Resonator Circuits for factoring high-dimensional vectors

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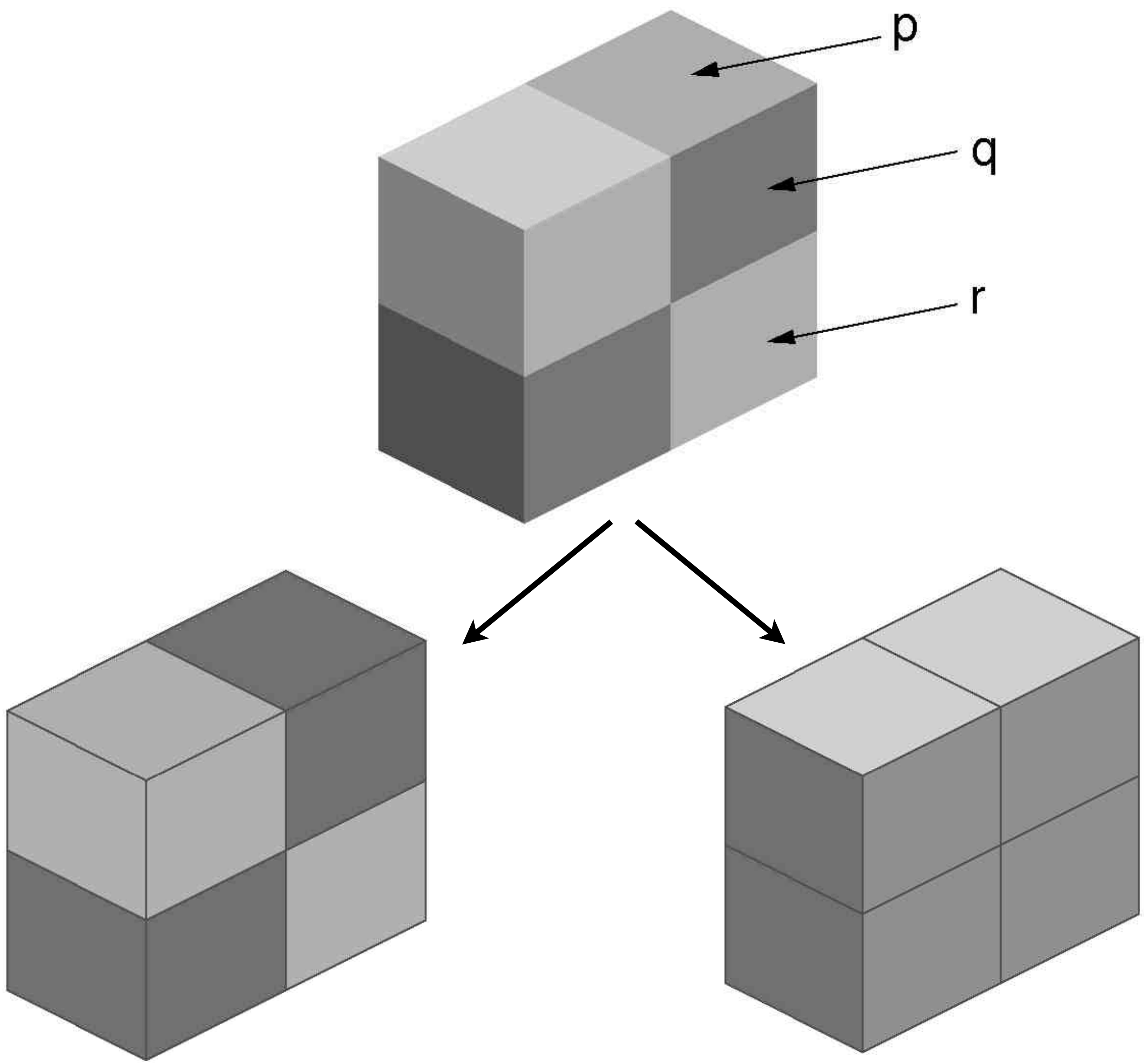
Redwood Center for Theoretical Neuroscience - April 2018

Factorization of shape and reflectance



reflectance

shading

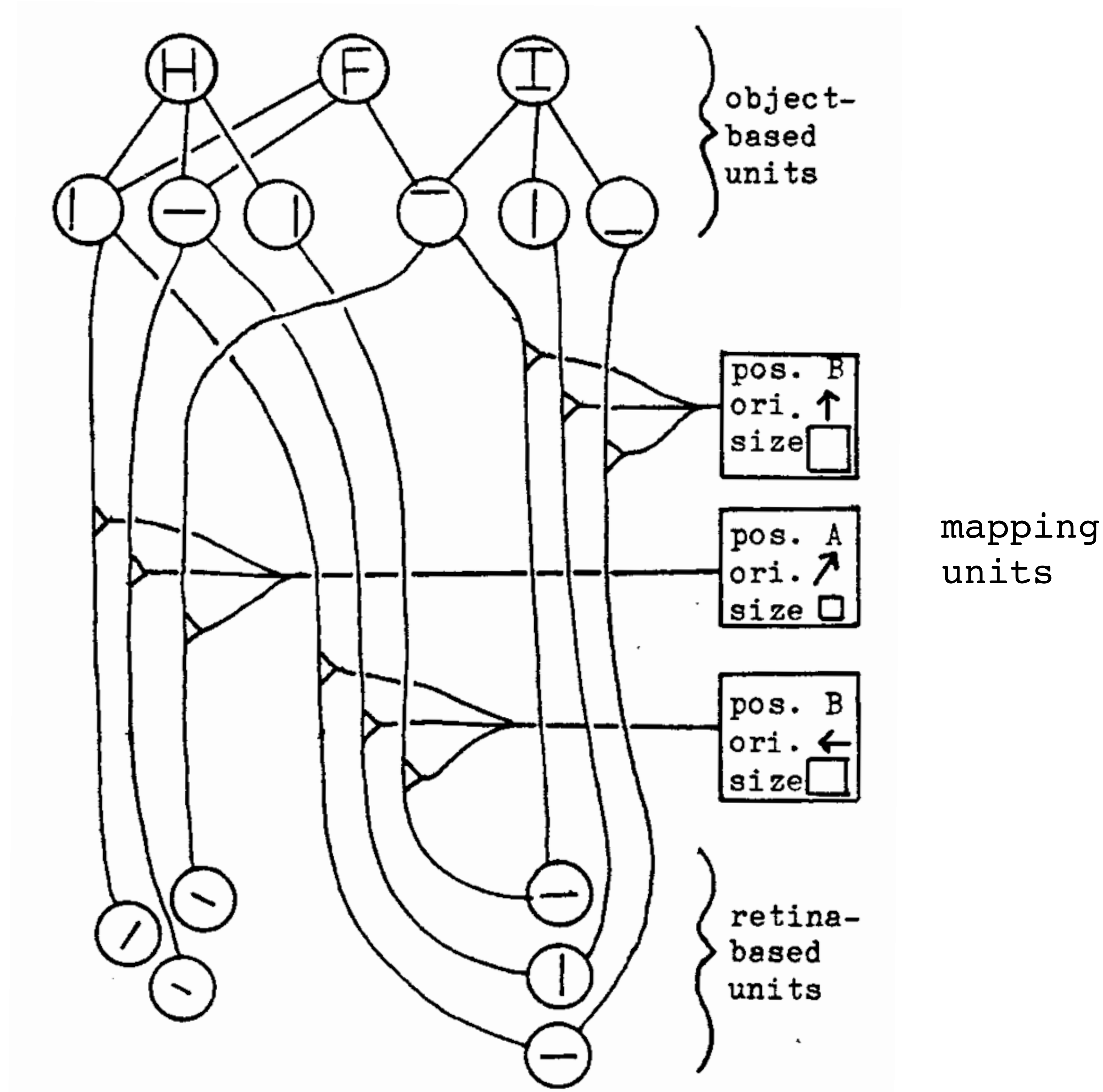
(Adelson, 2000)

