Quantum Algorithms for Second Order Cone Programming

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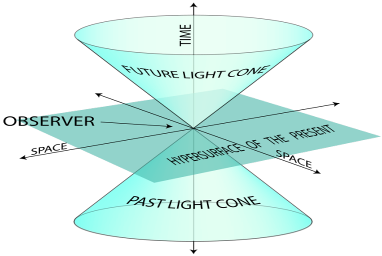
Second Order Cones

- Lorentz cone: The n-dimensional Lorentz cone, for $n \geq 1$ is defined as $\mathcal{L}^n := \{\vec{x} = (x_0; \vec{x}) \in \mathbb{R}^n \mid x_0 \geq \|\vec{x}\|\}$.



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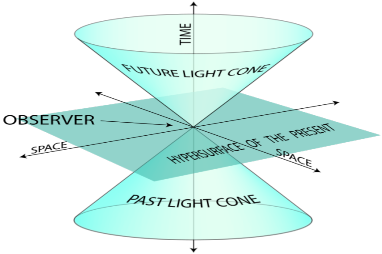

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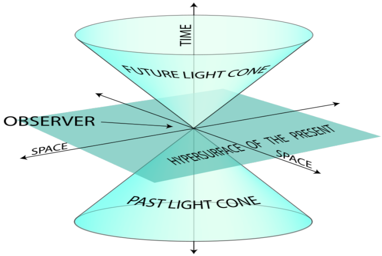

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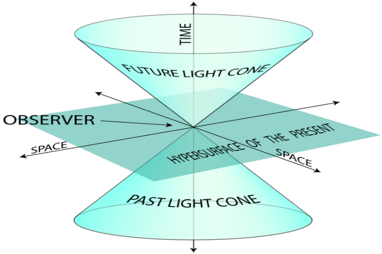

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- $\mathcal{L}^1 = \{x \in \mathbb{R} \mid x^2 \geq 0\}$.
- Second order cone programs (SOCPs) have constraints of the form $\vec{x} \in \mathcal{L}^n$.

Second Order Cone Programs

- A SOCP (Second Order Cone Program) is an optimization problem of the following form,

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\begin{aligned}
\min_{\vec{x}_1,\ldots,\vec{x}_r} \quad & \vec{c}_1^T \vec{x}_1 + \cdots + \vec{c}_r^T \vec{x}_r \\
\text{s.t.} \quad & A^{(1)}\vec{x}_1 + \cdots + A^{(r)}\vec{x}_r = \vec{b} \\
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- The sum of dimensions of the vectors, $n := \sum_i n_i$ is the dimension of the SOCP.

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- SOCPs generalize Linear Programs (LPs) and Convex Quadratic Programs (QPs).
- The running time for classical SOCP algorithms is given in terms of n, r and the duality gap ϵ.

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- The SVM reduces to an SOCP with variables $\mathbf{t} \in \mathcal{L}^{n+2}$ and $\xi_i \in \mathcal{L}^1$ with $r = n + m + 2$.

Main Results

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There is a quantum IPM for SOCPs with running time $O(n^{1.5} \sqrt{r} \frac{\kappa}{\delta^2} \log(1/\epsilon))$ where δ bounds the distance of the intermediate solutions from the cone boundary, κ is the condition number of intermediate matrices and ϵ is the duality gap.

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- Experimental results on random SVM instances: The quantum algorithm scales as $O(n^k)$ where $k \in [2.56, 2.62]$ with high probability while an external SOCP solver scales as $O(n^{3.31})$.

Jordan Algebras: The Spin factor

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- The identity element is $\vec{e} := (1; 0^n)$.

Jordan product, Arrow matrices

- The Jordan product is a linear operation, it has a matrix representation,

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- The Jordan product and the Arrow matrices can be extended blockwise,

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- The well structured arrow matrices make the linear systems that arise in the IPM for SOCPs simpler than those for SDPs.

