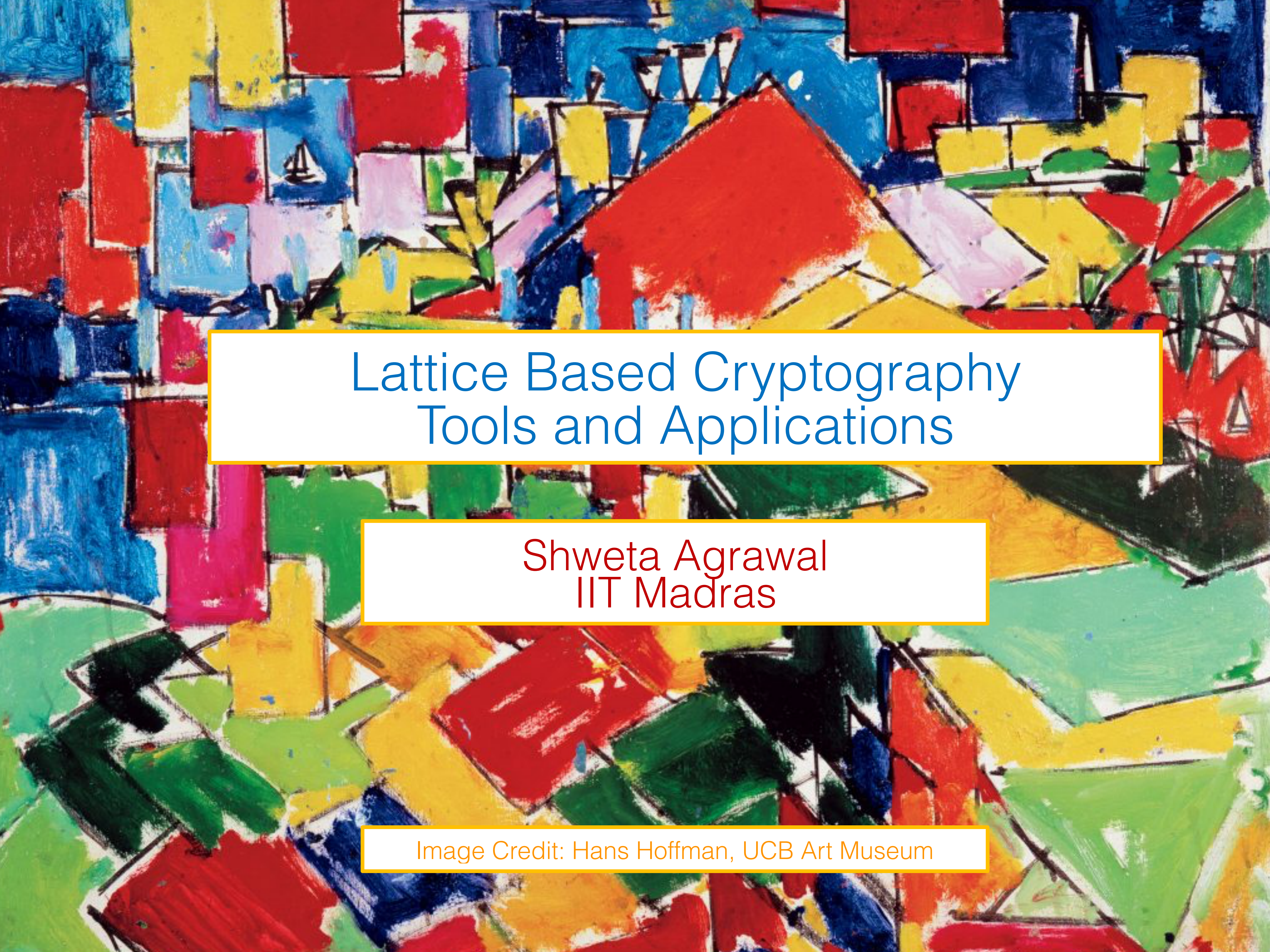

Lattice Based Cryptography
Tools and Applications

Shweta Agrawal
IIT Madras

Image Credit: Hans Hoffman, UCB Art Museum

Computing on Encrypted Data
Personalised Medicine

"The dream for tomorrow's medicine is to understand the links between DNA and disease — and to tailor therapies accordingly. But scientists have a problem: how to keep genetic data and medical records secure while still enabling the massive, cloud-based analyses needed to make meaningful associations."

Check Hayden, E. (2015). *Nature*, *519*, 400-401.

"You don't look anything like the long haired, skinny kid I married 25 years ago. I need a DNA sample to make sure it's still you."

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Doesn't FHE solve exactly this?

Access Control on Encrypted Data

Prof. Bob wants to store encrypted file so that:

- Other Professors or admin assistants of CS group can open it
- Encrypt file for each of them?
- If someone quits or new person joins? Re-encrypt ?
- Organizational nightmare !

Access Control on Encrypted Data

Prof. Bob wants to store encrypted file so that:

Access Control on Encrypted Data

Prof. Bob wants to store encrypted file so that:

What he really wants:
Encryption for formula

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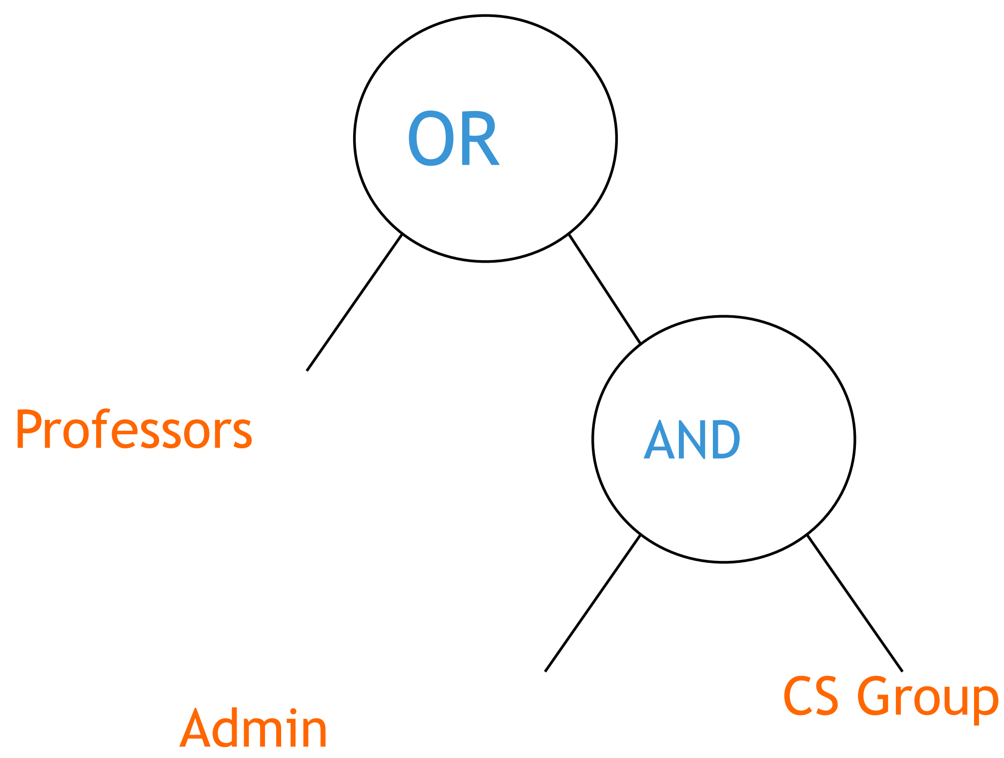

What do we want?

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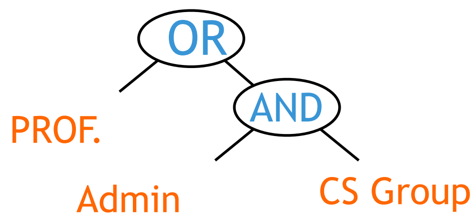

What do we want?

What do we want?

PROF OR {Admin AND CS}

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Key Authority

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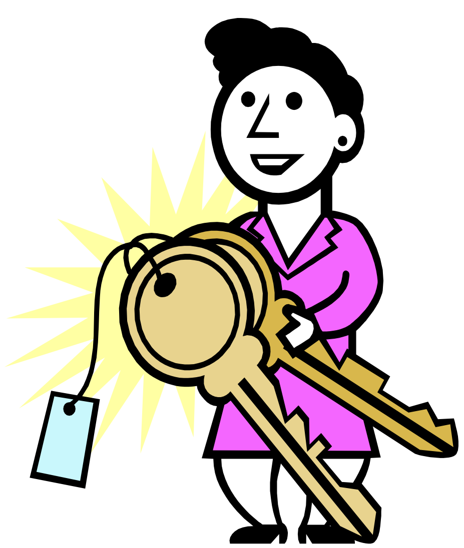

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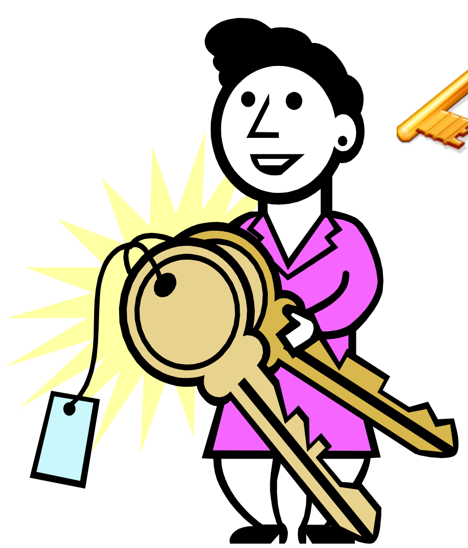

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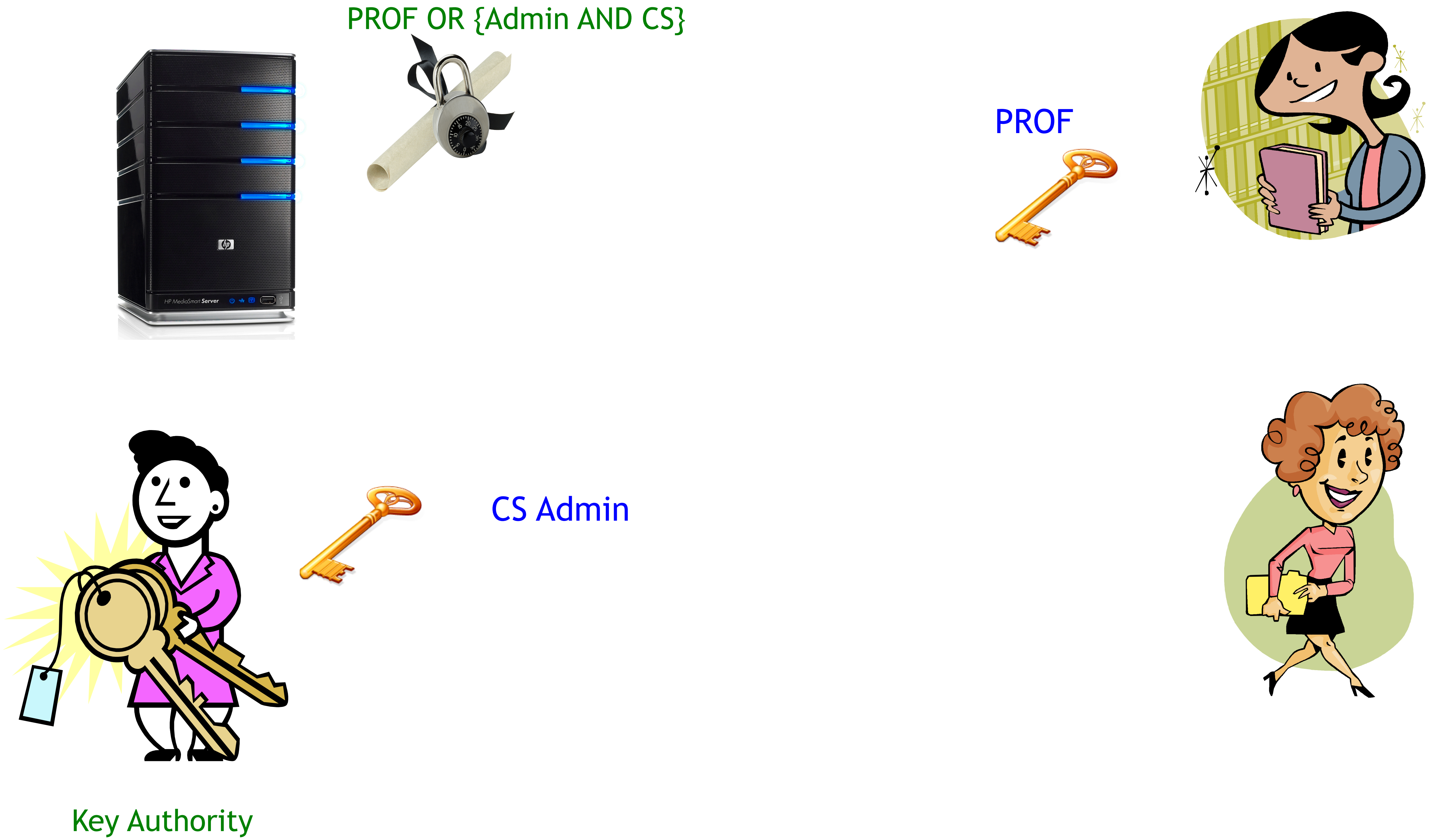

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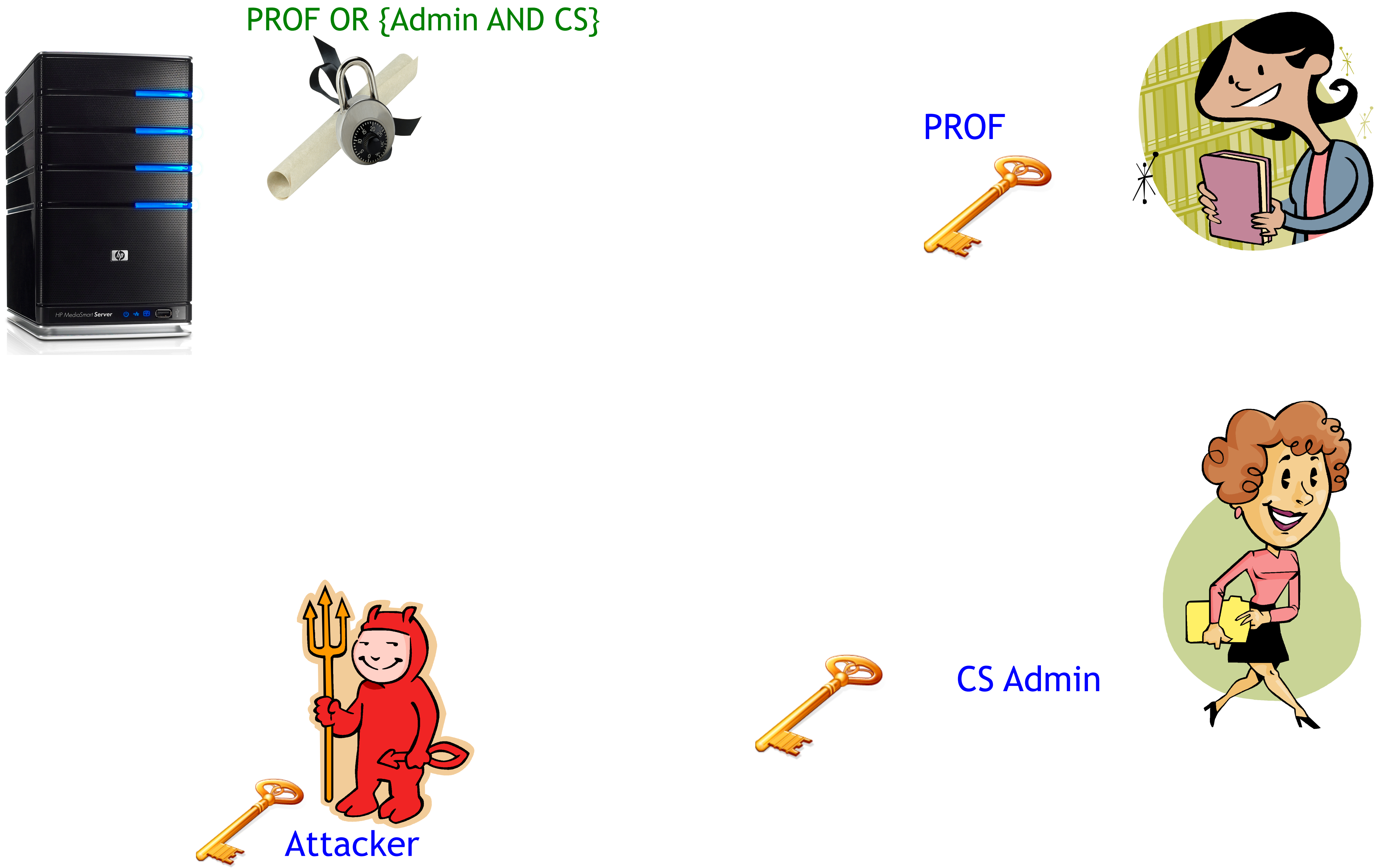

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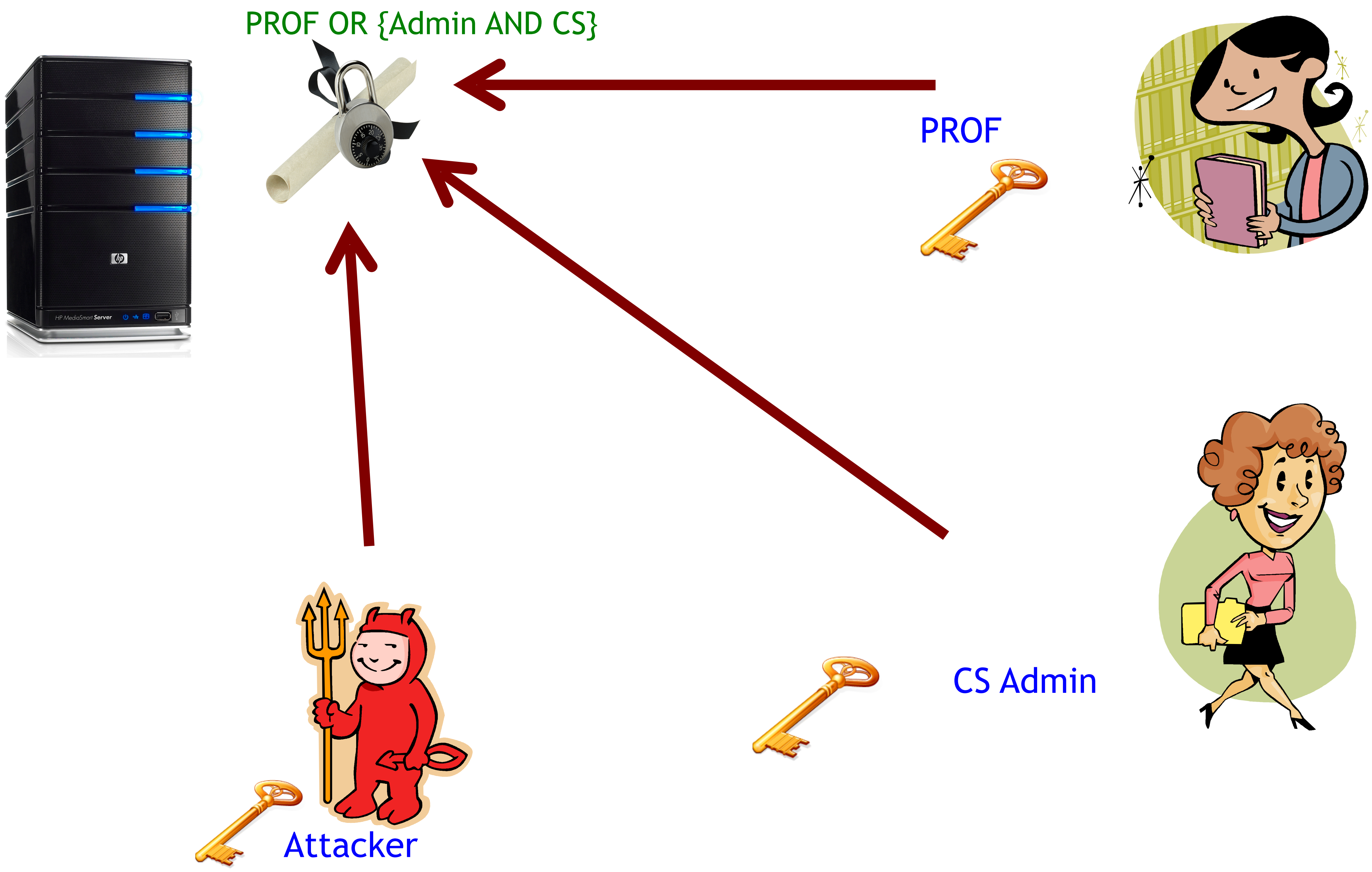

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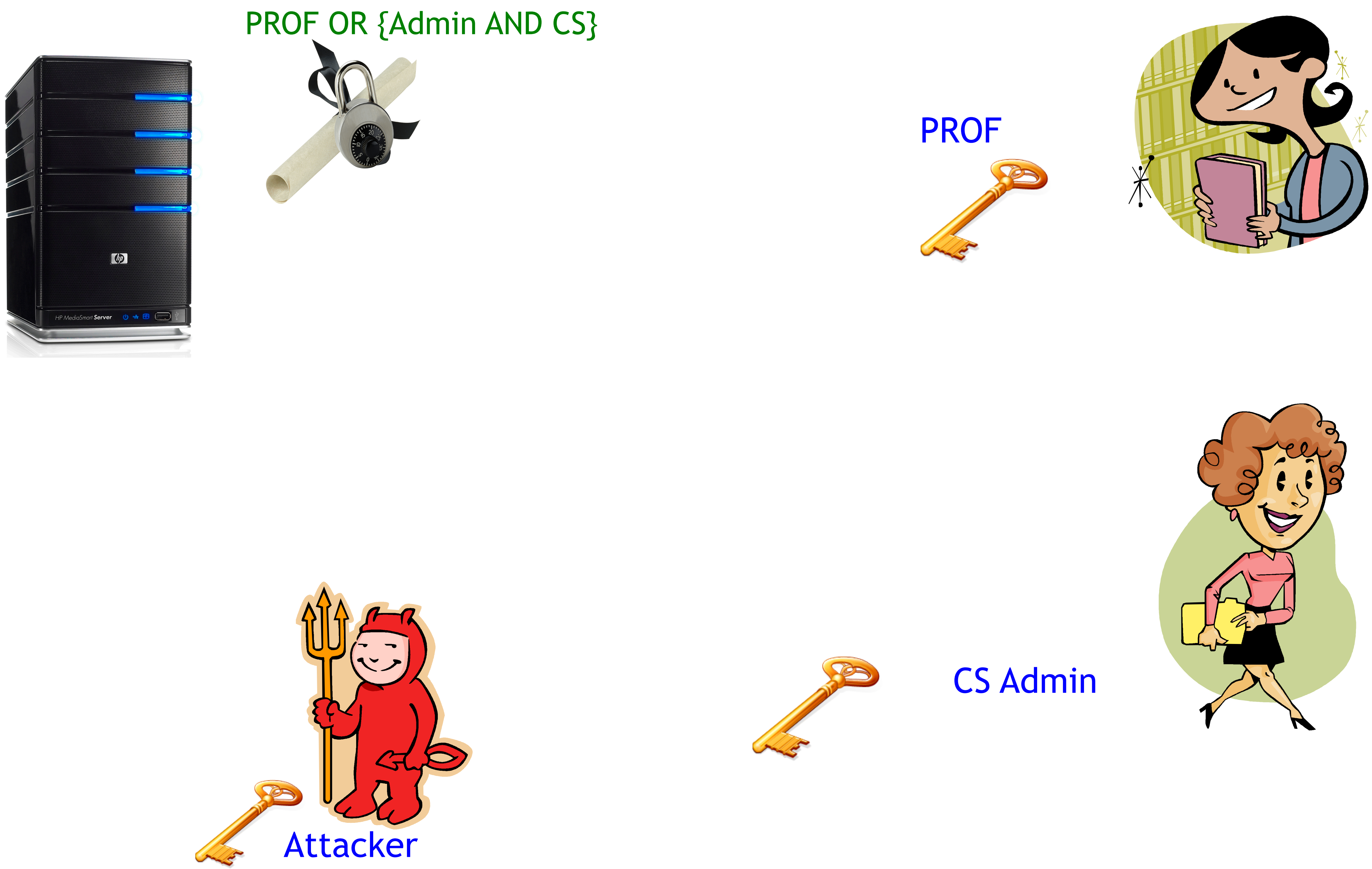

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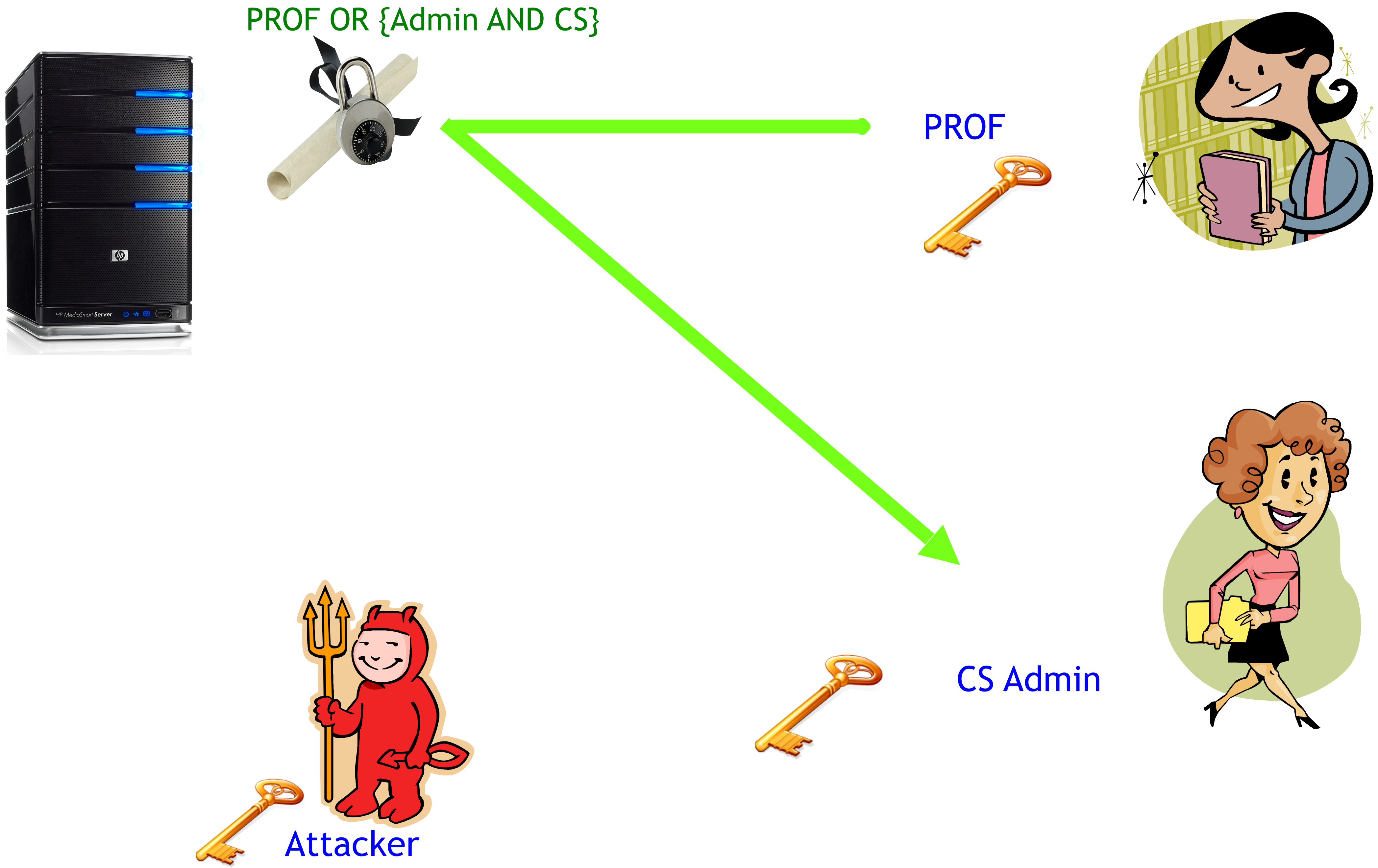

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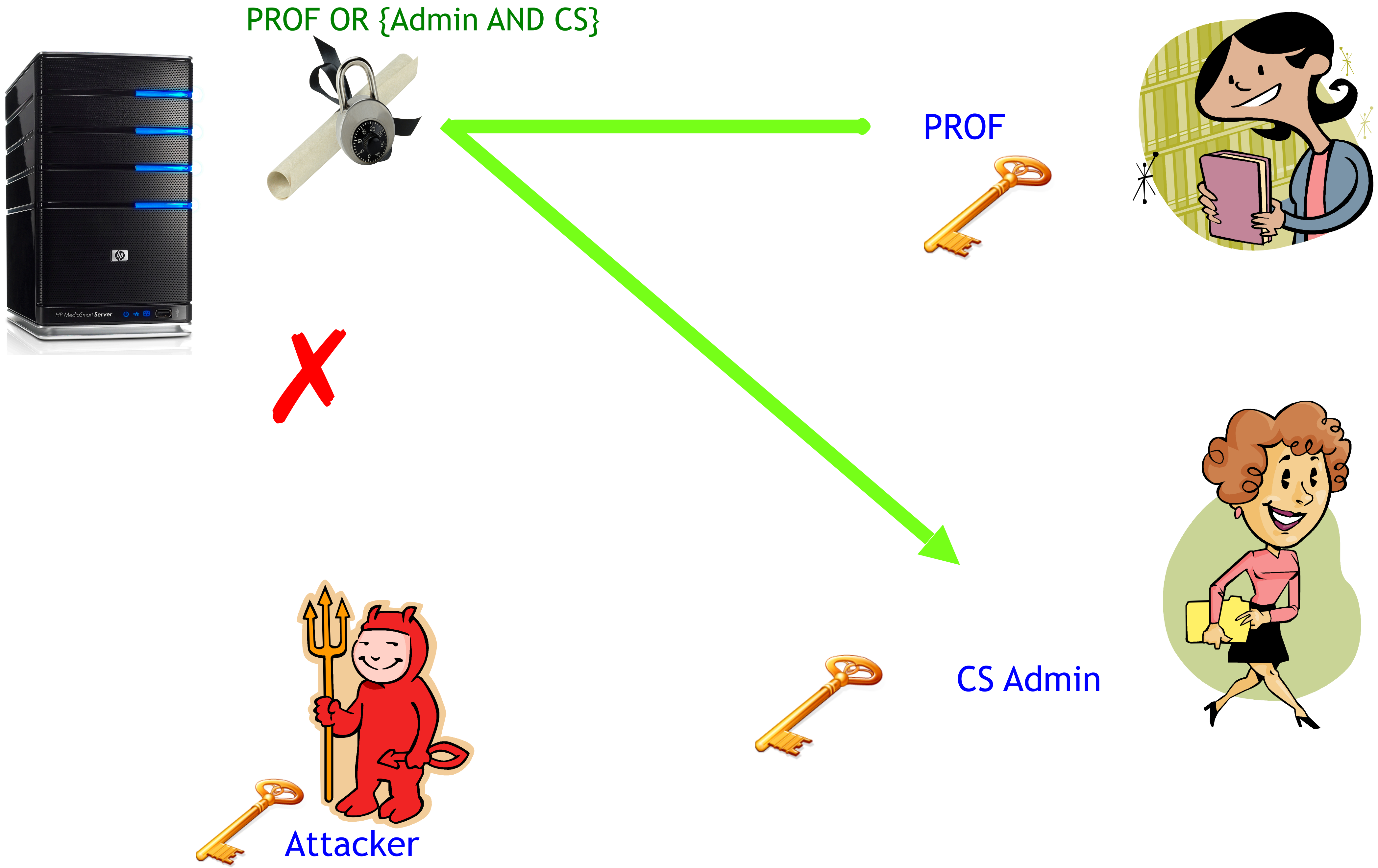

Need New Tools & Techniques!