Factorization of object shape and pose

The meaning of the triangular symbol in fig. 1 is quite complex. It stands for two rules:

1. Multiply the activity level in the retina-based unit by the activity level in the mapping unit and send the product to the object-based unit.

2. Multiply the activity level in the retina-based unit by the activity level in the object-based unit and send the product to the mapping unit.



Hinton (1981)

Computing with high-dimensional vectors

Pentti Kaneva

Concepts, variables, attributes are represented as high-dimensional vectors (e.g., 10,000 bits)

Three fundamental operations:

- multiplication (binding)
- addition (combining)
- permutation (sequencing)

Approximates a *field*

Kanerva P (2009) Hyperdimensional Computing: An Introduction to Computing in Distributed Representation with High-Dimensional Random Vectors. *Cognitive Computing, 1*: 139-159.

Plate, T. A. (1995). Holographic reduced representations. *IEEE Transactions on Neural networks*, *6*(3), 623-641.

Single neuron recording ⇒ Single neuron thinking

What the Frog's Eye Tells the Frog's Brain*

J. Y. LETTVIN†, H. R. MATURANA‡, W. S. McCULLOCH||, SENIOR MEMBER, IRE, AND W. H. PITTS||

Summary—**In this paper, we analyze the activity of single fibers in the optic nerve of a frog. Our method is to find what sort of stimulus causes the largest activity in one nerve fiber and then what is the exciting aspect of that stimulus such that variations in everything else cause little change in the response. It has been known for the past 20 years that each fiber is connected not to a few rods and cones in the retina but to very many over a fair area. Our results show that for**

factor. There are four types of fibers, each type concerned with a different sort of pattern. Each type is uniformly distributed over the

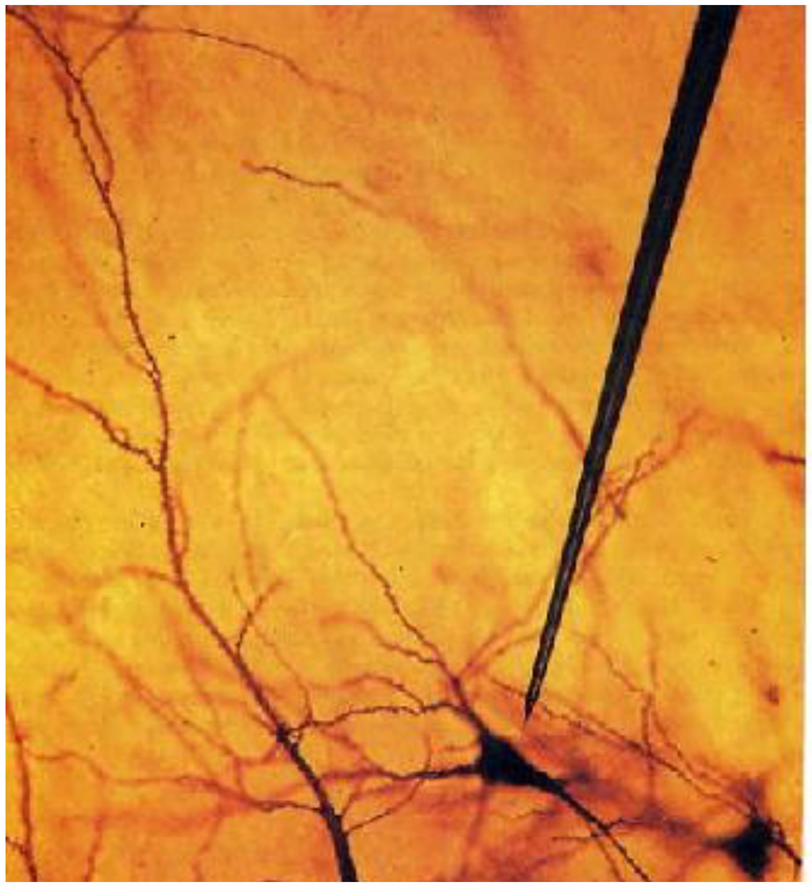
visual image in terms of local pattern independent of average illumination. We describe the patterns and show the functional and anatomical separation of the channels. This work has been done on the frog, and our interpretation applies only to the frog.

it moves like one. He can be fooled easily not only by a bit of dangled meat but by any moving small object. His sex life is conducted by sound and touch. His choice of paths in escaping enemies does not seem to be governed by anything more devious than leaping to where it is darker. Since he is equally at home in water and on

Anatomy of Frog Visual Apparatus

The retina of a frog is shown in Fig. 1(a). Between the rods and cones of the retina and the ganglion cells, whose axons form the optic nerve, lies a layer of con-

Single neuron recording ⇒ Single neuron thinking



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What the Frog's Eye Tells the Frog's Brain*

J. Y. LETTVIN†, H. R. MATURANA‡, W. S. McCULLOCH||, SENIOR MEMBER, IRE, AND W. H. PITTS||

Perception, 1972, volume 1, pages 371–394

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Single units and sensation: A neuron doctrine for perceptual psychology?

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Received 6 December 1972

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Abstract. The problem discussed is the relationship between the firing of single neurons in sensory pathways and subjectively experienced sensations. The conclusions are formulated as the following five dogmas:

either a more macroscopic or microscopic level, because behaviour depends upon the organized pattern of these intercellular interactions.
2. The sensory system is organized to achieve as complete a representation of the sensory stimulus as possible with the minimum number of active neurons.
3. Trigger features of sensory neurons are matched to redundant patterns of stimulation by experience as well as by developmental processes.
4. Perception corresponds to the activity of a small selection from the very numerous high-level neurons, each of which corresponds to a pattern of external events of the order of complexity of the events symbolized by a word.
5. High impulse frequency in such neurons corresponds to high certainty that the trigger feature is present.

The development of the concepts leading up to these speculative dogmas, their experimental basis, and some of their limitations are discussed.

Text classification:

Joshi, A., Halseth, J. T., & Kanerva, P. (2016). Language geometry using random indexing. In *International Symposium on Quantum Interaction* (pp. 265-274).

Sequence memory:

Frady, E. P., Kleyko, D., & Sommer, F. T. (2018). A theory of sequence indexing and working memory in recurrent neural networks. *Neural computation*, *30*(6), 1449-1513.

