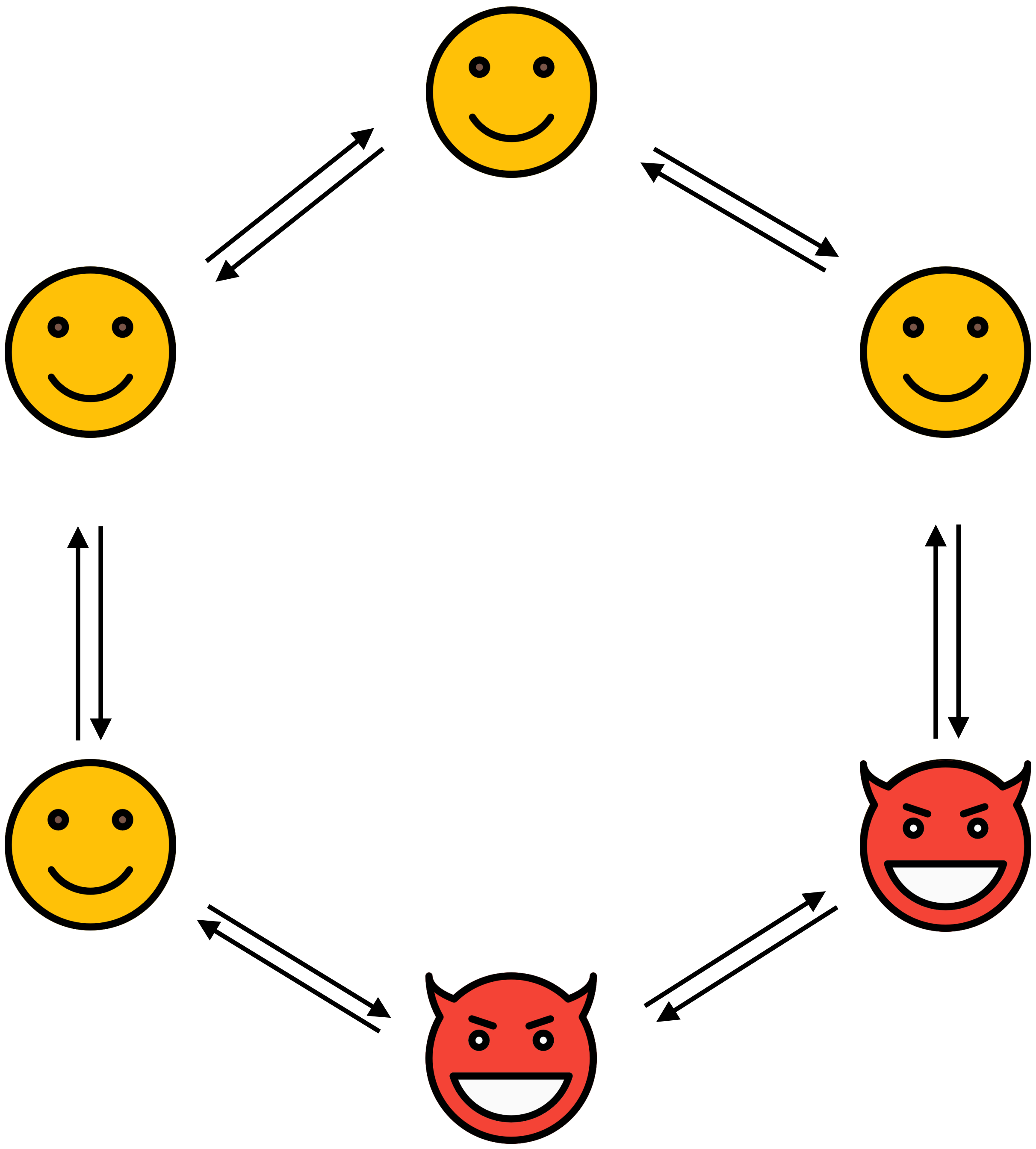

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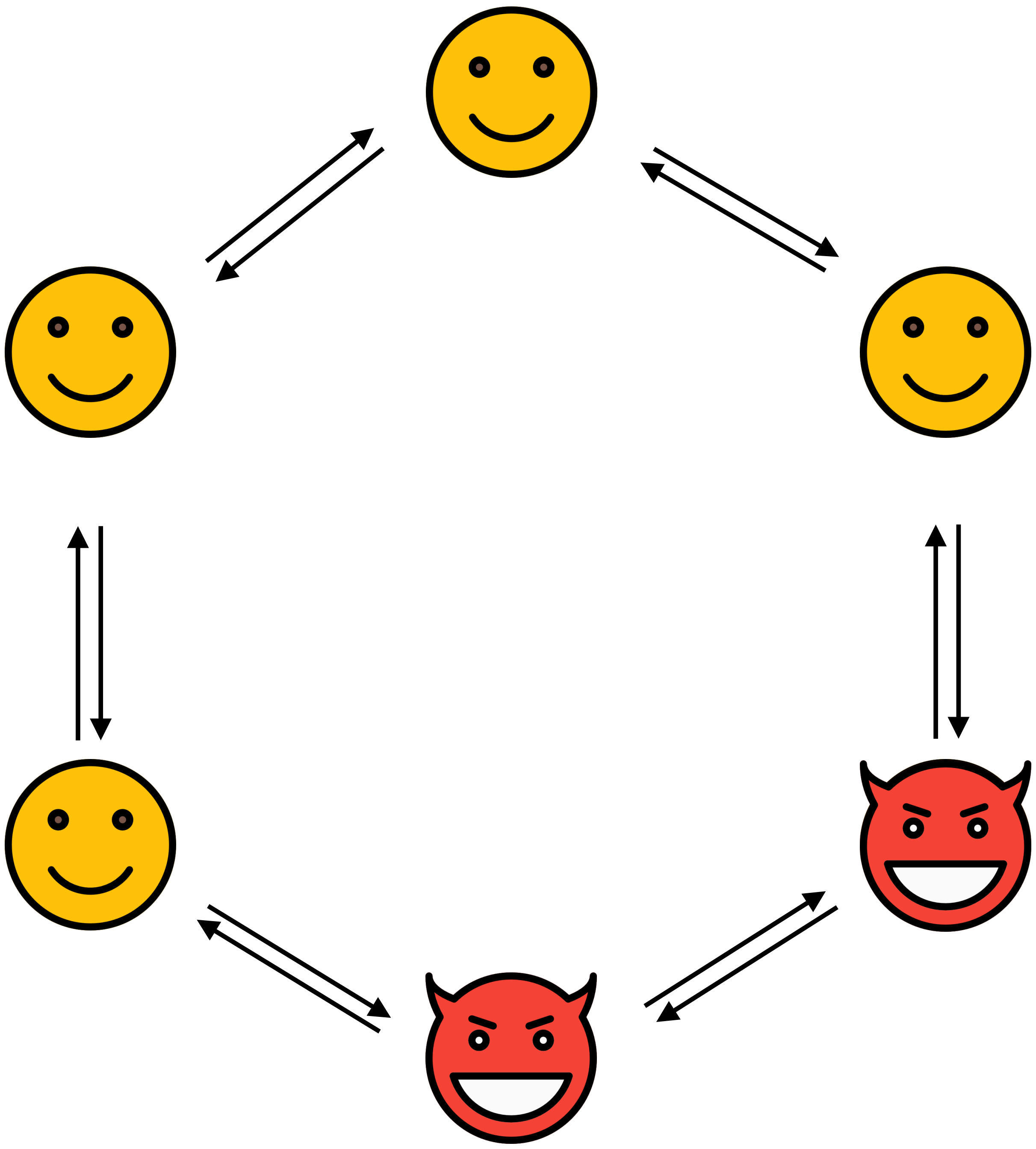