Main Tool: Lattice Trapdoors

Trapdoor Functions

Trapdoor Functions

Generate (f, T)

Trapdoor Functions

Generate (f, T)

$f : D \to R,$

Trapdoor Functions

Generate (f, T)

$f : D \to R$, One Way

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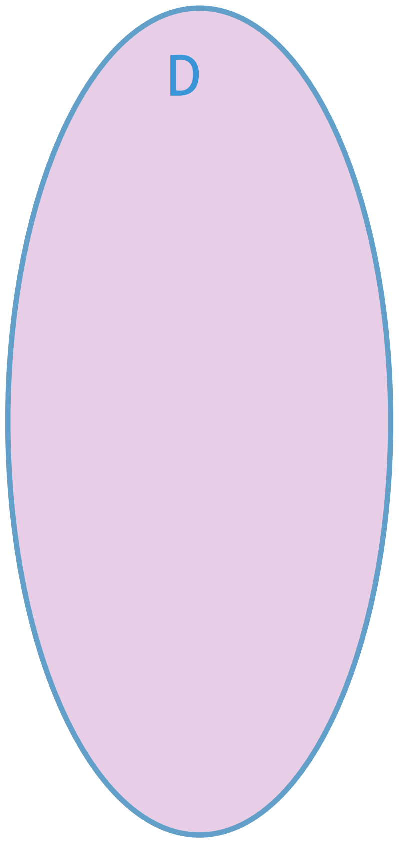

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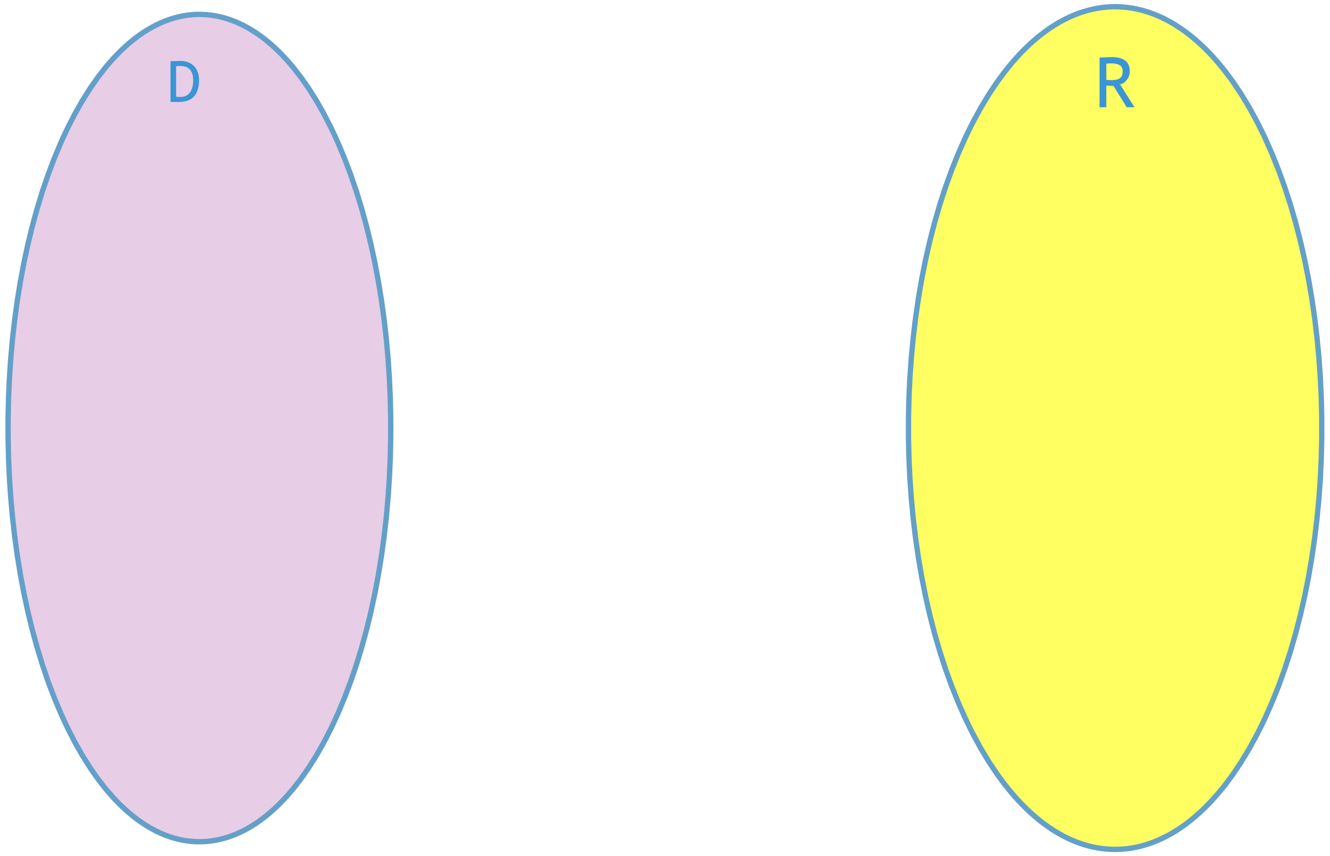

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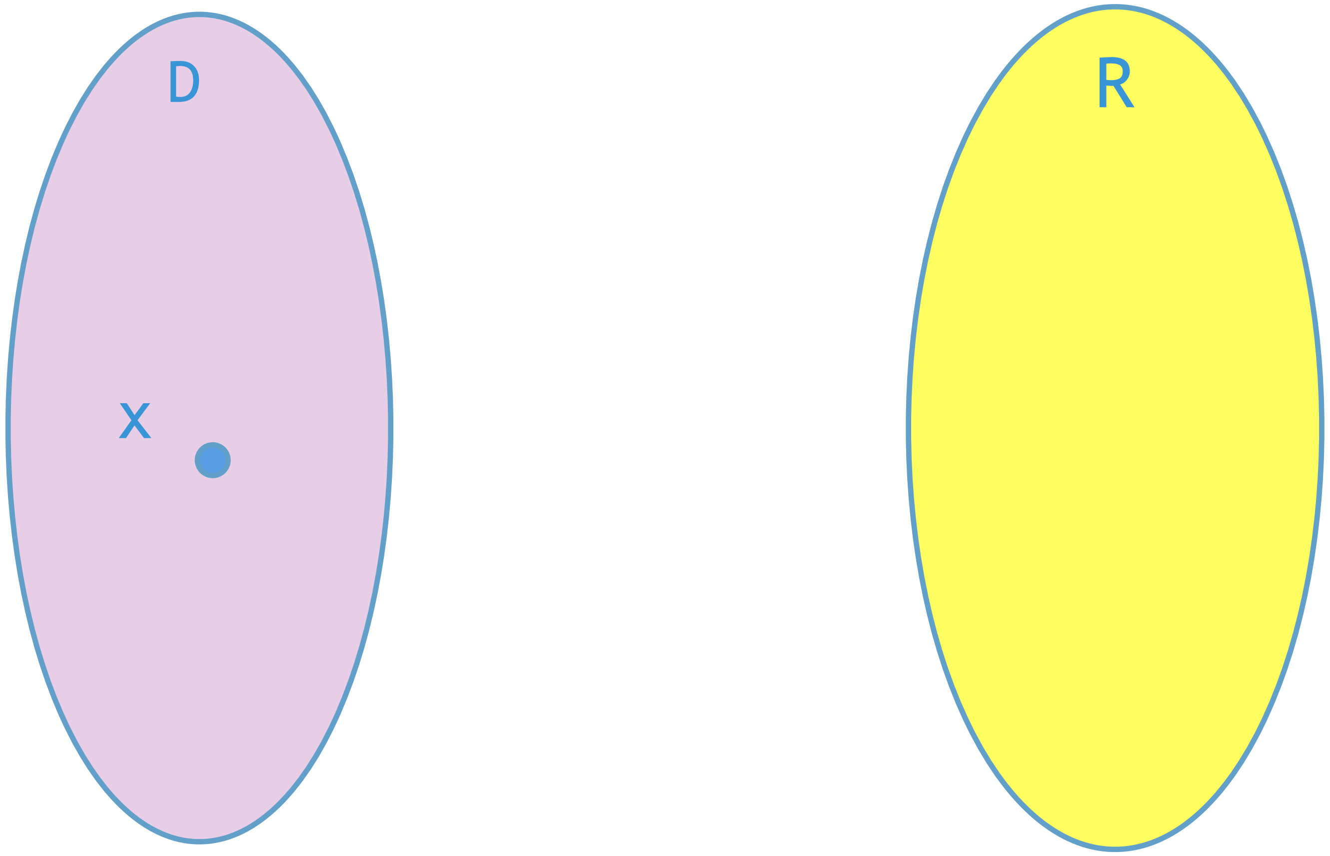

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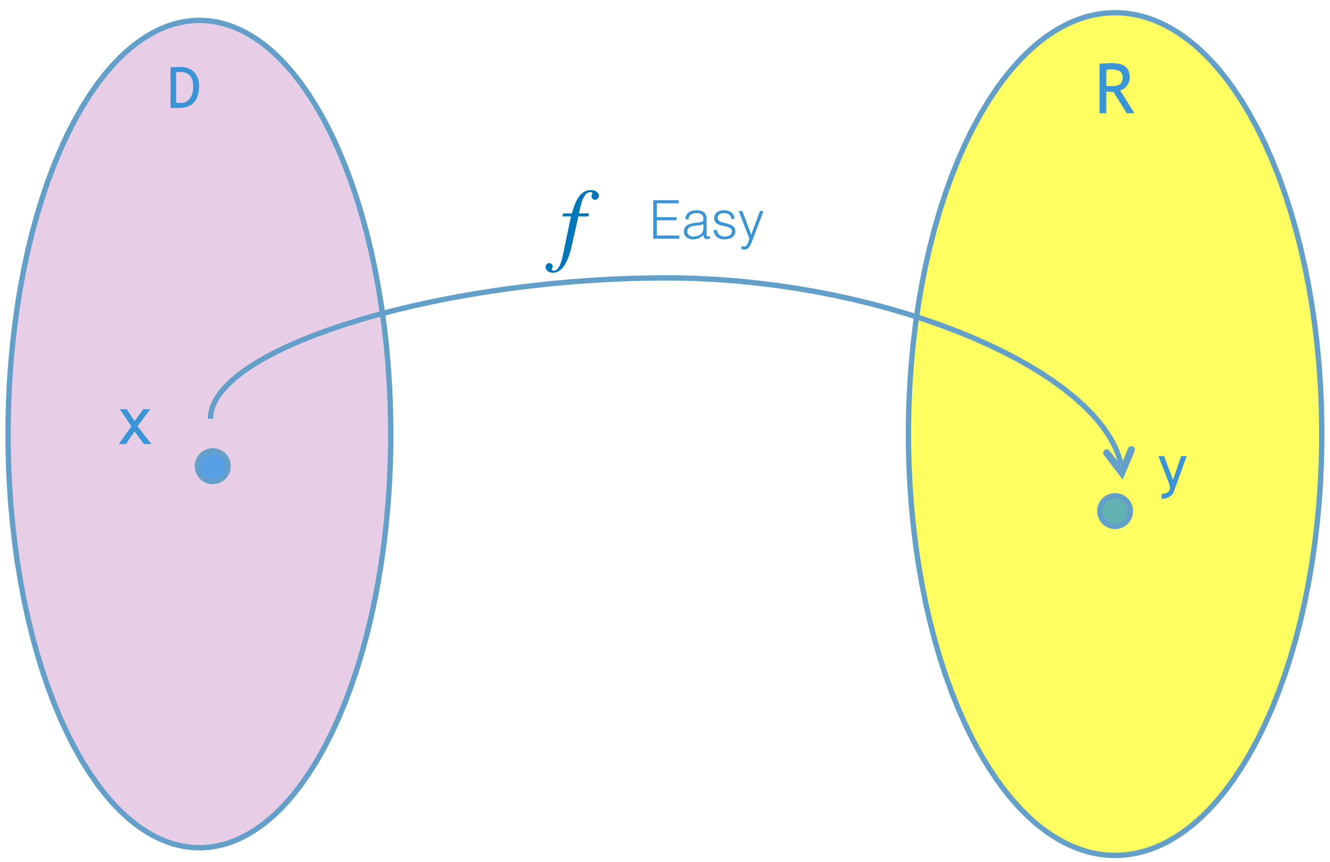

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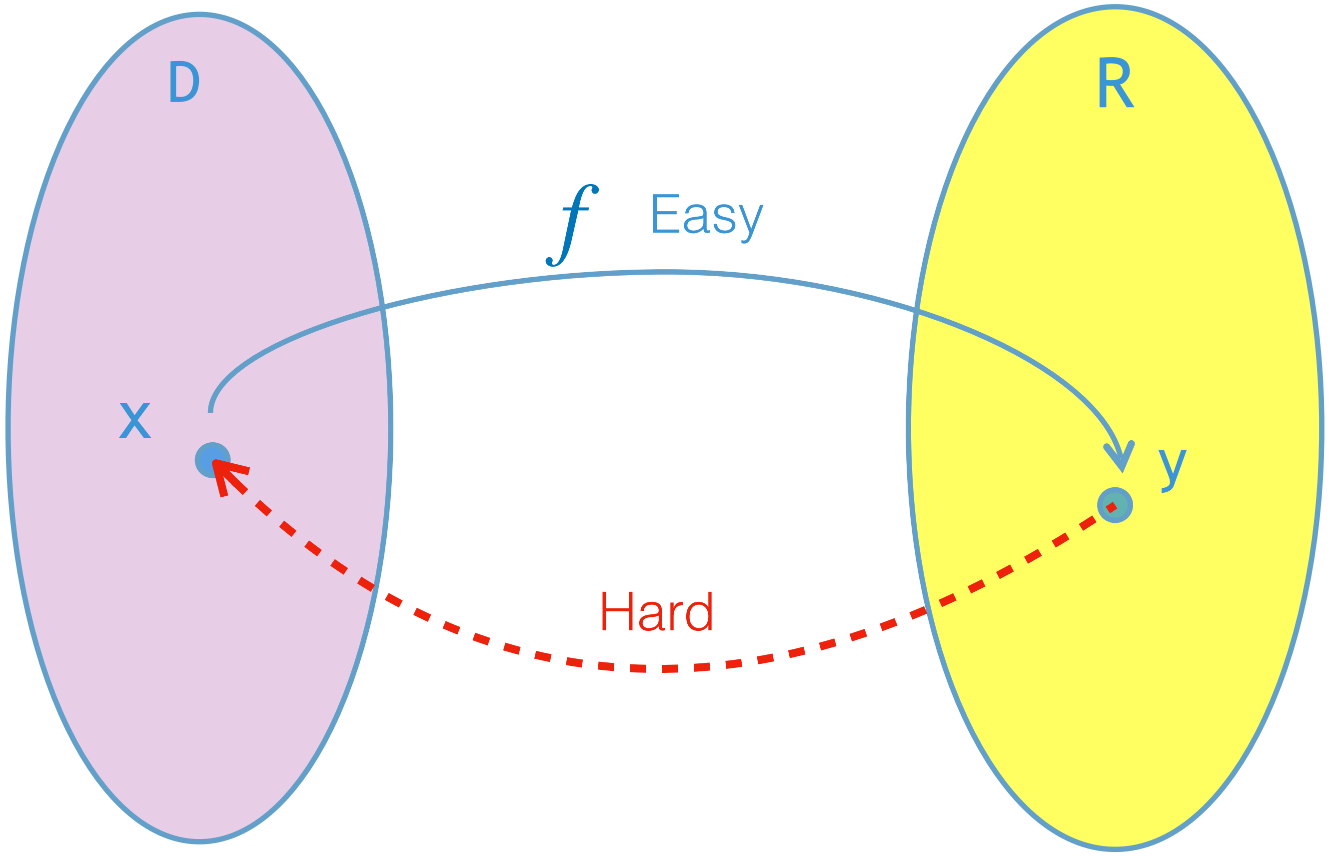

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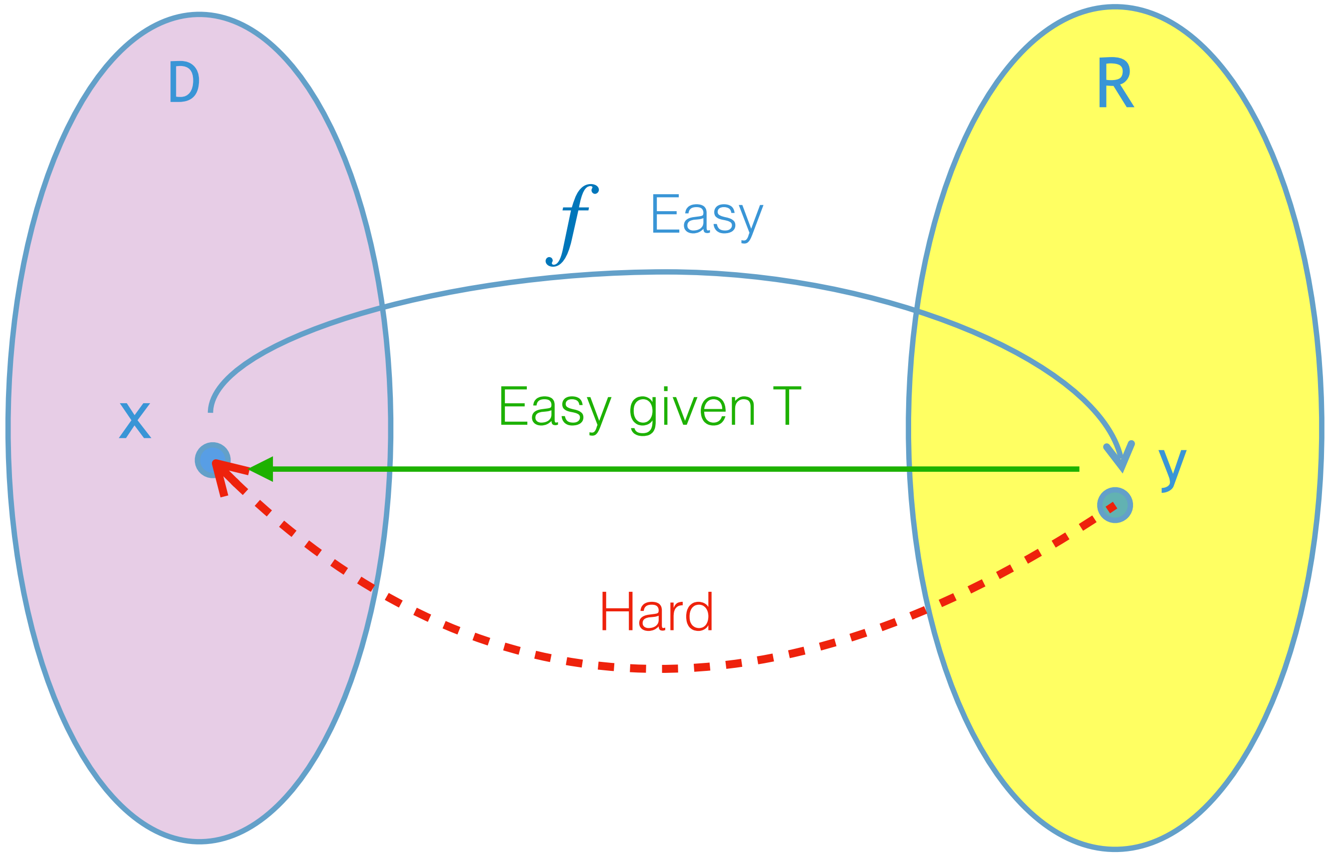
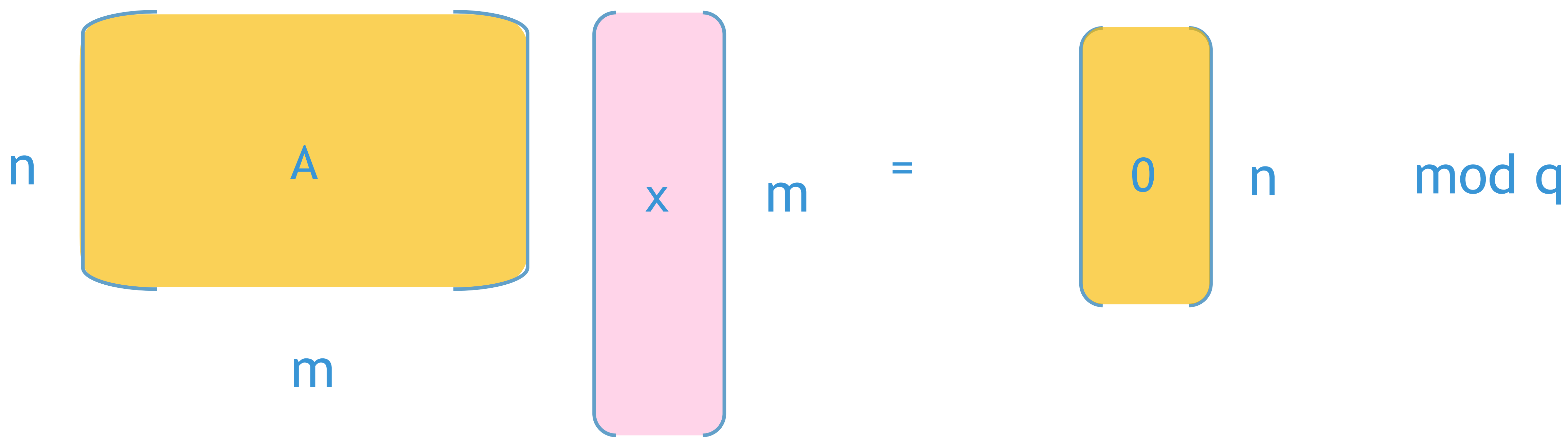


Short Integer Solution Problem

Let $\mathbf{A} \in \mathbb{Z}_q^{n\times m}$, $q = \mathsf{poly}(n)$, $m = \Omega(n \log q)$

Given matrix **A**, find "short" (low norm) vector **x** such that

$$\mathbf{A}\,\mathbf{x} = 0 \mod q \ \in \mathbb{Z}_q^n$$



Learning With Errors Problem

Distinguish "noisy inner products" from uniform

Fix uniform $s \in Z_q^n$

$a_1, b_1 = \langle a_1,s\rangle + e_1$ $a_2, b_2 = \langle a_2,s\rangle + e_2$ $\vdots$ $a_m, b_m = \langle a_m,s\rangle + e_m$	vs	a'_1, b'_1 a'_2, b'_2 $\vdots$ a'_m, b'_m
a_i uniform $\in Z_q^n$, $e_i \sim \phi \in Z_q$		a_i uniform $\in Z_q^n$, b_i uniform $\in Z_q$

Lattice Based One Way Functions

Public Key $\mathbf{A} \in \mathbb{Z}_q^{n \times m}$, $q = \text{poly}(n)$, $m = \Omega(n \log q)$

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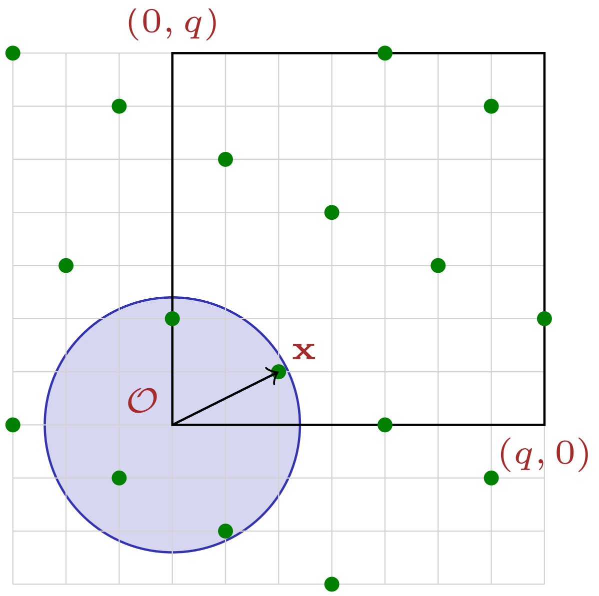
Based on SIS

$$f_{\mathbf{A}}(\mathbf{x}) = \mathbf{A}\,\mathbf{x} \mod q \in \mathbb{Z}_q^n$$

- Short x, surjective
- CRHF if SIS is hard [Ajt96…]



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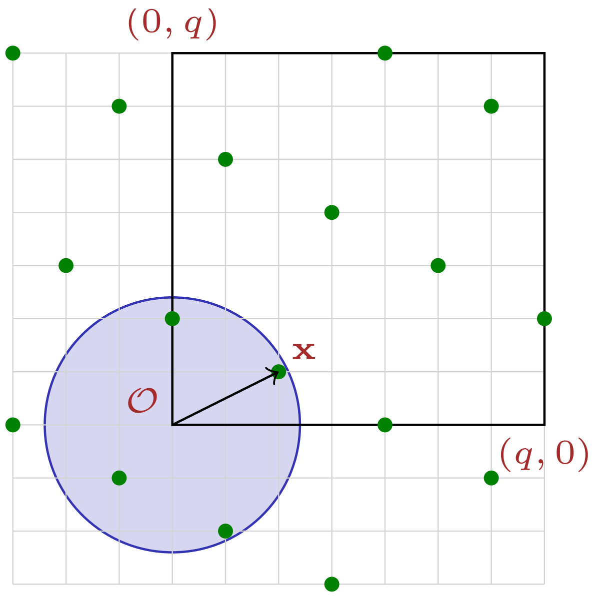


Based on LWE

$$g_{\mathbf{A}}(\mathbf{s}, \mathbf{e}) = \mathbf{s}^t\mathbf{A} + \mathbf{e}^t \mod q \in \mathbb{Z}_q^m$$

- Very short e, injective
- OWF if LWE is hard [Reg05…]

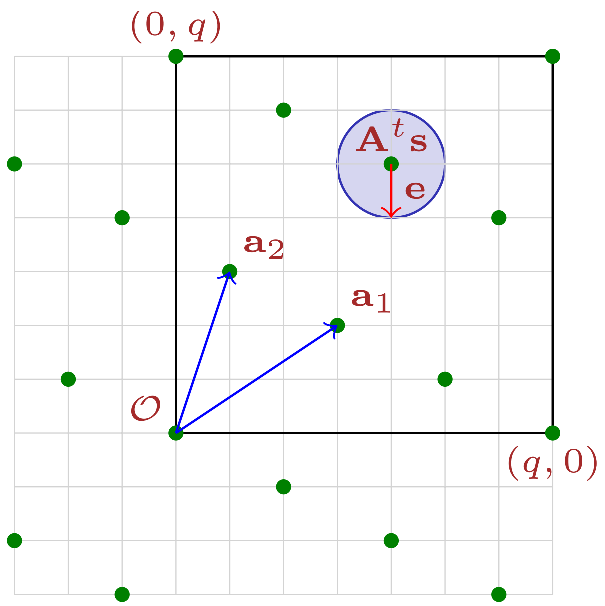
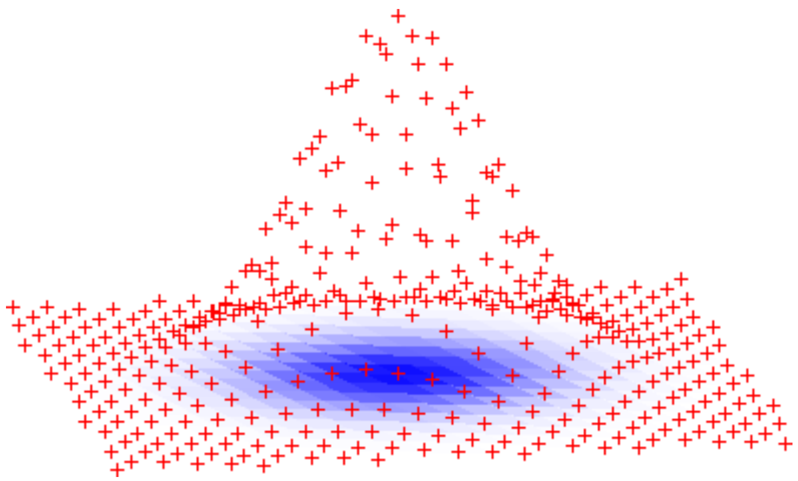


Image Credit: MP12 slides

Inverting functions for Crypto

- Given $\mathbf{u} = f_{\mathbf{A}}(\mathbf{x}) = \mathbf{A}\ \mathbf{x} \mod q$
- Sample
$$\mathbf{x}' \leftarrow = f_{\mathbf{A}}^{-1}(\mathbf{u})$$
with prob $\propto \exp(-\|\mathbf{x}'\|^2/\sigma^2)$



And

- Given $g_{\mathbf{A}}(\mathbf{s}, \mathbf{e}) = \mathbf{s}^t\mathbf{A} + \mathbf{e}^t \mod q$
- Find unique $(\mathbf{s}, \mathbf{e})$

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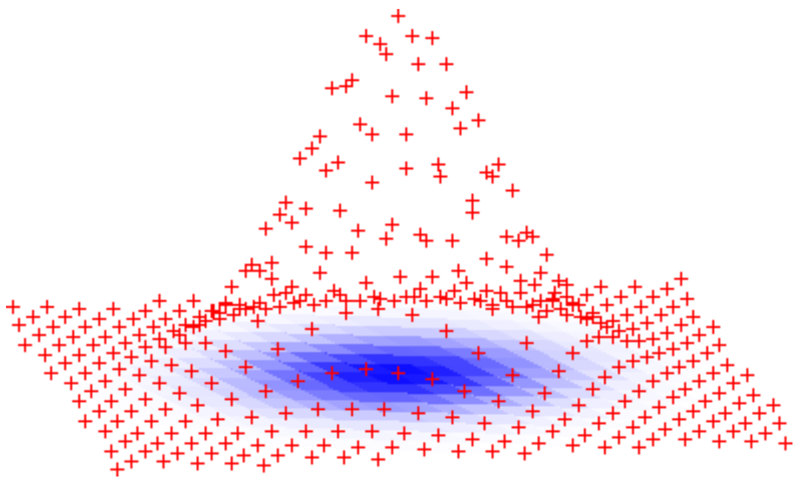

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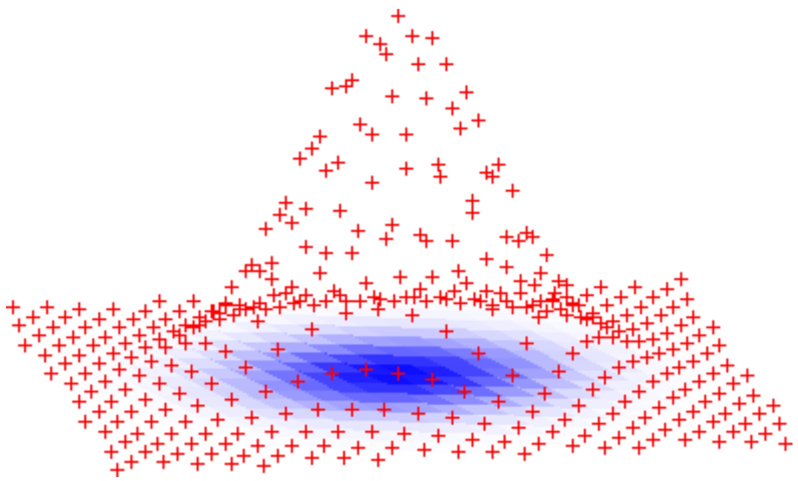

Preimage Sampleable Trapdoor Functions!