Scene analysis:

Weiss, E. (2018) Compositional scene representation with high-dimensional vectors. Ph.D. thesis. (UC Berkeley)

Song recognition:

Wong, J. (2018) Negative capacitance and hyperdimensional computing for unconventional low-power computing. Ph.D. thesis. (UC Berkeley)

Hardware implementation:

Wu, T. F., Li, H., Huang, P. C., Rahimi, A., Rabaey, J. M., Wong, H. S. P., ... & Mitra, S. (2018). Brain-inspired computing exploiting carbon nanotube FETs and resistive RAM: Hyperdimensional computing case study. In *Solid-State Circuits Conference-(ISSCC), 2018 IEEE International* (pp. 492-494).

Rahimi, A., Datta, S., Kleyko, D., Frady, E. P., Olshausen, B., Kanerva, P., & Rabaey, J. M. (2017). High-dimensional computing as a nanoscalable paradigm. *IEEE Transactions on Circuits and Systems I: Regular Papers*, *64*(9), 2508-2521.

Factorization in HD

Let $\mathbf{b} = \mathbf{x} \otimes \mathbf{y} \otimes \mathbf{z}$

$$\mathbf{x} \in \mathbb{X} := \{\mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_n\}$$
$$\mathbf{y} \in \mathbb{Y} := \{\mathbf{y}_0, \mathbf{y}_1, \ldots, \mathbf{y}_n\}$$
$$\mathbf{z} \in \mathbb{Z} := \{\mathbf{z}_0, \mathbf{z}_1, \ldots, \mathbf{z}_n\}$$

Problem: You are given $\mathbf{b}$, what are $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{z}$?

Solution: Resonate

$$\hat{\mathbf{x}}_{t+1} = g\Big(\mathbf{X}\mathbf{X}^\top(\mathbf{b} \otimes \hat{\mathbf{y}}_t^{-1} \otimes \hat{\mathbf{z}}_t^{-1})\Big)$$
$$\hat{\mathbf{y}}_{t+1} = g\Big(\mathbf{Y}\mathbf{Y}^\top(\mathbf{b} \otimes \hat{\mathbf{x}}_t^{-1} \otimes \hat{\mathbf{z}}_t^{-1})\Big)$$
$$\hat{\mathbf{z}}_{t+1} = g\Big(\mathbf{Z}\mathbf{Z}^\top(\mathbf{b} \otimes \hat{\mathbf{x}}_t^{-1} \otimes \hat{\mathbf{y}}_t^{-1})\Big)$$

$$\mathbf{X} = \begin{bmatrix} | & | & & | \\ \mathbf{x}_1 & \mathbf{x}_2 & \ldots & \mathbf{x}_n \\ | & | & & | \end{bmatrix}$$
$$\mathbf{Y} = \begin{bmatrix} | & | & & | \\ \mathbf{y}_1 & \mathbf{y}_2 & \ldots & \mathbf{y}_n \\ | & | & & | \end{bmatrix}$$
$$\mathbf{Z} = \begin{bmatrix} | & | & & | \\ \mathbf{z}_1 & \mathbf{z}_2 & \ldots & \mathbf{z}_n \\ | & | & & | \end{bmatrix}$$

$$g(x) = \operatorname{sgn}(x)$$

Consider the following energy function

$$E = -\mathbf{b} \cdot (\mathbf{x} \otimes \mathbf{y} \otimes \mathbf{z})$$

$$\mathbf{x} = \sum_{i=1}^{n} \alpha_i \, \mathbf{x}_i, \quad \mathbf{y} = \sum_{i=1}^{n} \beta_i \, \mathbf{y}_i, \quad \mathbf{z} = \sum_{i=1}^{n} \gamma_i \, \mathbf{z}_i$$

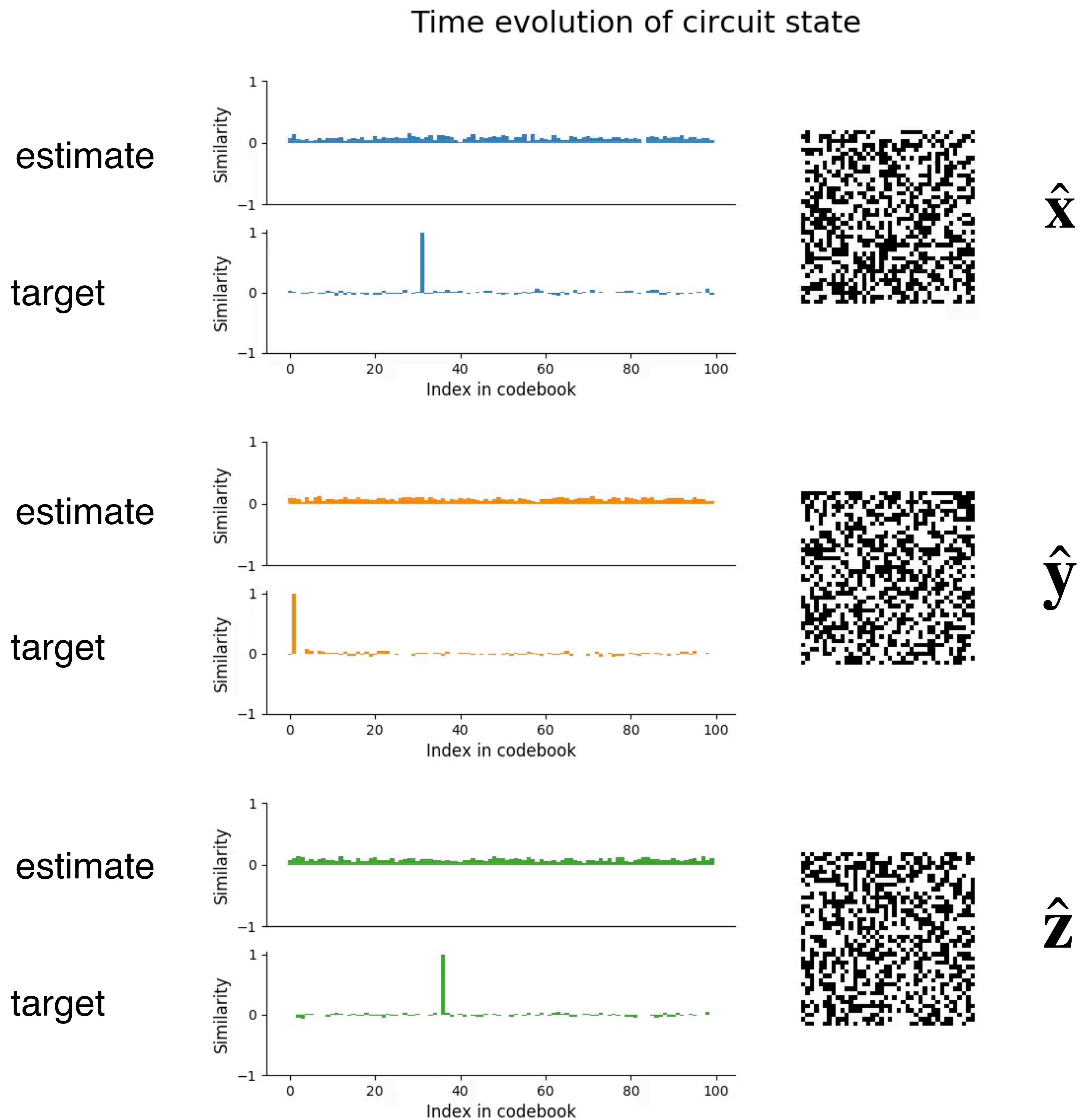
Consider the following energy function

1,000,000 combinations! (n=100)

