| | KEMs | Signatures | Open problems | Conclusion |
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Lattices and the NIST PQ-crypto standardization process

Damien Stehlé

ENS de Lyon

Berkeley, January 2020

| Introduction | KEMs | Signatures | Open problems | Conclusion |
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The NIST PQ-crypto standardization process

https://csrc.nist.gov/Projects/post-quantum-cryptography https://groups.google.com/a/list.nist.gov/forum/#!forum/pqc-forum

NIST wants to standardize digital signatures and key exchange mechanisms (KEMs), that are secure even against quantum computing

Main criteria:

- KEM secure against chosen ciphertext attacks (IND-CCA)
- KEM should provide keys with ≥ 256 bits
- Signatures secure against chosen message attacks (EUF-CMA)
- Secure with up to 2⁶⁴ decryption/signature queries
- [Level V] At least as secure as a key search for a 256-bit key block-cipher, such as AES256

| Introduction | KEMs | Signatures | Open problems | Conclusion |
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| Introduction 00000 | KEMs 000000 | Signatures 0000 | Open problems | Conclusion |
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- Dec. 2016: Call for proposals
- Nov. 2017: 82 candidates submitted
- Dec. 2017: 69 (49E+20S) passed the minimal criteria check
- Jan. 2019: end of 1st round of review, 26 (17E+9S) candidates for the 2nd round

What next? From the NIST website:

- 2020/2021 Round 3 begins or select algorithms
- 2022/2024 Draft standards available

From the mailing list (04/09/2019, D. Moody):

"NIST anticipates that there will be a 3rd round. We expect that sometime around June 2020 the 2nd round will end, and the 3rd round will begin. At that point, we will select a smaller number of algorithms to focus our attention on for standardization [...]."

Confirmed at the 7th ETSI-IQC Quantum-Safe Crypto Workshop

| Introduction | KEMs | Signatures | Open problems | Conclusion |
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| Introduction | KEMs | Signatures | Open problems | Conclusion |
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Lattice submissions

Out of the 26 Round-2 candidates, 12 (9E+3S) are based on lattices.

The others involve systems of multiquadratic equations, codes, Merkle trees and zero-knowledge proofs.

| KYBER | NewHope | Round5 | DILITHIUM |
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| FrodoKEM | NTRU | SABER | FALCON |
| LAC | NTRU Prime | Three Bears | qTESLA |
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69 authors in total (!)

Sociological conclusions:

- Lattice-based crypto enjoys the most focus, and the size and maturity of the community are high.
- Encryption schemes seem easier to design than signature schemes.

| Introduction | KEMs | Signatures | Open problems | Conclusion |
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| Introduction | KEMs | Signatures | Open problems | Conclusion |
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Damien Stehlé

| Introduction | KEMs | Signatures | Open problems | Conclusion |
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| Introduction | KEMs | Signatures | Open problems | Conclusion |
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We should make ourselves useful and help the economy

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We should make ourselves useful and help the economy

... bonus points if public research helps companies that avoid taxes

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This raises (or puts more emphasis on) some open problems \Rightarrow the purpose of this talk

| Introduction | KEMs | Signatures | Open problems | Conclusion |
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- Overview of the KEM candidates
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| Introduction | KEMs | Signatures | Open problems | Conclusion |
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| | KEMs | Signatures | Open problems | Conclusion | | |
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 $\underbrace{\textbf{Setup}}_{q \geq 2}: \text{ a ring } \mathcal{R} \text{ that is isomorphic to } (\mathbb{Z}^k, +) \text{ for some } k,$ $q \geq 2 \text{ and } \mathcal{R}_q := \mathcal{R}/q\mathcal{R}.$

KeyGen: pk is a matrix **A** over \mathcal{R}_q , sk is a vector **s** over \mathcal{R} s.t.

 $\|\mathbf{s}\|_{\infty} \ll q$ and $\|\mathbf{s}^T \cdot \mathbf{A}\|_{\infty} \ll q$

Enc: to encrypt a binary vector **m**, get **t** and **e** small, and return:

$$\mathbf{c} := \mathbf{A} \cdot \mathbf{t} + \mathbf{e} + \lfloor q/2 \rfloor \cdot \mathbf{m}$$

Dec: multiply and "round".

$$\mathbf{s}^T \cdot \mathbf{c} = \mathbf{s}^T \cdot \mathbf{A} \cdot \mathbf{t} + \mathbf{s}^T \cdot \mathbf{e} + \lfloor q/2 \rfloor \cdot \mathbf{s}^T \cdot \mathbf{m}$$
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Two instances of the setting