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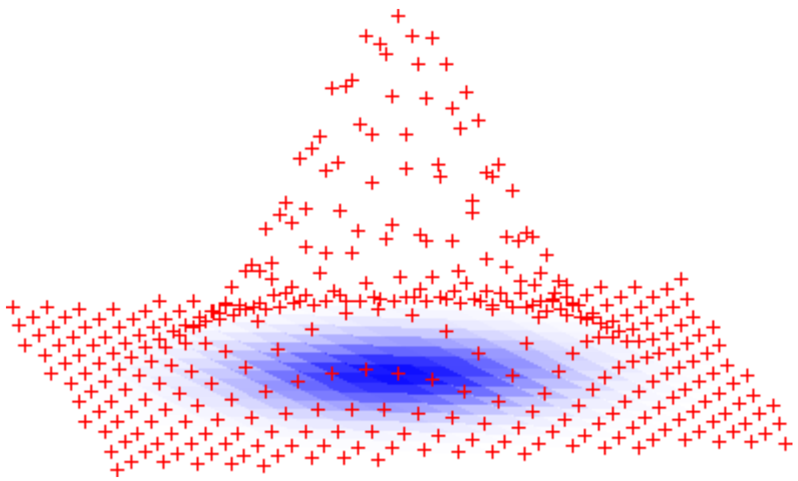

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Generate (x, y) in two equivalent ways

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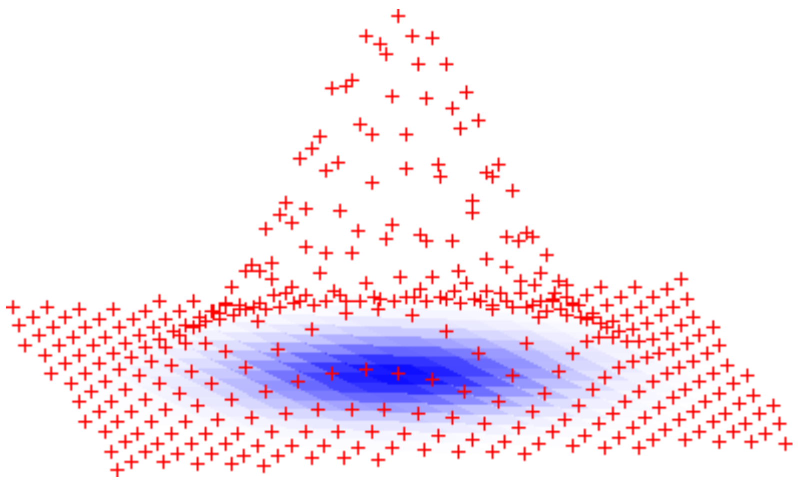

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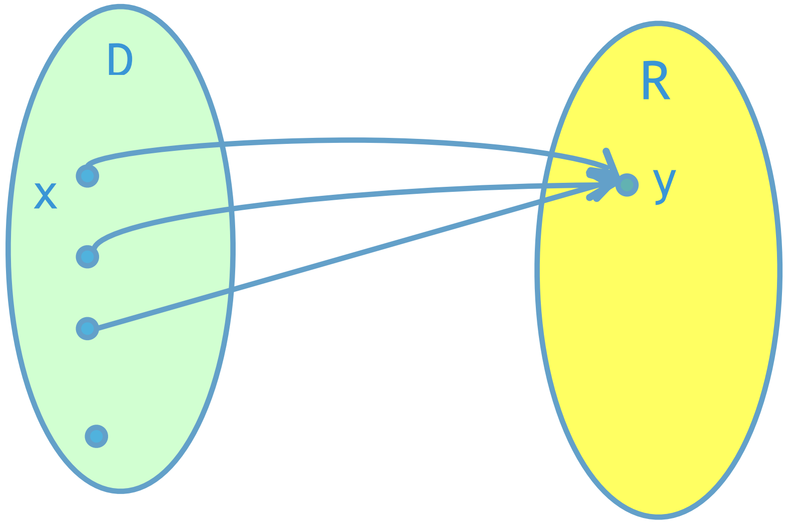

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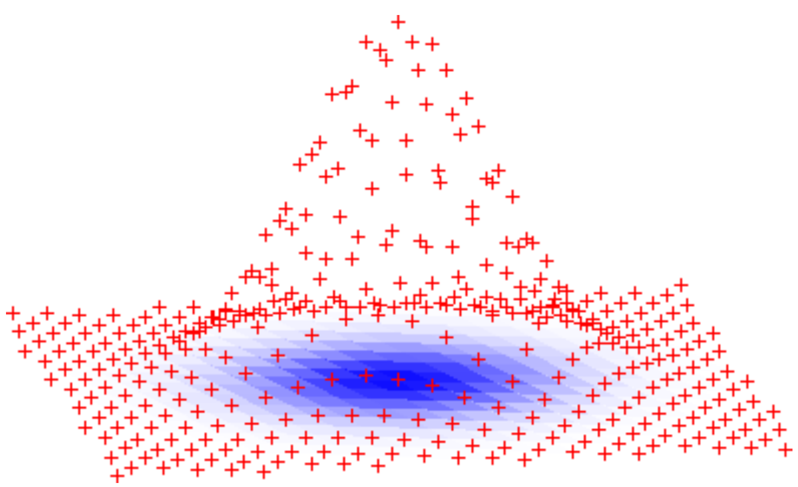

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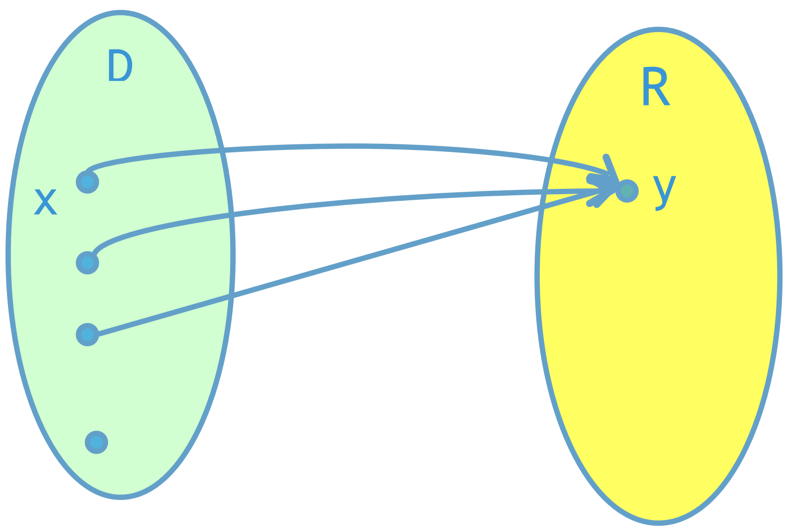

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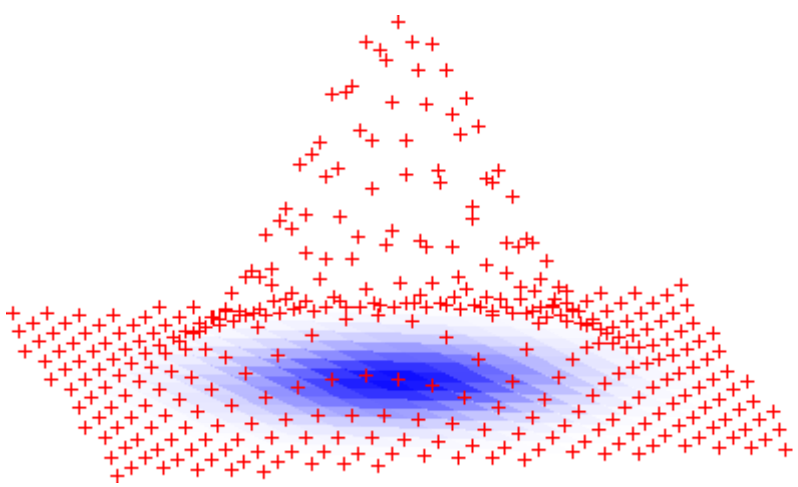

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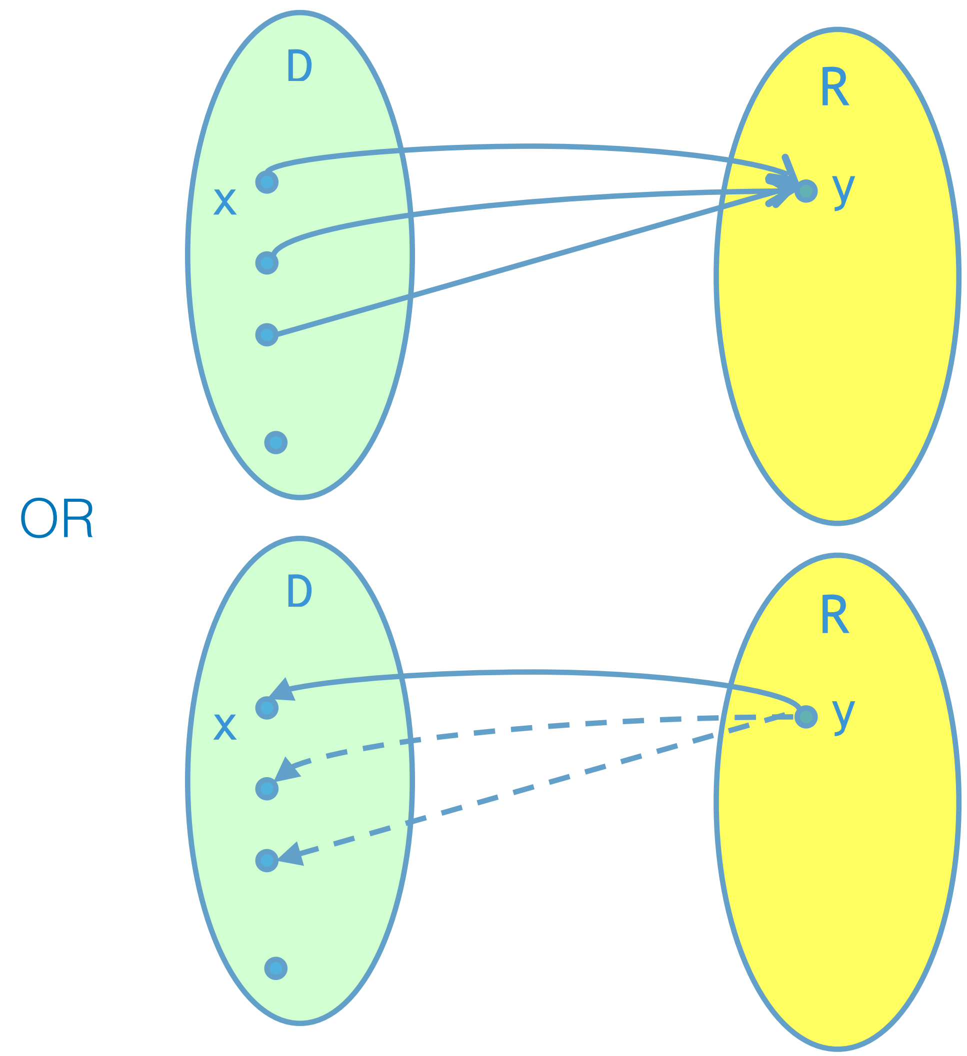

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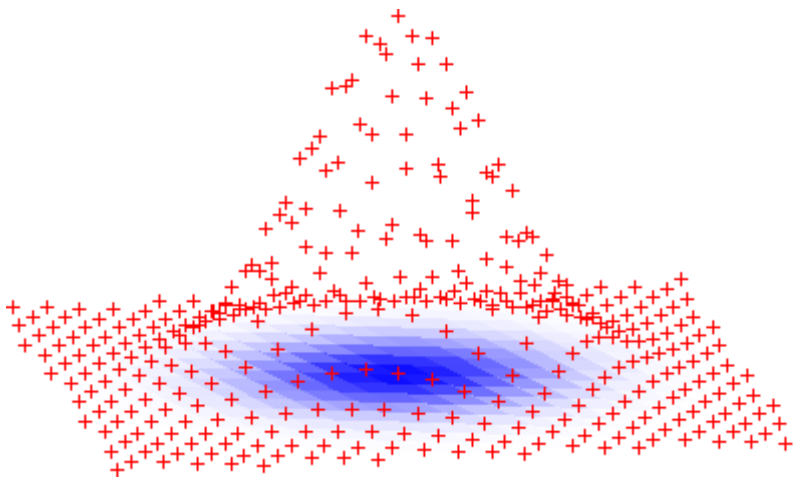

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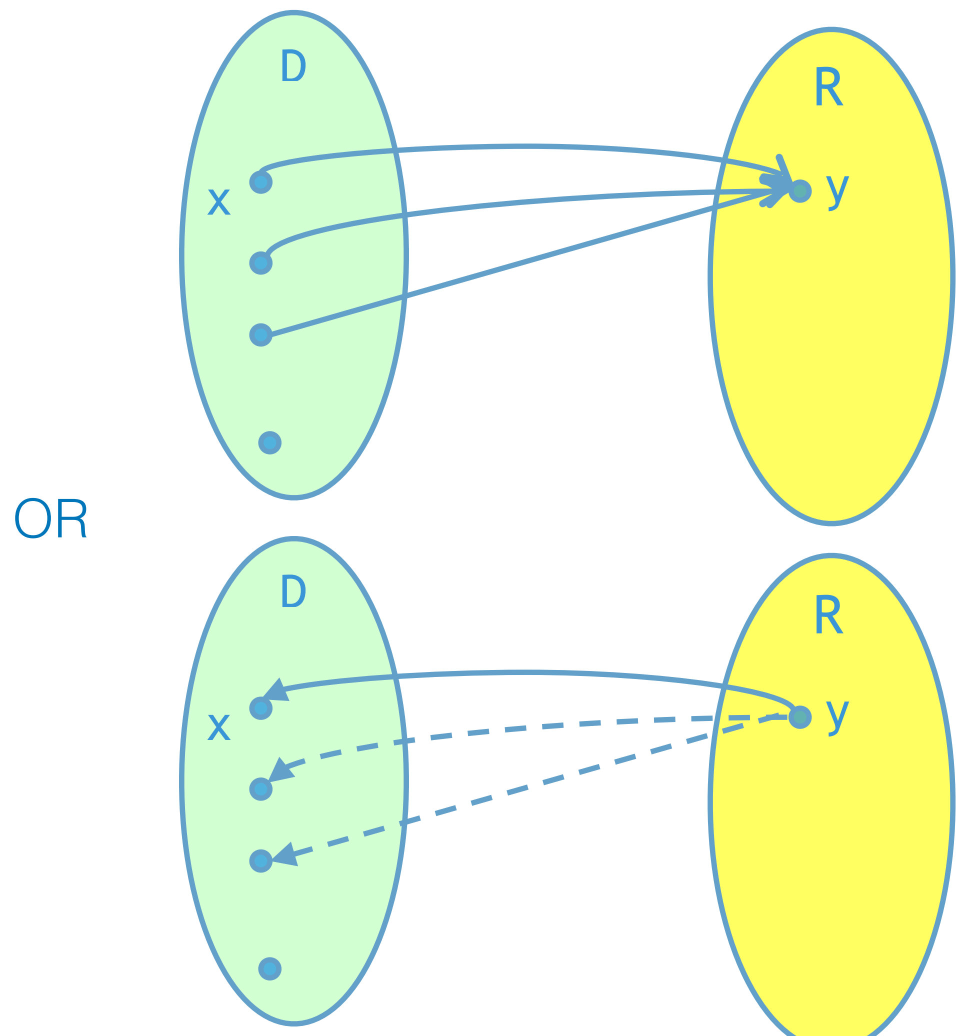

Same Distribution (Discrete Gaussian, Uniform) !

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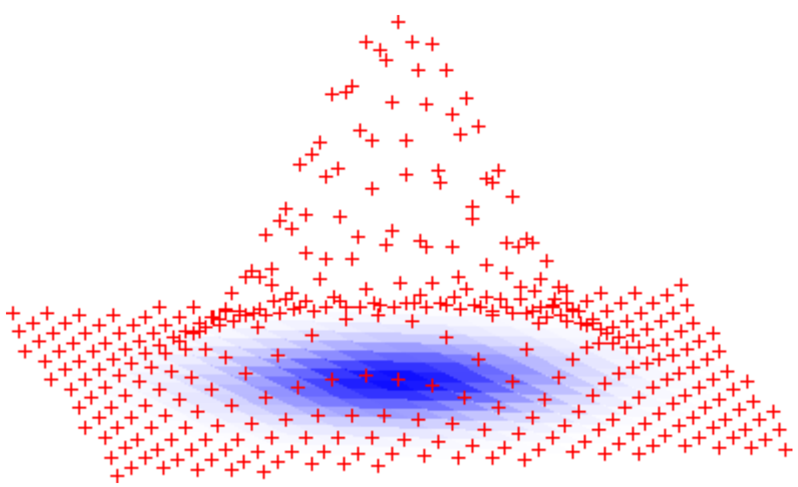

Latter distribution needs lattice trapdoors!

Preimage Sampleable Trapdoor Functions!

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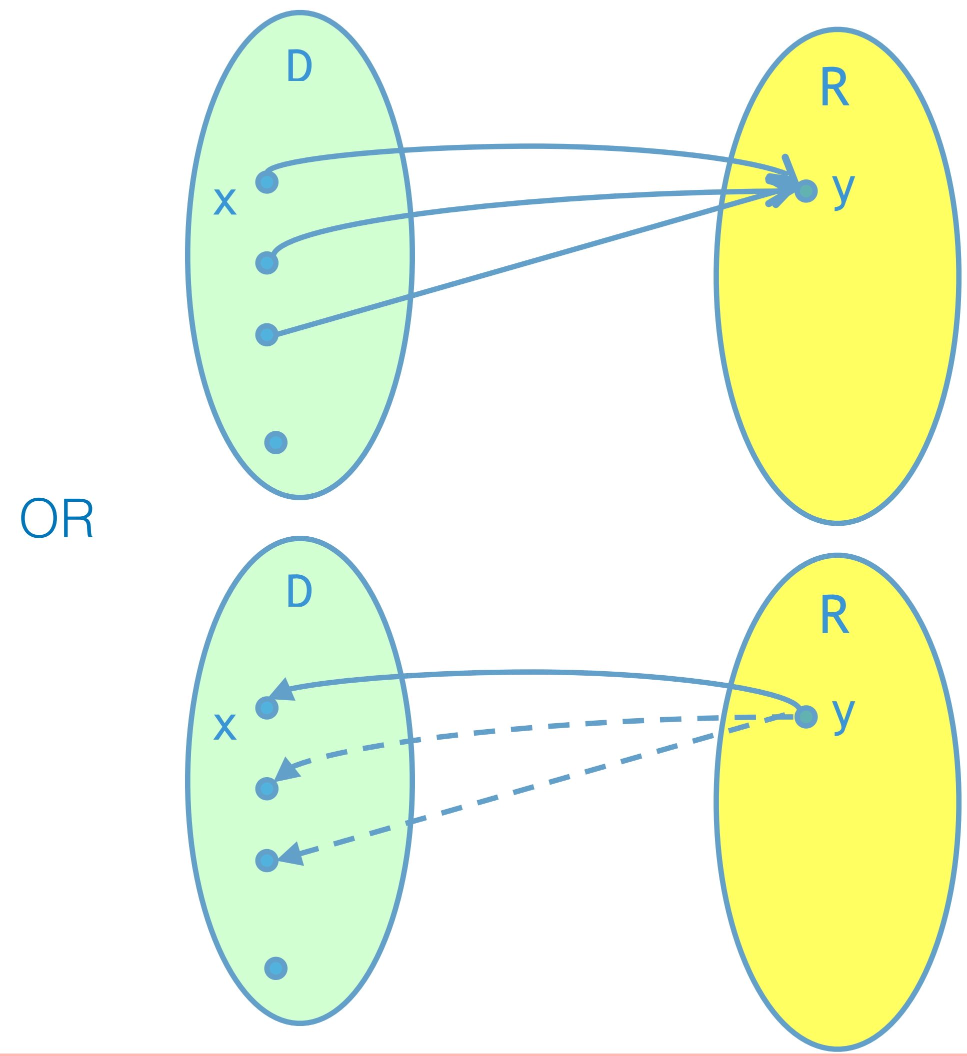
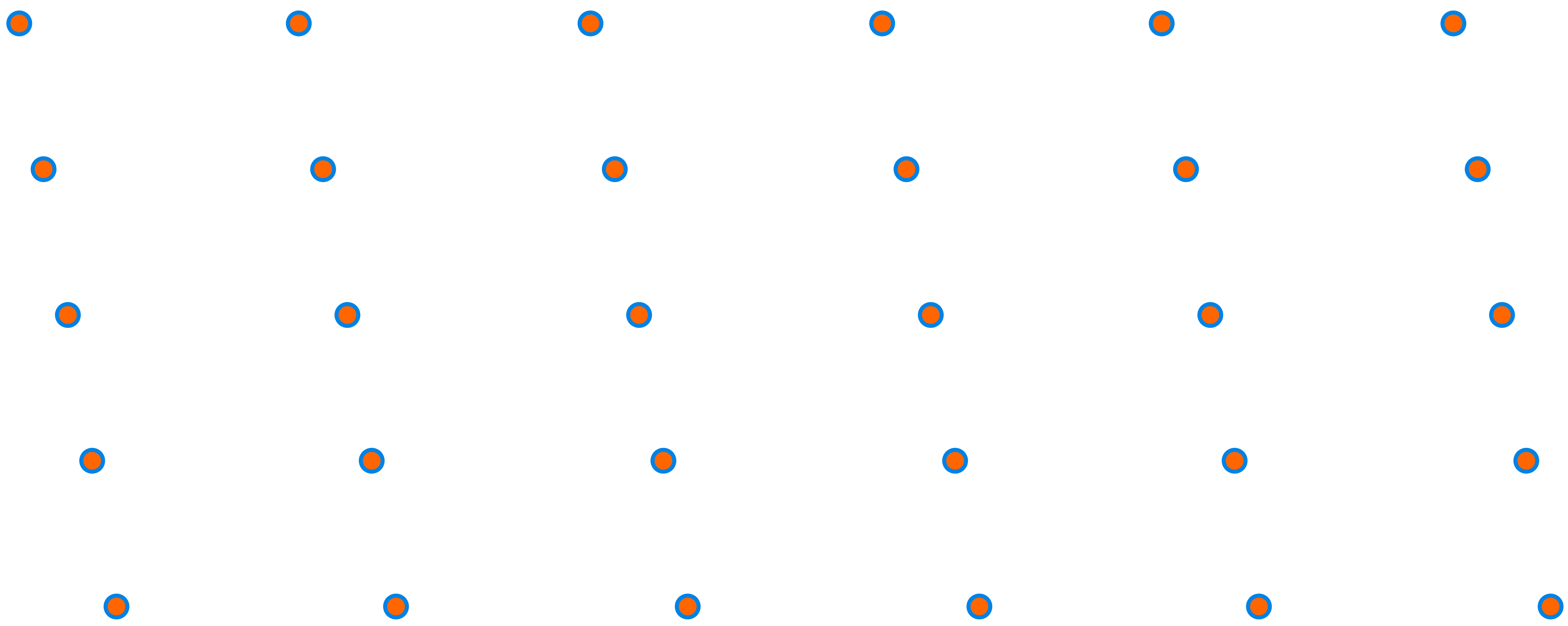


Same Distribution (Discrete Gaussian, Uniform) !


What do these trapdoors look like?

Lattice Trapdoors (Type 1): Geometric View



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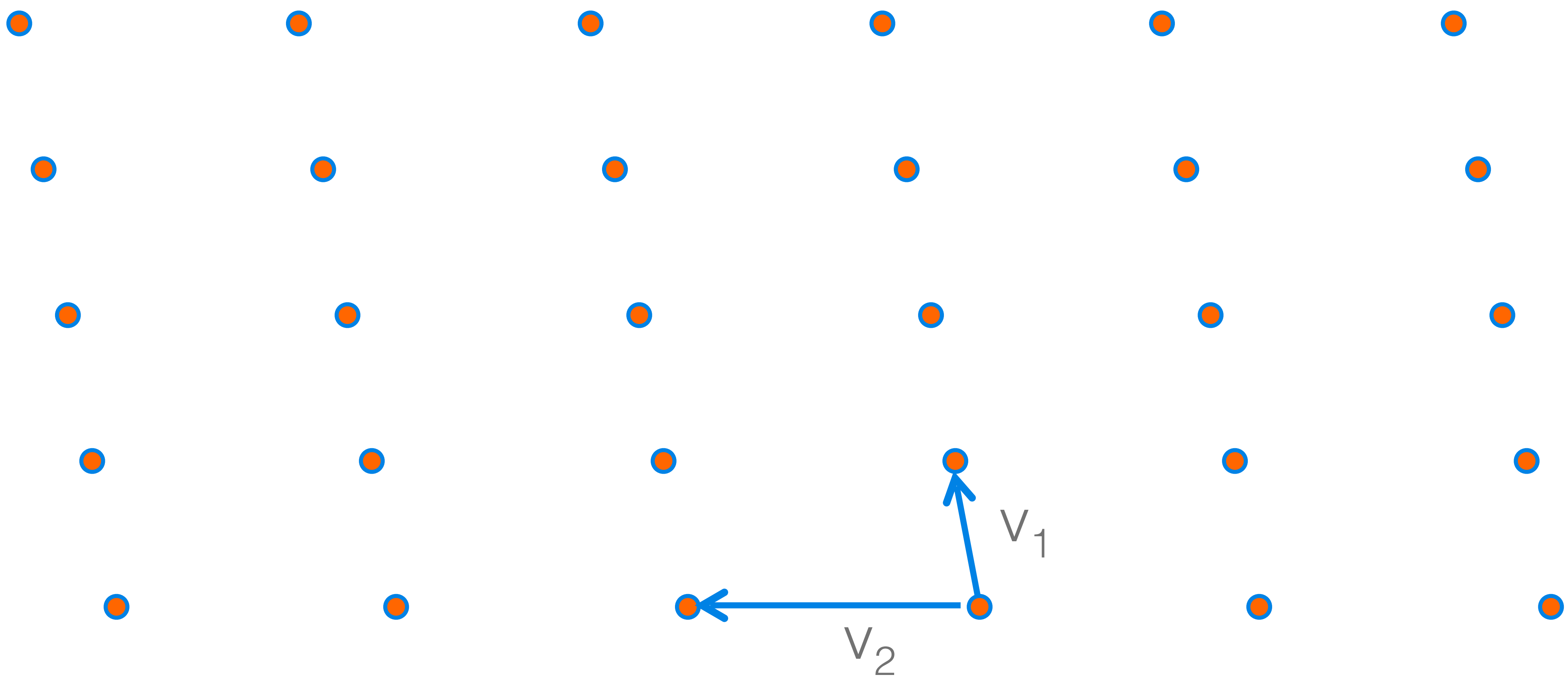

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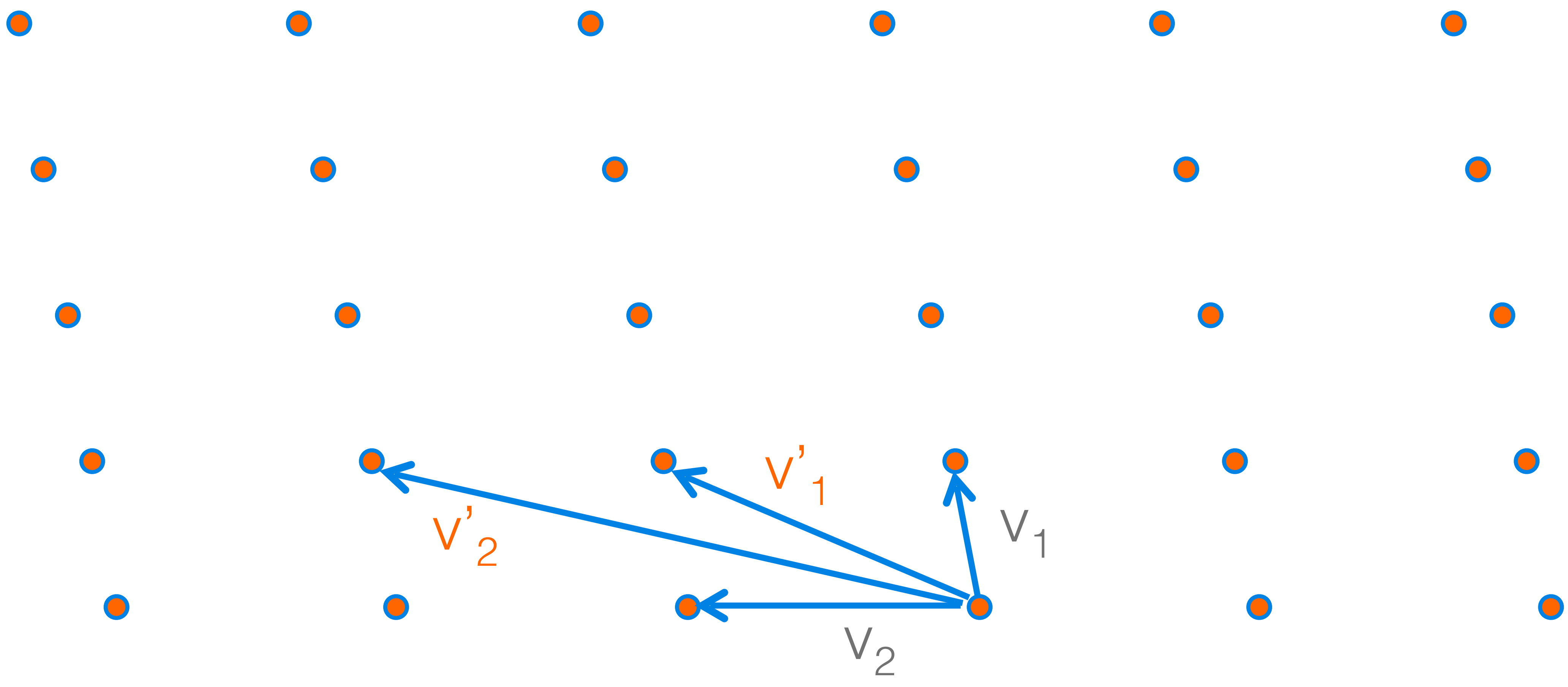

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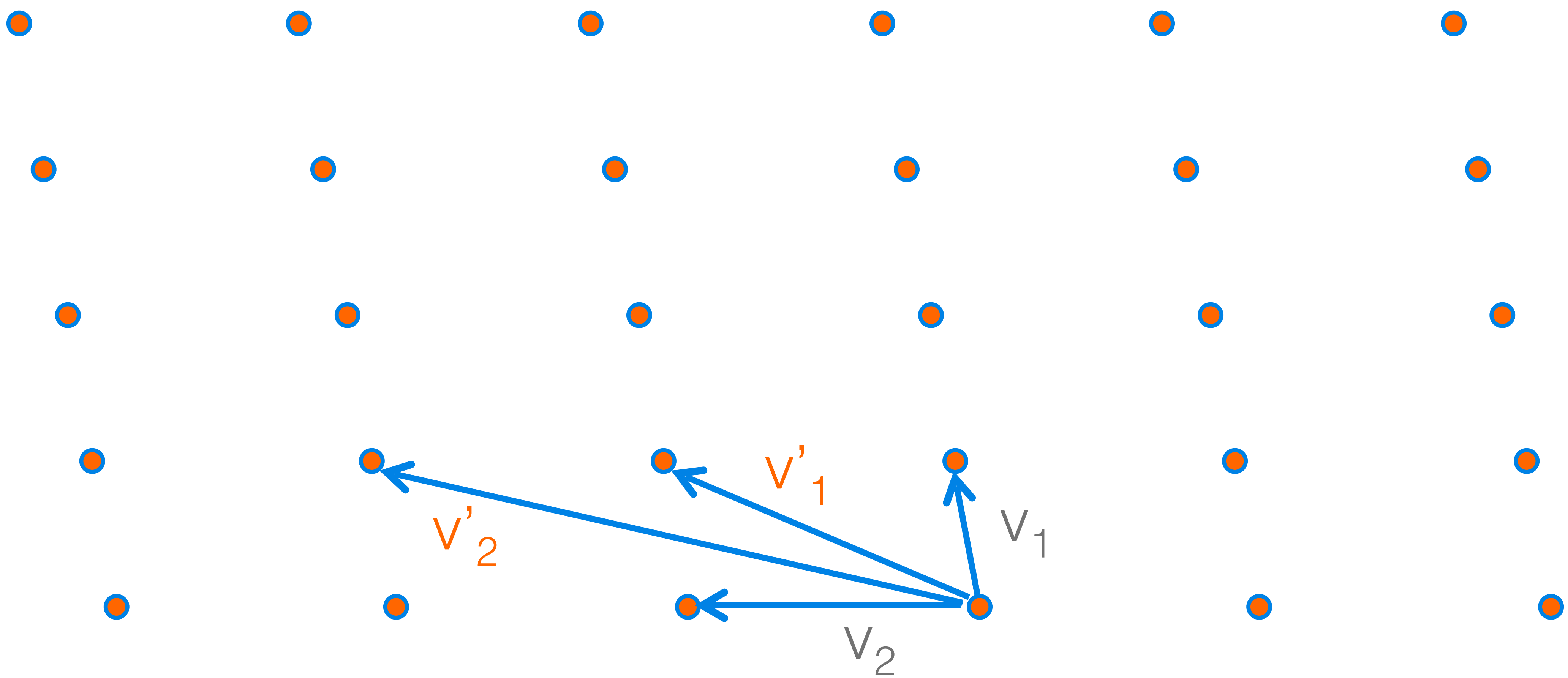
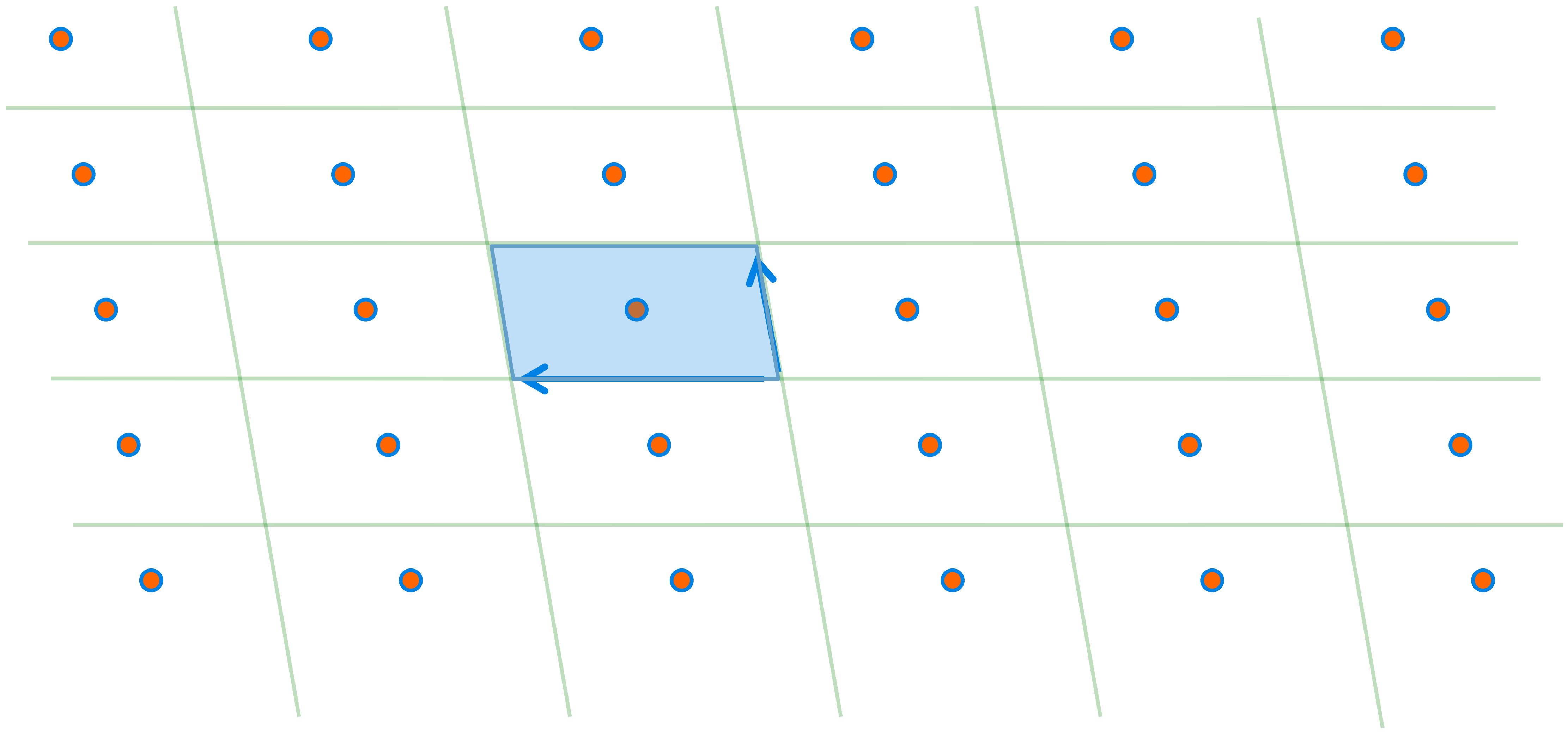