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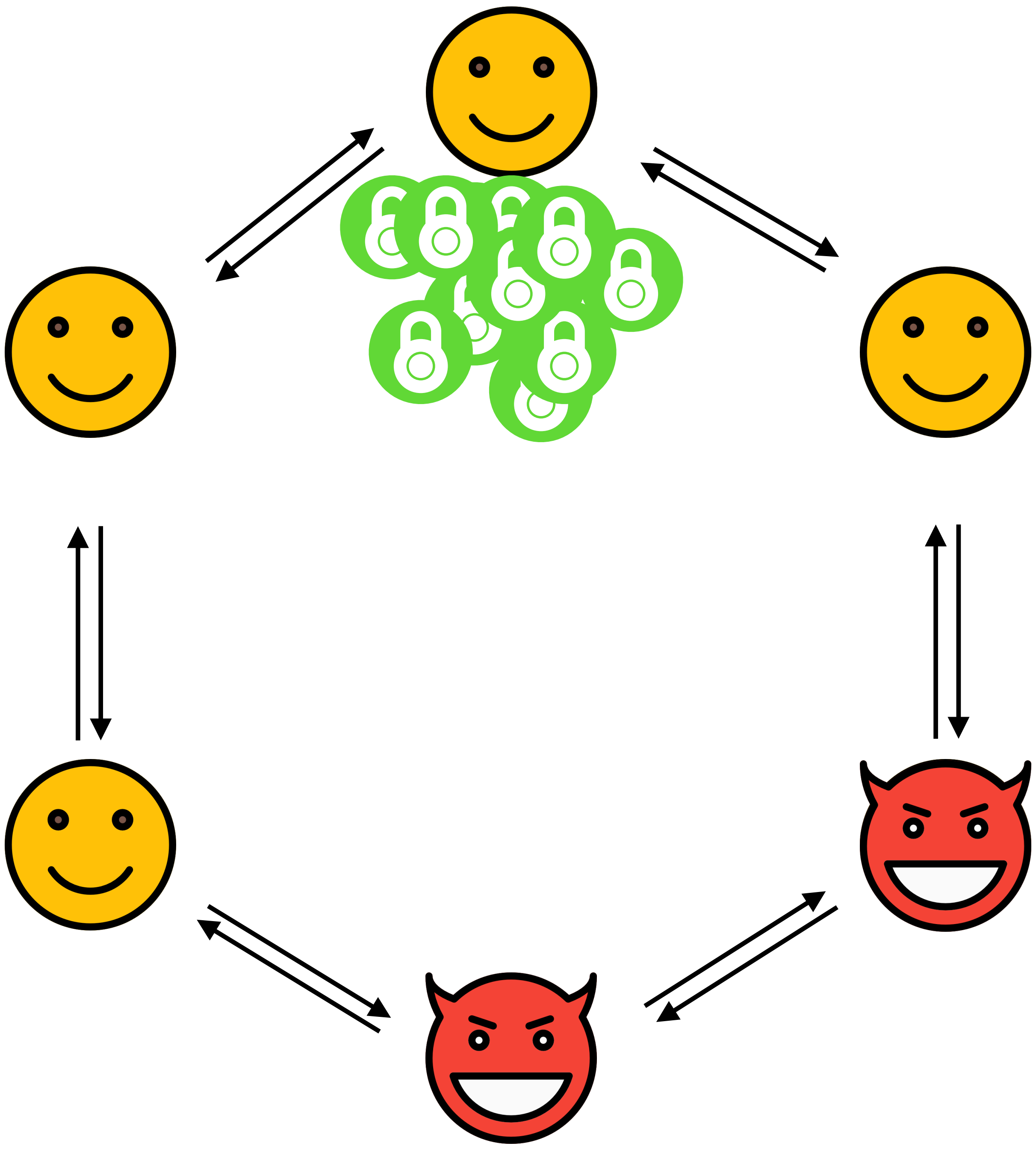

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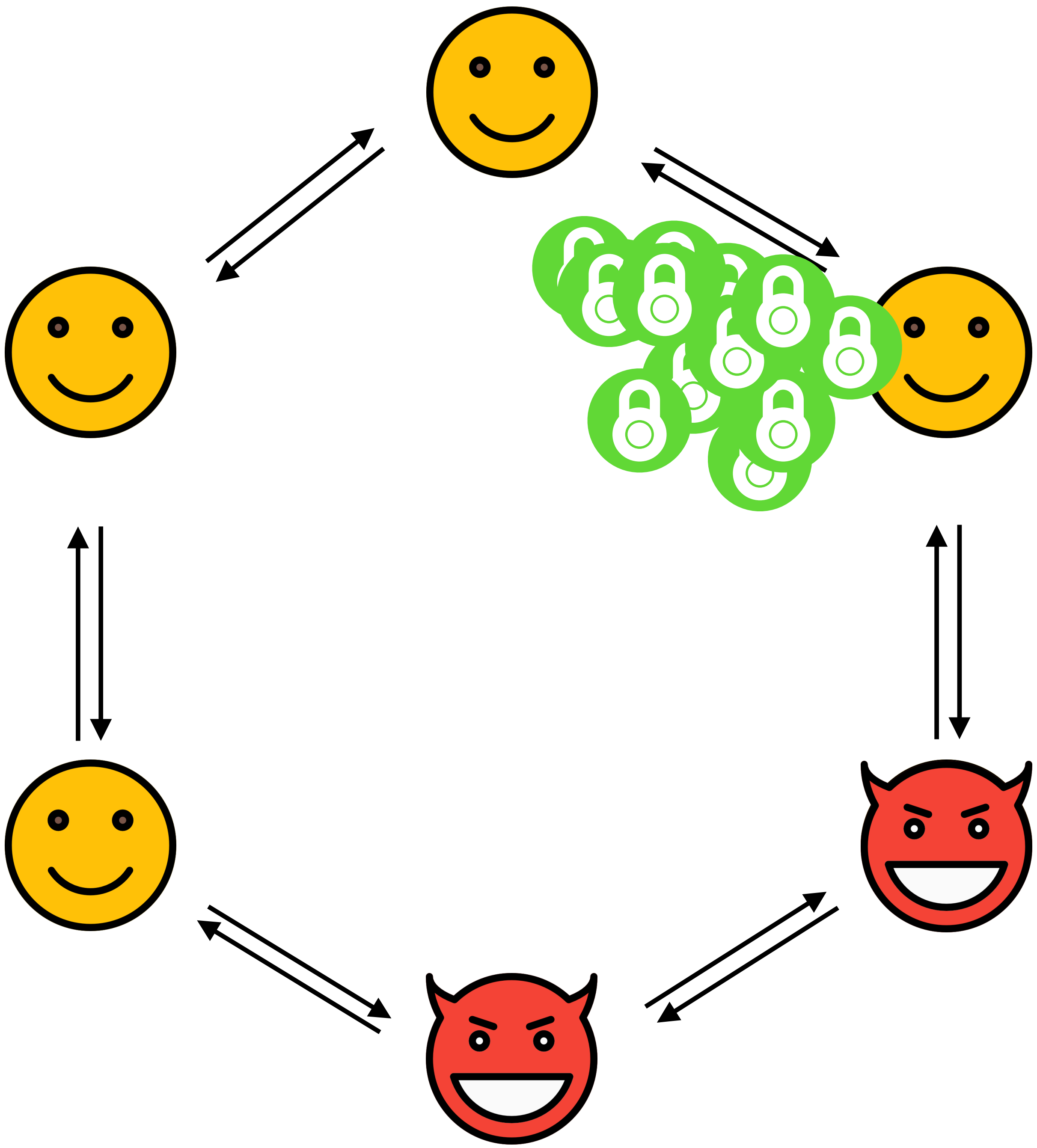

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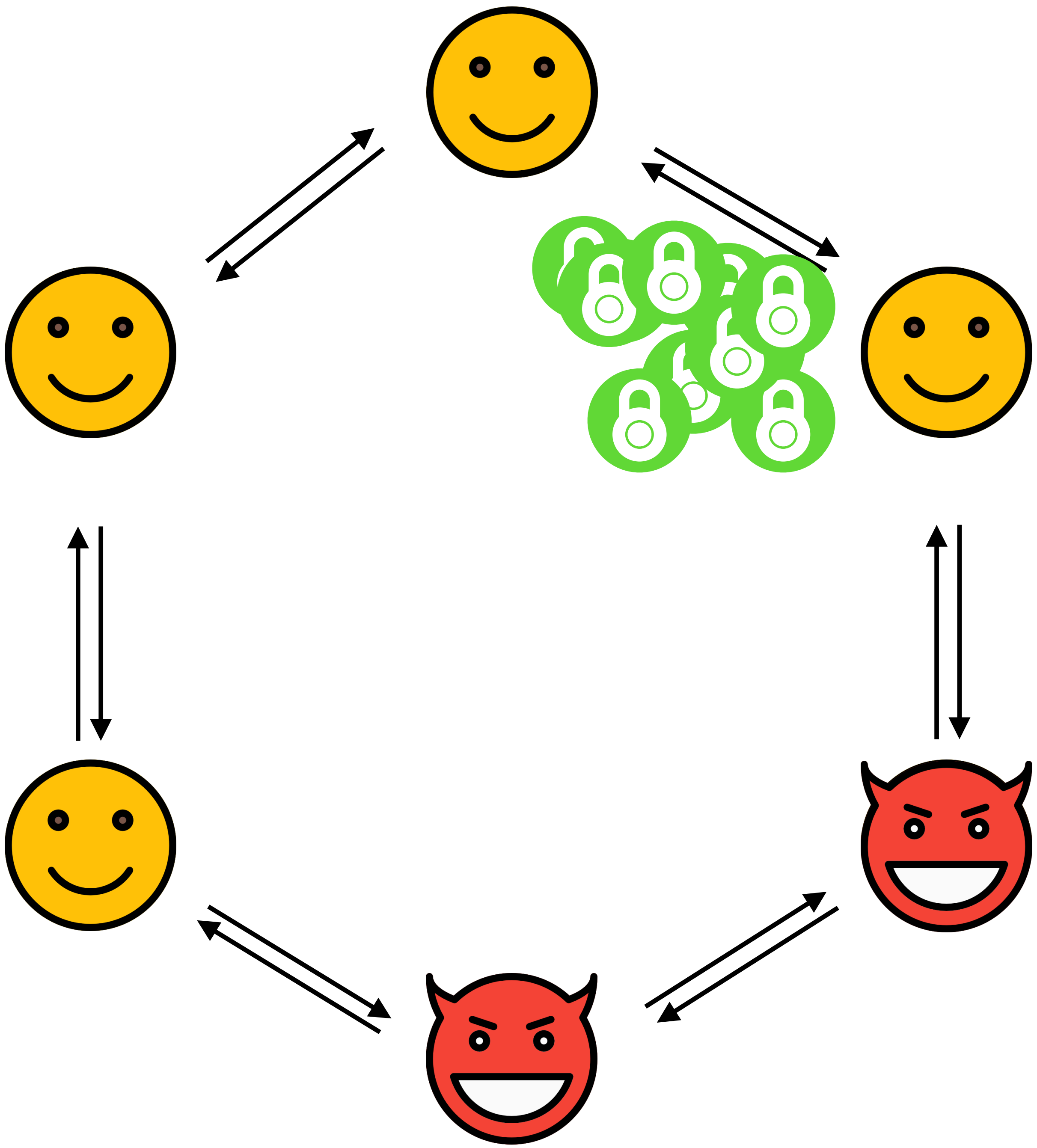