Interior Point Method for SOCP

- The central path for the SOCP is parametrized by $\nu > 0$ and is characterized by feasibility and complementary slackness conditions,

$$\begin{aligned} A\vec{x} &= \vec{b} \\ A^T\vec{y} + \vec{s} &= \vec{c} \\ \vec{x} \circ \vec{s} &= \nu\vec{e}, \end{aligned} \tag{4}$$

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- The central path converges to the optimal solution as $\nu \rightarrow 0$.
- A single iteration of the IPM finds $\Delta\vec{x}, \Delta\vec{y}, \Delta\vec{s}$ such that $\vec{x} + \Delta\vec{x}$, $\vec{y} + \Delta\vec{y}$ and $\vec{s} + \Delta\vec{s}$ are close to the central path for $\nu' = \sigma\nu$.

Interior Point Method for SOCP

- Linearizing the last equation and neglecting the term $\Delta\vec{x} \circ \Delta\vec{s}$ we get,

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- We thus obtain the Newton linear system for SOCPs,

$$\begin{bmatrix} A & 0 & 0 \\ 0 & A^T & I \\ \mathrm{Arw}(\vec{s}) & 0 & \mathrm{Arw}(\vec{x}) \end{bmatrix} \begin{bmatrix} \Delta\vec{x} \\ \Delta\vec{y} \\ \Delta\vec{s} \end{bmatrix} = \begin{bmatrix} \vec{b} - A\vec{x} \\ \vec{c} - \vec{s} - A^T\vec{y} \\ \sigma\nu\vec{e} - \vec{x} \circ \vec{s} \end{bmatrix} \tag{5}$$

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- Analysis shows that if (x, y, s) is in a neighborhood $\mathcal{N}$ of the central path at ν, then $\vec{x} + \Delta\vec{x}, \vec{y} + \Delta\vec{y}, \vec{s} + \Delta\vec{s}$ remains feasible and in $\mathcal{N}$ at ν'.

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- (Kerenidis, P. 18): The output of quantum linear system $|x\rangle = |A^{-1}b\rangle$ can be reconstructed in time $O(n \log n/\epsilon^2)$ queries to obtain $\tilde{x}$ such that $\|\tilde{x} - x\|_2 \leq \epsilon \|x\|_2$.

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- We can thus define $\|x\|_2$, $\|x\|_F$ and prove familiar inequalities like $\|x \circ y\|_F \leq \|x\|_2 \|y\|_F$.
- Matrix scaling $Y \rightarrow XYX$ has the Jordan algebra analog $2Arw^2(x) - Arw(x^2)$.

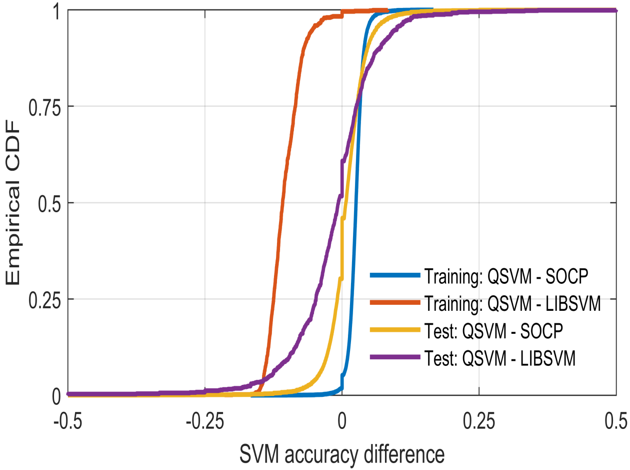
Experiments: Random SVM Instances

- Generate m points $\{\vec{x}_i \in \mathbb{R}^n \mid i \in [m]\}$ in the unit hypercube $[-1, 1]^n$.
- Generate a random unit vector $\vec{w} \in \mathbb{R}^n$ and assign labels to the points as $y^{(i)} = \text{sgn}(\vec{w}^T \vec{x}^{(i)})$.
- Corrupt a fixed proportion p of the labels, by flipping the sign of each $y^{(i)}$ with probability p.
- Shift the entire dataset by a vector $\vec{d} \sim \mathcal{N}(0, 2I)$, where $\mathcal{N}(\mu, \Sigma)$ denotes the multivariate normal distribution with mean μ and covariance Σ.
- Generate instances from $SVM(n, 2n, p)$ with n uniform in $[2, 2^9]$ and p uniform from $\{0, 0.1, \cdots, 0.9, 1\}$.