Multiple Bases

Parallelopipeds



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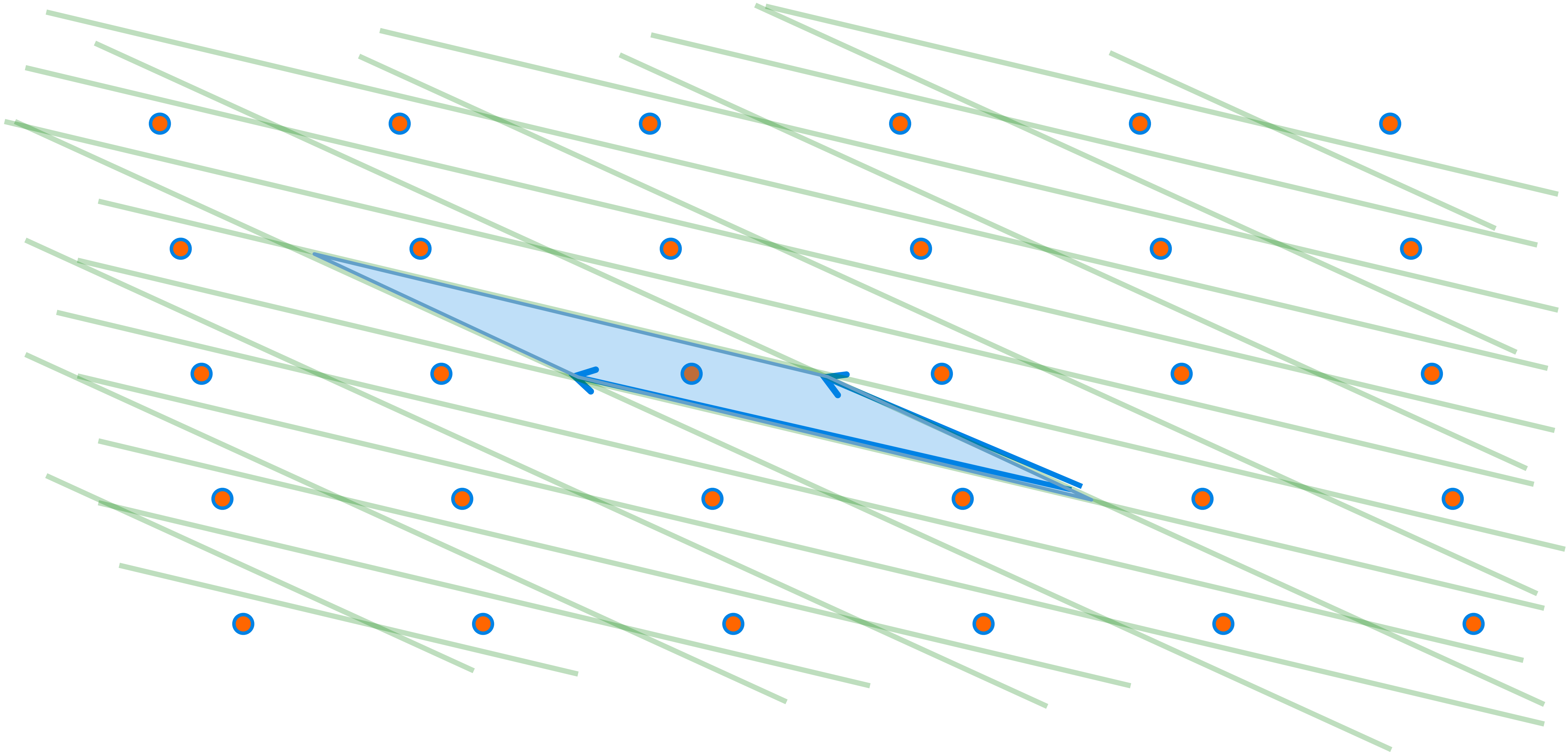
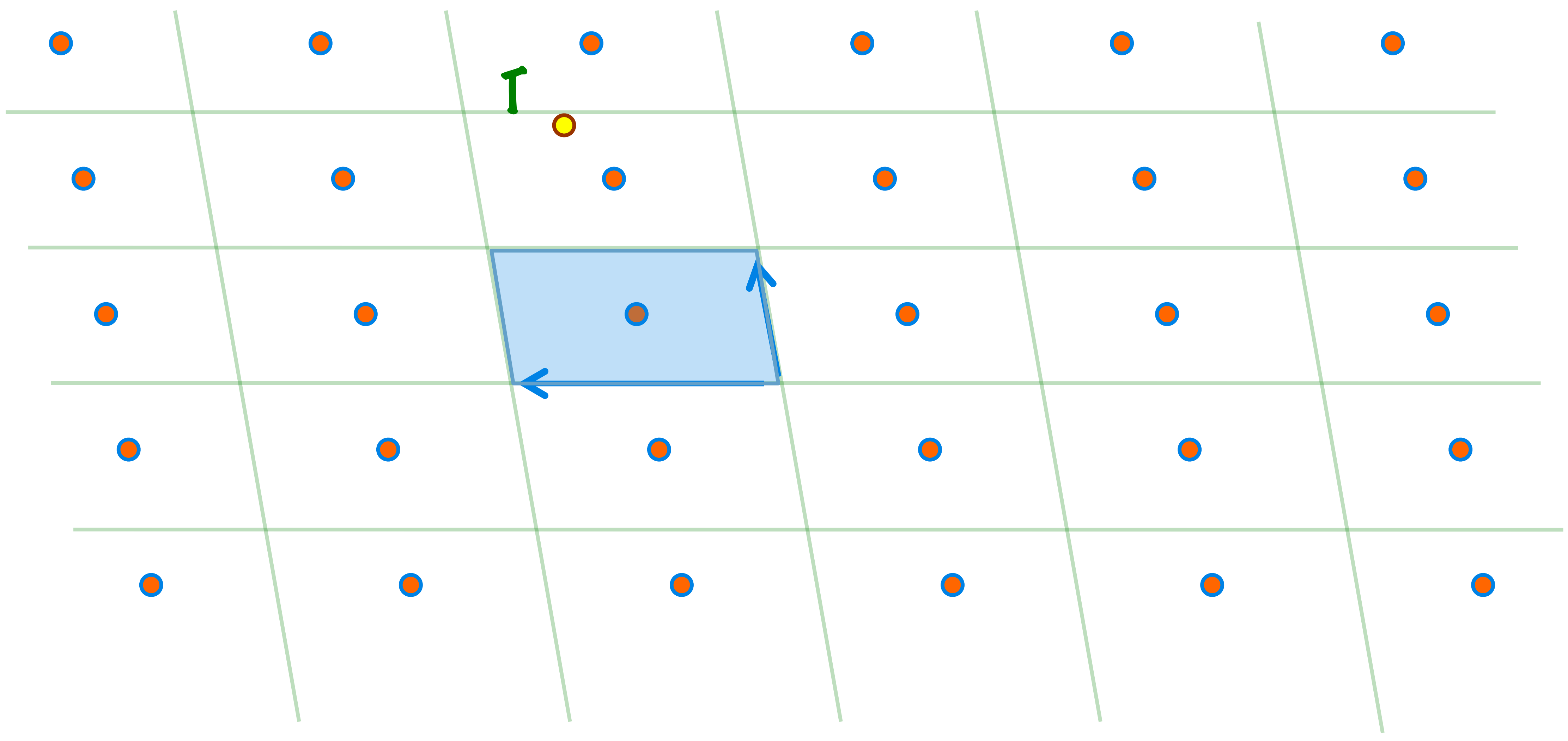


Good Basis



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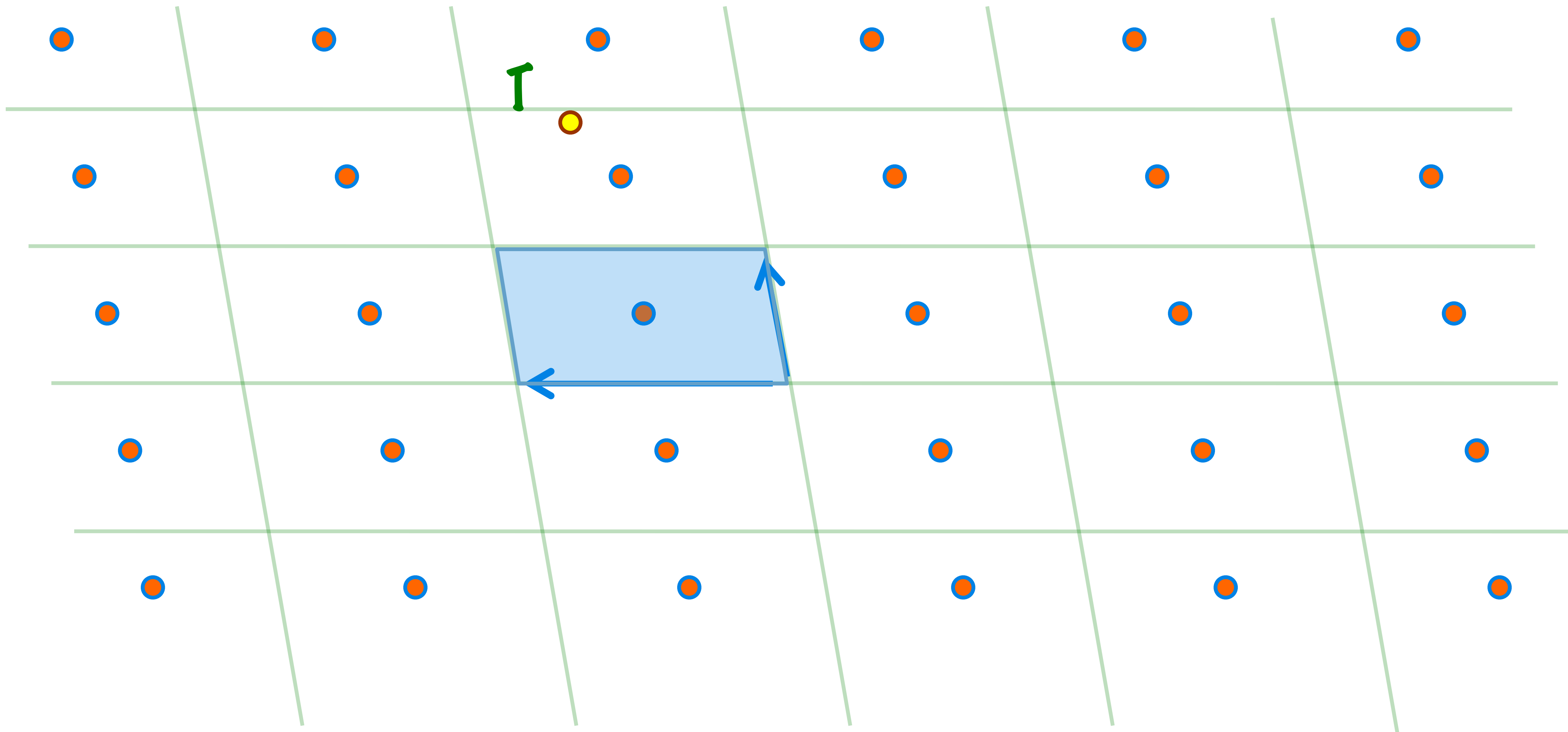

“Quite short” and “nearly orthogonal”

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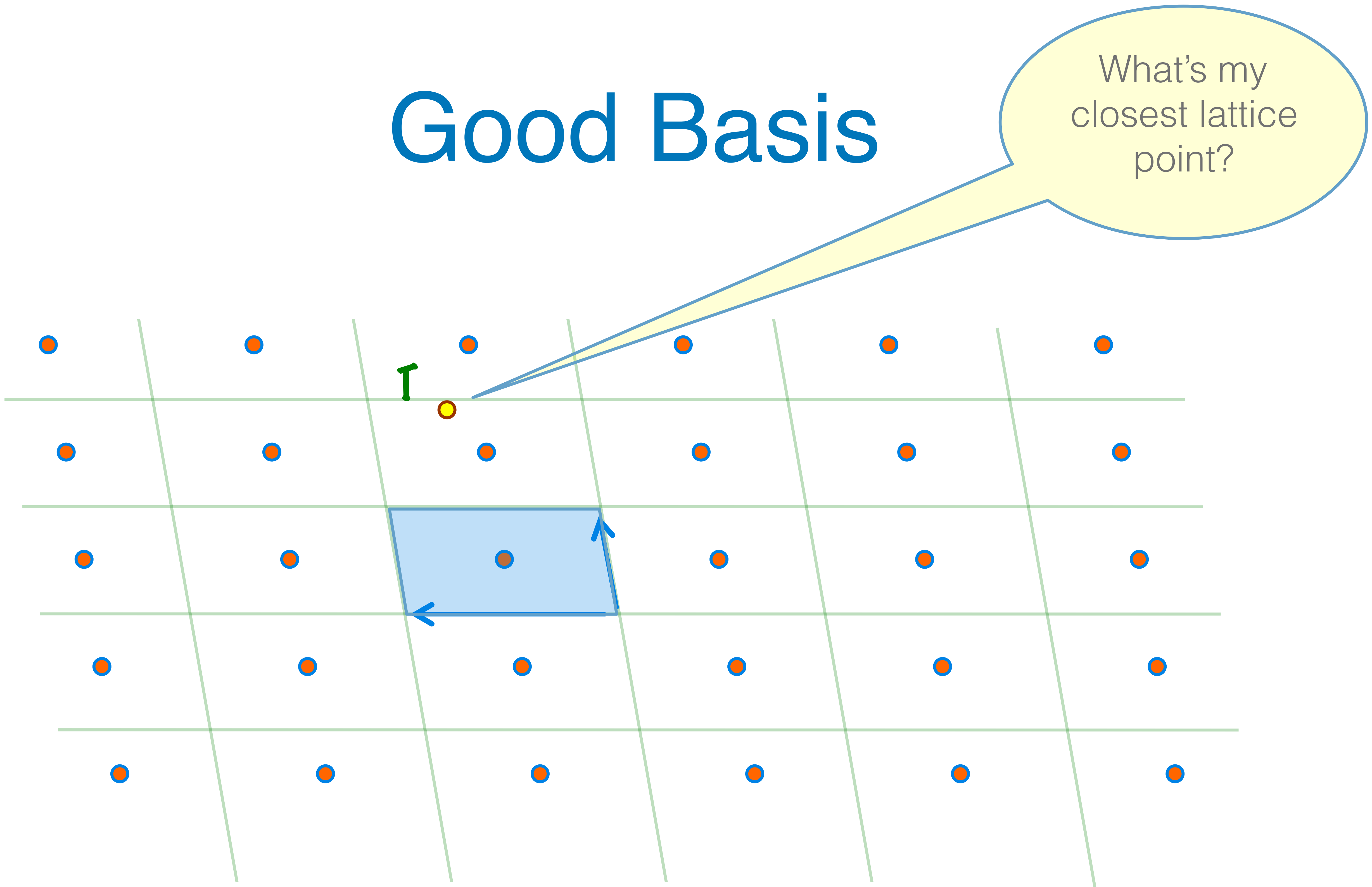

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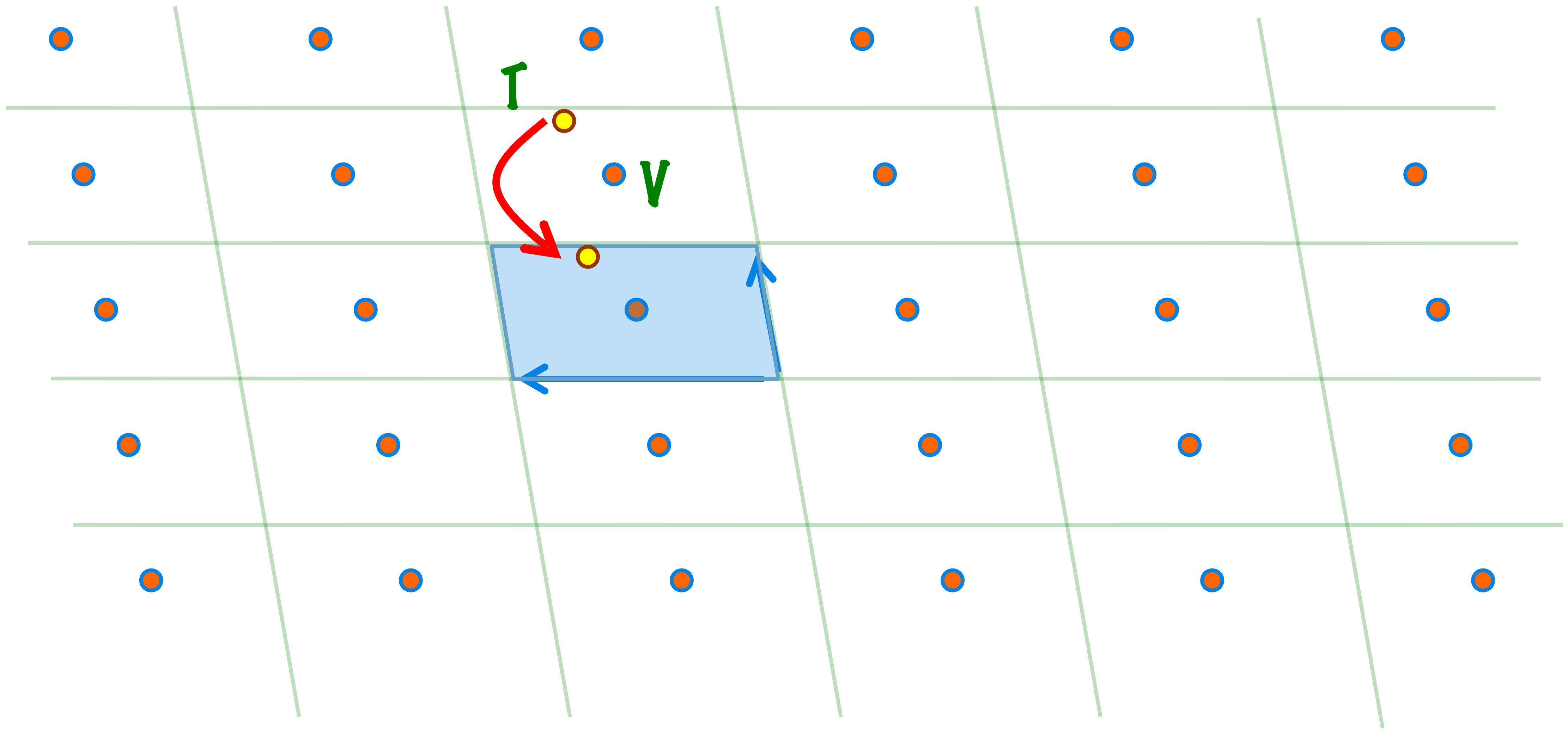

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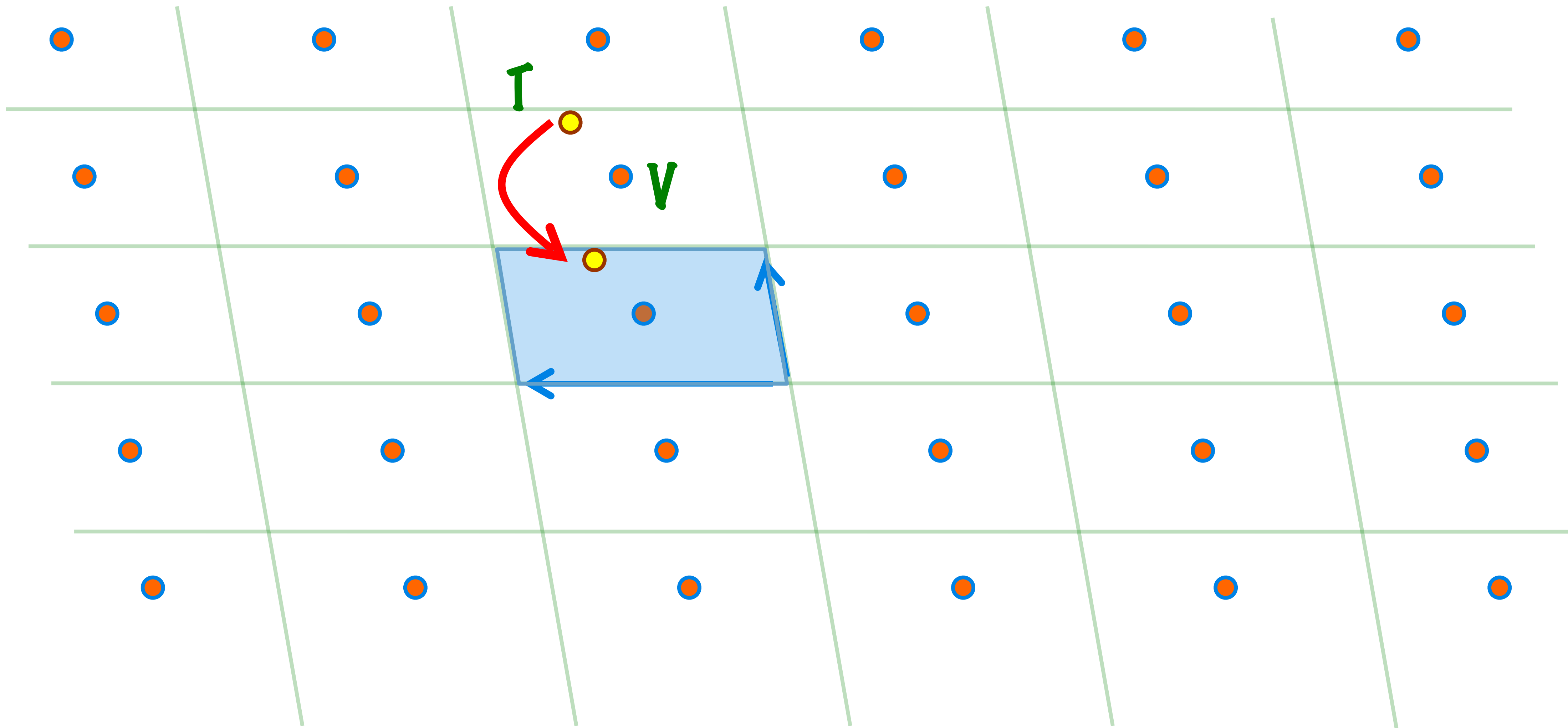

Output center of parallelopipid containing T

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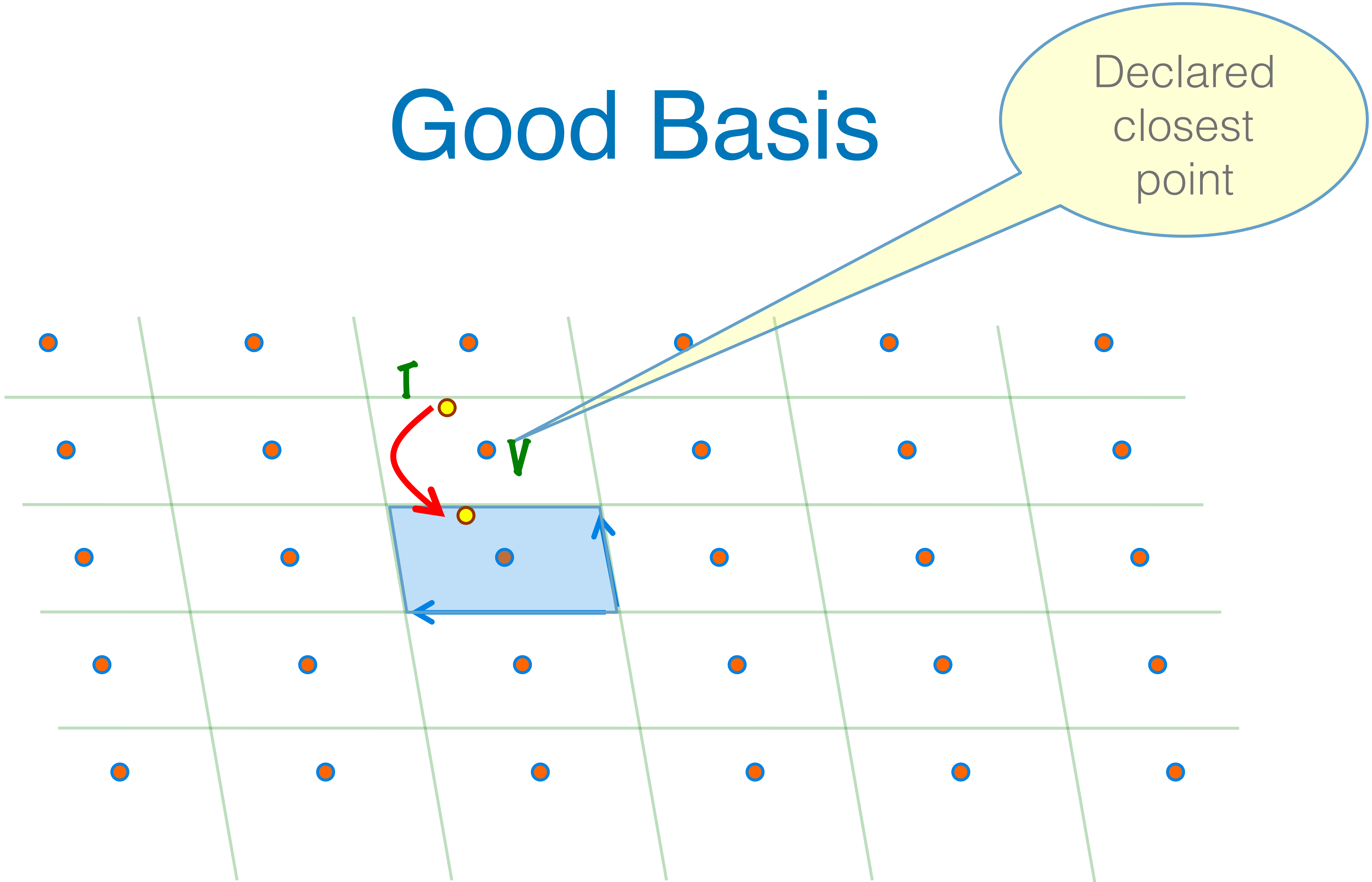

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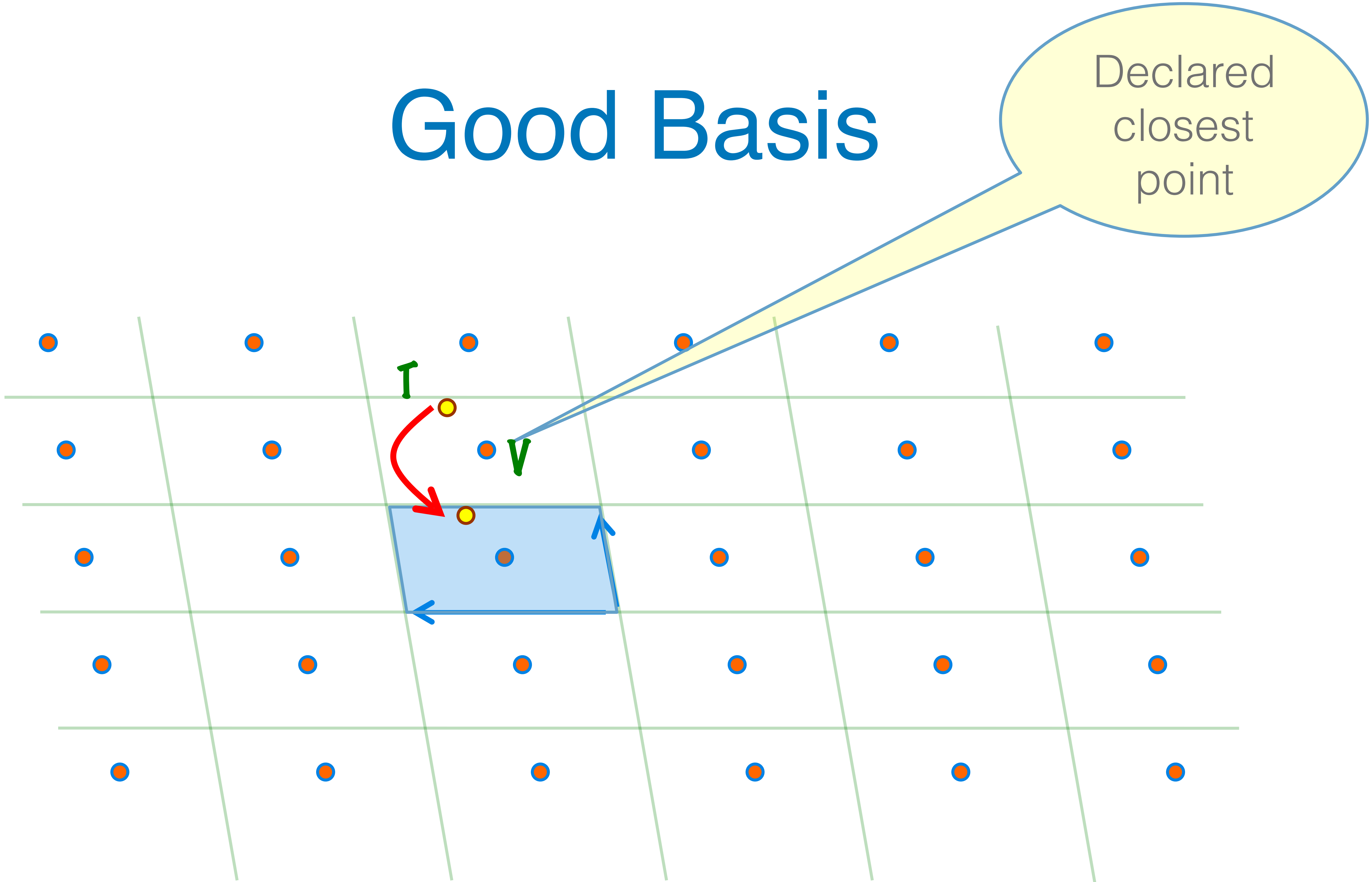