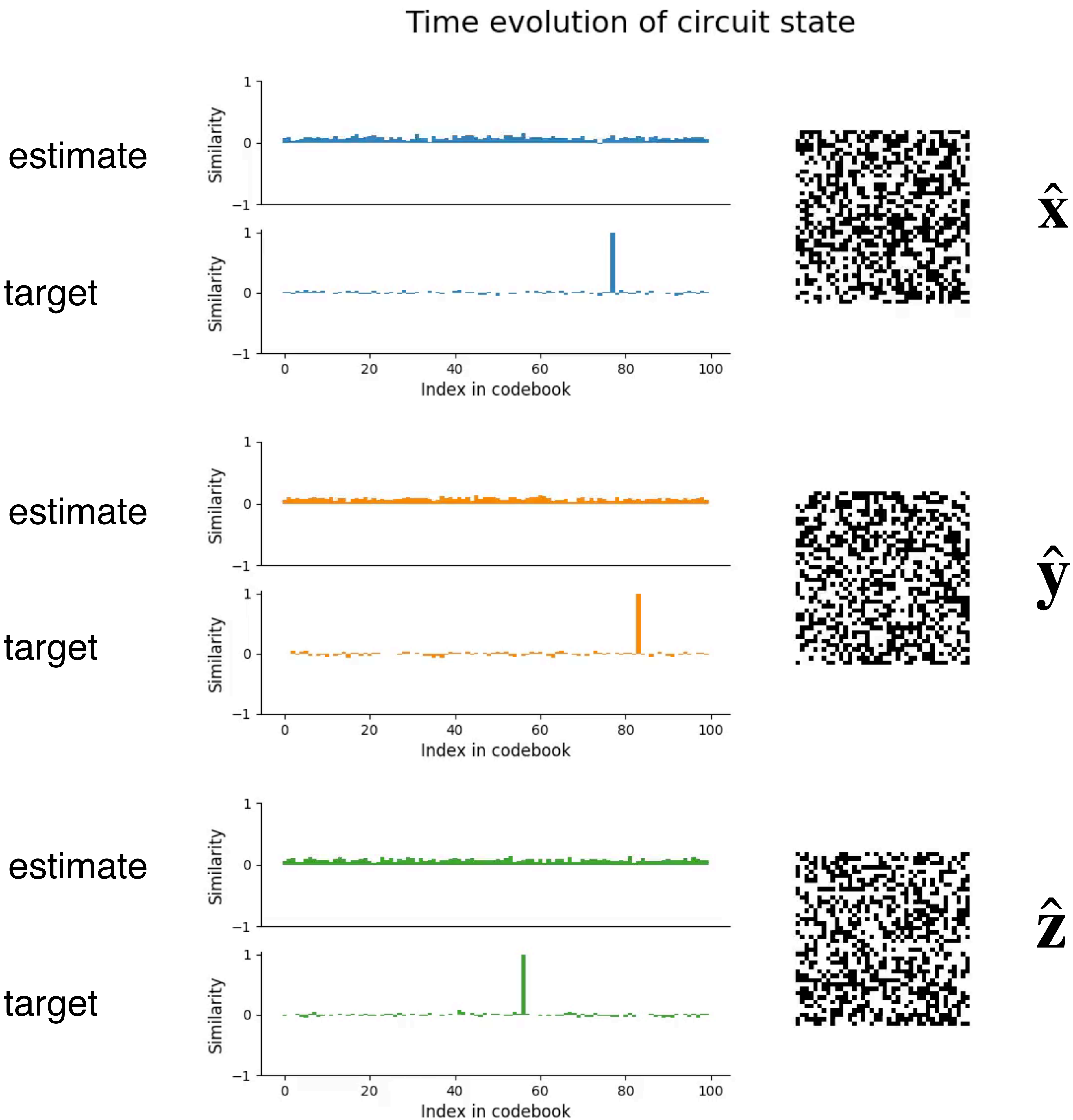
$$(\alpha_1\beta_1\gamma_1\,\mathbf{x}_1 \otimes \mathbf{y}_1 \otimes \mathbf{z}_1 \;+\; \ldots \;+\; \alpha_i\beta_j\gamma_k\,\mathbf{x}_i \otimes \mathbf{y}_j \otimes \mathbf{z}_k \;+\; \ldots \;+\; \alpha_n\beta_n\gamma_n\,\mathbf{x}_n \otimes \mathbf{y}_n \otimes \mathbf{z}_n)$$

$$E = -\mathbf{b} \cdot (\mathbf{x} \otimes \mathbf{y} \otimes \mathbf{z})$$

$$\mathbf{x} = \sum_{i=1}^{n} \alpha_i\,\mathbf{x}_i, \quad \mathbf{y} = \sum_{i=1}^{n} \beta_i\,\mathbf{y}_i, \quad \mathbf{z} = \sum_{i=1}^{n} \gamma_i\,\mathbf{z}_i$$



Time evolution of circuit state
estimate
target
Similarity
Index in codebook
$\hat{\mathbf{x}}$
$\hat{\mathbf{y}}$
$\hat{\mathbf{z}}$