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   - ✦ every player tests n random states 

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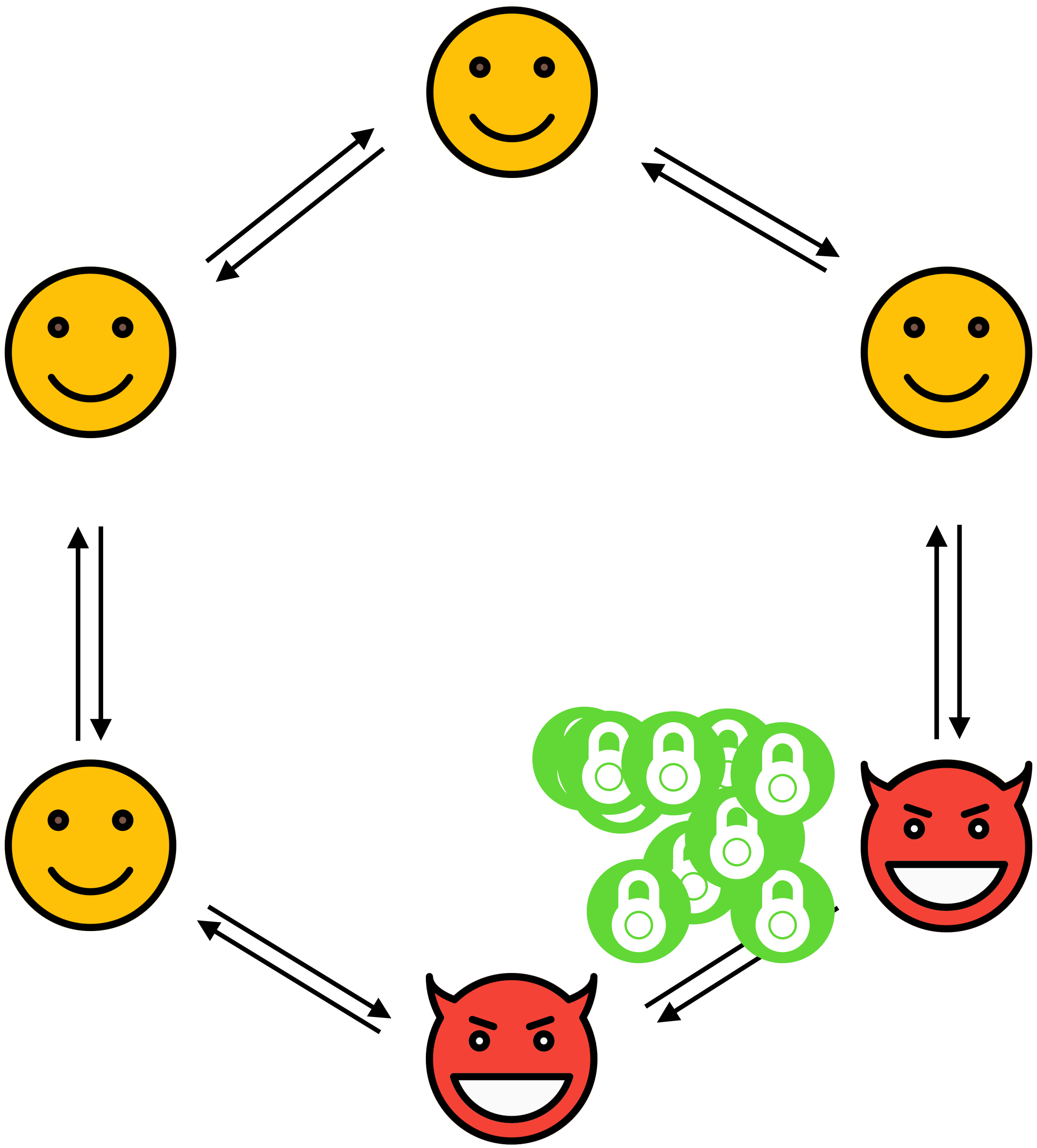

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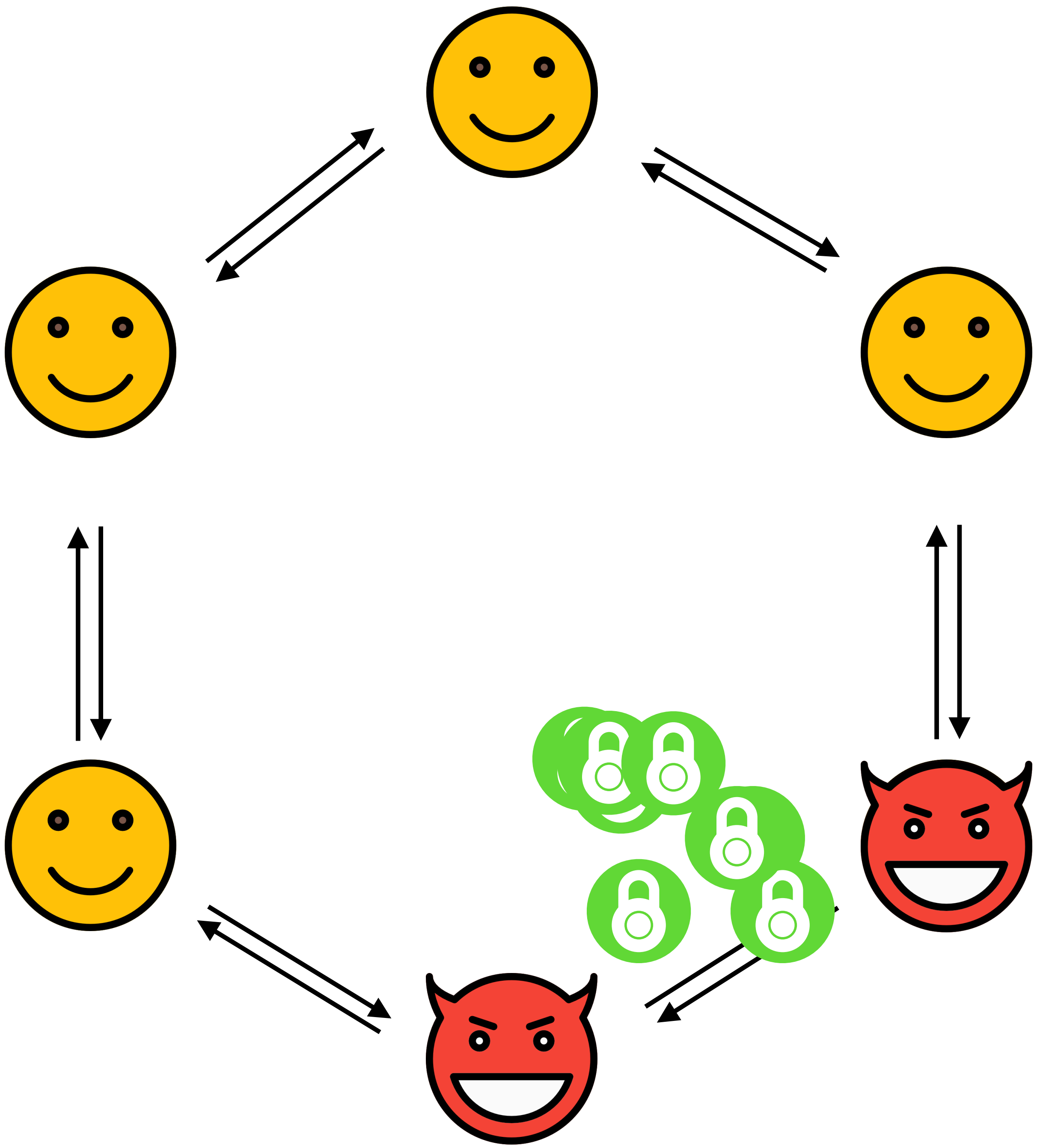

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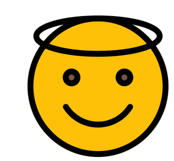
   ✦ every player tests n random states 