Experiments: Comparisons with classical algorithms

- We compare with classical algorithms on $SVM(n, 2n, p)$ instances for $\epsilon = 0.1$ where these algorithms achieve high accuracy.
- SOCP solver (ECOS) scales empirically as $O(n^{3.314})$, this is consistent with using a Strassen like algorithm with exponent 2.8.
- LIBSVM with linear kernel scales empirically as $O(n^{3.112})$, it is consistent with state-of-the-art alternate approaches to SVM.
- The running time $\frac{n^2 \kappa}{\delta^2}$ for the quantum IPM empirically scales as $O(n^{2.591})$ with a 95% confidence interval $[2.56, 2.62]$.

Experiments



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- The classification accuracy for the quantum algorithm is similar to that of the classical algorithms.

Conclusions

- Experiments indicate that the quantum IPM achieves a polynomial speedup for solving SOCPs with low and medium precision.
- For random SVM instances, it achieves a polynomial speedup with no detriment to the quality of the trained classifier.
- Similar results for the constrained portfolio optimization problem.
- Conclusion: Quantum optimization methods can achieve polynomial speedups for longer term algorithms.
- Open question: Improvements to the quantum IPM using tomography with ℓ_∞ guarantees?

Portfolio Optimization

- Portfolio optimization is the theory of optimal investment of wealth in assets that differ in expected return and risk [Markovitz 1952].
- Let $R(t) \in \mathbb{R}^m$ be returns for m assets over time epochs $t \in [T]$. Then, expected reward and risk can be estimated as,

$$\mu = \frac{1}{T} \sum_{t \in [T]} R(t)$$

$$\Sigma = \frac{1}{T-1} \sum_{t \in [T]} (R(t) - \mu)(R(t) - \mu)^T$$

- A portfolio is specified by $x \in \mathbb{R}^m$ with x_j being the investment in asset j.
- The expected reward and risk for x are $\mu^T x$ and $x^T \Sigma x$ respectively.

Portfolio Optimization

- Unconstrained portfolio optimization: Find portfolio that minimizes risk for a given reward.
- Constrained portfolio optimization: There are positivity $x_j \geq 0$ and budget constraints $Cx \geq d$. Introducing slack variables $s = Cx - d$, $s \geq 0$.
- The Constrained Portfolio Optimization problem reduces to SOCP,

$$\begin{aligned} \min \quad & x^T \Sigma x \\ \text{s.t.} \quad & \mu^T x = R \\ & Ax = b \\ & x \geq 0. \end{aligned} \tag{7}$$

- (Lloyd-Rebentrost). The unconstrained problem is a least squares problem and has a closed form solution using a single linear system solver.

Experiments: Constrained Portfolio Optimization

- cvxPortfolio dataset: Stocks for the S&P-500 companies for each day over a period of 9 years (2007-2016).
- Subsample 100 companies and consider random interval of t days where t is uniform on [10,500]. Add positivity constraints on portfolio.
- Quantum algorithm can be simulated by adding Gaussian noise of magnitude δ, the duality gap $\epsilon = 0.1$ due to market stochasticity.

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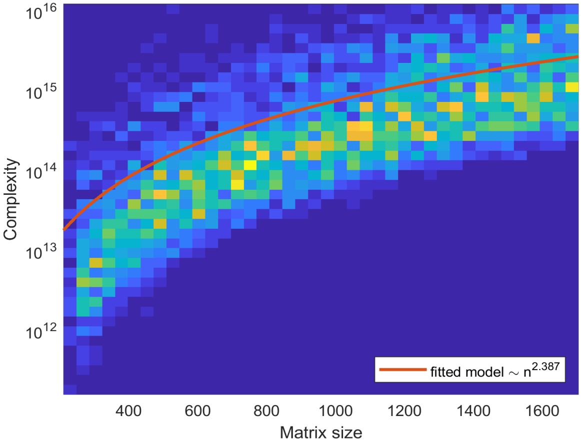

- Observed complexity for $\epsilon = 0.1$ and power law fit for random portfolio optimization instances.