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NTRU [HoPiSi98]

- Take $\mathcal{R} = \mathbb{Z}[x]/P$ for some large degree $P \in \mathbb{Z}[x]$
- Set h = g/f [q], with small f and g in \mathcal{R}
- When decrypting, we recover $f \cdot m$, from which we can get m

LPR [LPR10]

- Sample $\mathbf{A}_0 \in \mathcal{R}_q^{d imes d}$ uniformly, and $\mathbf{s}_0, \mathbf{e}_0 \in \mathcal{R}^d$ small
- Set $\mathbf{A}:=[\mathbf{A}_0 \mid -\mathbf{A}_0\cdot\mathbf{s}_0+\mathbf{e}_0]^{\mathcal{T}}\in \mathcal{R}_q^{(d+1) imes d}$ and $\mathbf{s}=[\mathbf{s}_0|1]$

• Set
$$\mathbf{m} = [\mathbf{0}|m]$$
, so that $\mathbf{s}^T \cdot \mathbf{m} = m$

J. Hoffstein, J. Pipher, J.H. Silverman; ANTS 1998. V. Lyubashevsky, C. Peikert, O. Regev; Eurocrypt 203

Damien Stehlé

| | KEMs | Signatures | Open problems | Conclusion |
|-------|-------|------------|---------------|------------|
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| | KEMs | Signatures | Open problems | Conclusion |
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| Introduction | KEMs | Signatures | Open problems | Conclusion |
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| Security | | | | |

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- If $\left(\textbf{A}, \textbf{A} \cdot \textbf{t} + \textbf{e}\right)$ looks random, the scheme is secure under CPAs
- Security of A: NTRU or LWE-like assumption
- Security of $\boldsymbol{A}\cdot\boldsymbol{t}+\boldsymbol{e}:$ LWE-like assumption

How to get a CCA-secure KEM?

- Use a generic transformation, in the (Q)ROM
- If the scheme is deterministic and perfectly correct:
 ⇒ use the [SXY18] transform (tight proof in QROM)
- Else use a variant of the Fujisaki-Okamoto transform [HHK17]

T. Saito, K. Xagawa, T. Yamakawa; Eurocrypt'18 D. Hofheinz, K. Hövelmanns, E. Kiltz; TCC'17.

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| | KEMs | Signatures | Open problems | Conclusion |
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For the public key, there are many options:

- Matrices over \mathbb{Z}_q
- A polynomial ring $\mathbb{Z}_q[x]/P$ for some polynomial $P \in \mathbb{Z}[x]$
- Matrices over such a polynomial ring
- In the last two cases, several P's have been considered
- Several q's can be considered

What is at stake?

- Impacts the underlying hardness assumption: LWE, P-LWE, M-LWE
- Does not seem to impact actual security
- Impacts efficiency

| | KEMs | Signatures | Open problems | Conclusion |
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Which distributions for the small vectors?

For s, t and e:

- Integer Gaussian or approximation thereof
- Centered binomial distribution
- Ternary distribution (possibly sparse)
- Deterministic **e**, obtained by rounding (for some p < q):

$$\mathbf{e} = -\frac{q}{p} \lfloor \frac{p}{q} \mathbf{A} \cdot \mathbf{t} \rfloor$$

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- If sparsity is pushed a lot, it impacts actual security ([H08], Round5)
- Small s, t and e ⇒ q can be set smaller (e.g., LAC), or perfect correctness can be obtained more easily (e.g., NTRUPrime)

N. Howgrave-Graham; Crypto'08.

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| Introduction | KEMs | Signatures | Open problems | Conclusion |
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Common strategy [ADPS15]:

- Forget about the CCA upgrade reduction loss (in the QROM)
- Express the selected hardness assumption as a lattice problem
- Assess how strong lattice reduction needs to be to break the scheme
- Convert the latter into a BKZ block-size [C13]
- Bound the cost from below by the cost of the best known SVP solvers in that dimension [BDGL16,L15]

And then try many parameters to minimize sizes/costs/simplicity under the constraint of a lower bound on the cost.