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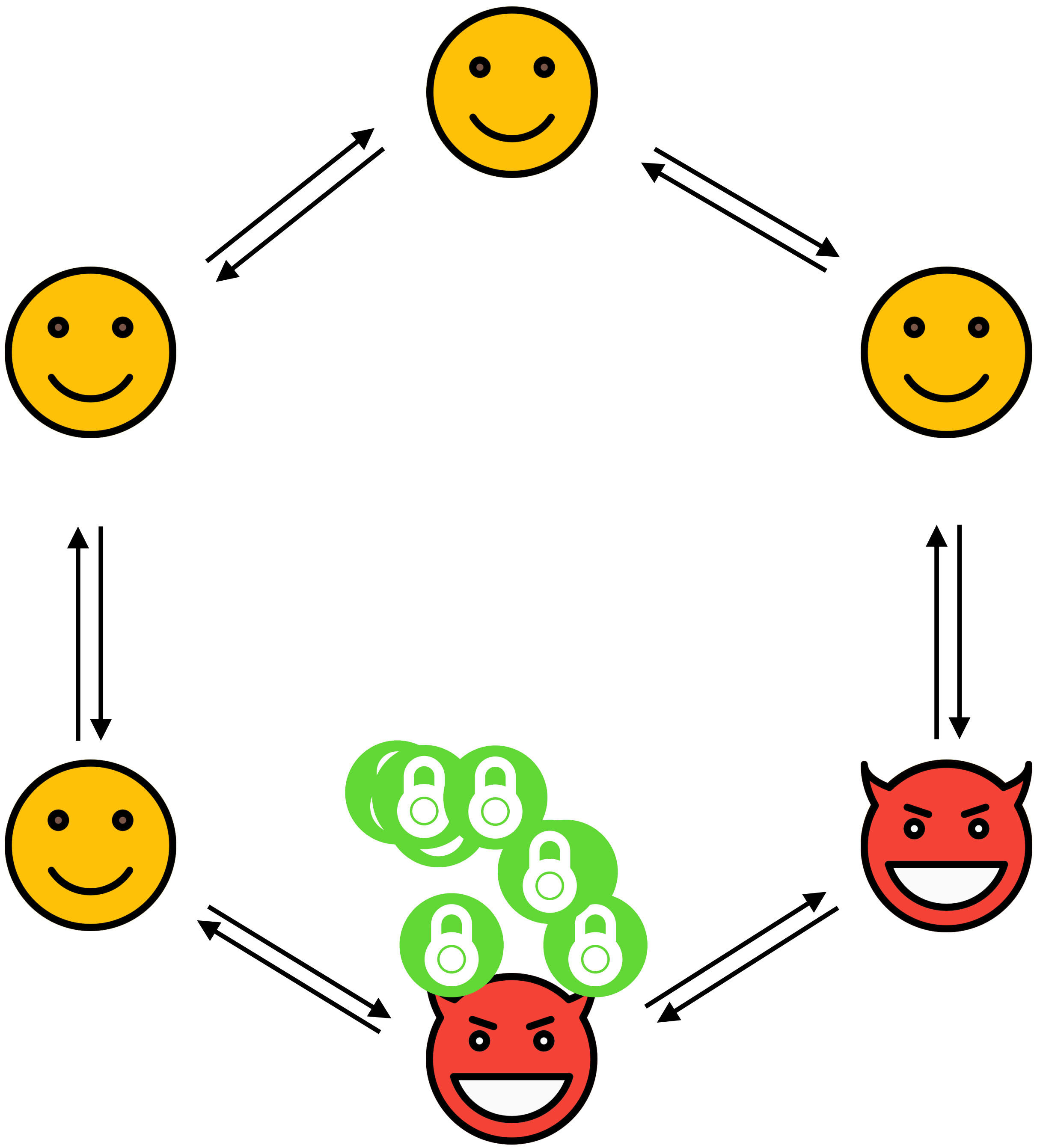

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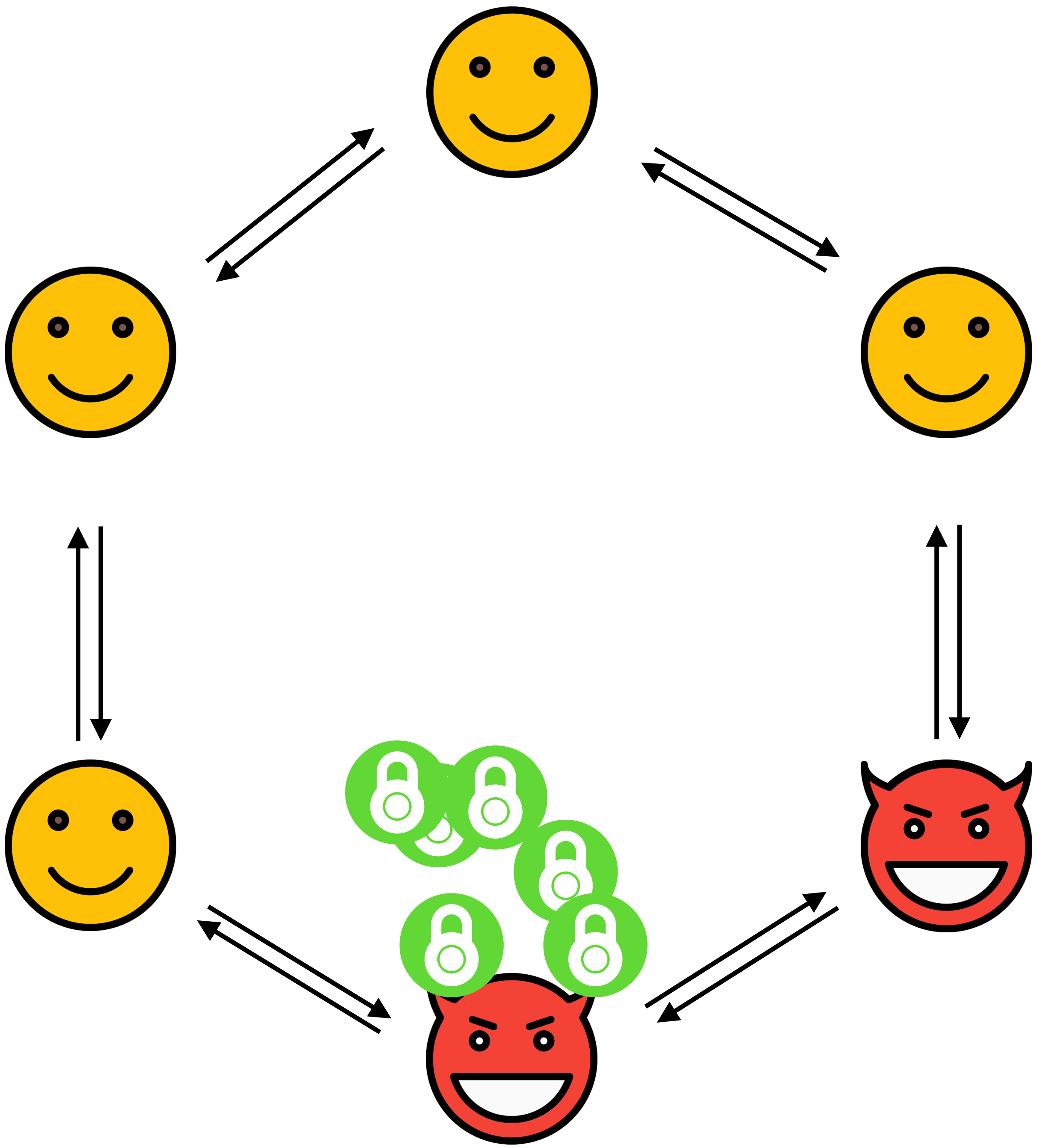