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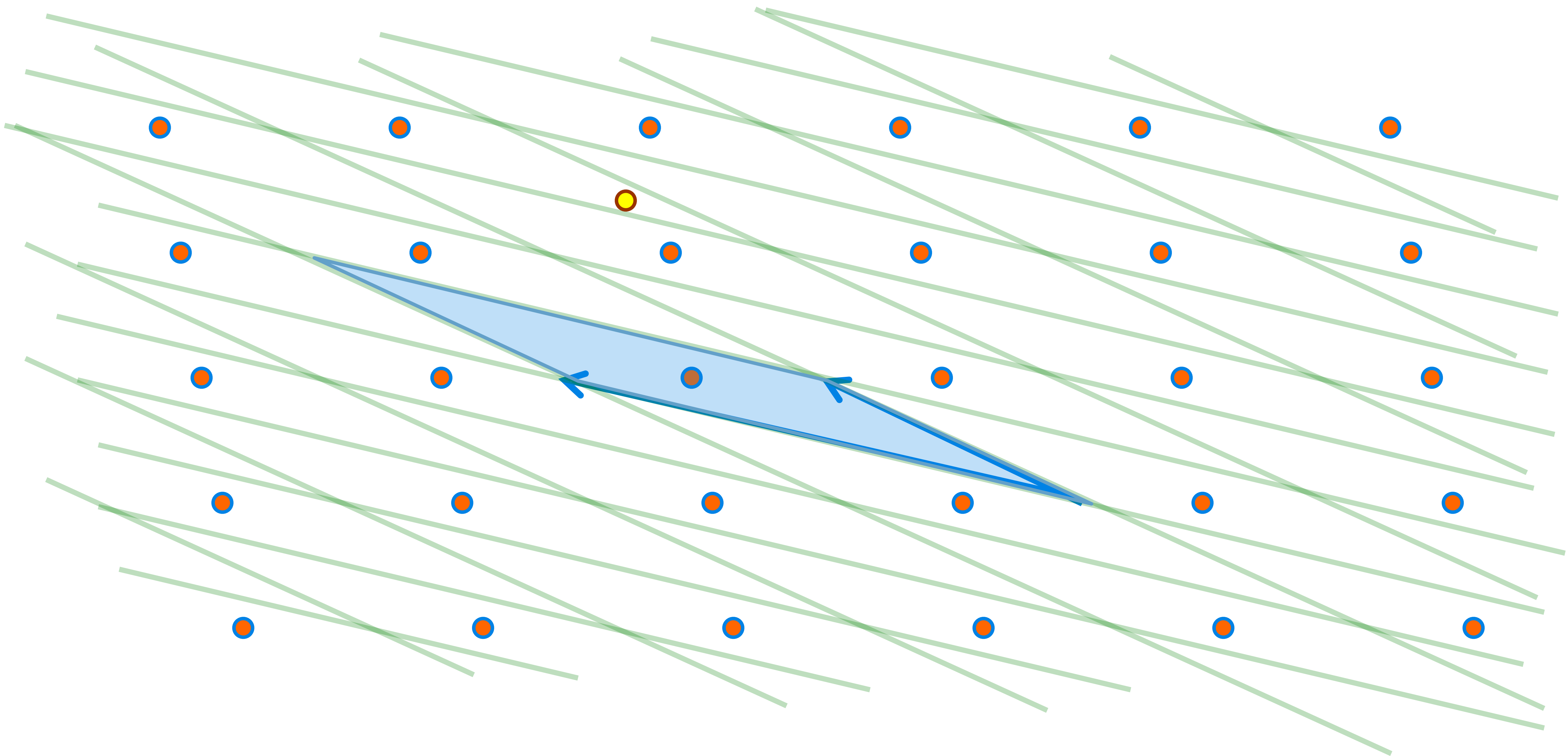
Pretty Accurate…

Bad Basis



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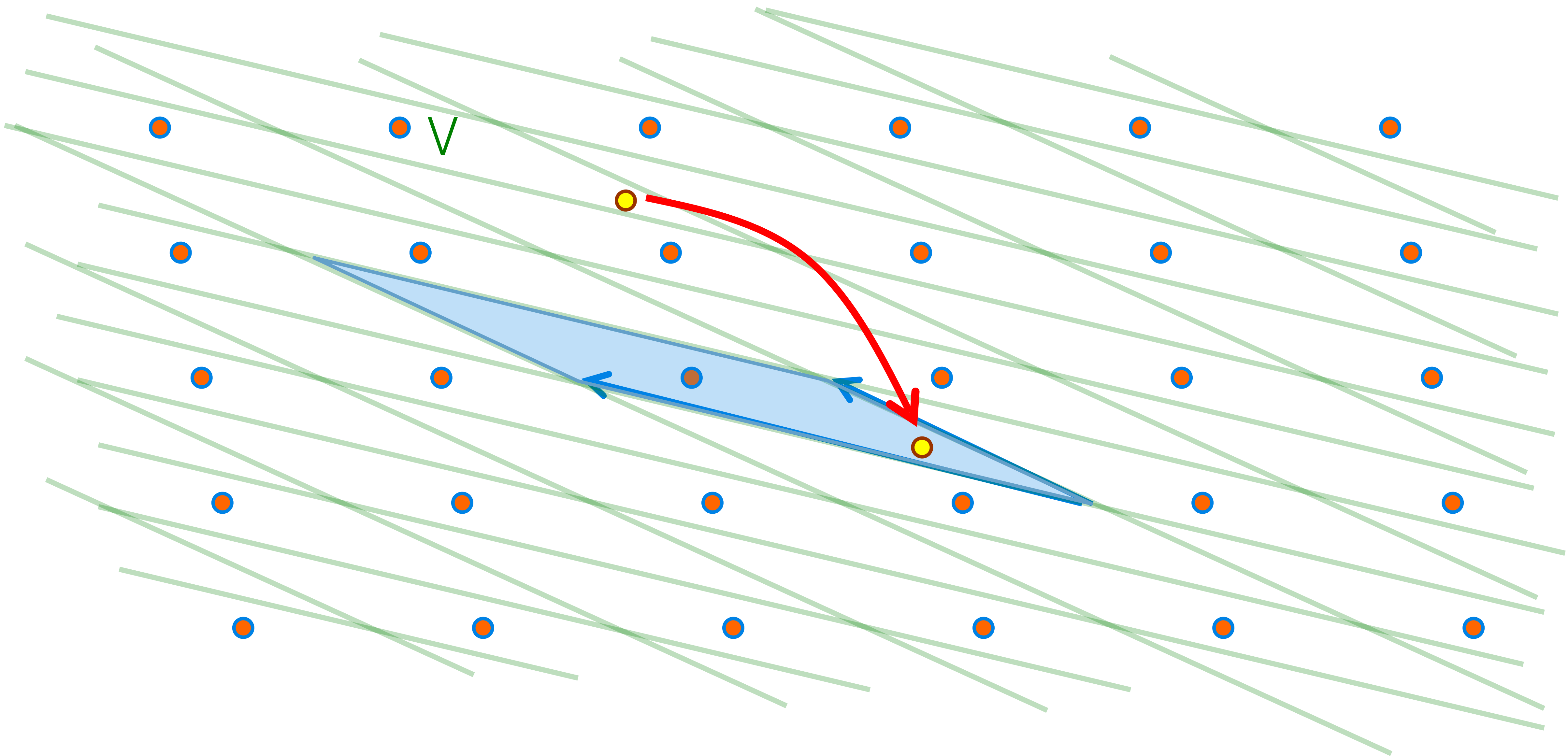

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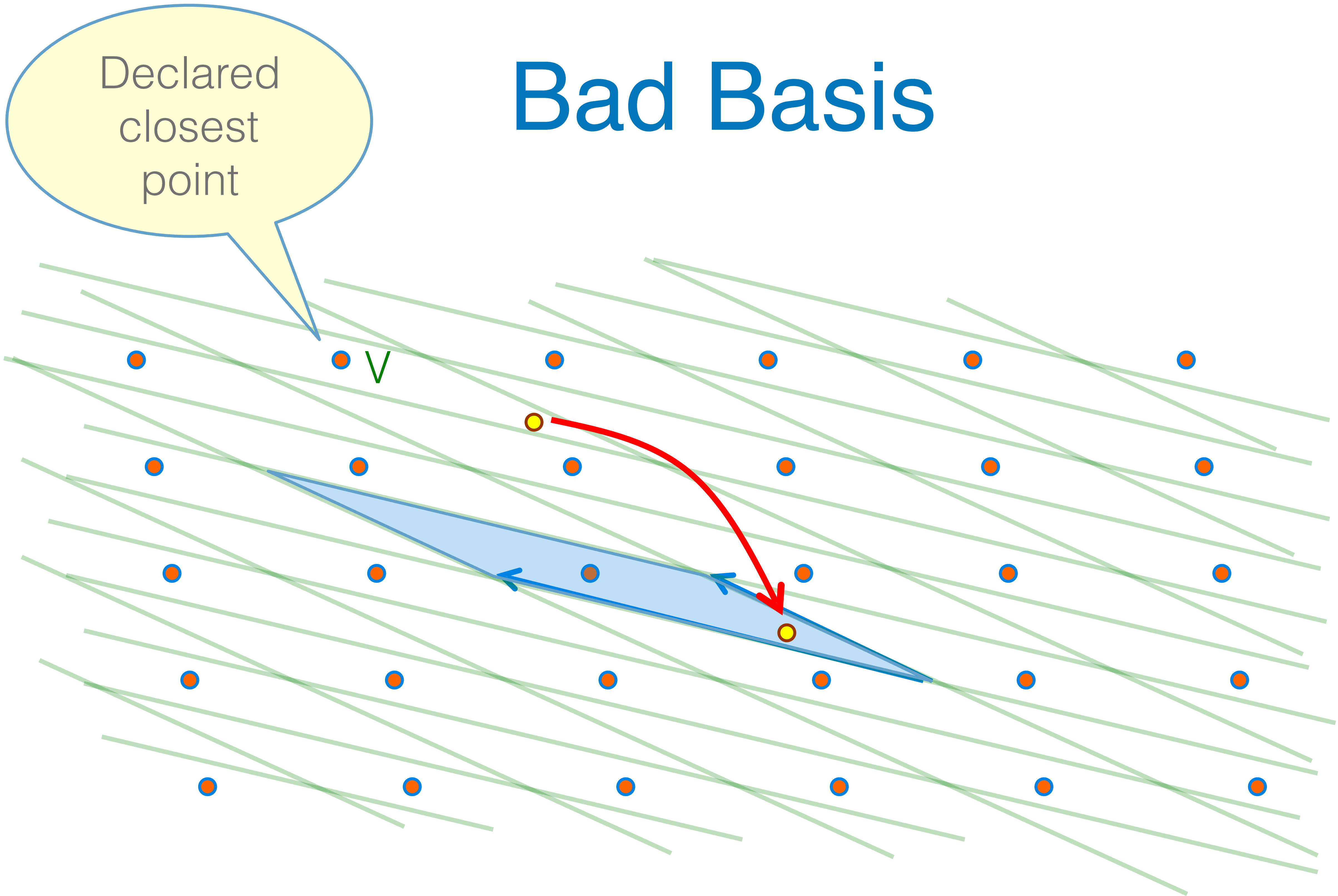

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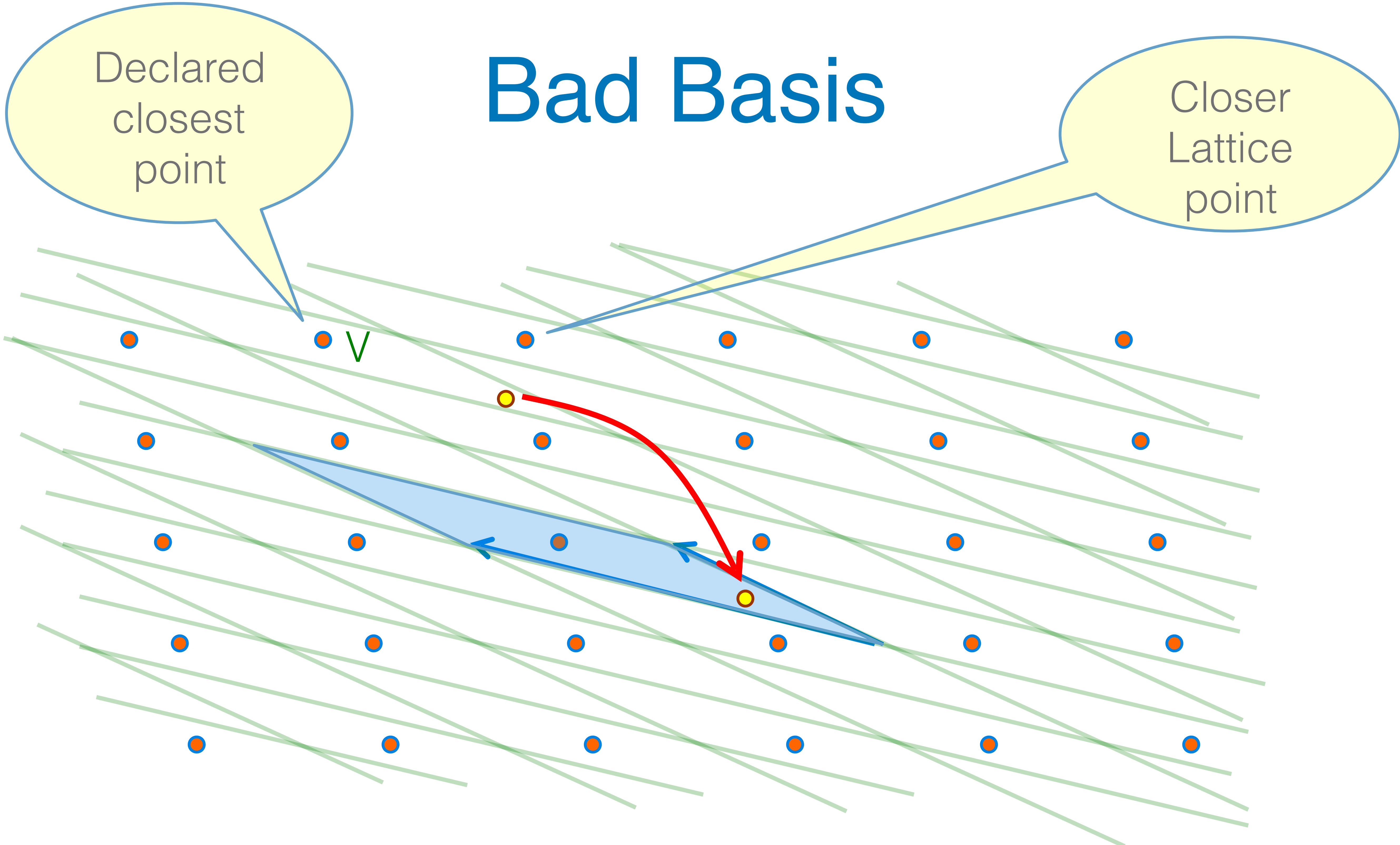

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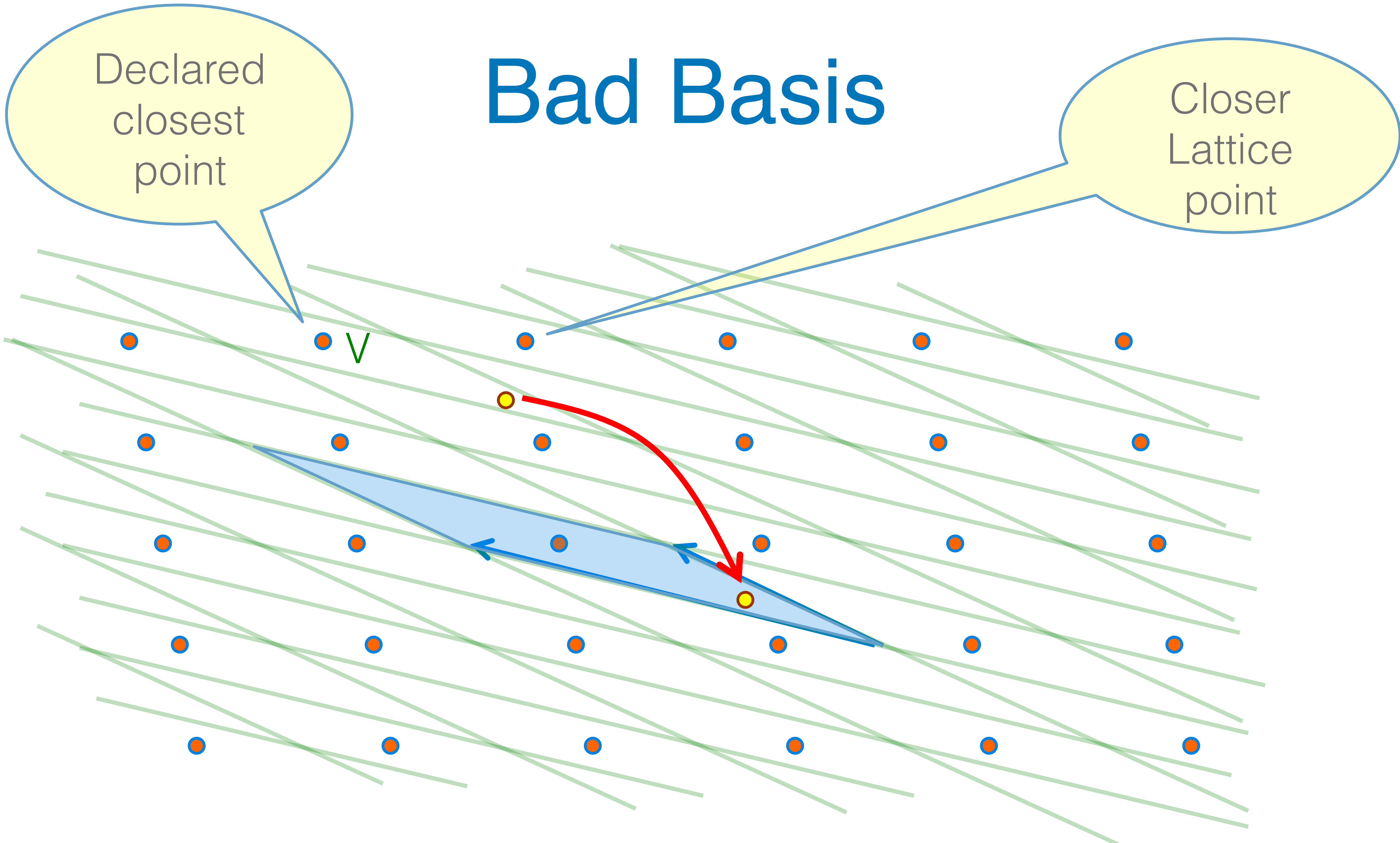
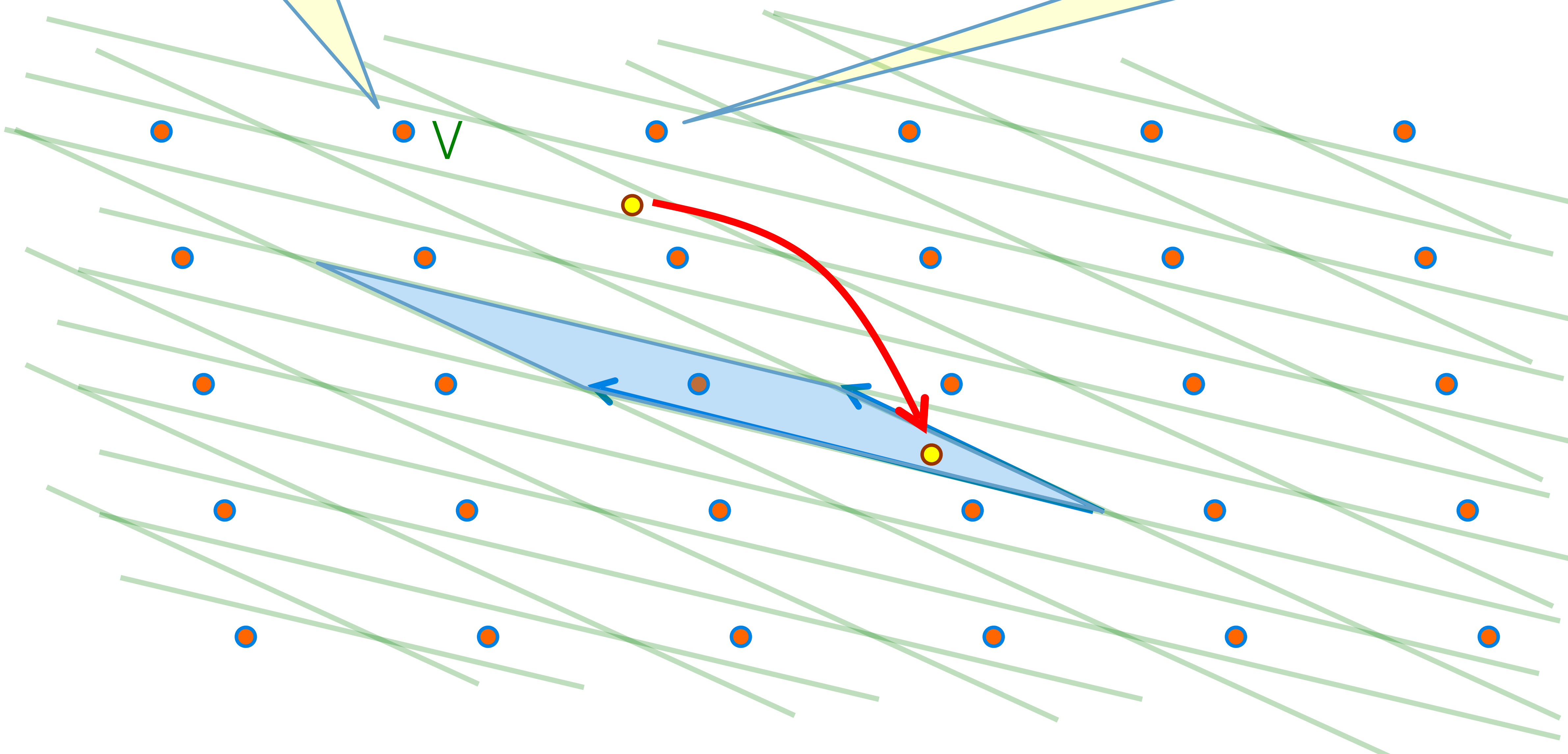


Output center of parallelopipid containing T

Bad Basis



Output center of parallelopipid containing T

Not So Accurate…

Basis quality and Hardness

- SVP, CVP, SIS (...) hard given arbitrary (bad) basis
- Some hard lattice problems are easy given a good basis
- Will exploit this asymmetry

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Use Short Basis as Cryptographic Trapdoor!

Lattice Trapdoors (Type 1)

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Inverting Our Function

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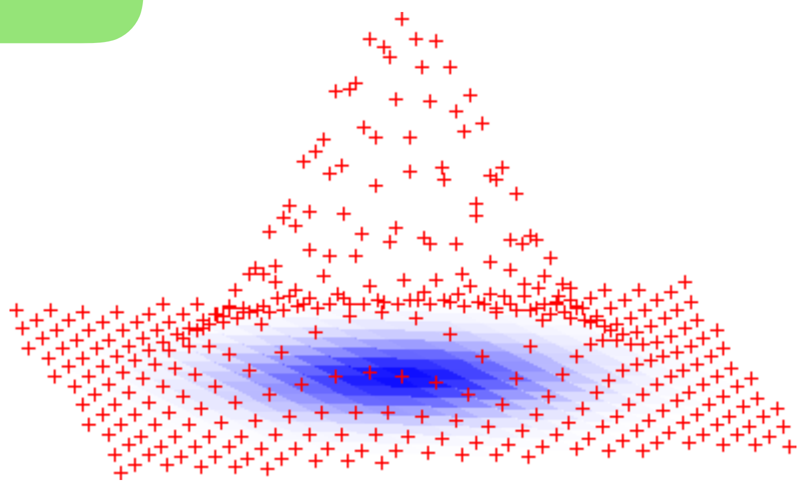

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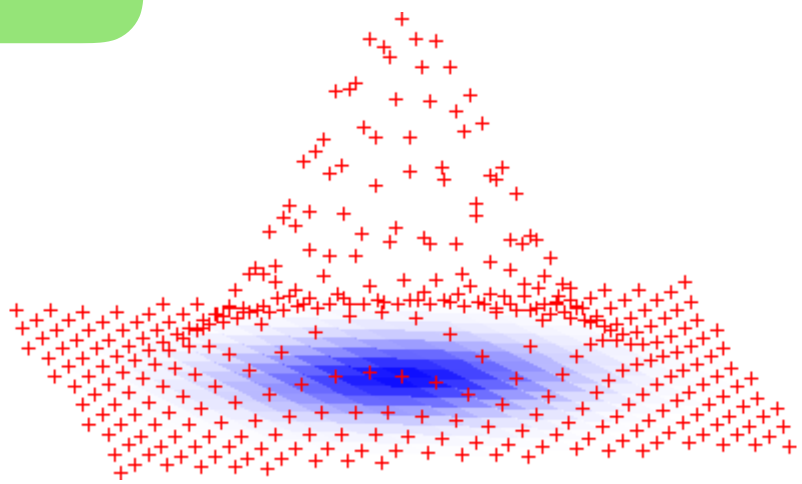

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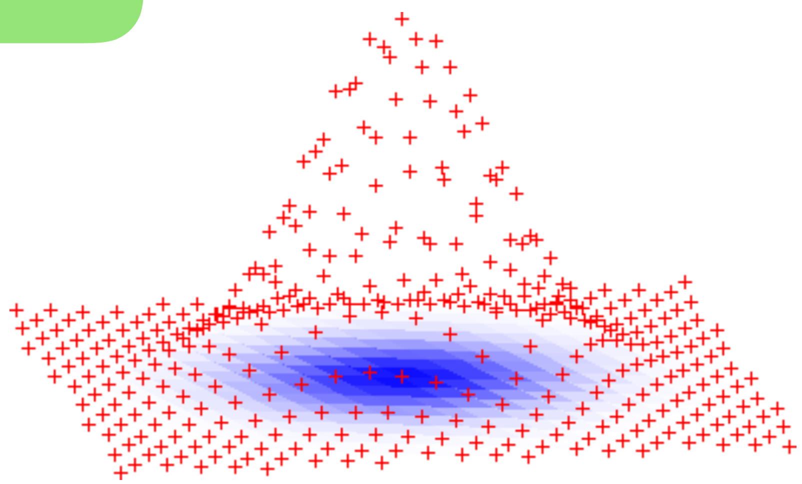

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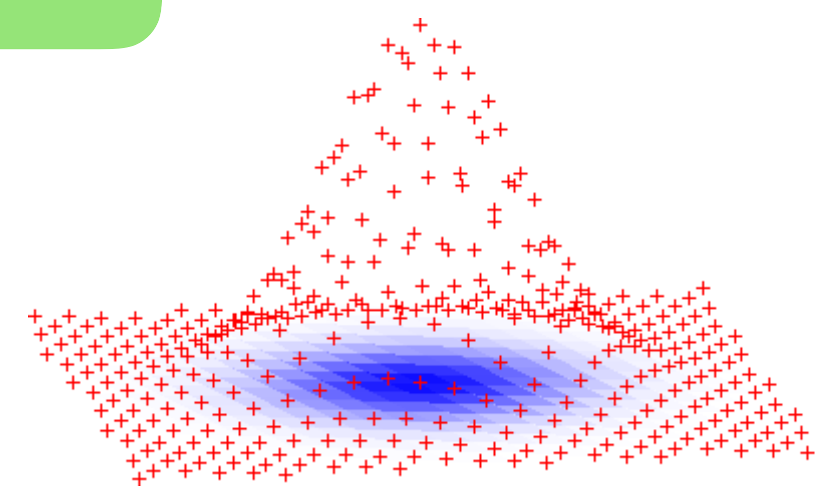

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Short basis for Λ lets us sample from $f_{\mathbf{A}}^{-1}(\mathbf{u})$
with correct distribution!

Two Questions

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1. How to use short basis

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   - Randomized Nearest plane Algorithm

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1. How to use short basis
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2. How to get short basis — this talk (almost)

Lattice Trapdoors (Type 2)

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Not a short basis but

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Image Credit: https://us.macmillan.com/podcasts/podcast/better-at-everything/

Type 2 Trapdoors [MP12]

Recall $f_{\mathbf{A}}(\mathbf{x}) = \mathbf{A}\ \mathbf{x} \mod q \in \mathbb{Z}_q^n$ and $g_{\mathbf{A}}(\mathbf{s}, \mathbf{e}) = \mathbf{s}^t\mathbf{A} + \mathbf{e}^t \mod q \in \mathbb{Z}_q^m$

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Design $f_{\mathbf{G}}^{-1},\ g_{\mathbf{G}}^{-1}$
for Gadget Matrix G
(fixed, public, offline)

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Randomize G ↔ A via
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1

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Transformation in Step 2 is the trapdoor!

Step 1: $f_{\mathbf{G}}^{-1}$, $g_{\mathbf{G}}^{-1}$ for Gadget G

Recall $f_{\mathbf{G}}(\mathbf{x}) = \mathbf{G}\ \mathbf{x} \mod q \in \mathbb{Z}_q^n$ and $g_{\mathbf{G}}(\mathbf{s}, \mathbf{e}) = \mathbf{s}^t\mathbf{G} + \mathbf{e}^t \mod q \in \mathbb{Z}_q^m$

Let $q = 2^k$ and $\mathbf{g} = [1, 2, 4, \cdots, 2^{k-1}] \in \mathbb{Z}_q^{1\times k}$

Invert LWE: find s s.t. $s \cdot \mathbf{g} + \mathbf{e} = [s + e_0, 2s + e_1, \cdots 2^{k-1}s + e_{k-1}]$

- Get lsb(s) from $2^{k-1}s + e_{k-1}$
- Then get next bit of s and so on.
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Invert SIS: sample Gaussian preimage $\mathbf{x}$ s.t. $u = \langle \mathbf{g}\ \mathbf{x} \rangle \mod q$

- For $i \in [0, \ldots, k-1]$, choose $x_i \leftarrow (2\mathbb{Z} + u)$, $u \leftarrow (u - x_i)/2 \in \mathbb{Z}$
- Let k= 2. $x_0 \leftarrow (2z_0 + u)$, $u \leftarrow (u - 2z_0 - u)/2 = -z_0$

$$x_1 \leftarrow (2z_1 - z_0)$$

$$\langle \mathbf{g}, \mathbf{x} \rangle = 2z_0 + u + 2(2z_1 - z_0) = u + 4z_1 = u \mod 4$$

Step 1: $f_{\mathbf{G}}^{-1},\ g_{\mathbf{G}}^{-1}$ for Gadget G

Note $\mathbf{g} = [1, 2, 4, \cdots, 2^{k-1}]$

S

$$\begin{pmatrix} 2 & & & & \\ -1 & 2 & & & \\ & -1 & 2 & & \\ & & & \ddots & \\ & & & & 2 \\ & & & & -1 & 2 \end{pmatrix} = 0 \bmod q$$

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Define gadget G : $\mathbf{G} = \mathbf{I}_n \otimes \mathbf{g}$

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$f_{\mathbf{G}}^{-1},\ g_{\mathbf{G}}^{-1}$ reduce to n parallel, offline calls to $f_{\mathbf{g}}^{-1},\ g_{\mathbf{g}}^{-1}$

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$$= \begin{bmatrix} \mathrm{B} & \mathrm{G - BR} \end{bmatrix}$$

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A is uniform by leftover hash lemma!

