MPC with Silent Preprocessing via Pseudorandom Correlation Generators

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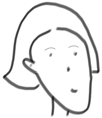
Based on joint works with Elette Boyle, Geoffroy Couteau, Niv Gilboa, Yuval Ishai, Peter Rindal, and Peter Scholl

Secure multi-party computation (MPC)

[Yao86; GMW87; BGW88; CCD88]



public function f

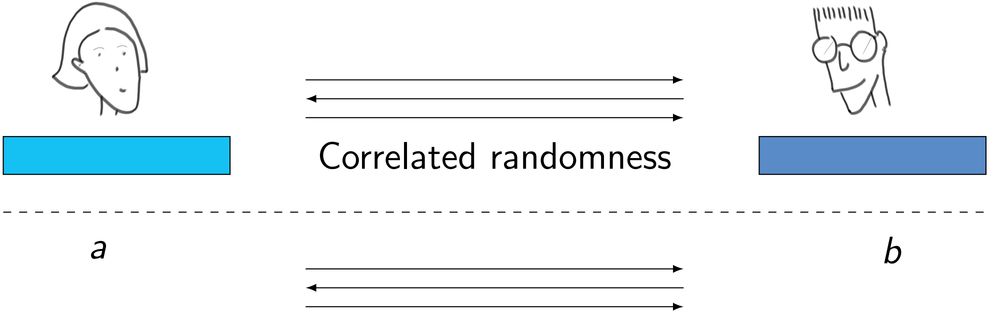
a

b

Goal: Parties learn $f(a,b)$ and *nothing more*

Secure MPC with preprocessing

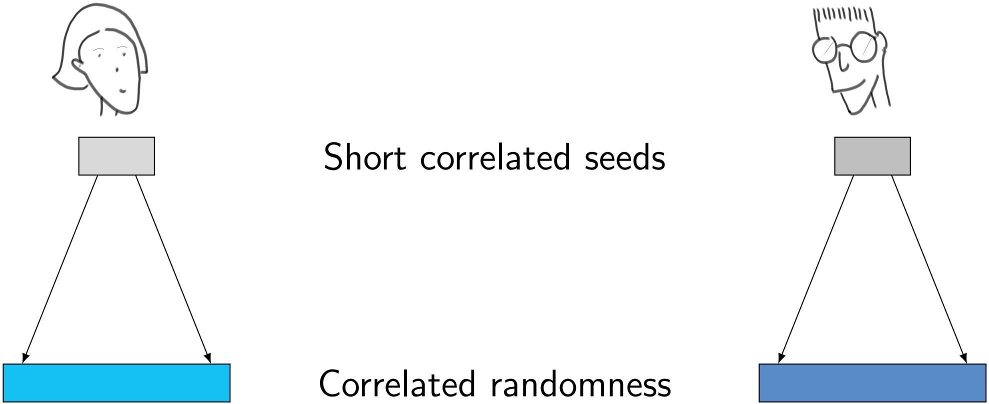
[Beaver91]



+ Fast online phase, security against dishonest majority
– Preprocessing expensive (communication & storage)

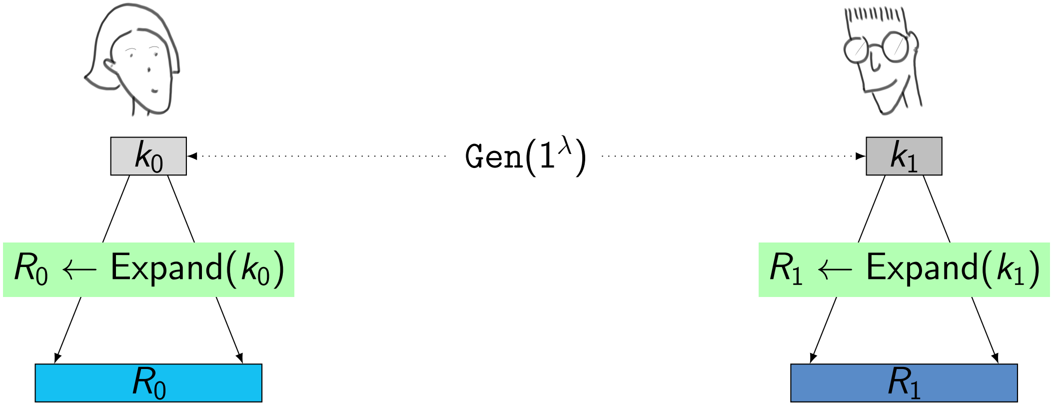
Pseudorandom correlation generator (PCG)