Time evolution of circuit state
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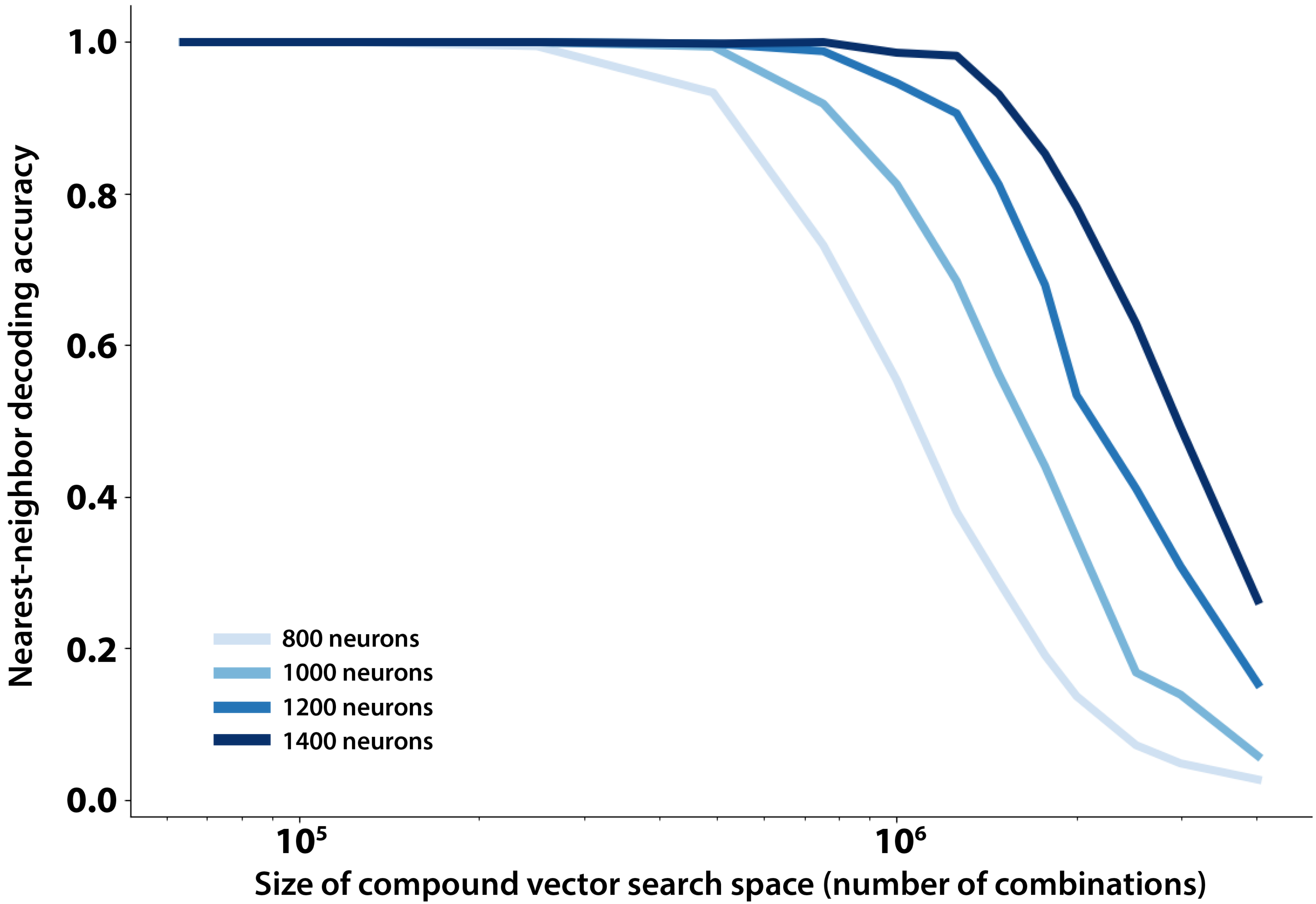
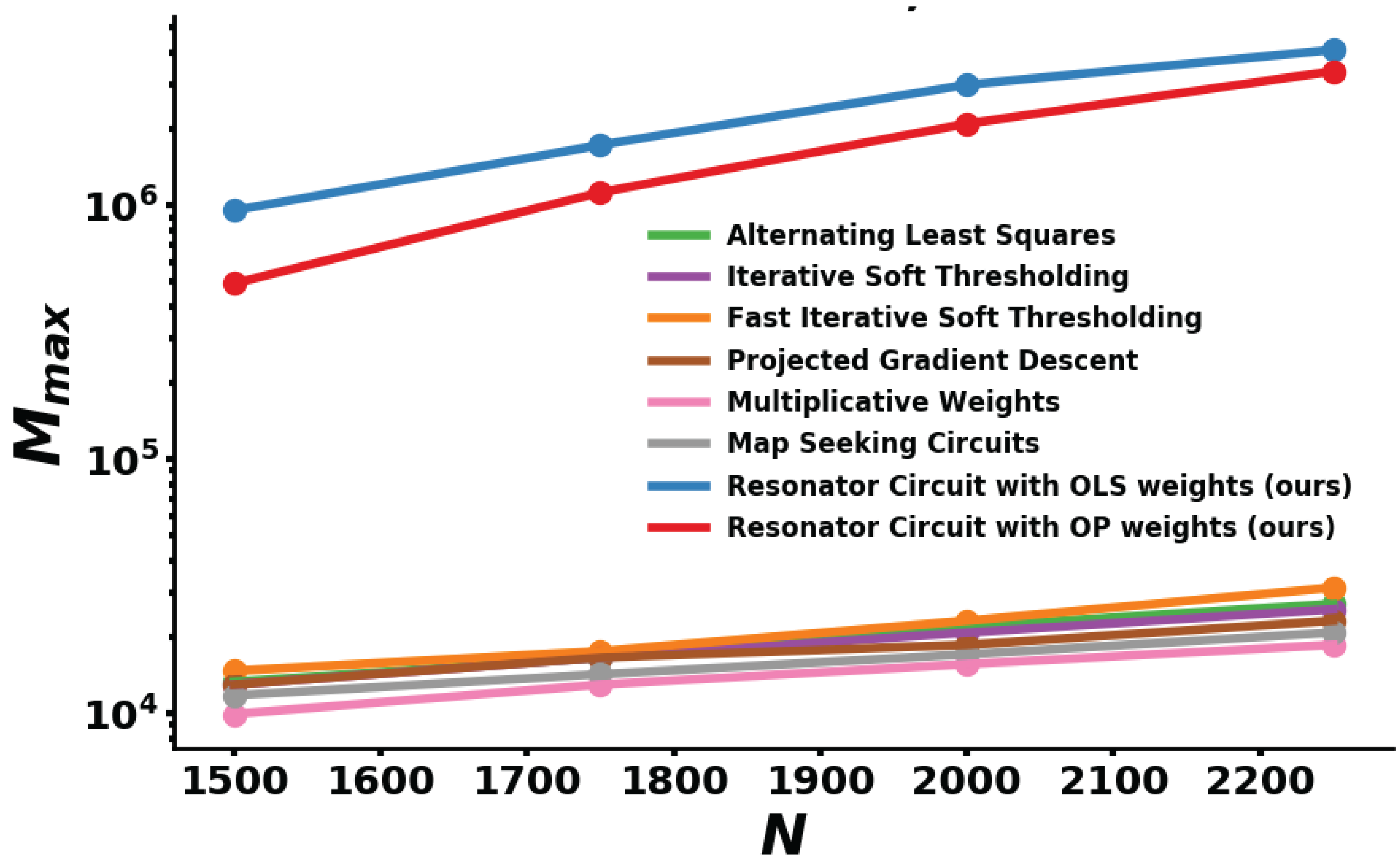
Search capacity increases with number of dimensions