Leftover Hash Lemma (oversimplified)

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Let $\mathbf{B} \in \mathbb{Z}_q^{n \times m'}$ uniform & $\mathbf{R} \in \mathbb{Z}_q^{m' \times n \log q}$ Gaussian

If $m' \approx n \log q$, then,

$$(\ \mathbf{B}, \mathbf{BR}\ ) \approx (\ \mathbf{B}, \mathbf{U}\ )$$

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Hence **A =** $\left[\ \mathrm{B} \ \middle|\ \mathrm{G} - \mathrm{BR}\ \right]$ uniform

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Have $\mathbf{A} =$

B	G - BR

Define: **R** is a trapdoor for **A** with tag $\mathbf{H} \in \mathbb{Z}_q^{n \times n}$,

If $$\mathbf{A} \cdot \begin{bmatrix} \mathbf{R} \\ \mathbf{I} \end{bmatrix} = \mathbf{H} \cdot \mathbf{G}$$

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Basis $\mathbf{S}$ for $\Lambda^{\perp}(\mathbf{G})$

&

Trapdoor R for A

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Want:

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Works if $\mathbf{e}^t \cdot \begin{bmatrix} \mathbf{R} \\ \mathbf{I} \end{bmatrix} \in [-q/4, q/4)$

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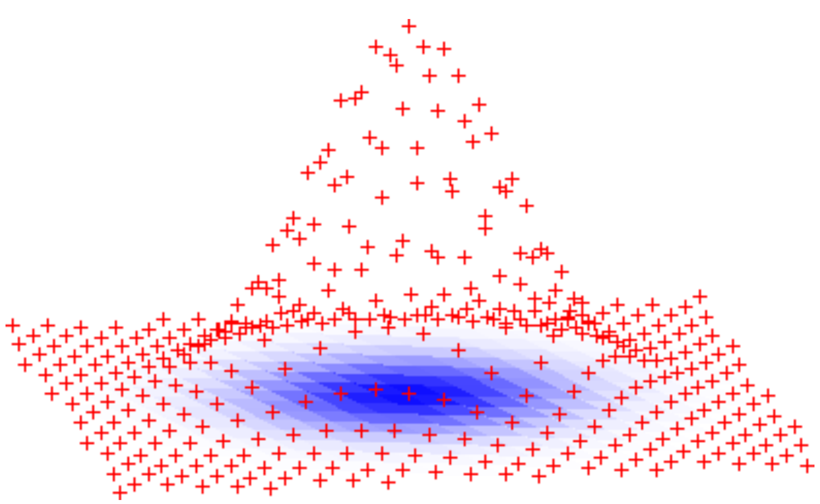

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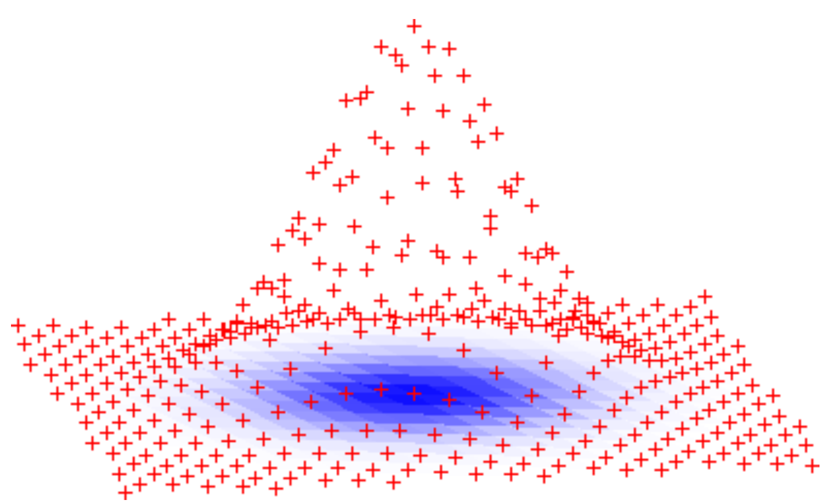

Compute:

Sample $\mathbf{z} \leftarrow f_{\mathbf{G}}^{-1}(\mathbf{u})$

Output $\mathbf{x} = \begin{bmatrix} \mathbf{R} \\ \mathbf{I} \end{bmatrix} \cdot \mathbf{z}$

Then,

$$\mathbf{A} \cdot \mathbf{x} = \mathbf{A} \cdot \begin{bmatrix} \mathbf{R} \\ \mathbf{I} \end{bmatrix} \cdot \mathbf{z} = \mathbf{G} \cdot \mathbf{z} = \mathbf{u}$$

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Are we done?

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Covariance of x leaks R!

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$$\Sigma := \mathbb{E}_{\mathbf{x}}\left[\mathbf{x}\cdot\mathbf{x}^t\right]=\mathbb{E}_{\mathbf{z}}\left[\mathbf{R}\cdot\mathbf{z}\mathbf{z}^t\cdot\mathbf{R}^t\right]\approx s^2\cdot\mathbf{R}\mathbf{R}^t.$$

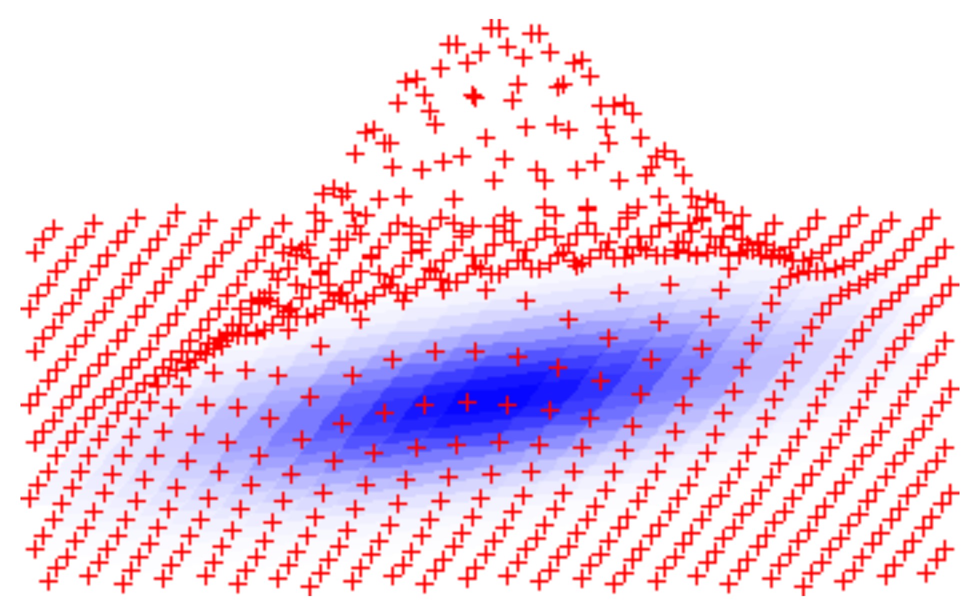

Image Credit: Chris Peikert

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Want to output spherical Gaussian!

Covariance Matrix $s^2\mathbf{I}$

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https://www.elegantthemes.com/

Fix using perturbation method [P'10]

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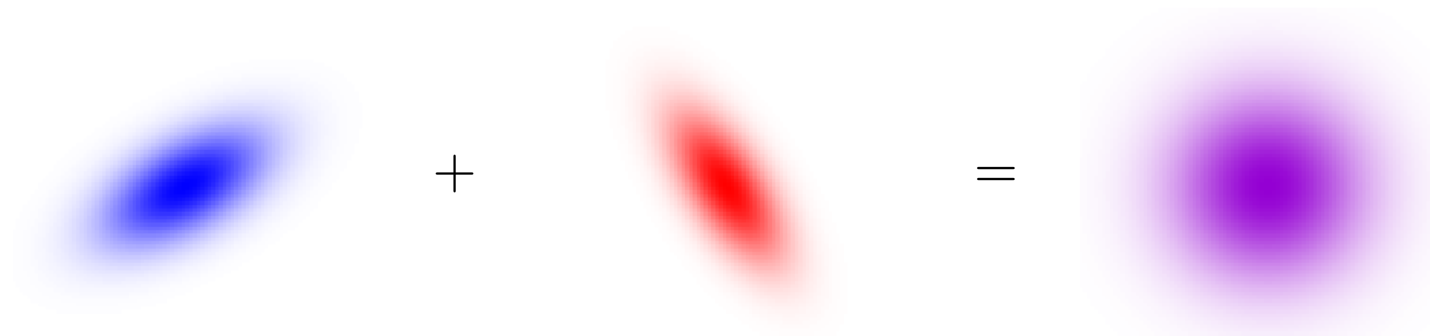
Want to output spherical Gaussian!
Covariance Matrix $s^2\mathbf{I}$

https://www.elegantthemes.com/

Fix using perturbation method [P'10]

Convolution of Gaussians



$$\mathbf{R}\mathbf{R}^t + (s^2\mathbf{I} - \mathbf{R}\mathbf{R}^t) = s^2\,\mathbf{I}$$

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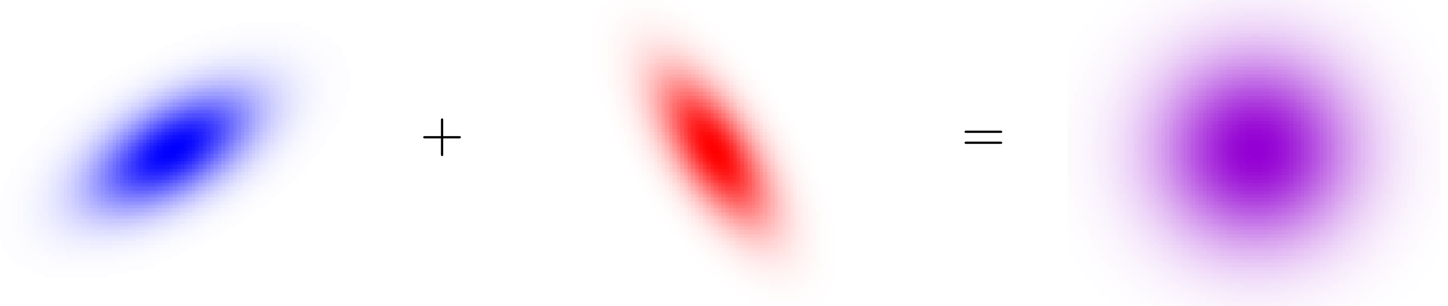

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To fix covariance:

- Generate perturbation vector **p** with covariance $(s^2\mathbf{I} - \mathbf{R}\mathbf{R}^t)$
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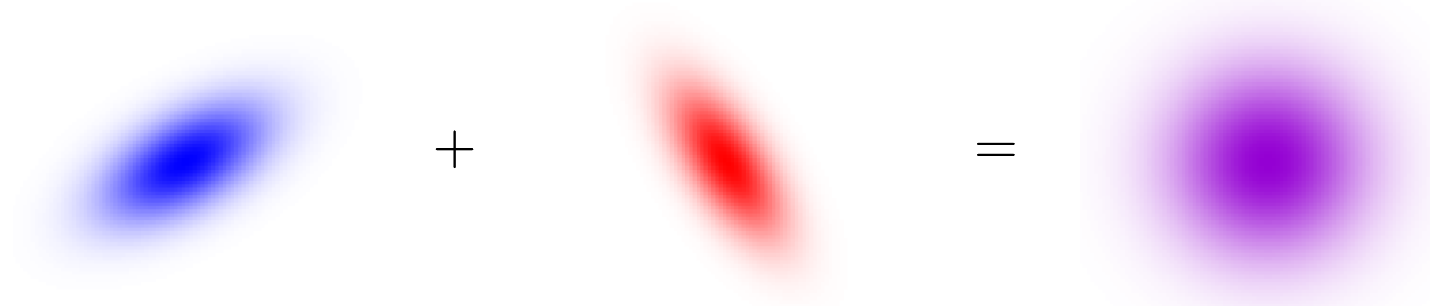

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Check

$$\mathbf{A} \cdot \begin{bmatrix}\mathbf{R}\\ \mathbf{I}\end{bmatrix} = \mathbf{G}$$

$$\mathbf{A} \cdot \mathbf{x} = \mathbf{A}\mathbf{p} + \mathbf{A}\begin{bmatrix}\mathbf{R}\\ \mathbf{I}\end{bmatrix} \cdot \mathbf{z}$$

$$= \mathbf{A}\mathbf{p} + \mathbf{G}\mathbf{z} = \mathbf{u}$$

Takeaway for Applications

Let $\mathbf{B} \in \mathbb{Z}_q^{n \times m'}$, uniform $\mathbf{R} \in \mathbb{Z}_q^{m' \times n \log q}$, Gaussian

Let $\mathbf{A} = \left[\begin{array}{c|c} \mathrm{B} & \mathrm{G - BR} \end{array}\right]$

Then, $\mathbf{A}$ uniform, admits LWE and SIS inversion

$$f_{\mathbf{A}}^{-1},\ g_{\mathbf{A}}^{-1}$$

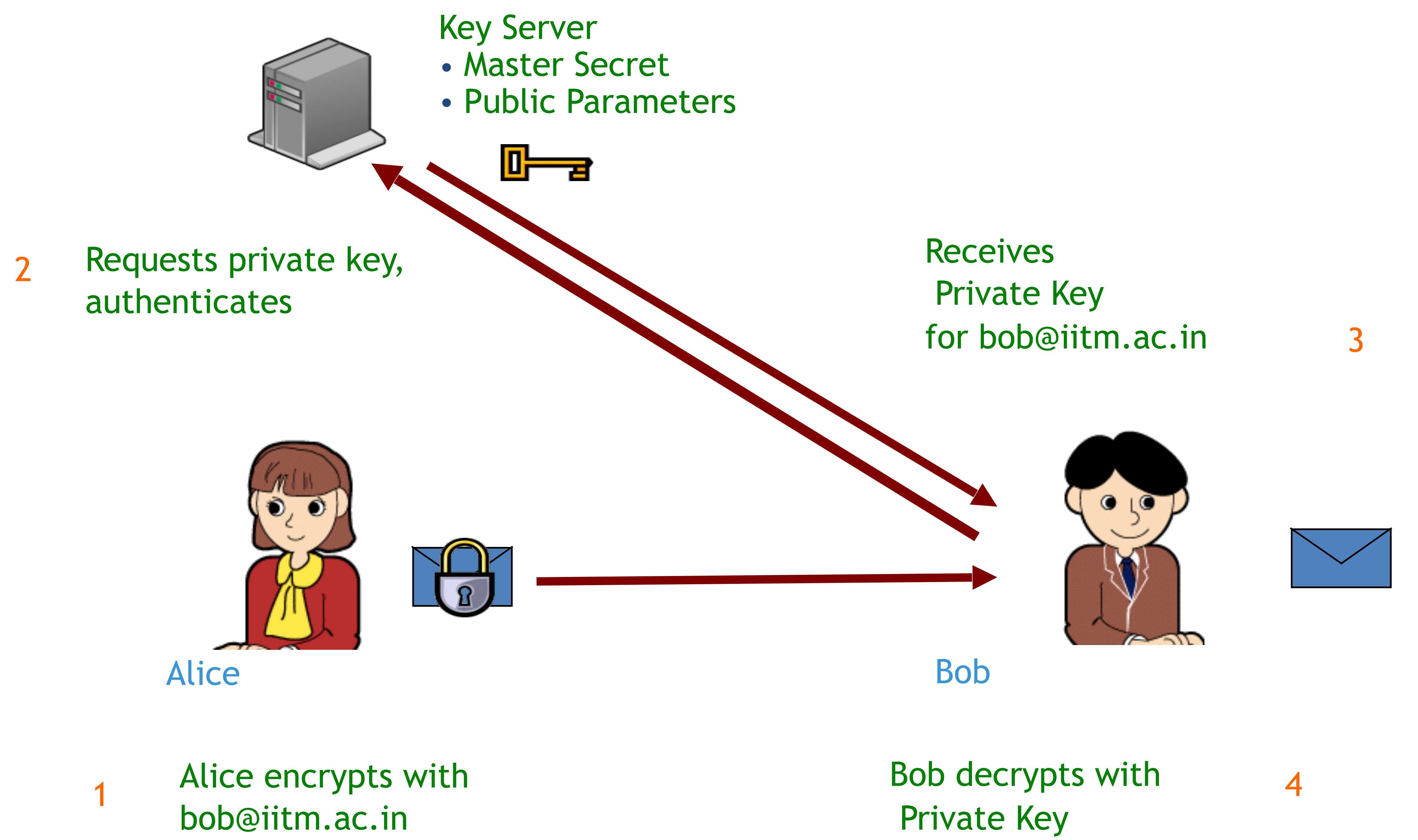
Applications

Identity Based Encryption (IBE)

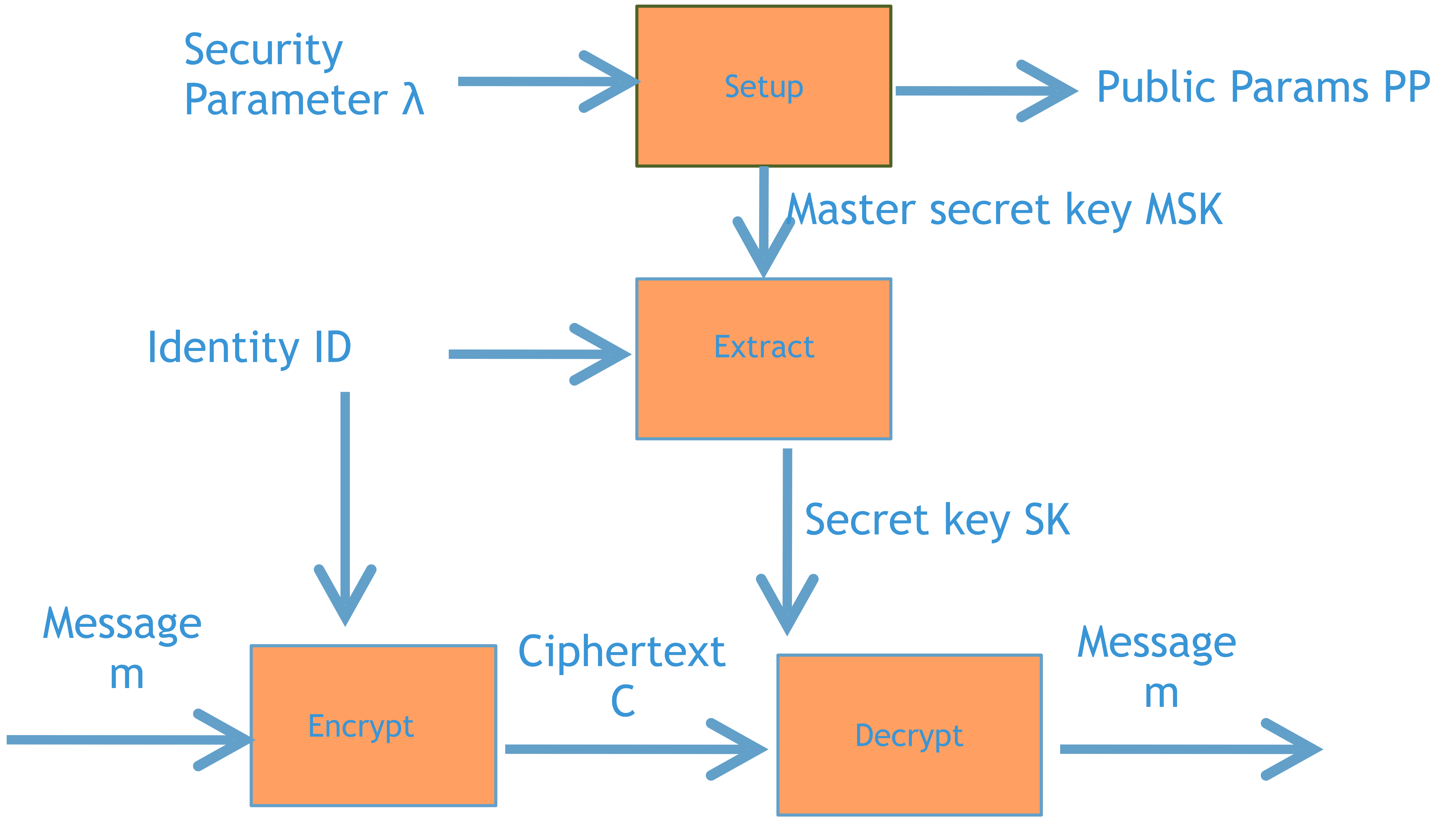
In short…………

Public Key Encryption in which ANY arbitrary string can be public key!

IBE: How does it work?



Identity Based Encryption



Regev PKE

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- Recall A (e) = u mod q hard to invert

- Secret: e, Public : A, u

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$$\begin{Bmatrix} A \end{Bmatrix} \begin{bmatrix} e \end{bmatrix} \equiv \begin{bmatrix} u \end{bmatrix} \bmod q$$

- Encrypt (A, u) :
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- Decrypt (e) :
  - $e^T c_0 - c_1 = \text{msg} + \text{noise}$

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Encryption matrix A

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Small only if e is small

Broad structure IBE

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Identity Based Encryption [CHKP10]

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Let $|id|=2$

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- Parameters

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- Master secret key : basis for A_0

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- Parameters $\{A_0\}$ $\{A_1^0\}$ $\{A_1^1\}$ $\{A_2^0\}$ $\{A_2^1\}$ u

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- Secret Key for (id=01) : short e such that F_{01} e = u mod q

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- Secret Key for (id=01) : short e such that F_{01} e = u mod q

Where $F_{01} = [A_0 | A_1^0 | A_2^1]$ (one block per bit!)

Identity Based Encryption [CHKP10]

Let $|id|=2$

- Parameters A_0 A_1^0 A_1^1 A_2^0 A_2^1 u

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- Figure out how to compute trapdoor for "extended" matrix $[T_1 | T_2 | T_3]$

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- Figure out how to compute trapdoor for "extended" matrix $[T_1|T_2|T_3]$

- Encrypt (b, id=01): Uses regev PKE on matrix F_{01}

Identity Based Encryption [CHKP10]

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- **Secret Key** for (id=01) :   low norm vector e such that

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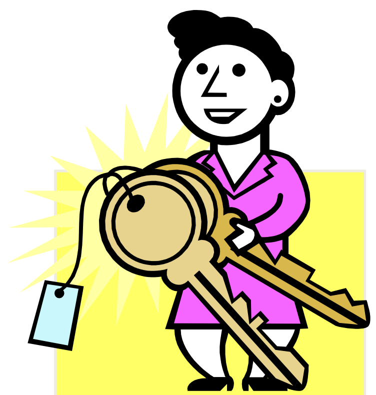
  - Compute $e^T c_0 - c_1 = \text{noise} + \text{msg} \bmod q$

IBE Security



Challenger Ch.

Adversary Ad.

IBE Security

Get instance of hard problem **H**

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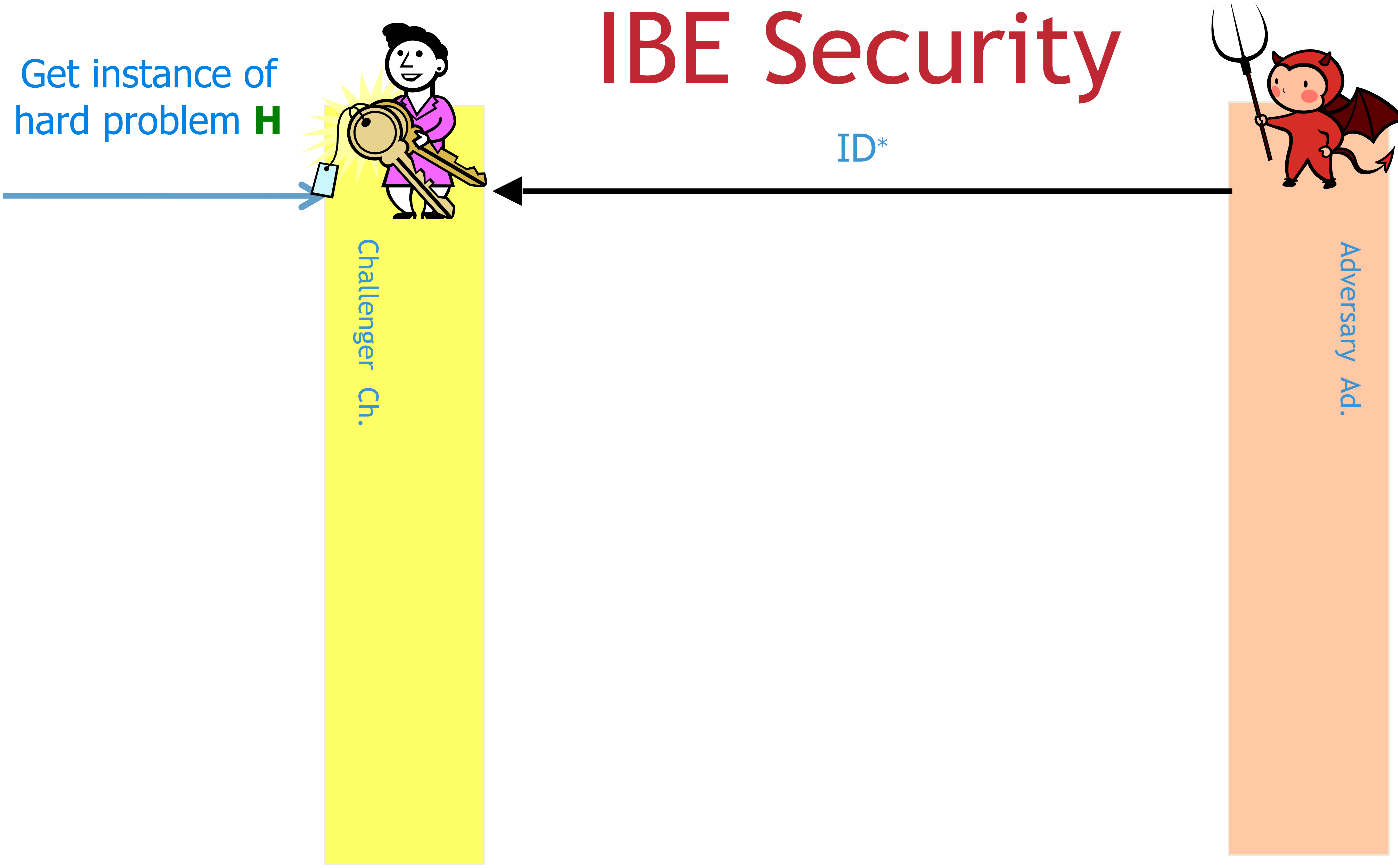

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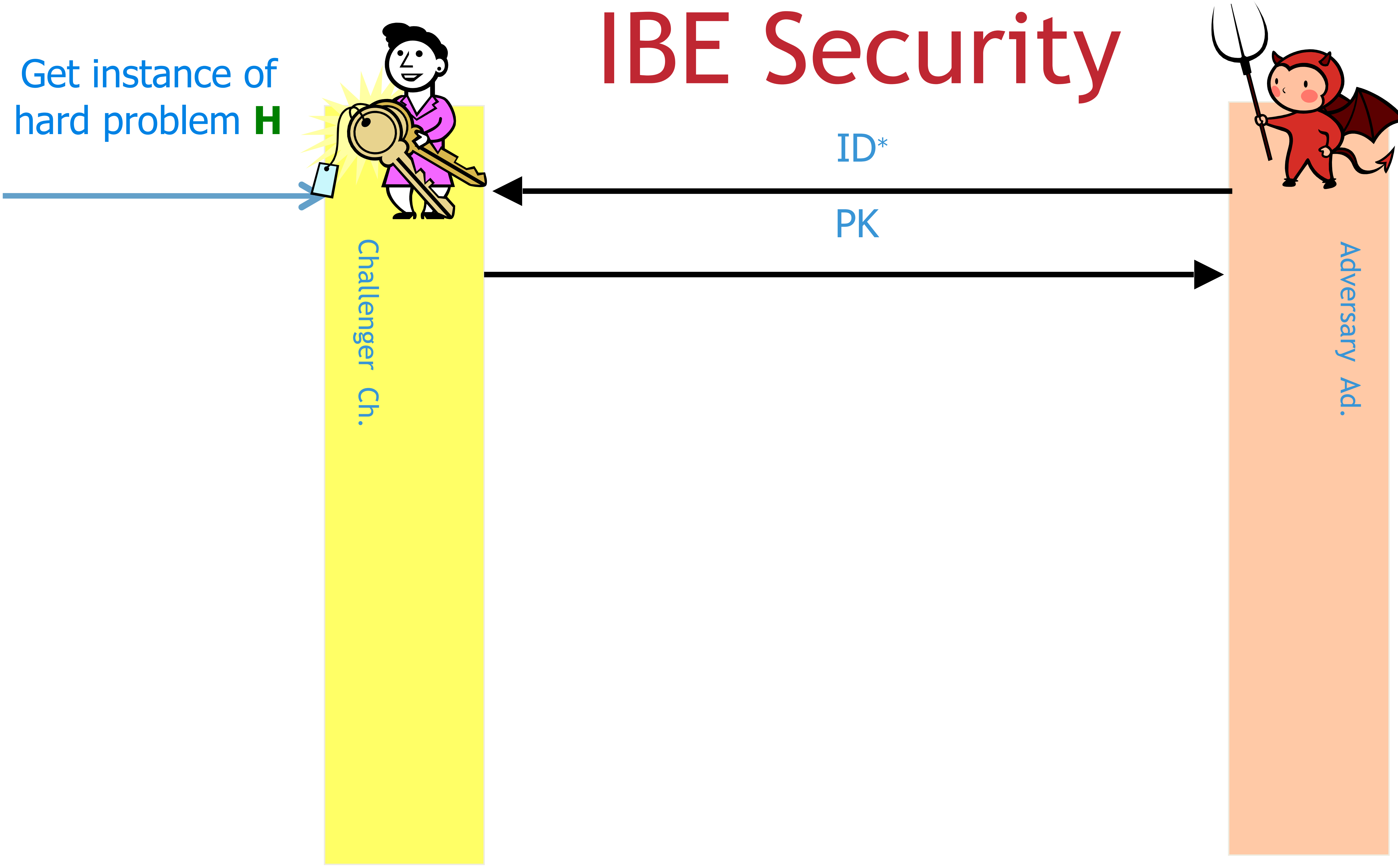

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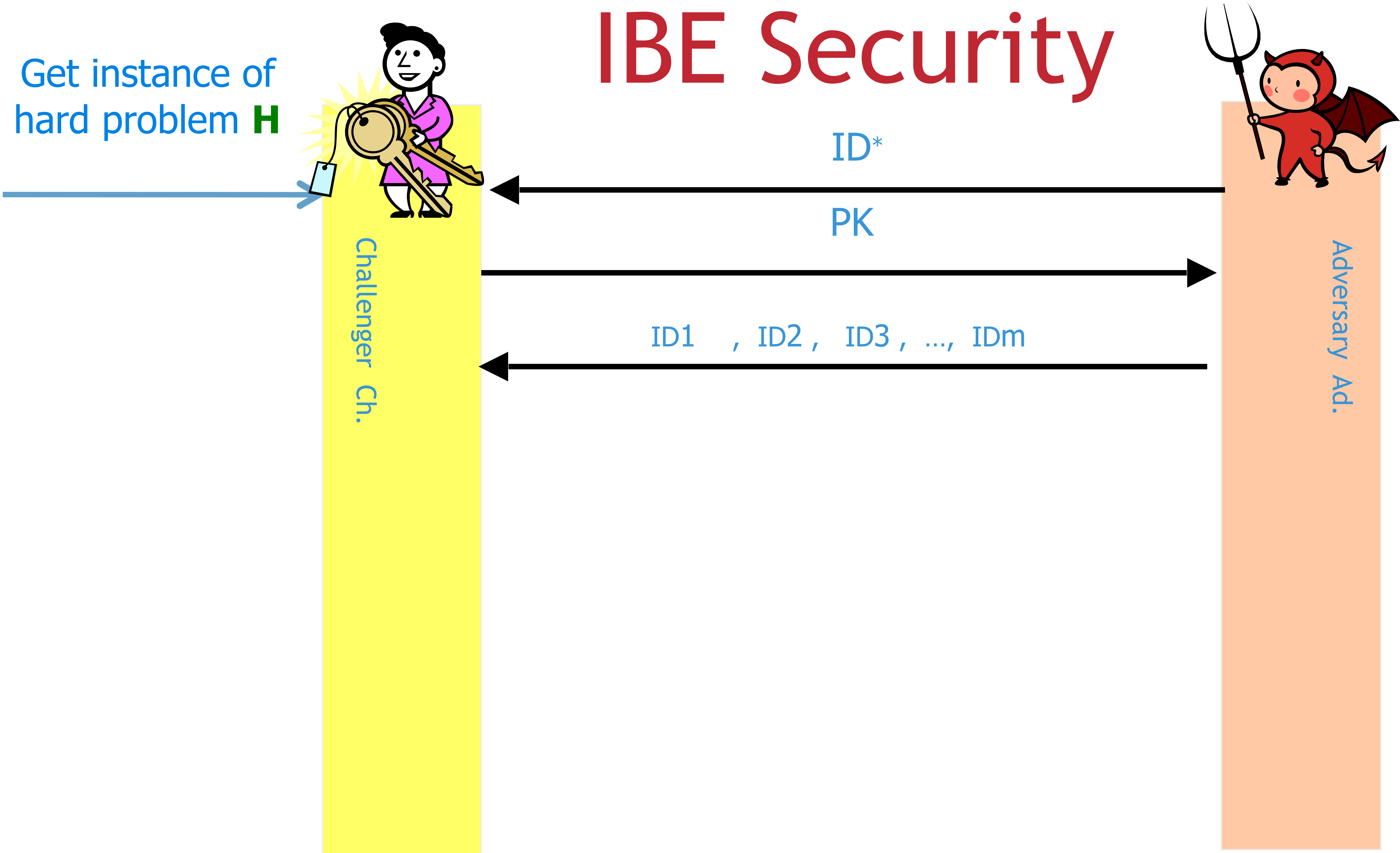