[BCGI18; BCGI**K**S19]



Pseudorandom correlation generator (PCG)

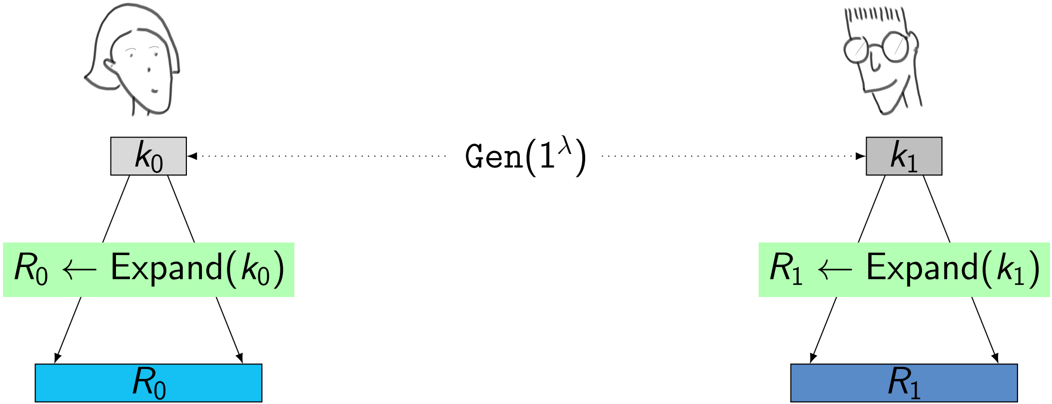
[BCGI18; BCGI**K**S19]



Correctness: $R_0 \sim R_1$

Pseudorandom correlation generator (PCG)

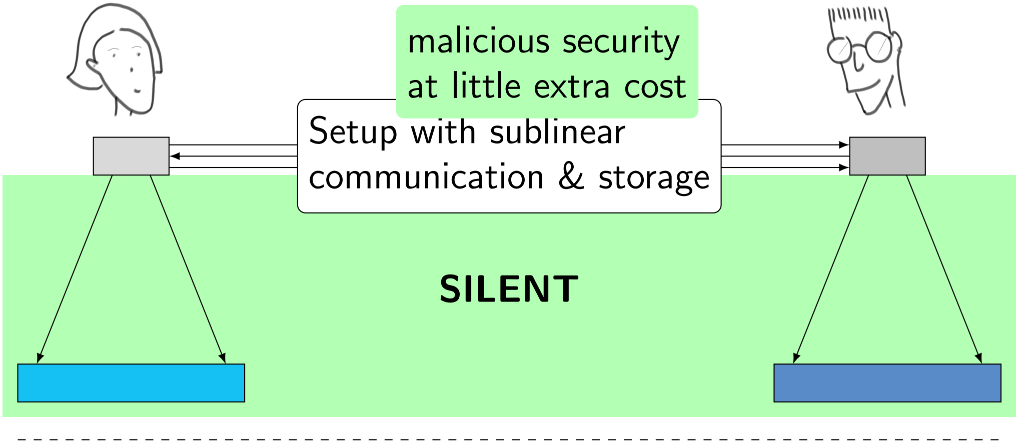
[BCGI18; BCGIKS19]



Security: $(k_0, R_1) \approx_c (k_0, [R_1 \mid R_0 \sim R_1])$

Secure MPC with *silent* preprocessing

[BCGIKS19]



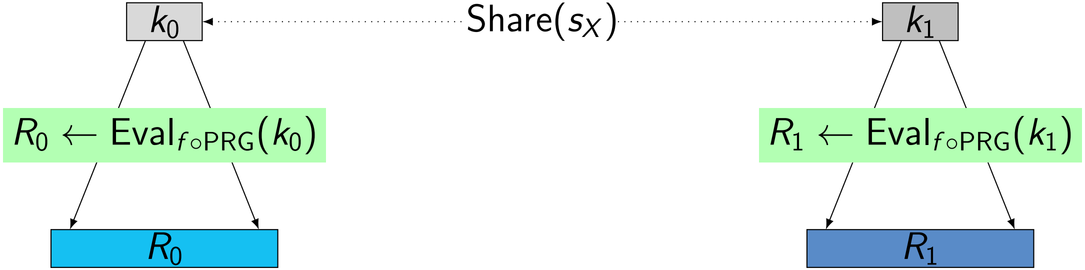
Generic construction of PCGs

[BCGI**K**S19]