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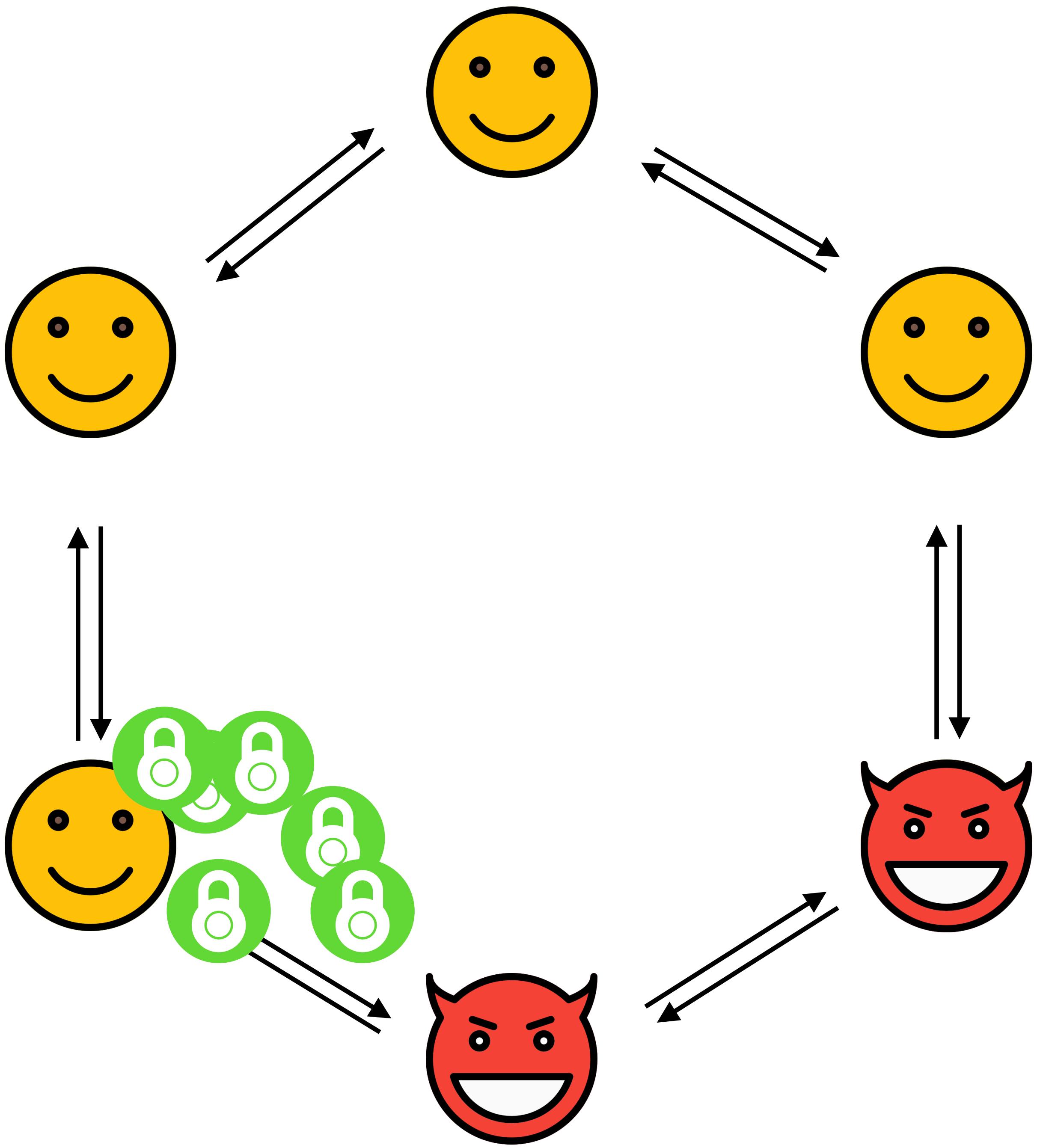

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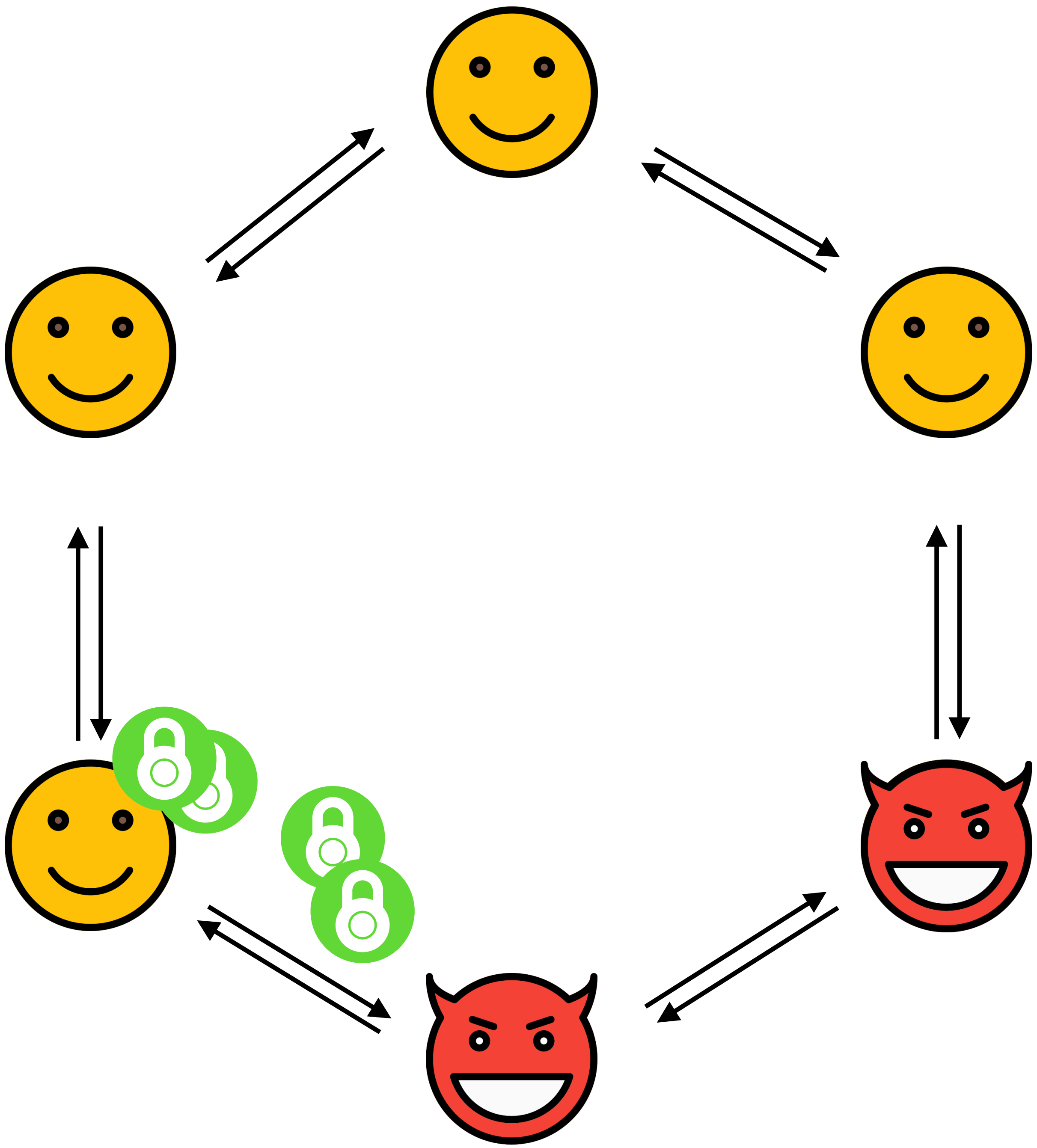

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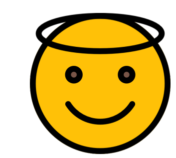
   ✦ every player tests n random states 

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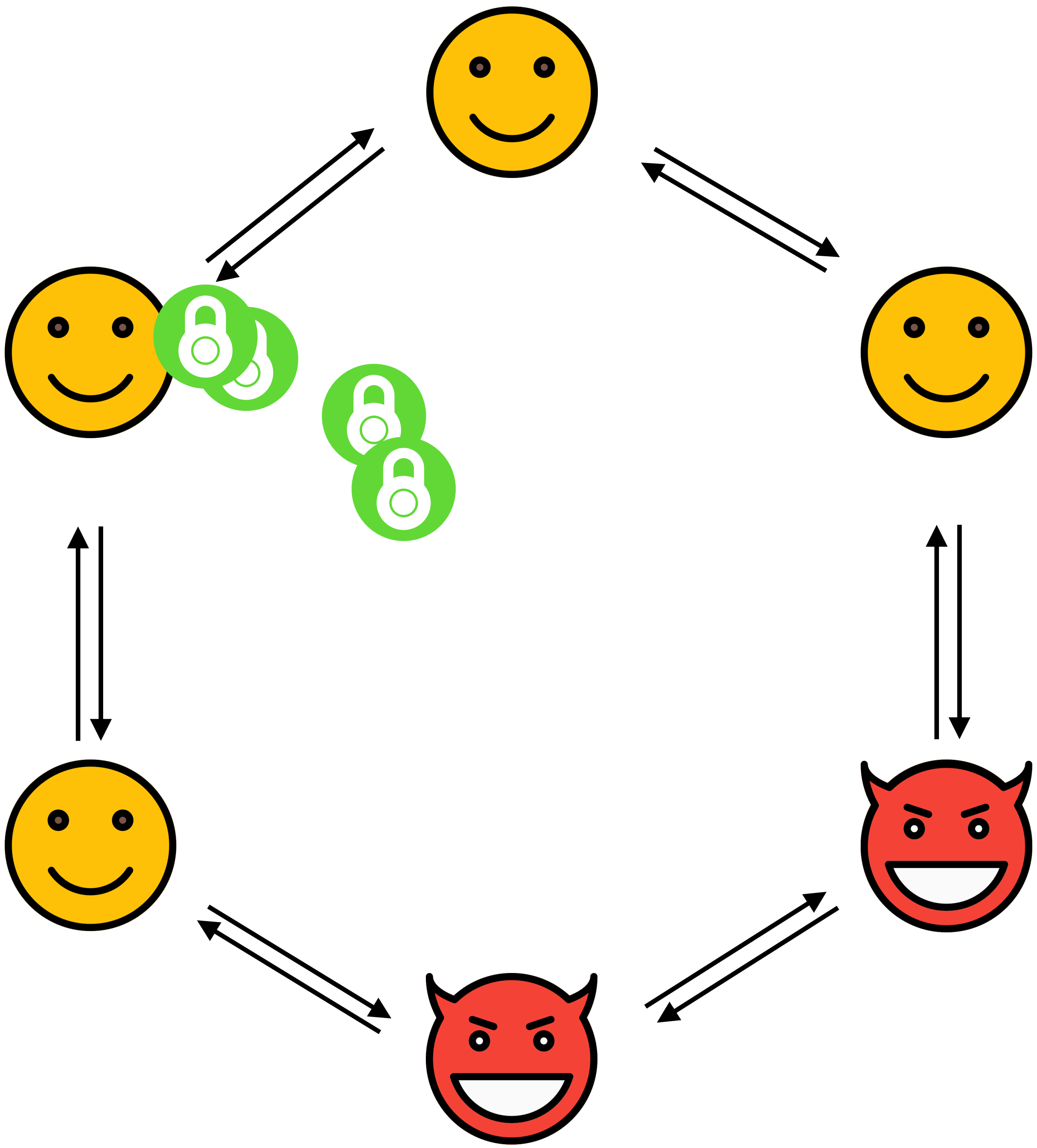