E. Alkim, L. Ducas, T. Pöppelmann, P. Schwabe; USENIX'16.

Y. Chen; ENS PhD thesis, 2013.

A. Becker, L. Ducas, N. Gama, T. Laarhoven; SODA'16.

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| Introduction | KEMs | Signatures | Open problems | Conclusion |
|--------------|--------|------------|---------------|------------|
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| Roadmap | | | | |

- Overview of the KEM candidates
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| Introduction | KEMs | Signatures | Open problems | Conclusion |
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| Falcon | | | | |

Combine NTRUSign and the ${\scriptstyle [GPV08]}$ framework, and optimize

Public key is $h = g/f \in \frac{\mathbb{Z}_q[x]}{x^n+1}$ with $n \in \{512, 1024\}$.

Secret key is a small basis $[f,g]^T, [F,G]^T$ of the module lattice

$$\left\{ [a,b]^T \in \left(\frac{\mathbb{Z}[x]}{x^n+1}\right)^2 : a \cdot h - b = 0 \ [q] \right\}$$

A signature is a Gaussian sample over a lattice coset.

- Compact signatures
- Somewhat complex to implement
- Hardness relies on solving an inhomogeneous version of SIS:

Given h and y, find a small s.t.: $a \cdot h \approx y$ [q]

C. Gentry, C. Peikert, V. Vaikuntanathan; STOC'08.

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| Introduction | KEMs | Signatures | Open problems | Conclusion |
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Dilithium and Tesla

Schnorr's discrete-log signature, mapped to the lattice setting [L12]

Public key is made of $\mathbf{A} \in R_a^{k \times \ell}$ and $\mathbf{t} = \mathbf{A} \cdot \mathbf{s}_1 + \mathbf{s}_2$.

Secret key is (s_1, s_2) .

Signing consists in proving knowledge of (s_1, s_2) .

- Larger signatures
- Easier to implement (no Gaussians, no use of subfields, no floating-p. numbers)
- Hardness relies on solving an inhomogeneous version of SIS:

Given **A** and **t**', find **z** small s.t.: $\mathbf{A} \cdot \mathbf{z} \approx \mathbf{t}'$ [q]

V. Lyubashevsky; Eurocrypt'12.

| Introduction | KEMs | Signatures | Open problems | Conclusion |
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Efficiency comparison

| | vk | sig | KeyGen | Sign | Verify | q-sec |
|------------|------|------|--------|------|--------|-------|
| Falcon512 | 0.9k | 0.7k | 26M | 1.3M | 160k | 103 |
| Falcon1024 | 1.8k | 1.3k | 78M | 2.7M | 200k | 230 |
| Dilithium3 | 1.5k | 2.7k | 370k | 1.6M | 380k | 128 |
| Dilithium4 | 1.8k | 3.4k | 470k | 1.4M | 510k | 158 |
| qTESLA-I | 15k | 2.6k | 2.4M | 3.1M | 670k | 139 |
| qTESLA-III | 38k | 5.7k | 14M | 8.5M | 1.8M | 279 |

Reference C implementations For a single signature Sizes in bytes, runtimes in cycles Average sizes and runtimes, approximations to 2 significant digits

| Introduction 00000 | KEMs 000000 | Signatures 0000 | Open problems | Conclusion 000 |
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| Roadman | | | | |

- Overview of the KEM candidates
- Overview of the signature candidates
- Some raised problems that I like

Partly based on

```
http://crypto-events.di.ens.fr/LATCA/program/alperin-sheriff.pdf
```

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http://www.h2020prometheus.eu/dissemination/blog/
assessing-security-lattice-based-submissions-10-questions-nist-should-be-asking
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| | KEMs | Signatures | Open problems | Conclusion |
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| Rest know | vn algorithms | : | | |

When setting parameters, one should consider the best known practical algorithms. What are they, and how do they extrapolate?

- Have sieving and enumeration be pushed as far as possible?
- The best algorithms asymptotically are all heuristic.
 ⇒ Can we prove, support, dispute these heuristics?

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- Improve cost lower bounds? (e.g., counting SVP-solver calls)
- Will sieving still outperform enumeration for larger dimensions, considering its memory requirements?
- How do we put a price tag on a massive quantum computation?

M.R. Albrecht, V. Gheorghiu, E.W. Postlethwaite, J.M. Schanck; eprint 2019/1161.
E. Kirshanova, E. Mårtensson, E.W. Postlethwaite, S.R. Moulik; Asiacrypt'19.
M.R. Albrecht, L. Ducas, G. Herold, E. Kirshanova, E.W. Postlethwaite, M. Stevens; Eurocrypt'19.
Y. Aono, P.Q. Nguyen, T. Seito, J. Shikata; Crypto'18.