General additive correlations:

R_0 R_1 $R_0 + R_1 = f(X)$

Feasibility: PRG + Homomorphic secret sharing



Landscape of PCGs

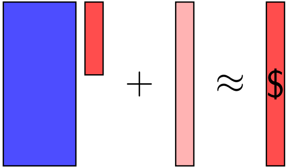
“Gentryland”	LWE+:	General additive [BCGI**K**S19]
“Cryptomania”	DDH + PRG*:	Log-space [BCGIO17]
	LWE + PRG*:	Bounded depth* [BCGI**K**S19]
“Lapland”	LPN:	Vector OLE [BCGI18]
		OT, Constant-degree [BCGI**K**S19]
	Ring-LPN:	OLE [BCGI**K**S20]
“Minicrypt”	OWF:	Linear [GI99; CDI05]
		Truth tables [BCGI**K**S19]

*low-degree

*concretely efficient

Learning with errors vs. learning parity with noise

LWE:

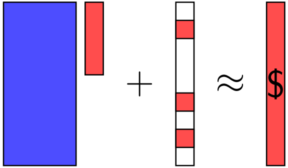


$p > 2$

s over $\mathbb{Z}_p$

$\|e\|_\infty$ small

LPN:

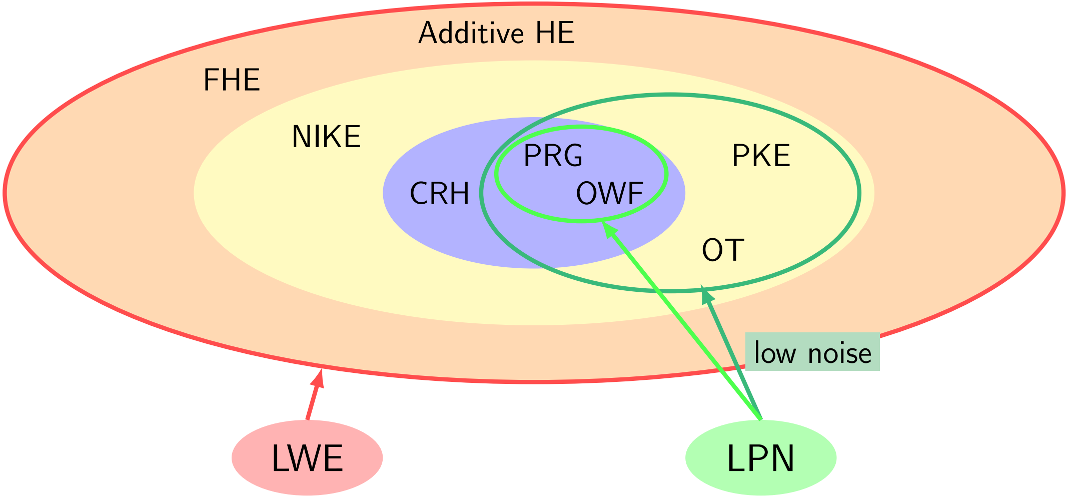


$p = 2$ (*here:* $p \geq 2$)