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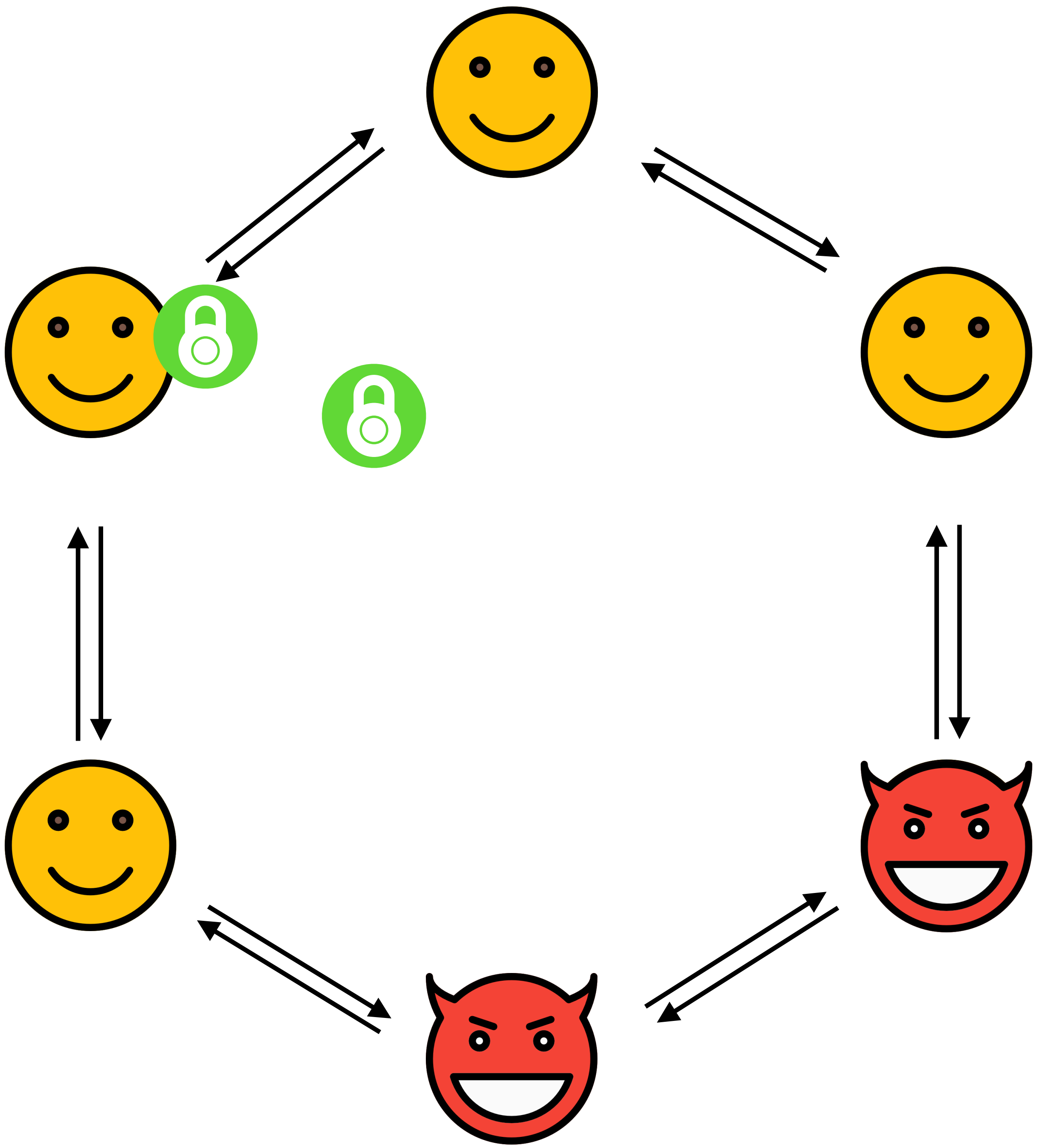

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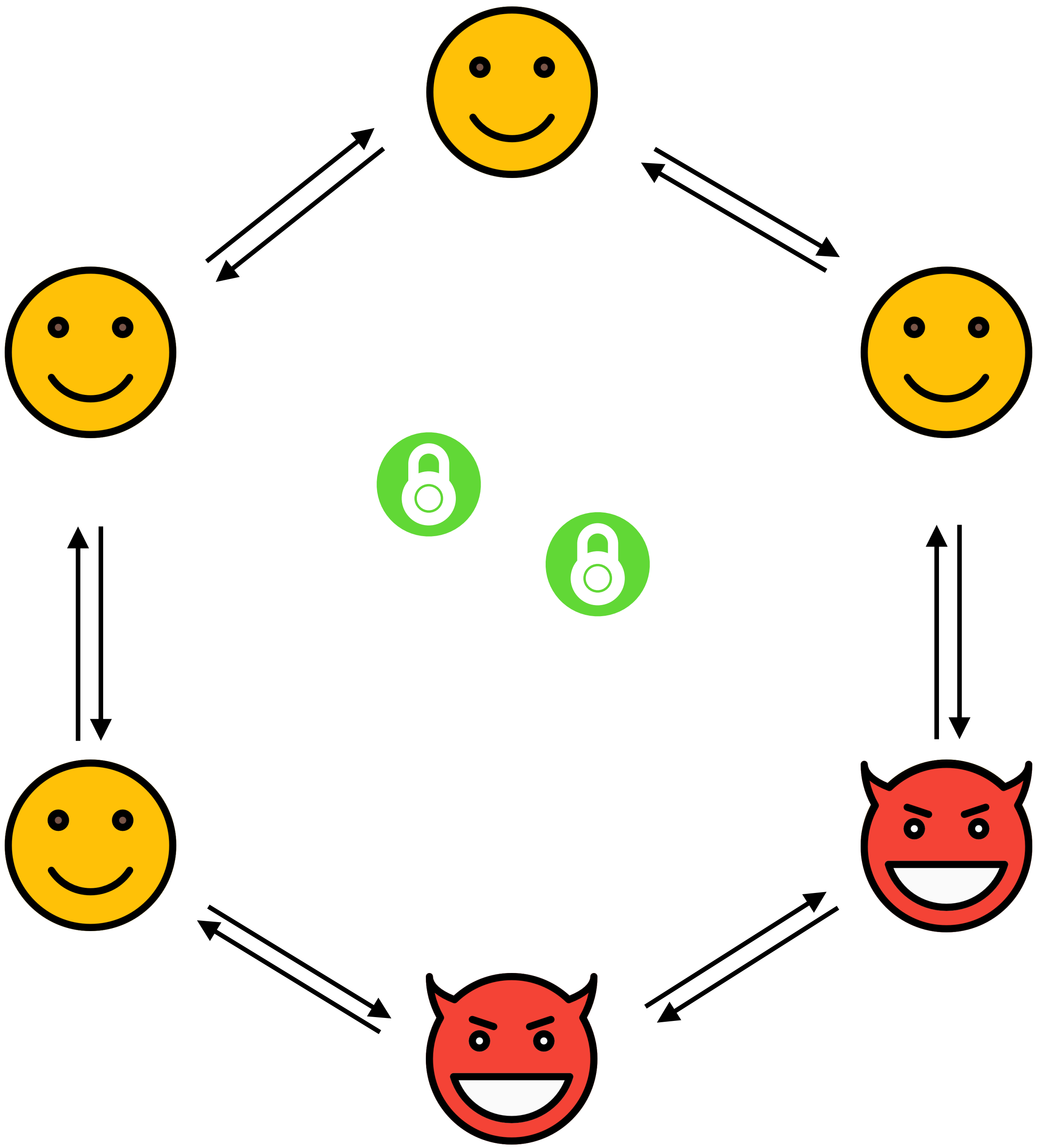