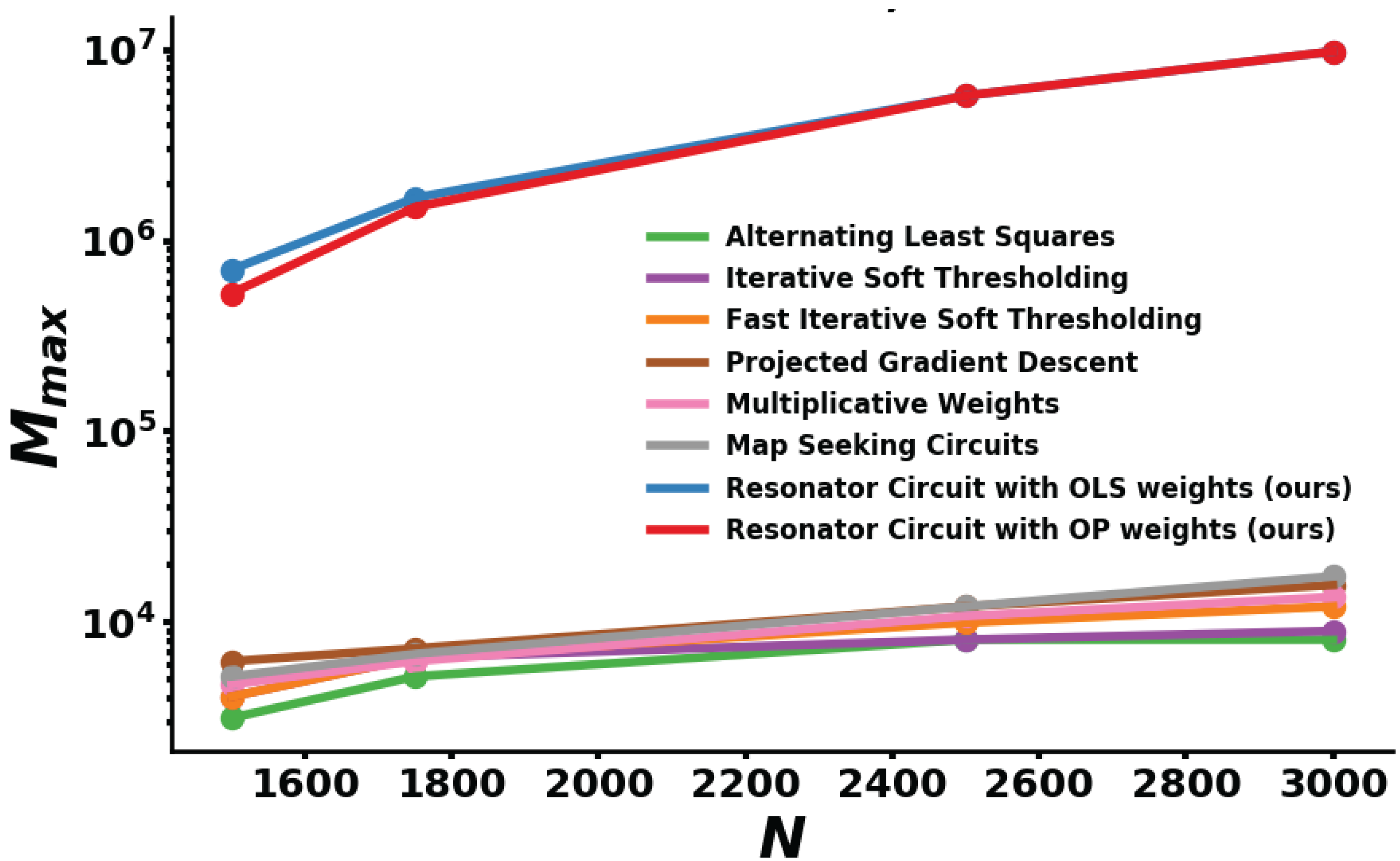


Operational capacity



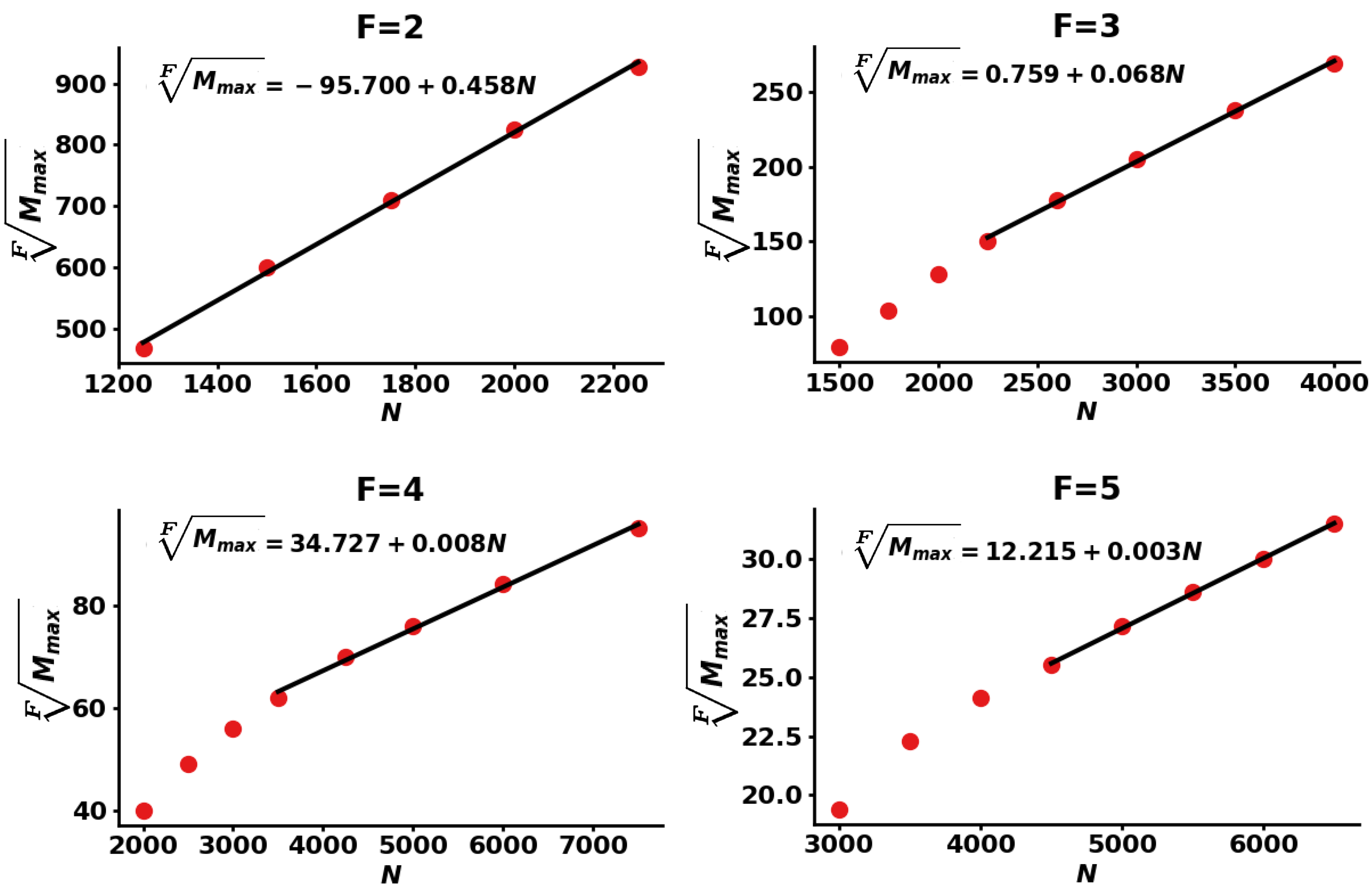
Three factors, $F = 3$

Operational capacity



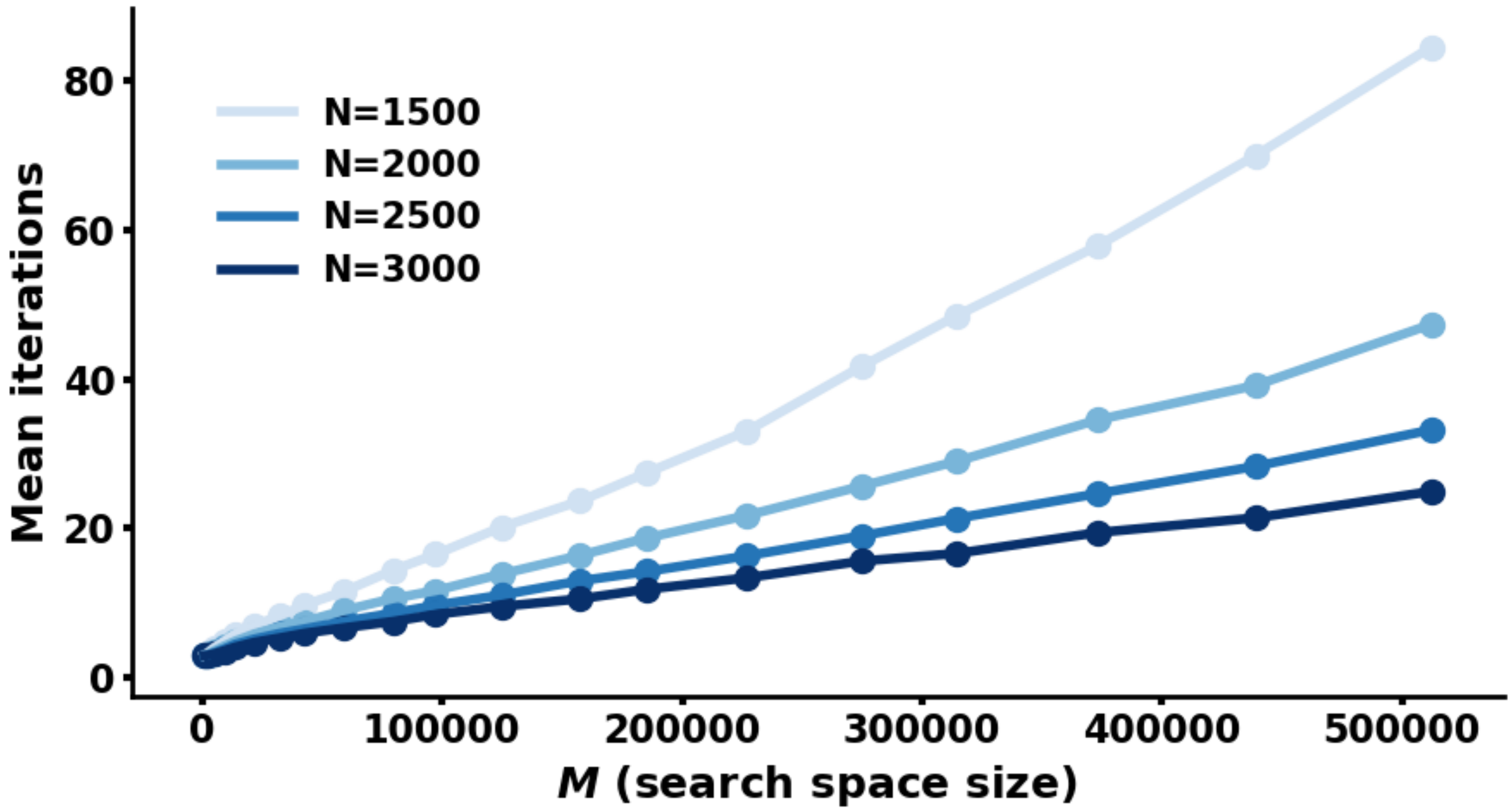
Four factors, $F = 4$