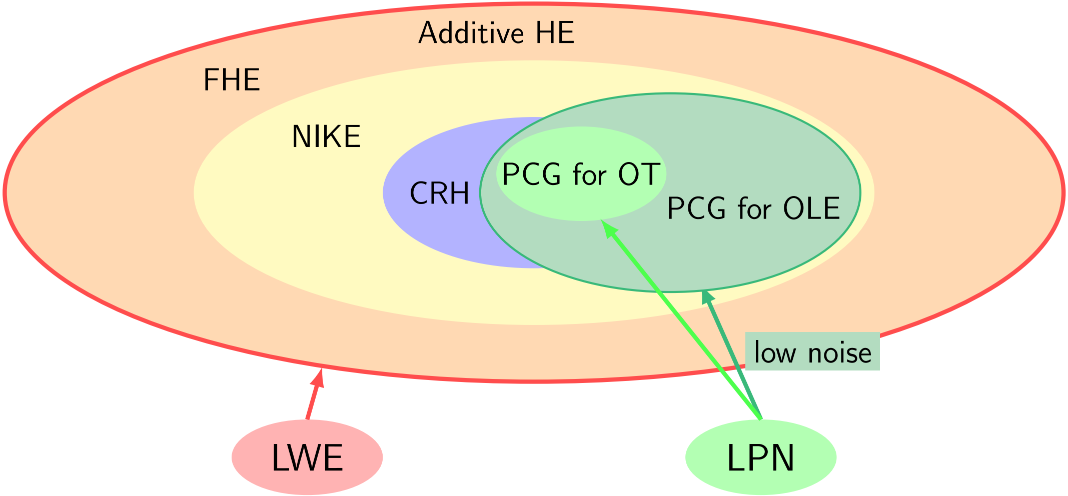
s over $\mathbb{Z}_p$

$\mathrm{HW}(e)$ small

Cryptography from LWE vs. LPN

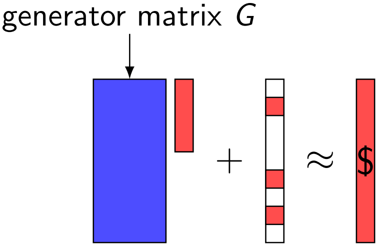


Cryptography from LWE vs. LPN

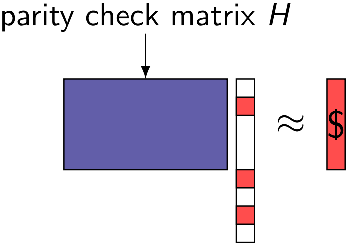


A simple PRG from LPN

LPN:



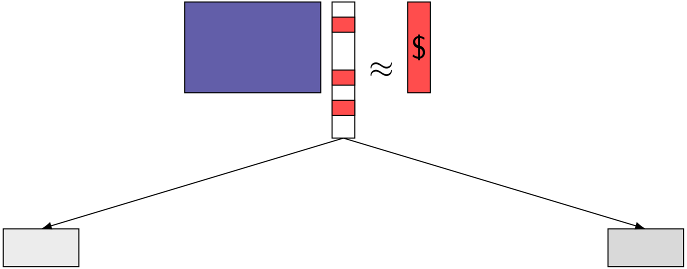
Dual-LPN:



limited to quadratic stretch

arbitrary polynomial stretch

Why LPN is a perfect match for PCGs



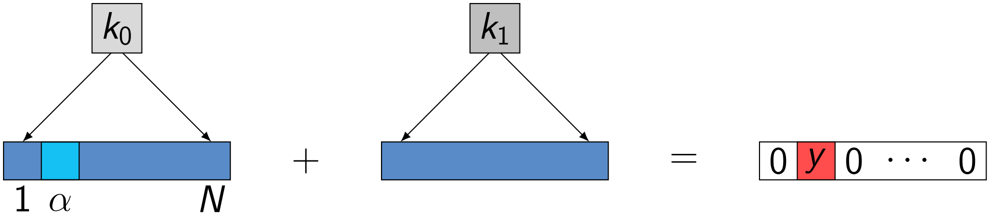
- Sparse vector can be distributed via compressed secret shares
- LPN assumption is linear $\rightsquigarrow$ homomorphic properties

How to distribute a sparse vector efficiently

[GI14]

Point Function: $\mathsf{F}^{\alpha}\colon \{1,\ldots,N\} \to \mathbb{F}_{2^\lambda}, \mathsf{F}^{\alpha}(x) = \begin{cases} y & \text{, if } x = \alpha \\ 0 & \text{, else} \end{cases}$

Distributed Point Function:

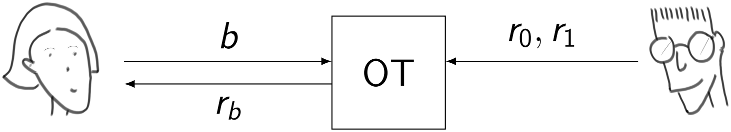


- Efficient constructions from OWFs [GI14; BGI16]
- Efficient distributed setup [Ds17]