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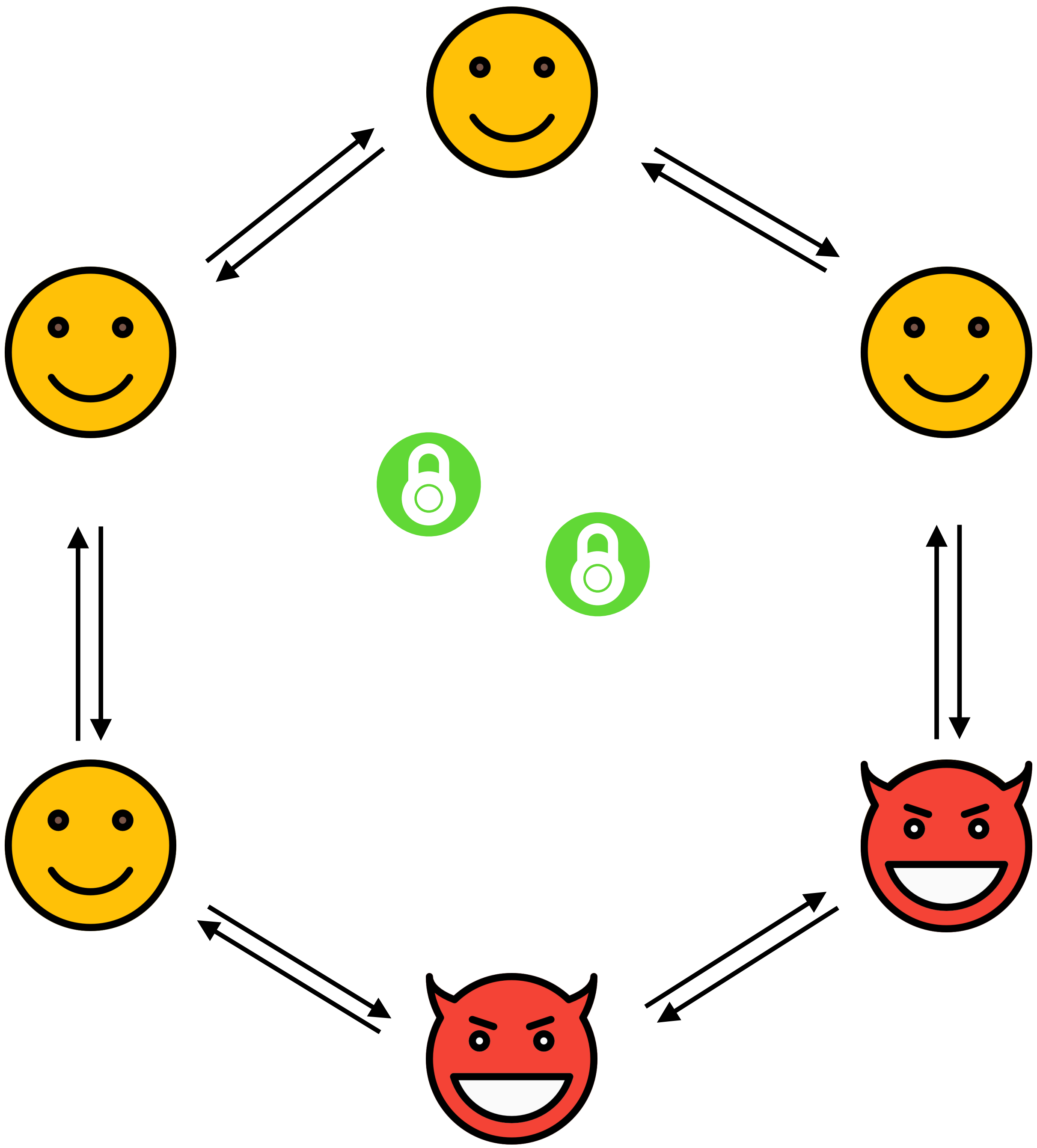

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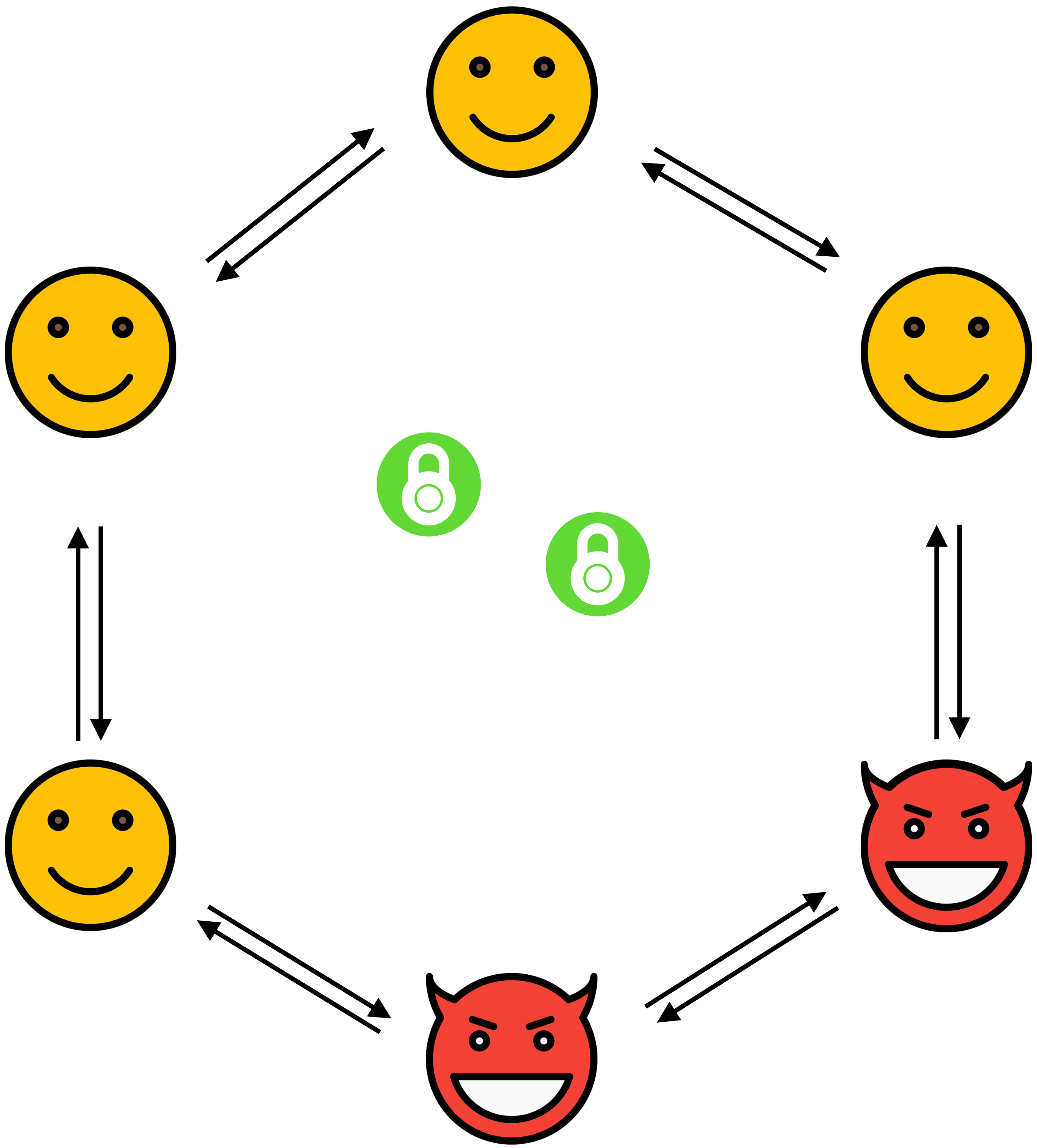