Operational capacity - scaling law

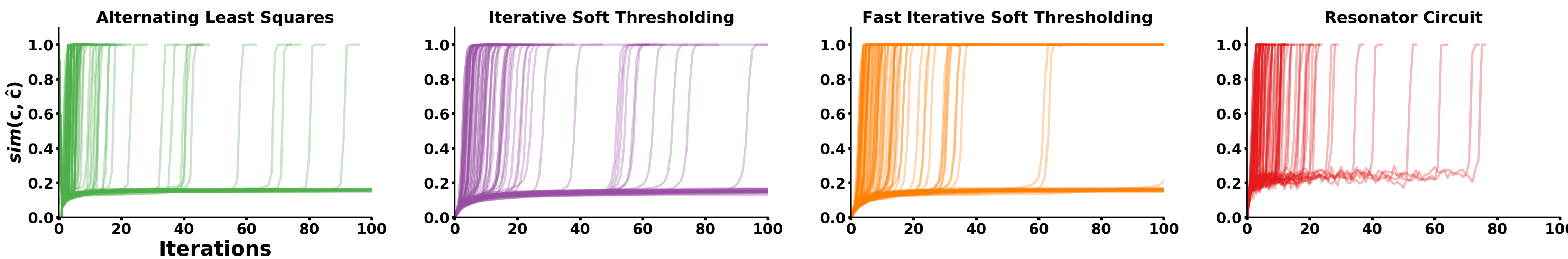


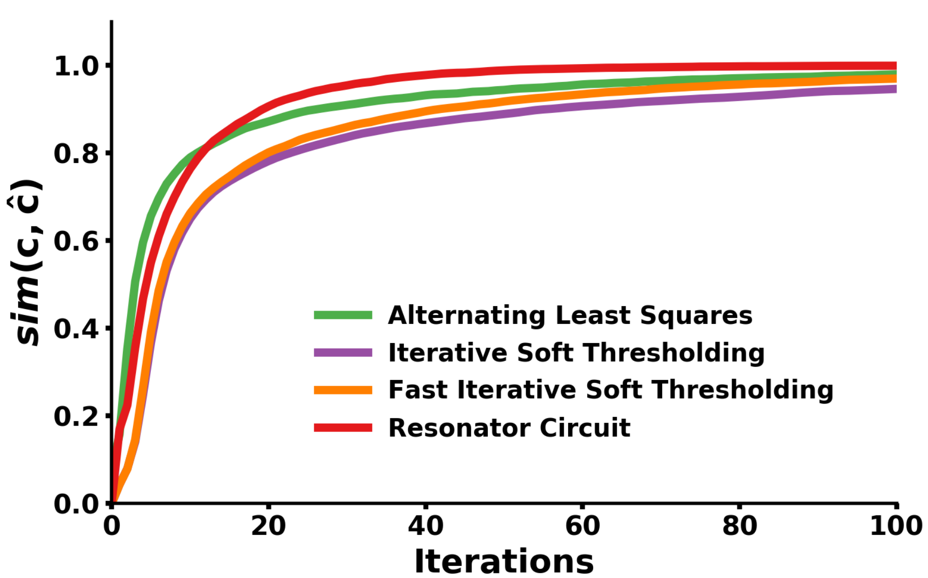
$$\sqrt[F]{M_{max}} = \beta_0 + \beta_1 N \implies M_{max} = (\beta_0 + \beta_1 N)^F = \mathcal{O}(\beta_1^F N^F)$$