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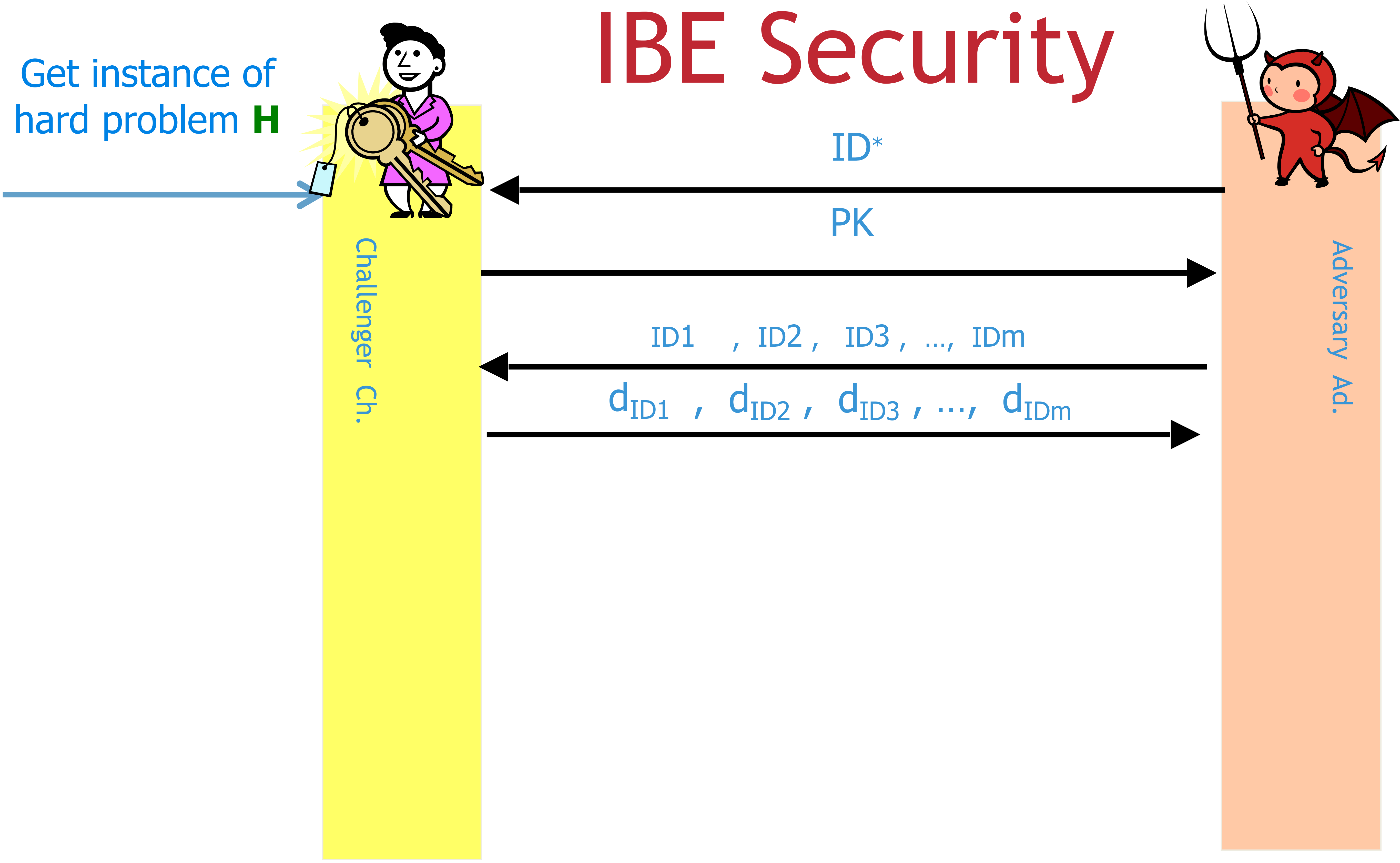

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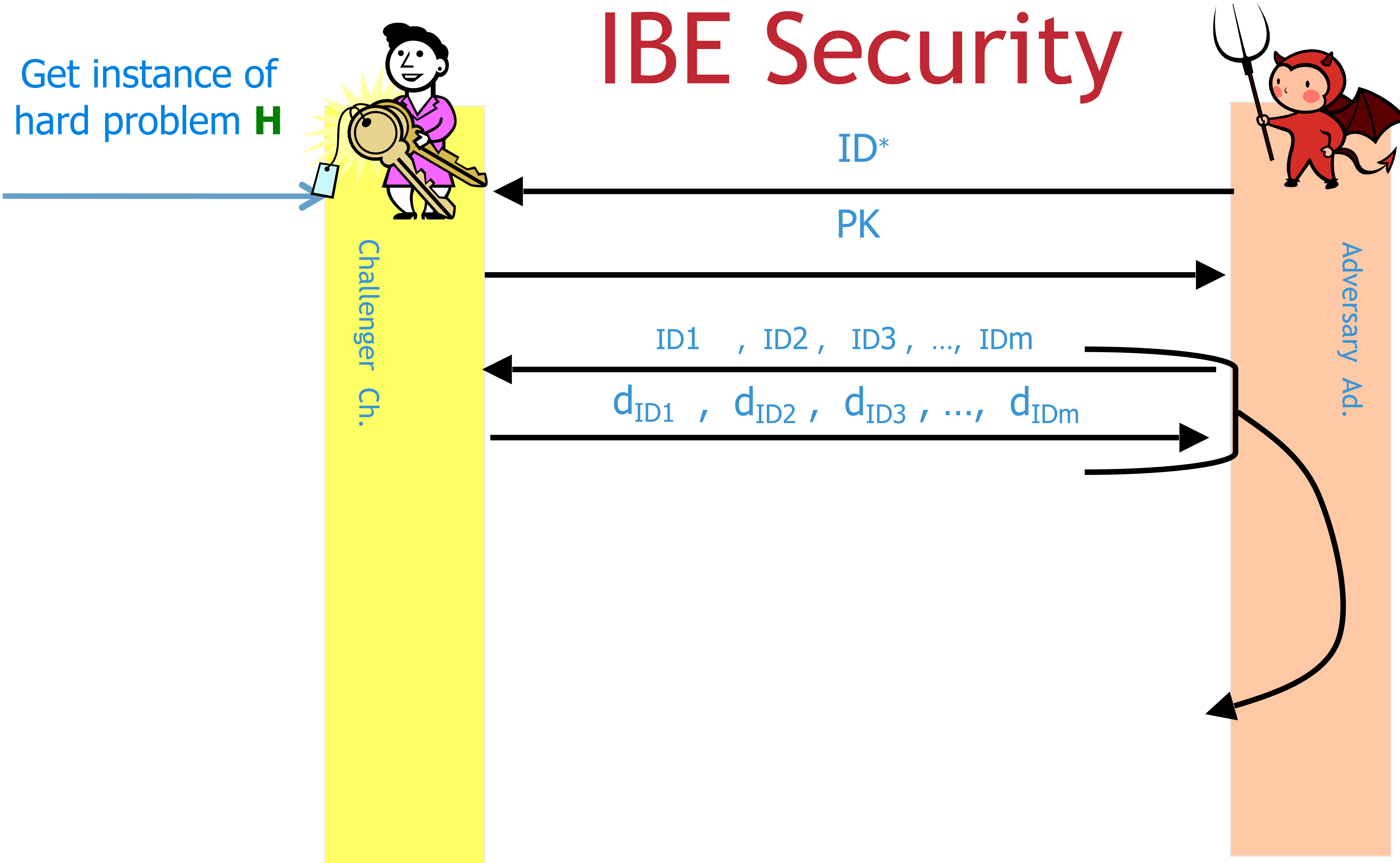

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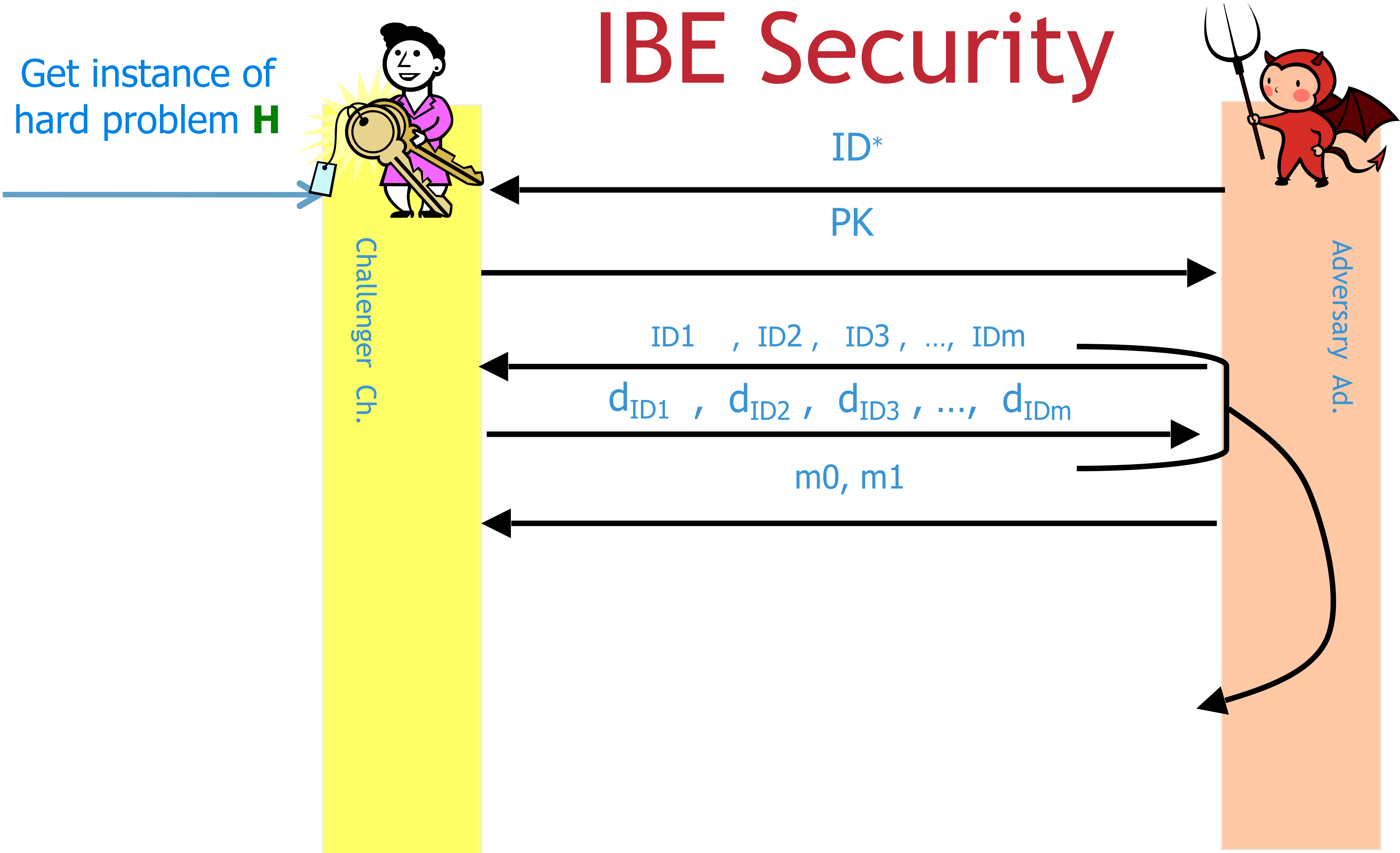

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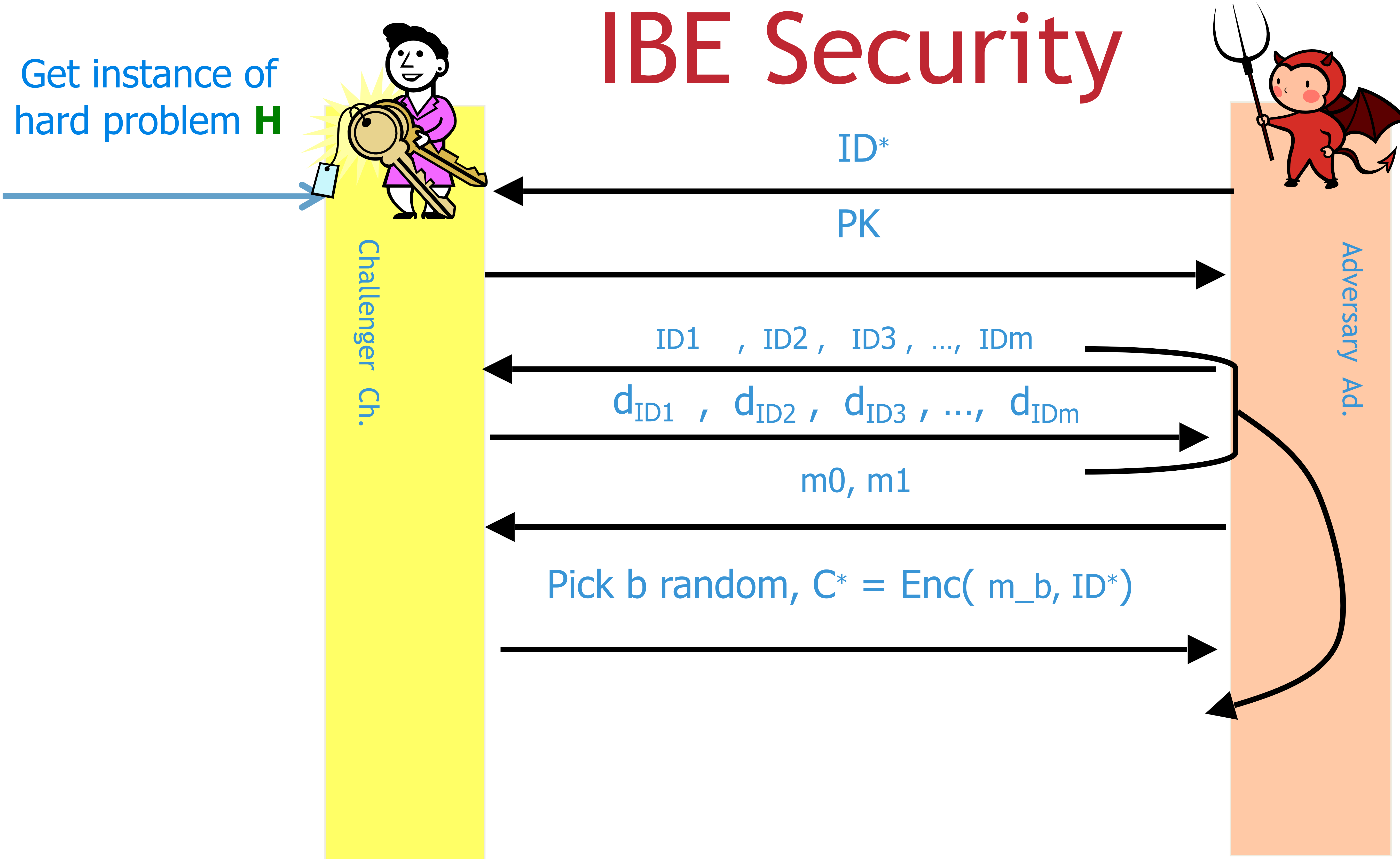

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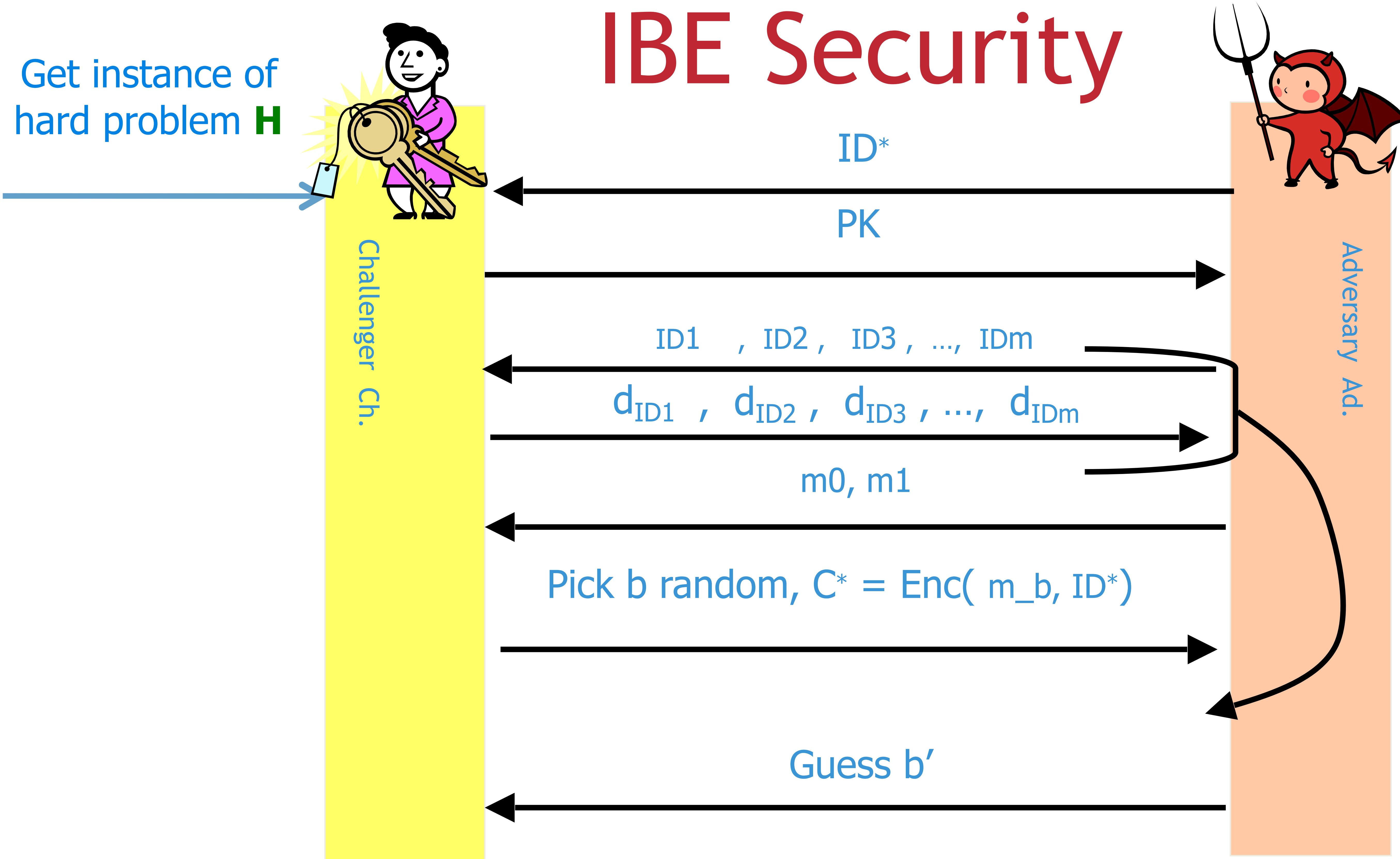
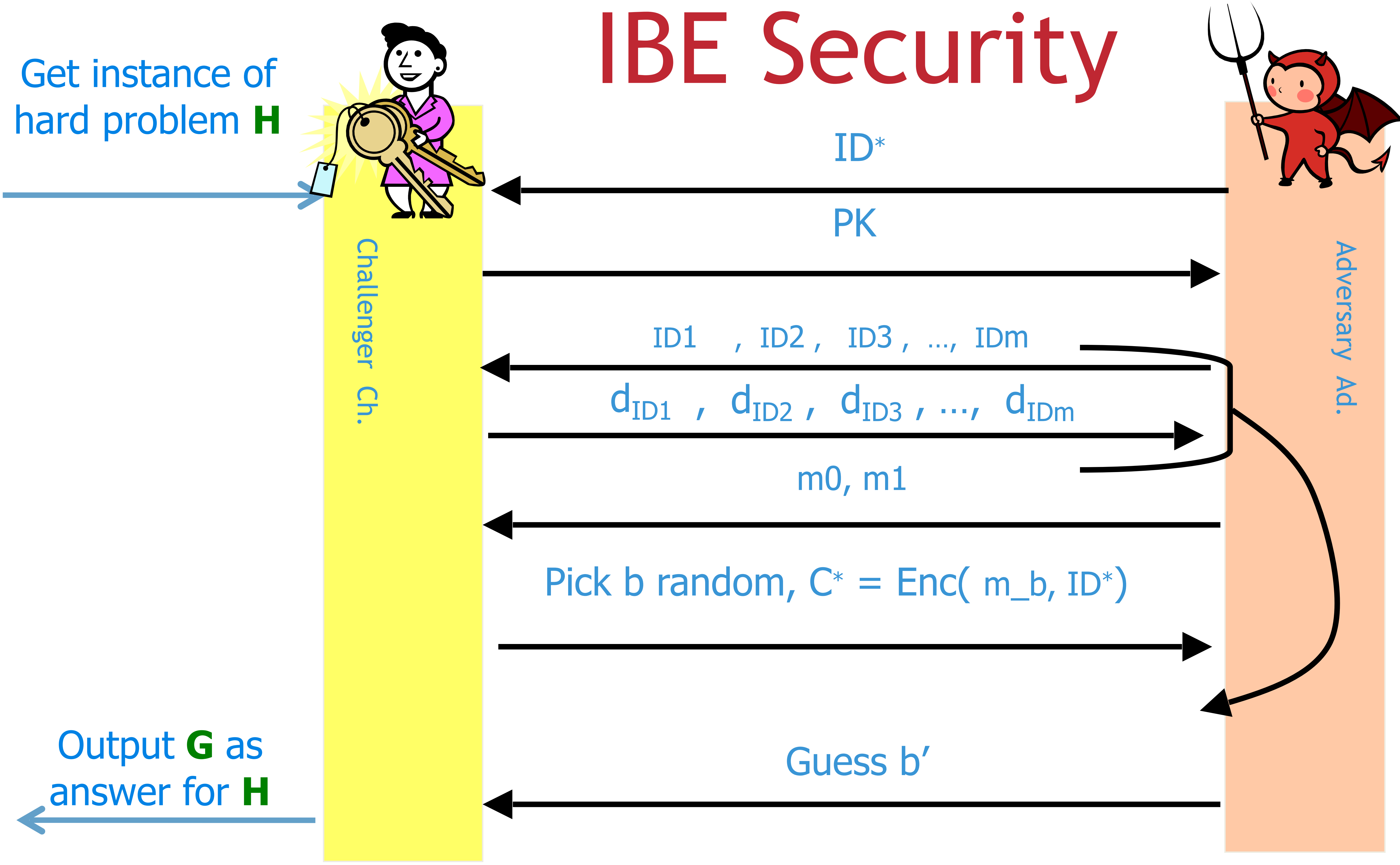
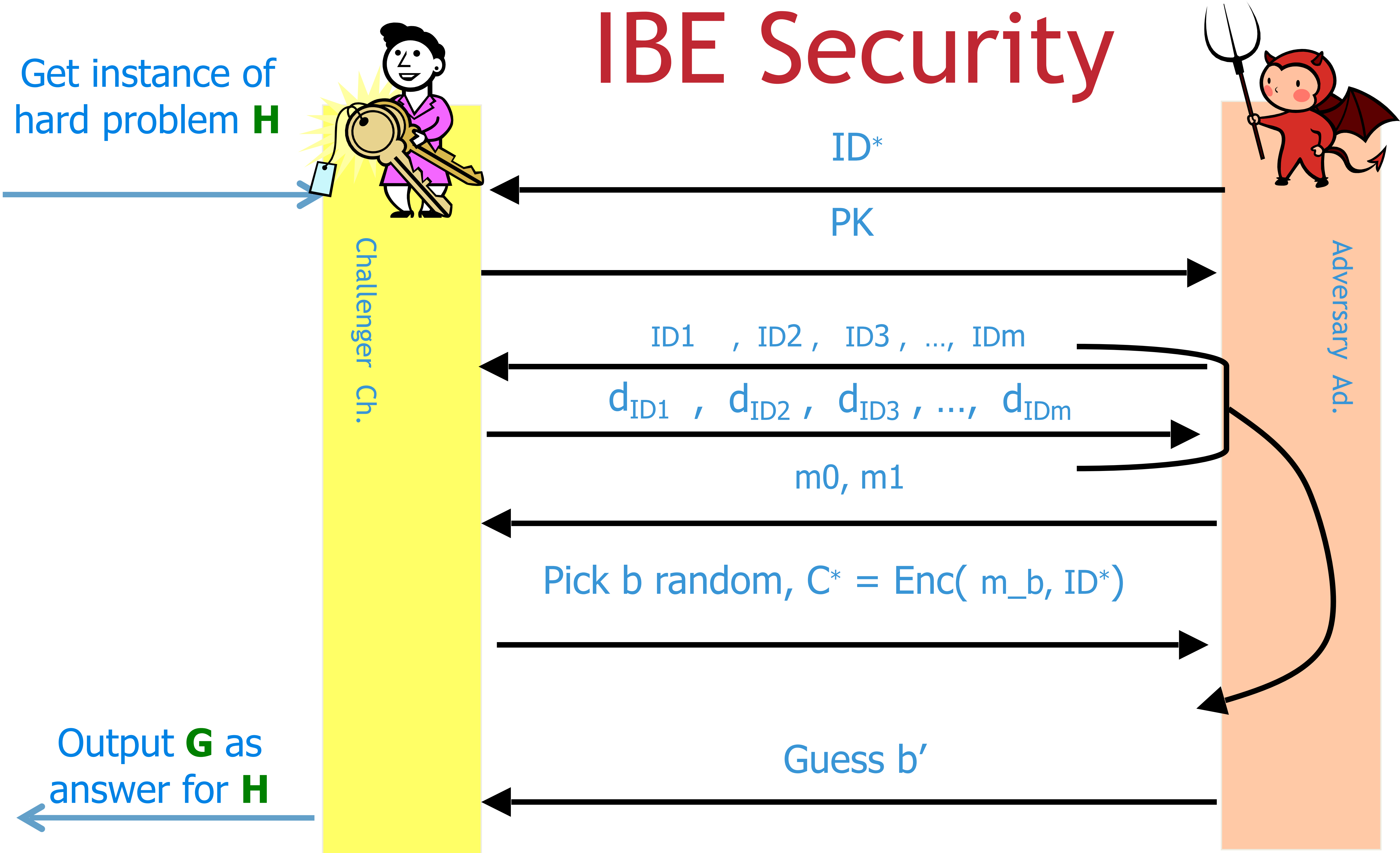



IBE Security
Get instance of hard problem H
Challenger Ch.
Adversary Ad.
ID*
PK
ID1 , ID2 , ID3 , …, IDm
dID1 , dID2 , dID3 , …, dIDm
m0, m1
Pick b random, C* = Enc(m_b, ID*)
Guess b'
Output G as answer for H

IBE Security



Attacker wins if | Pr[b=b’] - ½ | is non-negligible

Security Model: Key Points

- Ch. needs to be able to answer private key queries of Ad.

- Ch. should not be able to answer query for id^* (hence can't have master trapdoor)

- Ch. should be able to generate challenge ciphertext so that Ad's answer is useful.

Simulation

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- Let challenge identity $id^* = 11$

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- Cannot compute basis of $F_{11} = [\, A_0 | \, A_1^1 | A_2^1]$

Efficient Identity Based Encryption [ABB10]

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Parameters: $\left\{ A_0 \right\}$ $\left\{ A_1 \right\}$ $\left\{ G \right\}$ u

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Independent of |id|!

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$$\text{Let } F_{id} = [A_0 \mid A_1 + id \times G]$$

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KeyGen for identity id :

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key

$$F_{id} \; e \equiv u \mod q$$

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Know how to compute trapdoor for "extended" matrix $[A_0 \mid \textit{any}\,]$

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Encryption for id’ = Regev PKE on matrix F_{id}

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Encryption for id' = Regev PKE on matrix F_{id}

- Pick random vector s
- Let $F_{id} = [A_0 \mid A_1 + id \times G]$
- $C = u^T s + \text{noise} + \text{msg}$
- $C' = F_{id}^T s + \text{noise}$

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Encryption for id' = Regev PKE on matrix F_{id}

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Fixed size

- $C = u^T s + \text{noise} + \text{msg}$

- $C' = F_{id}^T s + \text{noise}$

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$C_0 = u^T s + \text{noise} + m$ and $C_1 = F_{id}{}^T s + \text{noise}$

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Efficient Identity Based Encryption [ABB10]

$C_0 = u^T s$ + noise + m and $C_1 = F_{id}{}^T s$ + noise

Decryption : Regev decryption

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- $e^T C_1 = (F_{id}\, e)^T s$ + noise
- Since $F_{id}\, e = u \bmod q$, we have

w = m + noise from which we can recover m.

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Random low norm matrix

Efficient Identity Based Encryption [ABB10]

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- $F_{id} = [A_0 \mid A_0 R + (id - id^*) G]$

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Random low norm matrix

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- Have basis for G
- Let $A_1 = [A_0 R - id^* \times G]$
- $F_{id} = [A_0 | A_0 R + (id - id^*) G]$
- Need to find basis for F_{id} given basis for G

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Let $\mathbf{A} =$

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Then, $\mathbf{A}$ uniform, admits LWE and SIS inversion

$$f_{\mathbf{A}}^{-1},\ g_{\mathbf{A}}^{-1}$$

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MP12

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$$f_{\mathbf{A}}^{-1},\ g_{\mathbf{A}}^{-1}$$

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$$f_{\mathbf{A}}^{-1}, \; g_{\mathbf{A}}^{-1}$$

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- Can find basis for F_{id} given basis for G !

Developed in ABB10

Efficient Identity Based Encryption [ABB10]

MP12

Let $\mathbf{B} \in \mathbb{Z}_q^{n \times m'}$, uniform $\mathbf{R} \in \mathbb{Z}_q^{m' \times n \log q}$, Gaussian

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Efficient Identity Based Encryption [ABB10]

Real System

Simulation

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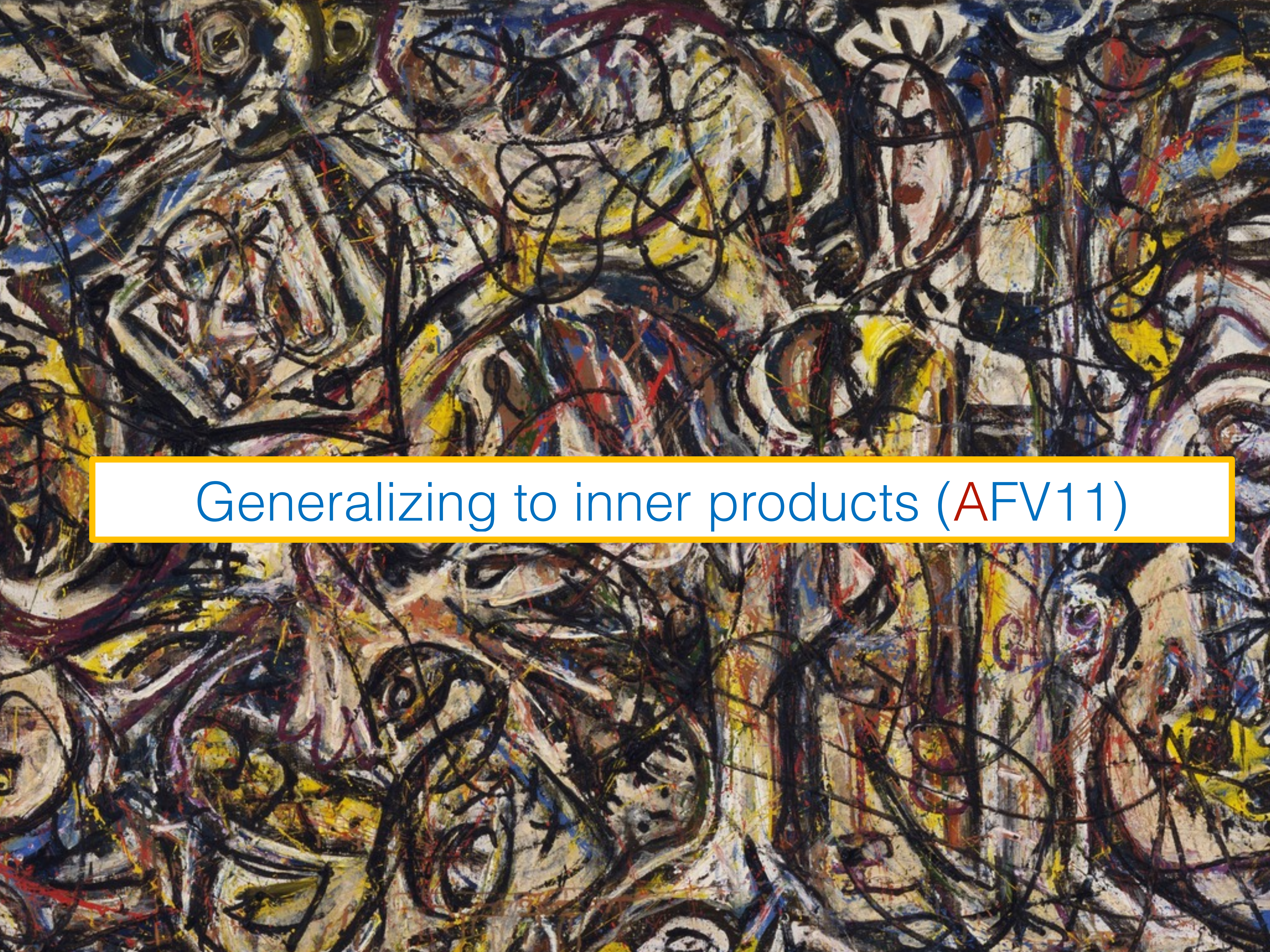
Trapdoor for G ➔ Key for $id \neq id^*$

Indistinguishable since R is random!