Part I: PCG for oblivious transfer from LPN

Oblivious transfer (OT)

[Rab81; EGL85]

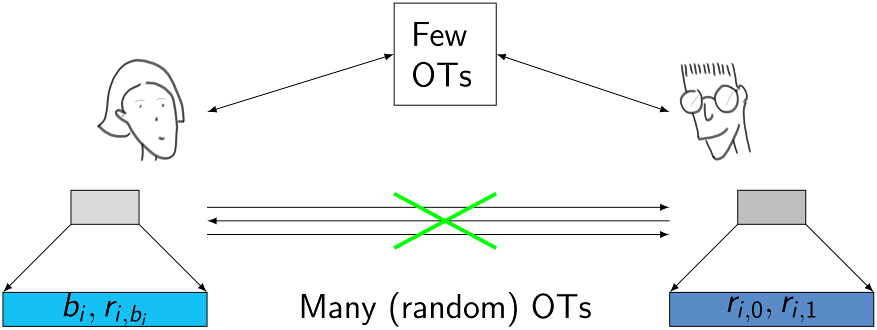


Security: Alice learns only r_b, Bob doesn't learn b

GMW Protocol: Secure MPC with 2 OTs per AND-Gate

Problem: OT is expensive ("public-key primitive")

OT extension



OT extension: Few base OTs + "cheap crypto" [Bea96; IKNP03]
Silent OT extension: Local expansion [BCGI**K**S19; BCGI**K**RS19]

Comparison of OT extension protocols

128-bit security

Reference	Rounds	Comm. per random OT	Silent	Active	Based on
[Bea96]	2	poly	✗	✗	OWF
[IKNP03; ALSZ13; KOS15]	$3^{\star}$	128	✗	✓	crh
[KK13] (short strings)	3	≈ 78	✗	✗	crh
[BCGI**K**S19]	$\log N$	$0 - 3$	✓	✗	LPN, $\text{crh}^{\star\star}$
[BCGI**K**RS19]	$2^{\star}$	0.1	✓	✓	LPN, $\text{crh}^{\star\star}$

*Fiat-Shamir for active security, **correlated-input secure hash function

[GMMM18]: RO $\not\Rightarrow$ 2-round OT extension

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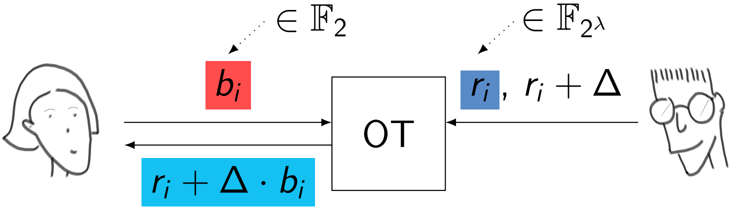
[BCGI**K**RS19]	2*	0.1	✓	✓	LPN, crh**

*Fiat-Shamir for active security, **correlated-input secure hash function

- Semi-honest 2-PC w/ 4.2 bits per AND, 30× less than [DKSSZZ17]
- Improves PSI, malicious MPC
- Useful for non-interactive secure comp. [IKOPS11; AMPR14; MR17]

Correlated OT



Correlated OT + correlation robust hash function ⇒ OT [IKNP03]

As vectors: $\triangleq$ Subfield vector oblivious linear evaluation

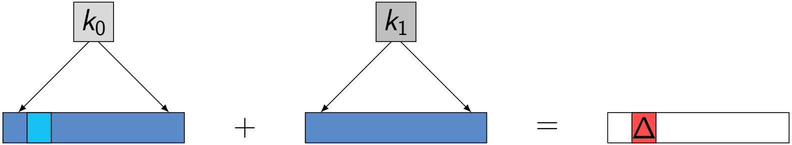
$$\Delta \cdot \mathbf{b} = \mathbf{r} + \Delta \cdot \mathbf{b} + \mathbf{r}$$

Overview: PCG for correlated OT

[BCGI**K**S19]

Idea:

1. Via distributed point functions:



2. Via addition:

3. Via LPN:

1a. Towards 2-round setup

[SGRR19; BCGI**K**RS19]

Problem: DPF require $\log N$ rounds for distributed setup!