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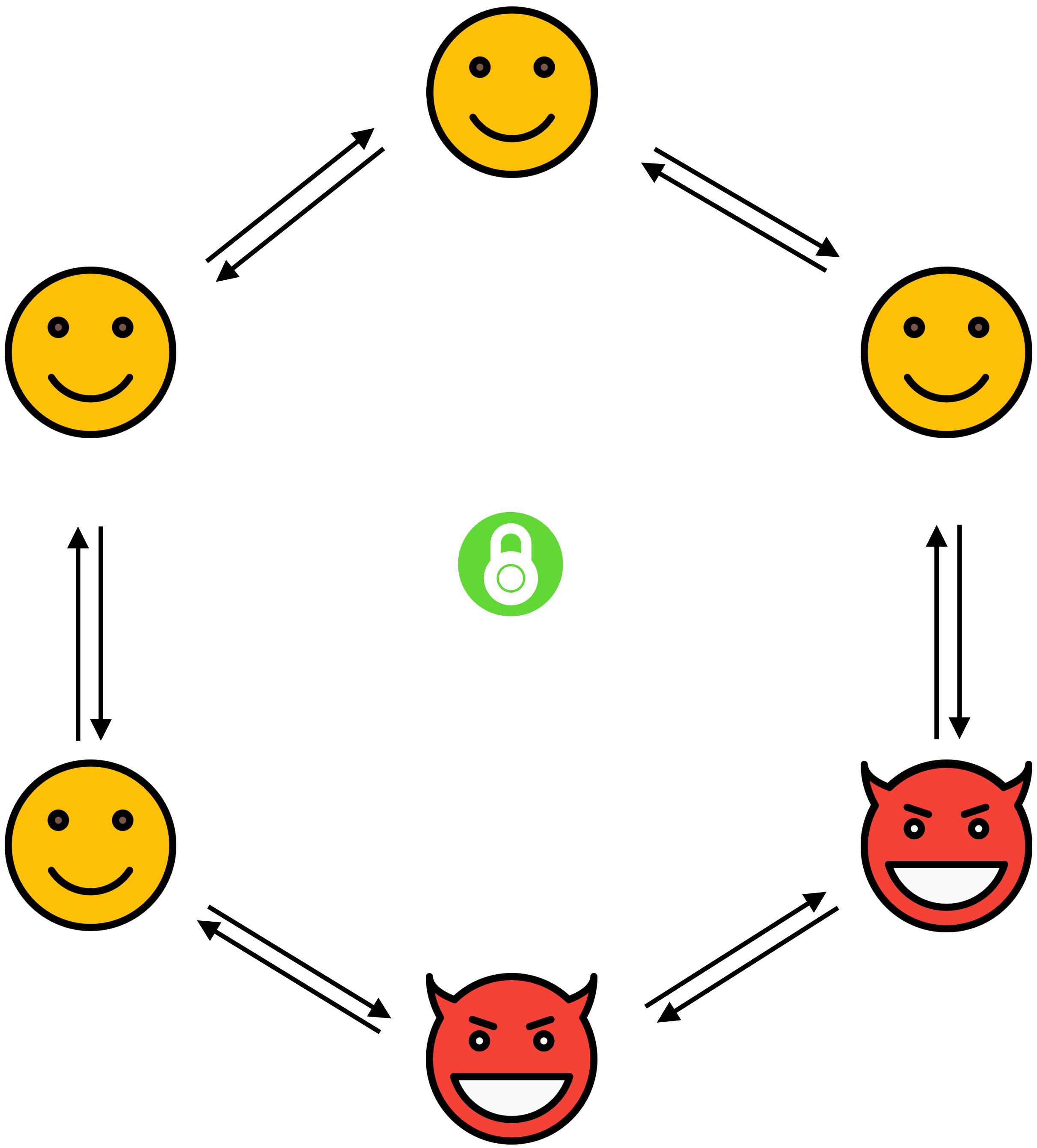

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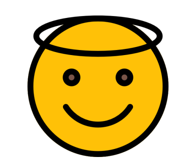
   - every player tests n random states 
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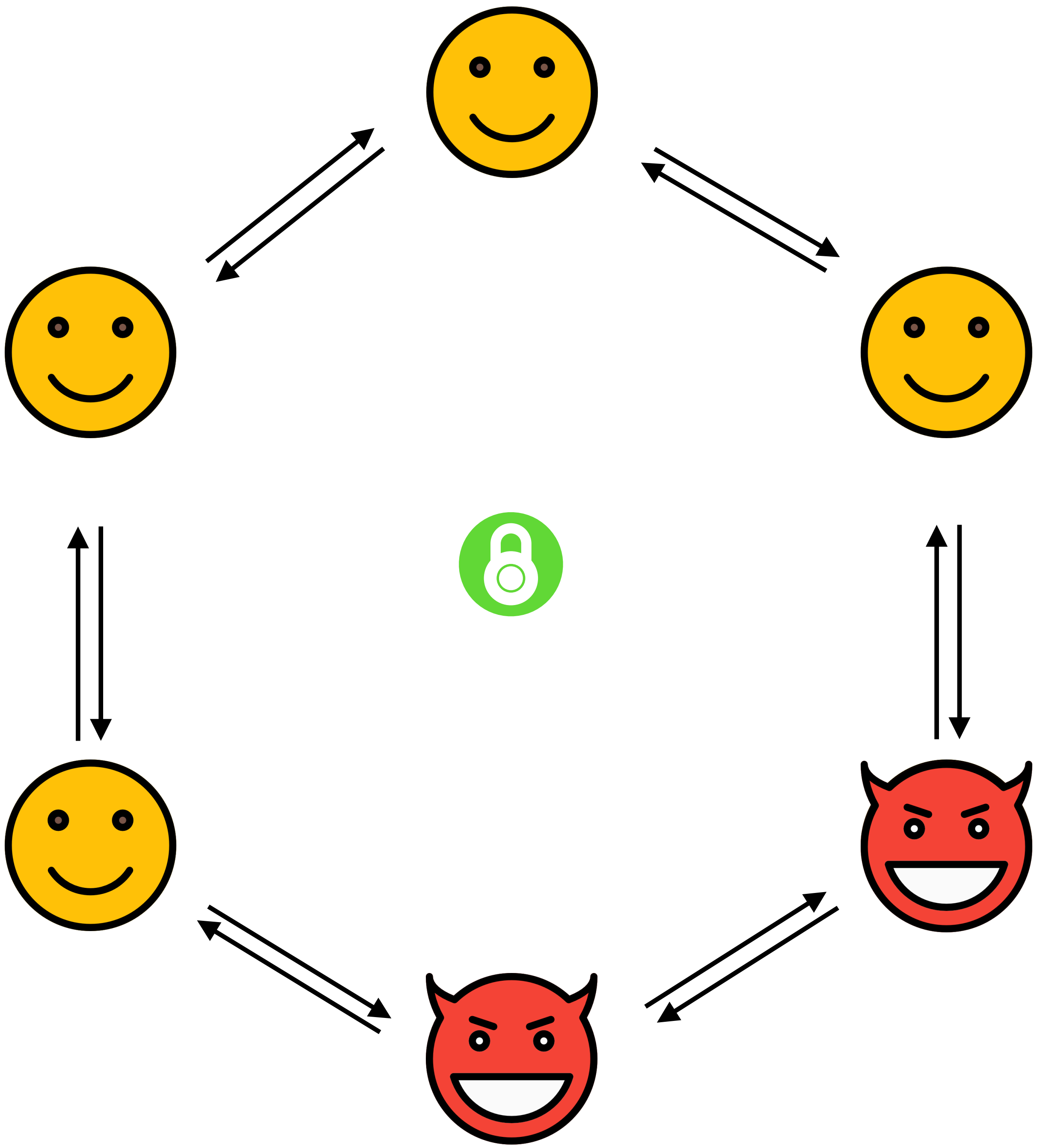

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Computation

$|\psi\rangle$ $G|\psi\rangle$

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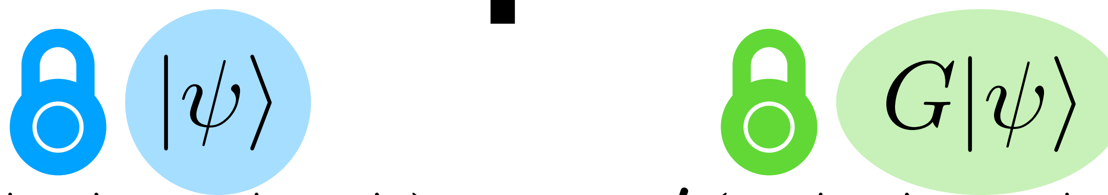

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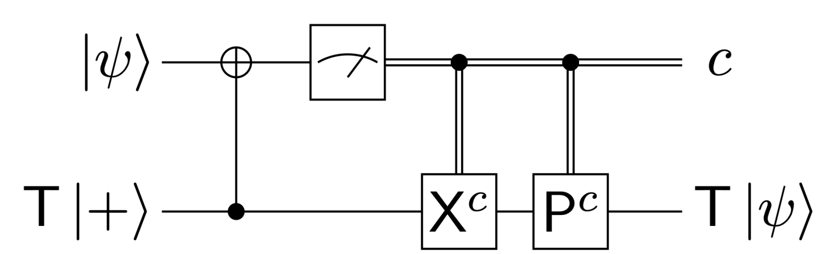

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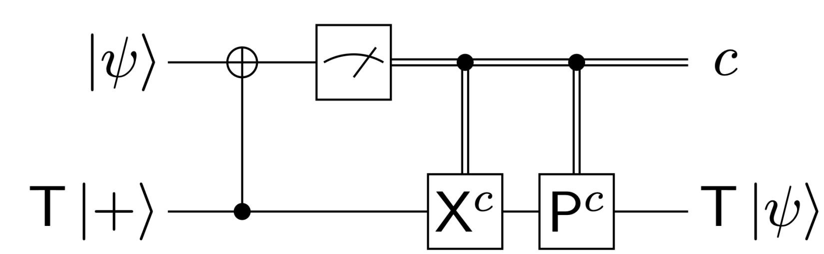

- Computational-basis measurement

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  for d the $\{\mathsf{CNOT},\mathsf{T}\}$-depth of the quantum computation

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Thank you!