Generalizing to inner products (AFV11)



Generalizing to Inner Product (KSW08)

Key : $y = (y_1, \ldots, y_n)$

CT : $x = (x_1, \ldots, x_n)$

$$\text{Function } f(x, y) = \begin{cases} 1 & \text{If } \langle x \,.\, y\rangle = 0 \\ 0 & \text{otherwise} \end{cases}$$

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Supports:

- OR –- Bob OR Alice $OR_{A,B}(z) = 1$ if $z = A$ OR $z = B$

$$p(z) = (A - z)(B - z)$$

- CNF/DNF formulas of bounded size

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Ciphertext Hides Attributes x_i

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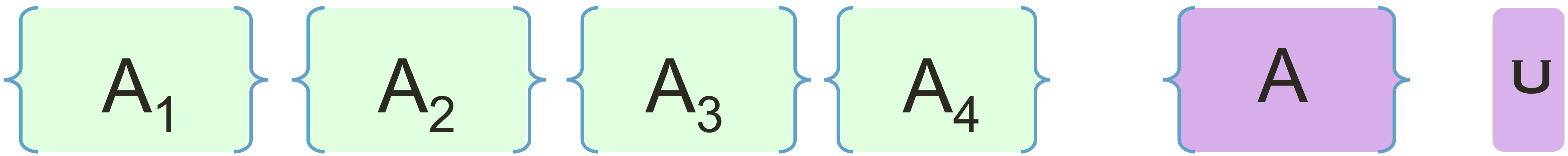

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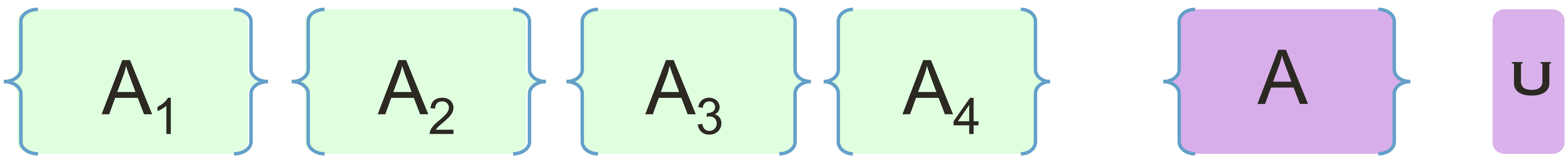

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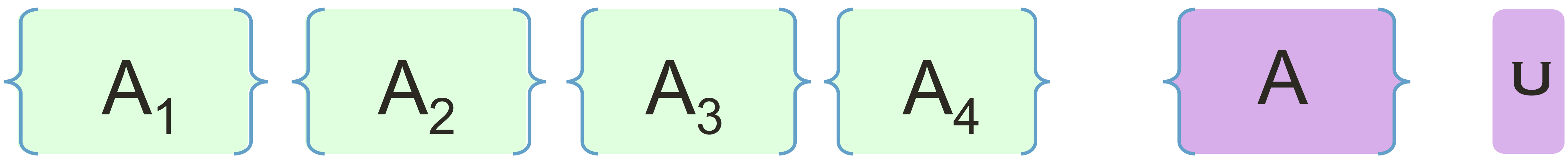

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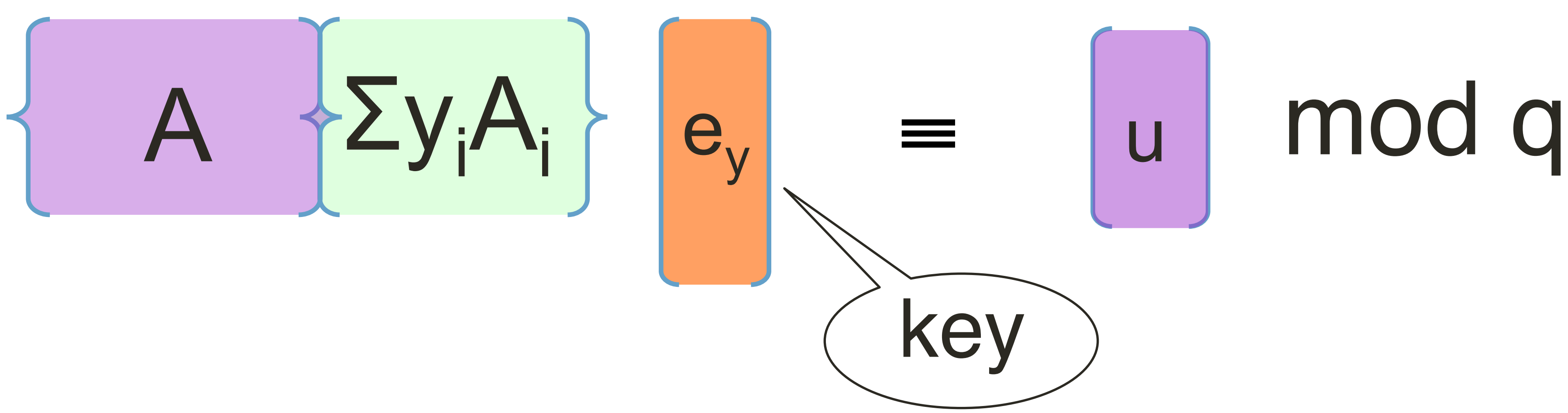

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Generalizing to Inner Product (AFV11)

Decryption

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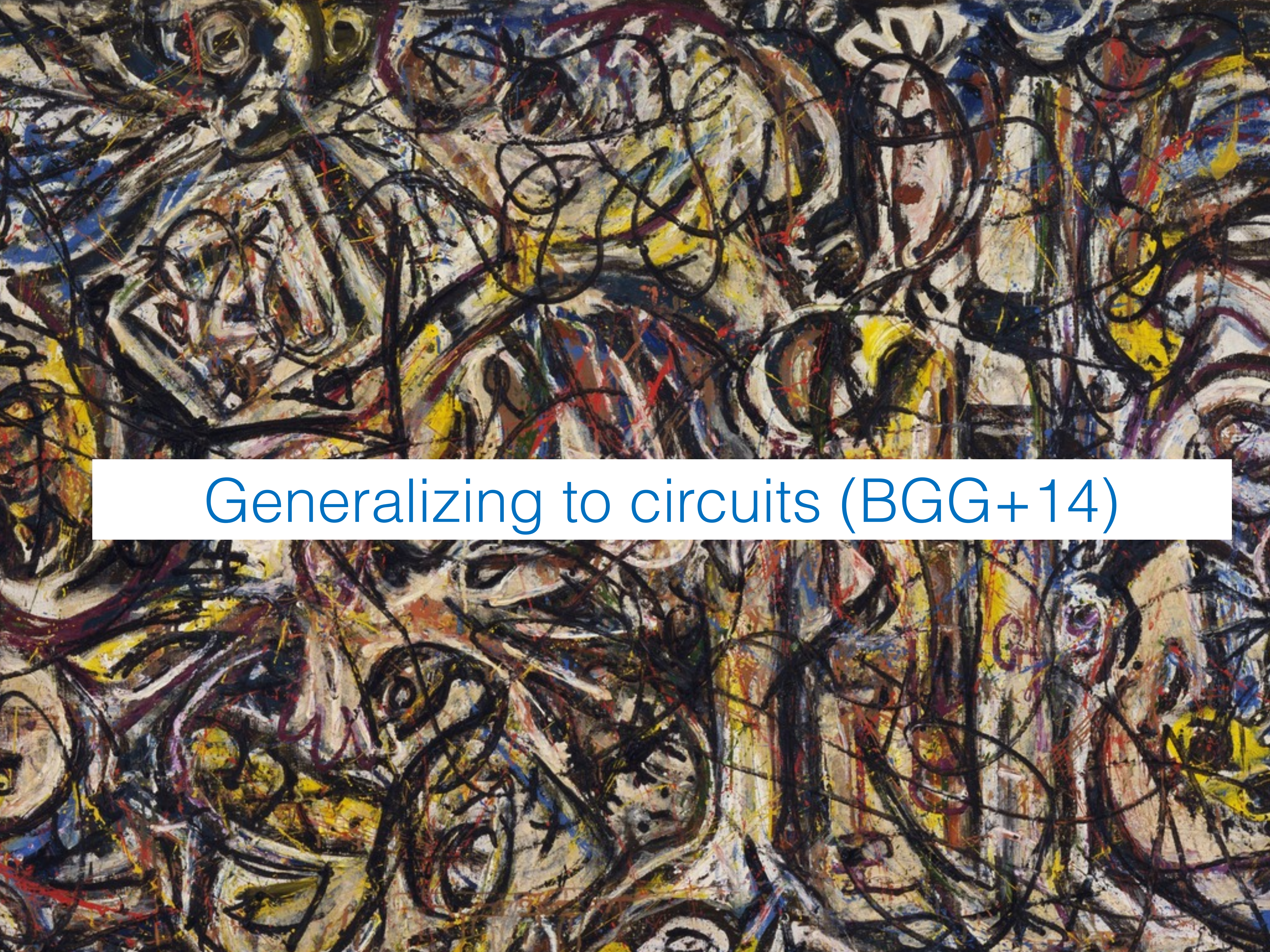
But this is what we have the key for !

Perform Regev Decryption.

Generalizing to circuits (BGG+14)



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Handling Multiplication [BGG+14]

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Then $C_{x1\ x2} = R^T C_1 + x_1 C_2$

$= (A_{12} + x_1 x_2 G)^T s + \text{noise}$

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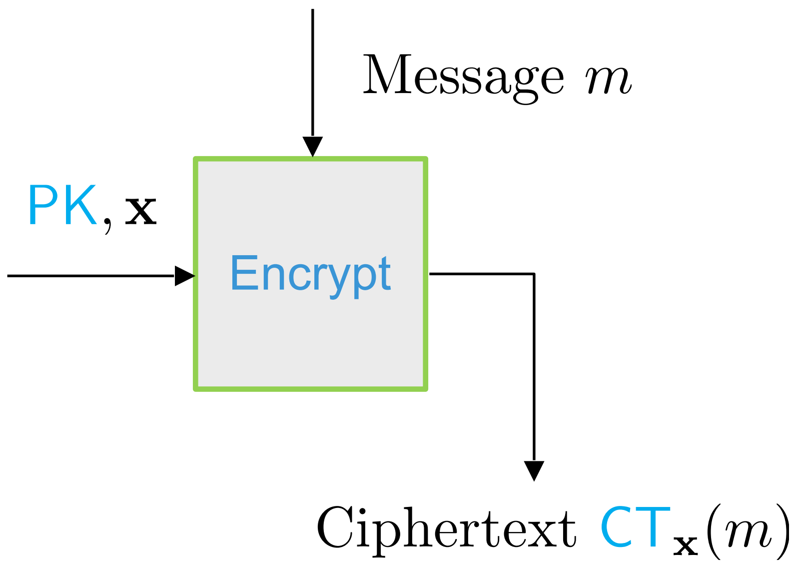

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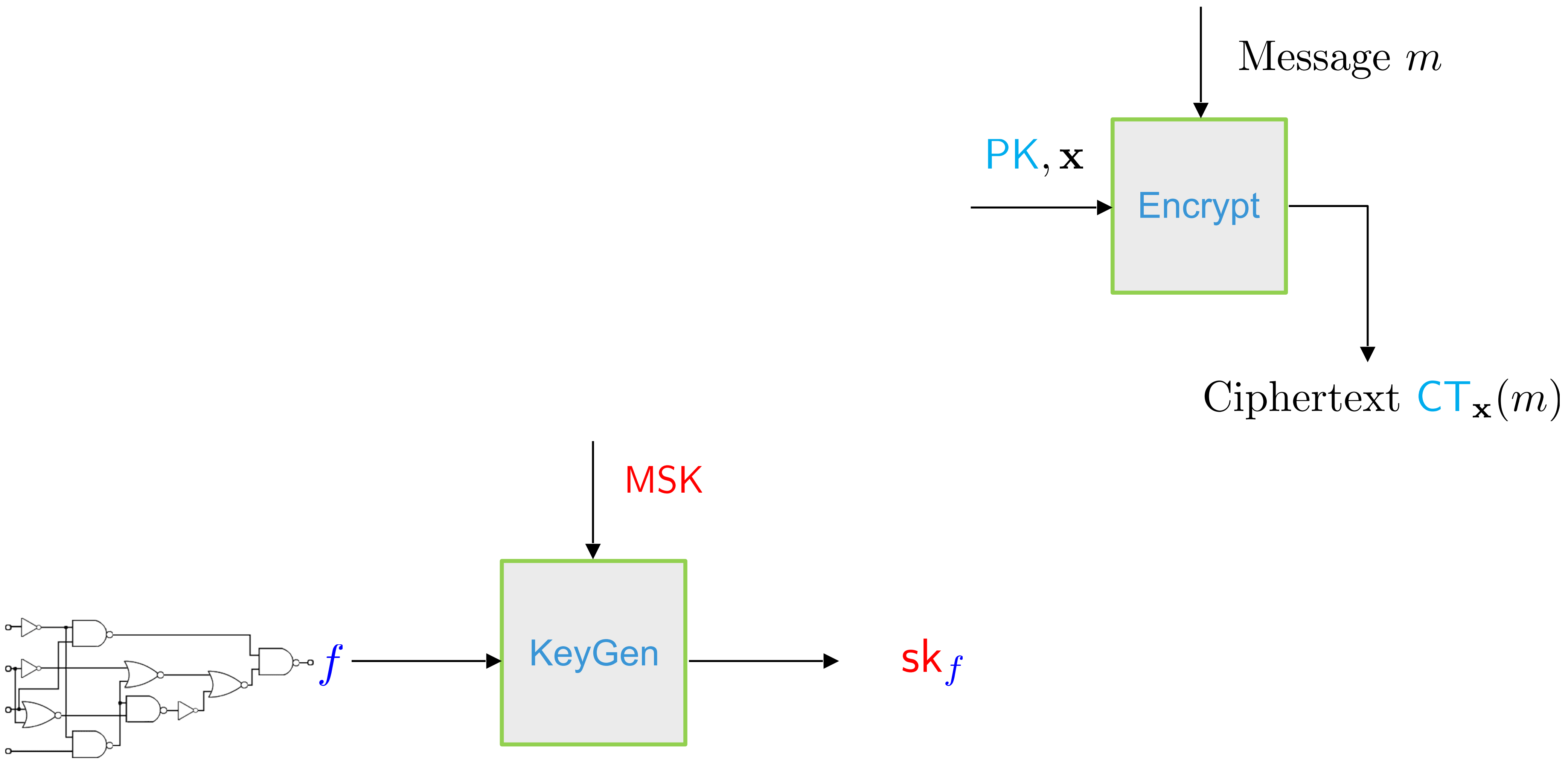

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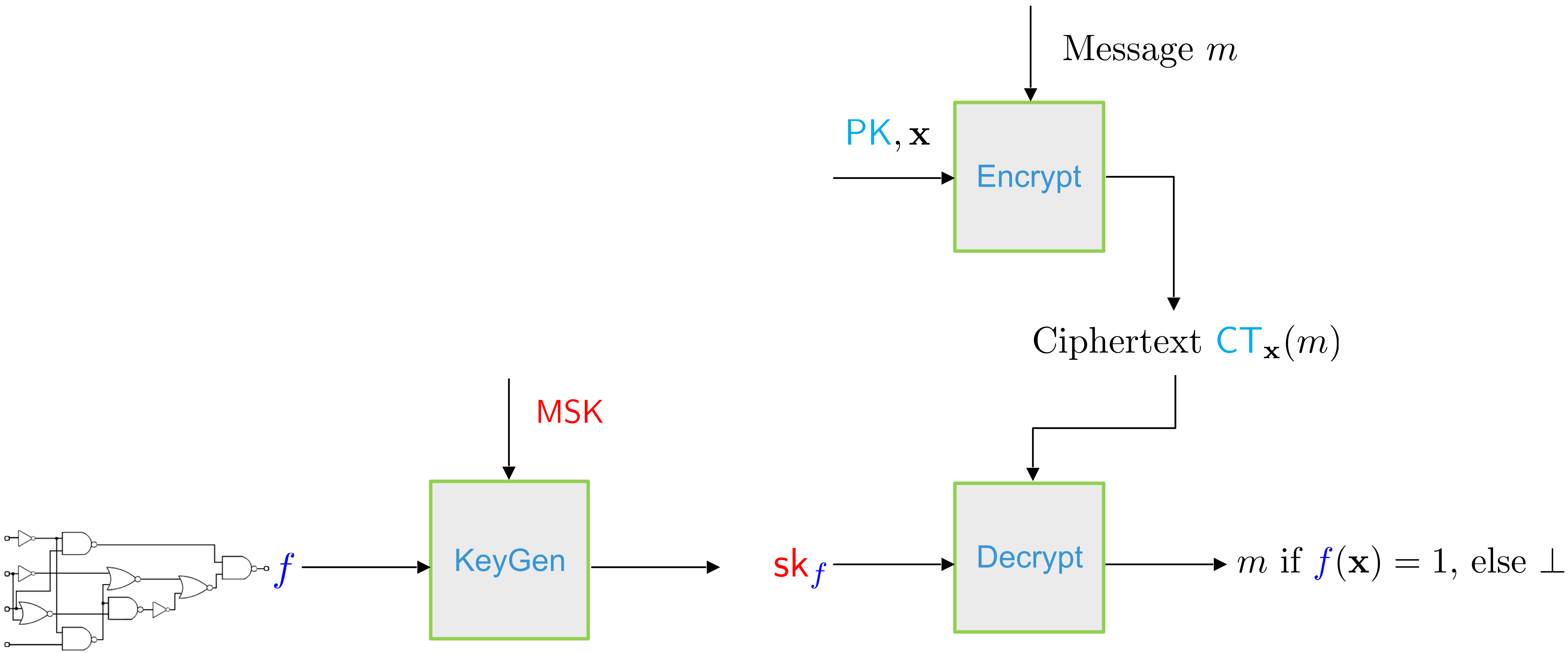

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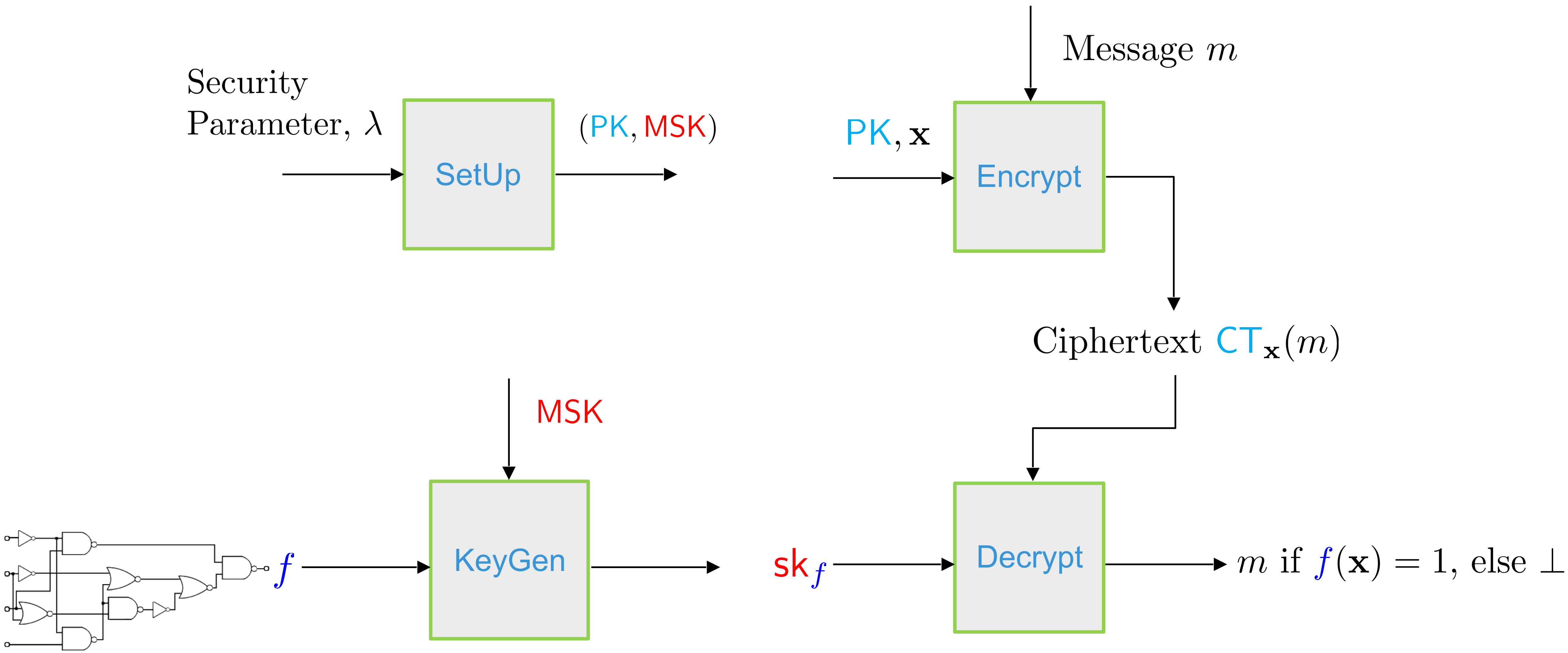

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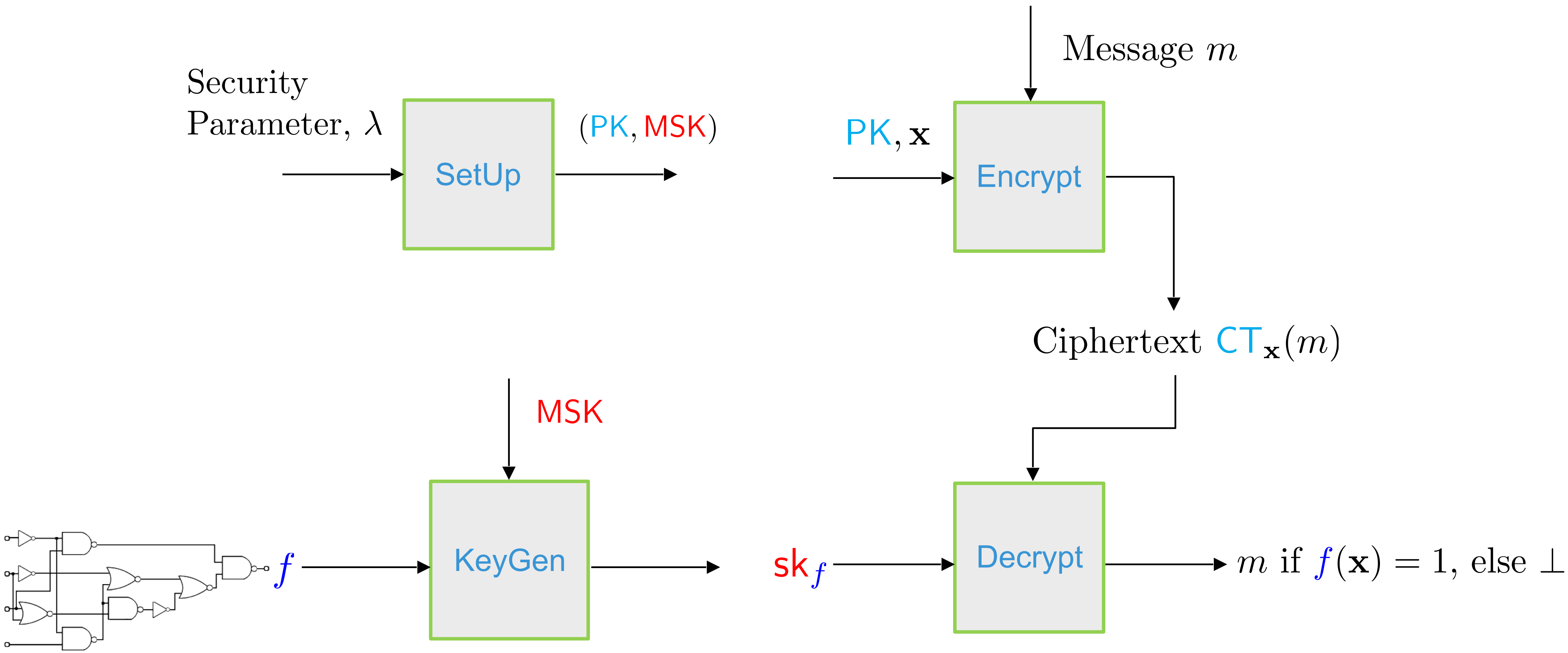
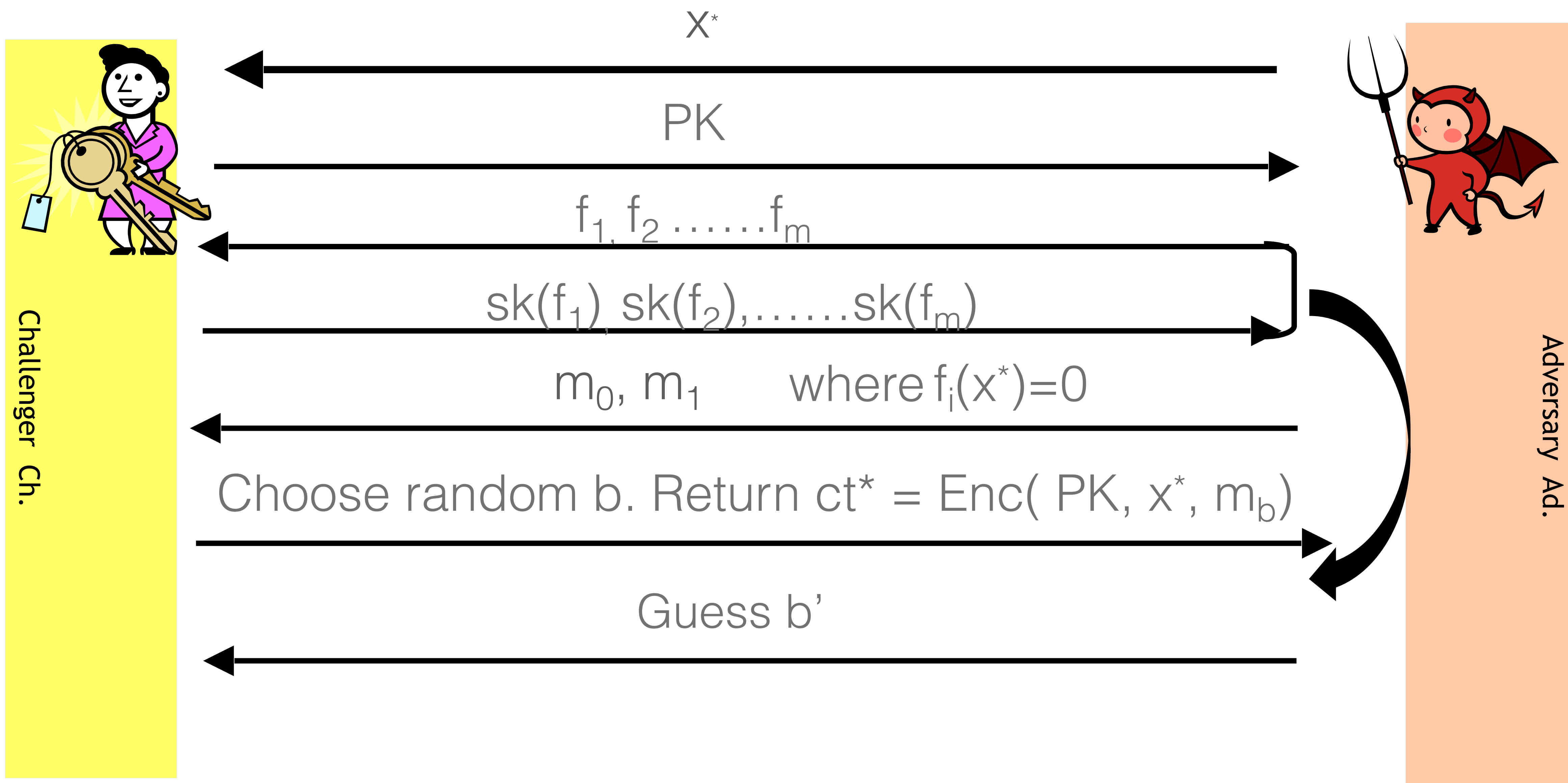


Attribute based Encryption (ABE) [SW05]

Security Definition



Attacker wins if | Pr[b=b'] - ½ | is non-negligible

Security: Challenges

- Challenger needs to be able to answer private key queries of Adversary: much more complex!
- Challenger can't have master trapdoor(Trapdoor for A)
- Must embed LWE challenge into challenge ciphertext

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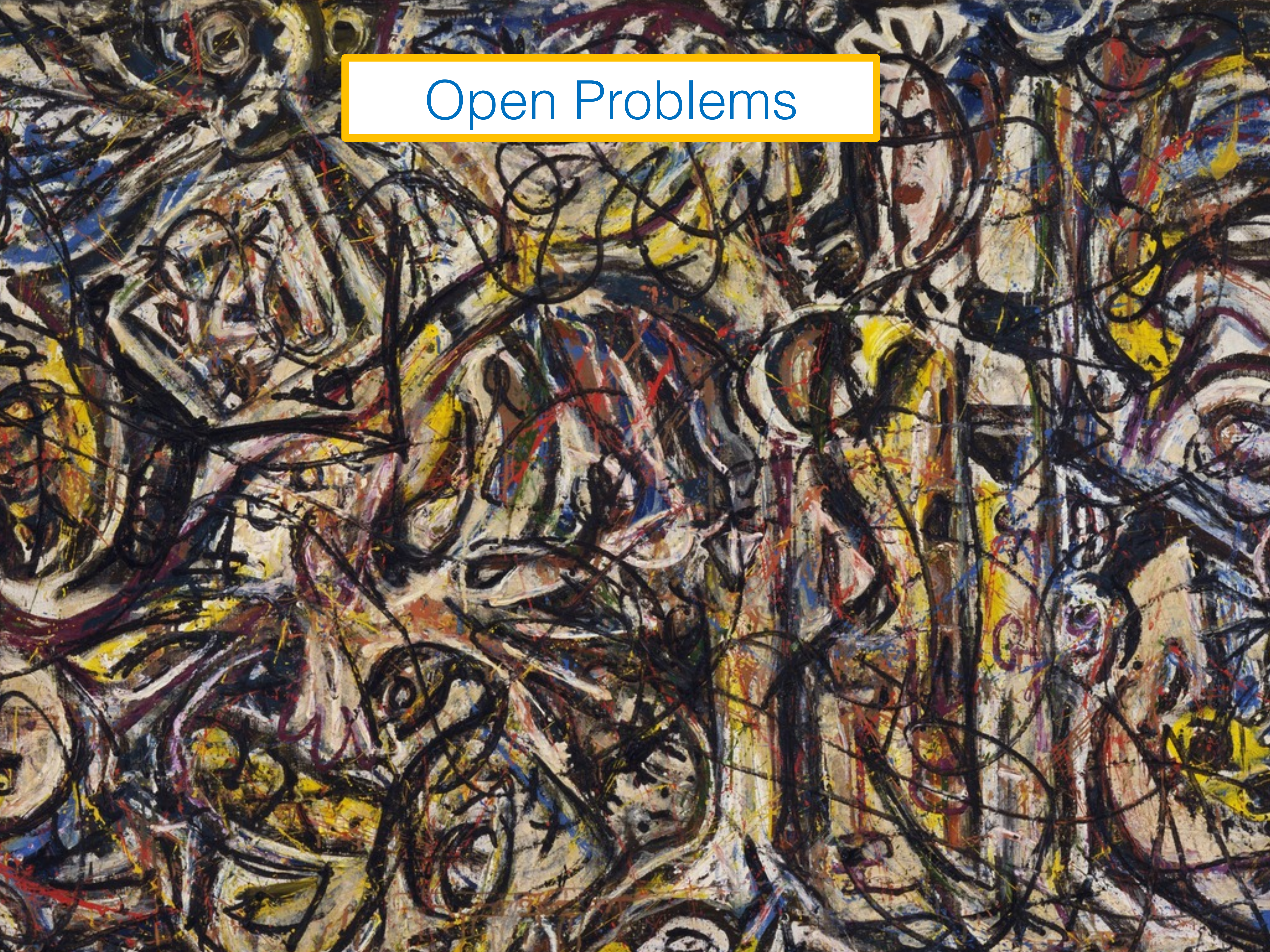
Let $\mathbf{A} \in \mathbb{Z}_q^{n \times m'}$ uniform, $\mathbf{R} \in \mathbb{Z}_q^{m' \times n \log q}$ small

Then $[\,A \mid AR - H\,G\,]$

admits LWE and SIS inversion.

Open Problems



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Thank You!

Image Credits : Hans Hoffman, Jackson Pollock