Observation:

- Receiver knows **b**

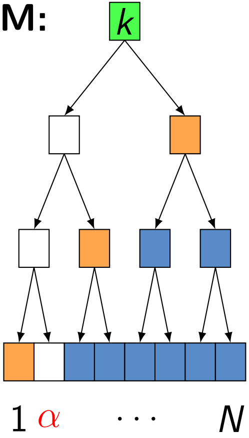
$\rightsquigarrow$ Receiver knows the point α, where PF $\neq 0$

$\rightsquigarrow$ Puncturable pseudorandom functions sufficient!

1b. Puncturable pseudorandom function

[BGI13; BW13; KPTZ13]

Via GGM:



$|k| = \lambda$, $|k^\star| = \lambda \log N$

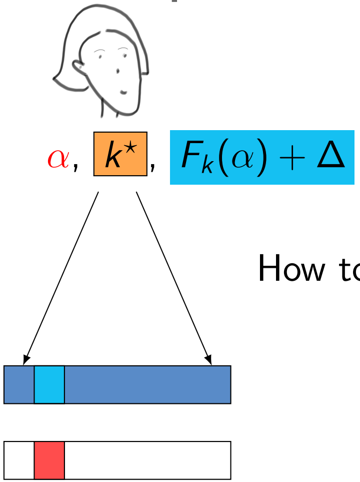
Puncturable PRF (PPRF):

$F_k \colon \{1, \ldots, N\} \to \mathbb{F}_{2^\lambda}$

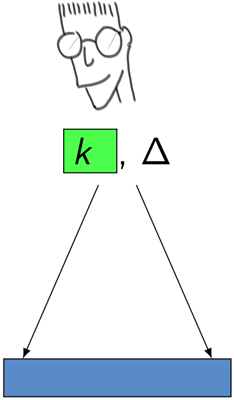
- k $\rightsquigarrow$ $F_k(x)$ for all x
- $k^\star$ $\rightsquigarrow$ $F_k(x)$ for all $x \neq \alpha$

1c. PCG for unit vector via PPRF

[SGRR19; BCGI**K**RS19]

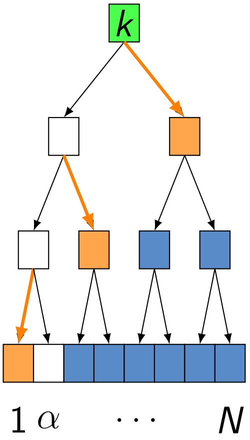


How to set up $k^\star, k$?



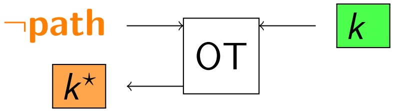
1d. 2-Round setup for unit vector

[SGRR19; BCGI**K**RS19]



Strategy: (based on [Ds17])

- Sender chooses k
- Receiver receives $k^\star$ via chosen OTs:

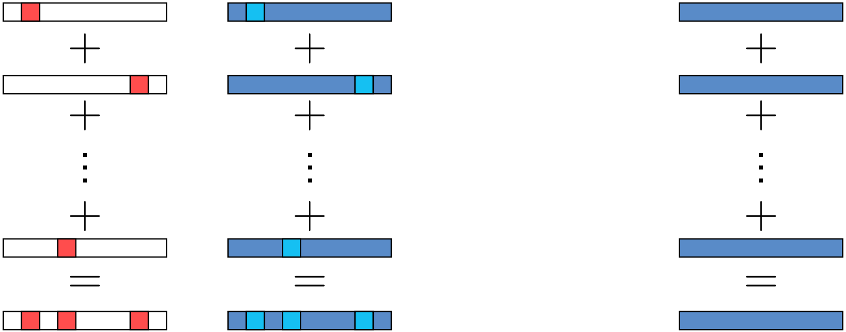


- **Note:** OTs can be executed *in parallel!*

2. From unit to sparse vectors

[BCGI18; BCGI**K**S19]

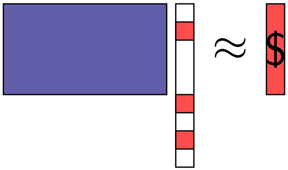
Repeat t times:



Alternative: Concatenation + LPN with *regular* noise

3. From sparse to pseudorandom vectors

[BCGI18; BCGIKS19]



Main challenge: Parity check matrix is big!