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| Introduction 00000 | KEMs 000000 | Signatures 0000 | Open problems 00●0000 | Conclusion 000 |
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| The choice o | f the assump | tion | | |

When designing the scheme, what should be the hardness assumption that is put forward to claim security?

Asymptotically, many of the assumptions are polynomial-time equivalent. For concrete parameters, most of this vanishes.

- Is LWE as OK as LWR?
- M-LWE versus P-LWE?
- How more aggressive is the NTRU assumption, compared to the P-LWE/M-LWE assumptions?
- What about the ThreeBear Integer M-LWE problem?

M.R. Albrecht, A. Deo; Asiacrypt'17.

| | KEMs | Signatures | Open problems | Conclusion |
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| Stretchin | a assumption | e | | |
| JUELUIIII | g assumptions | 2 | | |

Many schemes rely on standard assumptions but with unusual parameter settings. How far can we stretch the assumptions?

- \bullet Assess the hardness of SIS with a large $\ell_\infty\text{-norm}$ bound $\,$ (Dilithium).
 - Does [MiPe13] extend to this setting?
 - How can we exploit it, algorithmically?
- Can we exploit various shapes of noises in lattice algorithms?
- Concrete resistance of NTRU schemes against [KF17]?
- Concrete resistance of ternary noise schemes against [KF15]?

- P. Kirchner, P.-A. Fouque; Eurocrypt'17.
- P. Kirchner, P.-A. Fouque; Crypto'15.
- N. Howgrave-Graham; Crypto'08.

D. Micciancio, C. Peikert; Crypto'13.

| | KEMs | Signatures | Open problems | Conclusion |
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D. Micciancio, C. Peikert; Crypto'13.

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| Choice o | f polynomial r | ing | | |

For NTRU, P-LWE and M-LWE, many polynomials are possible. Does this choice impact security?

- Can cyclotomic polynomials be showed bad in any way?
- Among them, are some worse than others?
- Can the other polynomials selected for NIST candidates be showed bad in some way?
- What do attacks on Ideal-SVP say? Can they be extended to P-LWE/M-LWE?

C. Lee, A. Pellet-Mary, D. Stehlé, A. Wallet; Asiacrypt'19.

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| Introduction | KEMs | Signatures | Open problems | Conclusion |
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| Impact of | decryption e | errors | | |

Most of the candidates have imperfect correctness. What is the impact of decryption errors?

- Determine the precise interplay between the polynomial ring structure and the probability of incorrect decryption.
- Assess the probability of having weaker secret keys.
- What is the cost of thwarting these attacks via the security proofs? Are these optimal?

Q. Guo, T. Johansson, J. Yang; Asiacrypt'19. J.-P. D'Anvers, Q. Guo, T. Johansson, A. Nilsson, F. Vercauteren, I. Verbauwhede; PKC'19.

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| OROM n | roofs? | | | |

The Fujisaki-Okamoto upgrade to CCA security incurs a large loss. Does it have to?

Decrease the dependency of the distinguishing advantage of the CPA scheme as a function of

- the decryption error probability;
- the number of decryption queries;
- the distinguishing advantage of the CCA upgrade.

Or show that this is not possible!

Tighter QROM proofs for Dilithium under 'standard' assumptions?

N. Bindel, M. Hamburg, K. Hövelmanns, A. Hülsing, E. Persichetti; TCC'19.

R. Steinfeld, A. Sakzad, D. Stehlé, V. Kuchta, S. Sun; Stay tuned!

Q. Liu, M. Zhandy; Crypto'19.

J. Don, S. Fehr, C. Majenz, C. Schaffner; Crypto'19.

E. Kiltz, V. Lyubashevsky, C. Schaffner; Eurocrypt'18.

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Other aspects not covered in this talk

Among others...

- Non-lattice candidates
- Efficient implementations
- Proved implementations
- Resistance to side-channel attacks
- How to measure (and cost) simplicity?

| Introduction | KEMs | Signatures | Open problems | Conclusion |
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| Post-NIST | post-quant | um crypto! | | |

- Have we reached a ceiling for basic asymmetric primitives?
 - Signatures do not seem as explored as encryption.
 - Dilithium and Falcon have very different designs and performances.
 - Find trade-offs? Improvements? [CPSWX19]
 - By how far are standard model schemes out of the game?
 - What else would we want to deploy at very efficiently?
 - Identity-based encryption?
 - Zero-knowledge proofs? Voting, anonymous credentials?
 - Computing on personal data? HE, FE, MPC?

QUESTIONS?

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