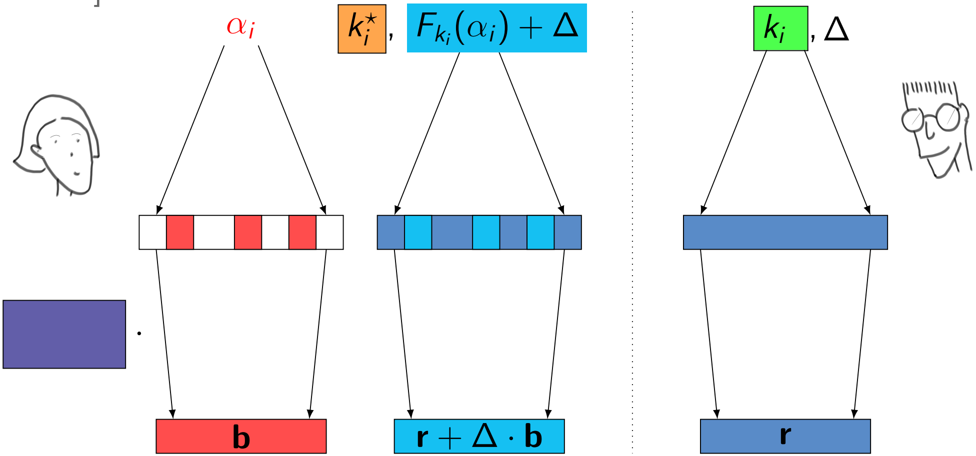
- use quasi-cyclic codes $\rightsquigarrow$ multiplication in $\widetilde{\mathcal{O}}(N)$

Security

- Similar to PQ cryptosystems BIKE, HQC [AAB+19; ABB+19]

PCG for correlated OT from LPN - Recap

[BCGIKS19]



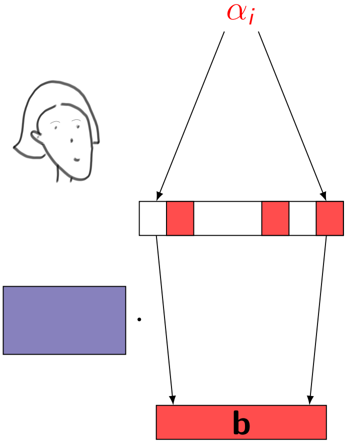
From correlated OT to chosen OT

1. **Break correlations:**
   - *Locally* apply crh [IKNP03]

⇝ MPC with 2-round silent preprocessing

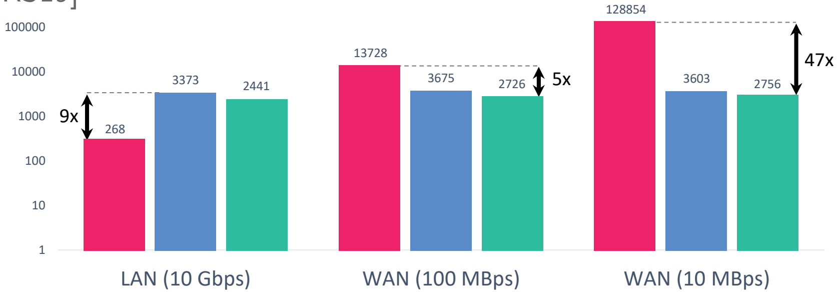
2. **Derandomization:**
   - Depends only on **b**
   - Can be sent along with first message

⇝ 2-round OT extension



Runtimes (ms) for 10 million random OTs

[BCGIKRS19]

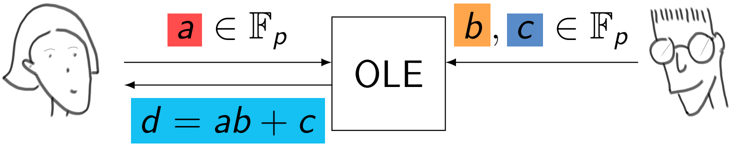


[IKNP03] vs 2-round silent vs 3-round hybrid

- Total communication: 160 MB vs 145 kB vs 127 kB

Part II: PCGs for OLE from LPN and ring-LPN

Oblivious linear evaluation (OLE)



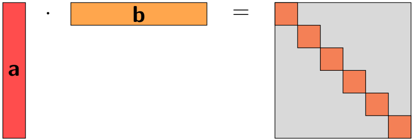
- Generalization of OT to $\mathbb{F}_p$
- 2 OLEs can be locally transformed into a multiplication triple

$$\mathbf{a} * \mathbf{b} = \mathbf{d} - \mathbf{c}$$

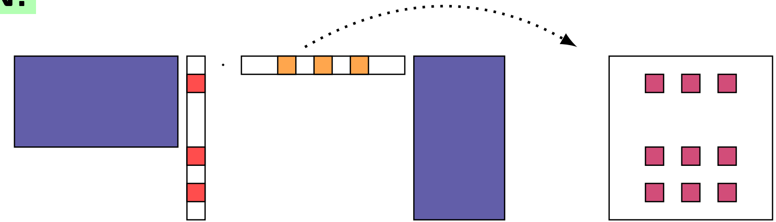
Towards PCG for OLE from LPN

[BCGIKS19,BCGIKS20]

Idea: Rewrite $\mathbf{a} * \mathbf{b}$ and use linearity of LPN



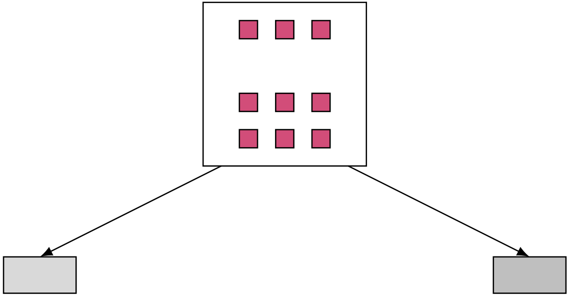
Via LPN:



PCG for OLE via LPN

[BCGI**K**S19,BCGI**K**S20]

Via DPF:



Problem: Dimension ($\rightsquigarrow$ computational cost) *quadratic* in N

A different perspective

[BCGIKS20]

Observations:

- Generalizes to more dimensions
- Better efficiency via choosing H such that $H * H$ *compressible*

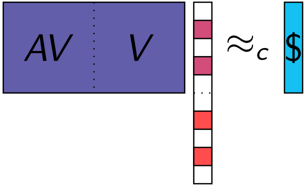
More efficient PCG for OLE from ring-LPN

[BCGIKS20]

public: $a(X)$; sparse: $s(X)$, $e(X)$

$$a(X) \cdot s(X) + e(X) \approx_c u(X) \text{ over } \mathbb{Z}_p[X]/\varphi(X)$$

If $\varphi(X)$ (of degree N) fully splits over $\mathbb{Z}_p[X]$:



More efficient PCG for OLE from ring-LPN

[BCGI**K**S20]

public — sparse

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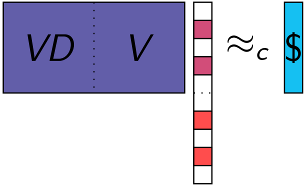

$\rightsquigarrow$ N OLEs over $\mathbb{Z}_p$ in $\widetilde{\mathcal{O}}(N)$ computation time

Efficiency of our PCG construction for OLE

[BCGI**K**S20]

To generate 1 Mio OLEs over $\mathbb{Z}_q$ (q composite of 62-bit primes):

Reference	Amount	Seed size	Communication	OLEs/second
[KPR18]	32 MB	32 MB	$>$ 1 GB	30 K
[BCGI**K**S19]	17 GB	3 GB	6 GB	6 K*
[BCGI**K**S20]	32 MB	1.25 MB	7 MB	100 K*

*expansion only, estimated costs

- Setup with malicious security
- Generalizes to authenticated multiplication triples at $\approx \times 2$ cost!

Conclusion

PCGs for OT from LPN [BCGI**K**S19; BCGI**K**RS19]

- Random OT: *practical*, almost *zero communication*
- 2-Round OT extension (malicious security, implementation)

PCGs for OLE [BCGI**K**S20]

- More efficient instantiation based on *fully splittable ring-LPN*

Open problems/ Ongoing work:

- Optimize OT: Better codes
- Efficient PCGs for more correlations
- Better understanding of LPN-flavored assumptions

Thank you!