Search efficiency

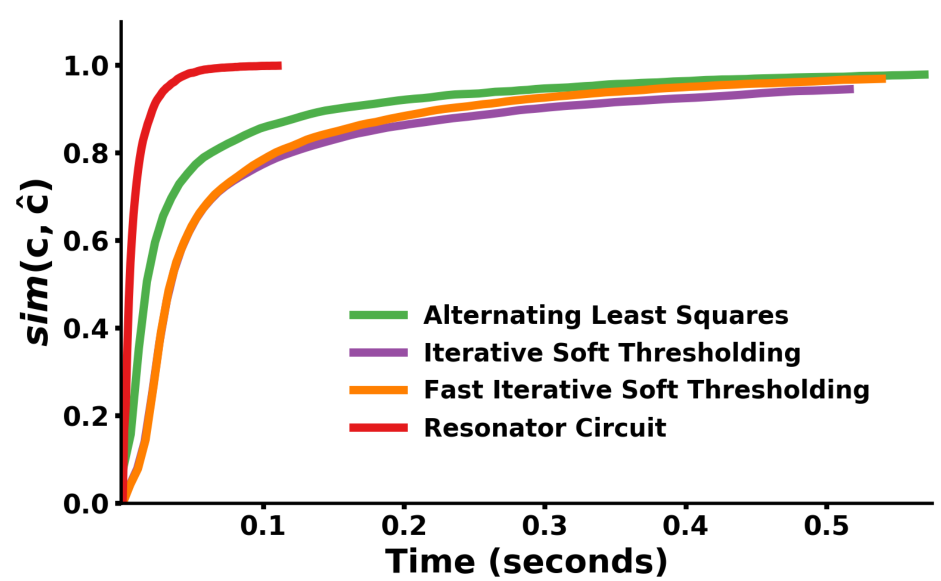


Search efficiency



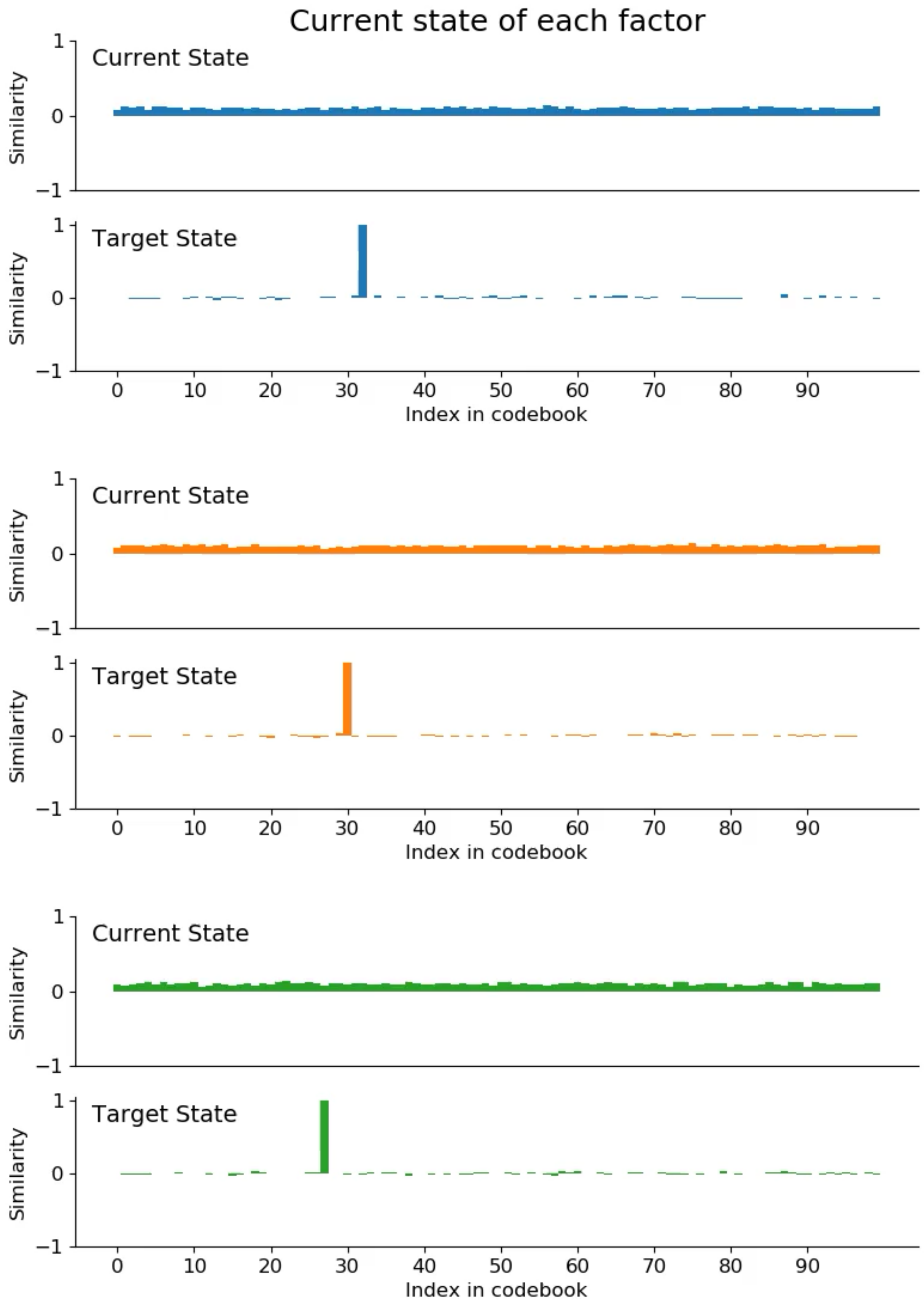


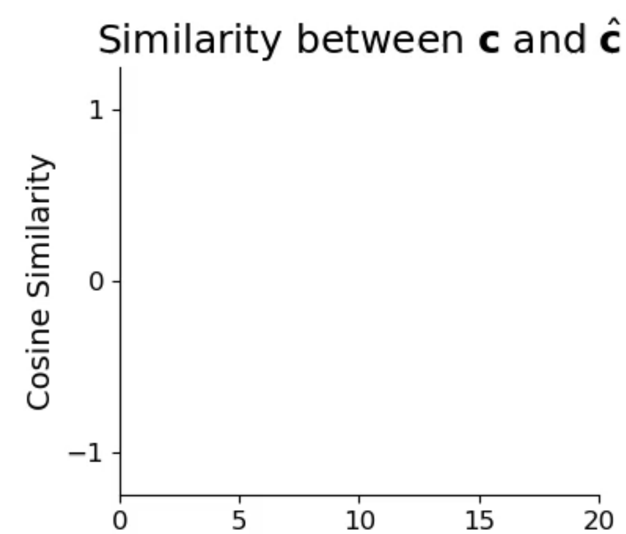
(b) Avg. cosine similarity vs. iteration number (only trials with accuracy 1.0)



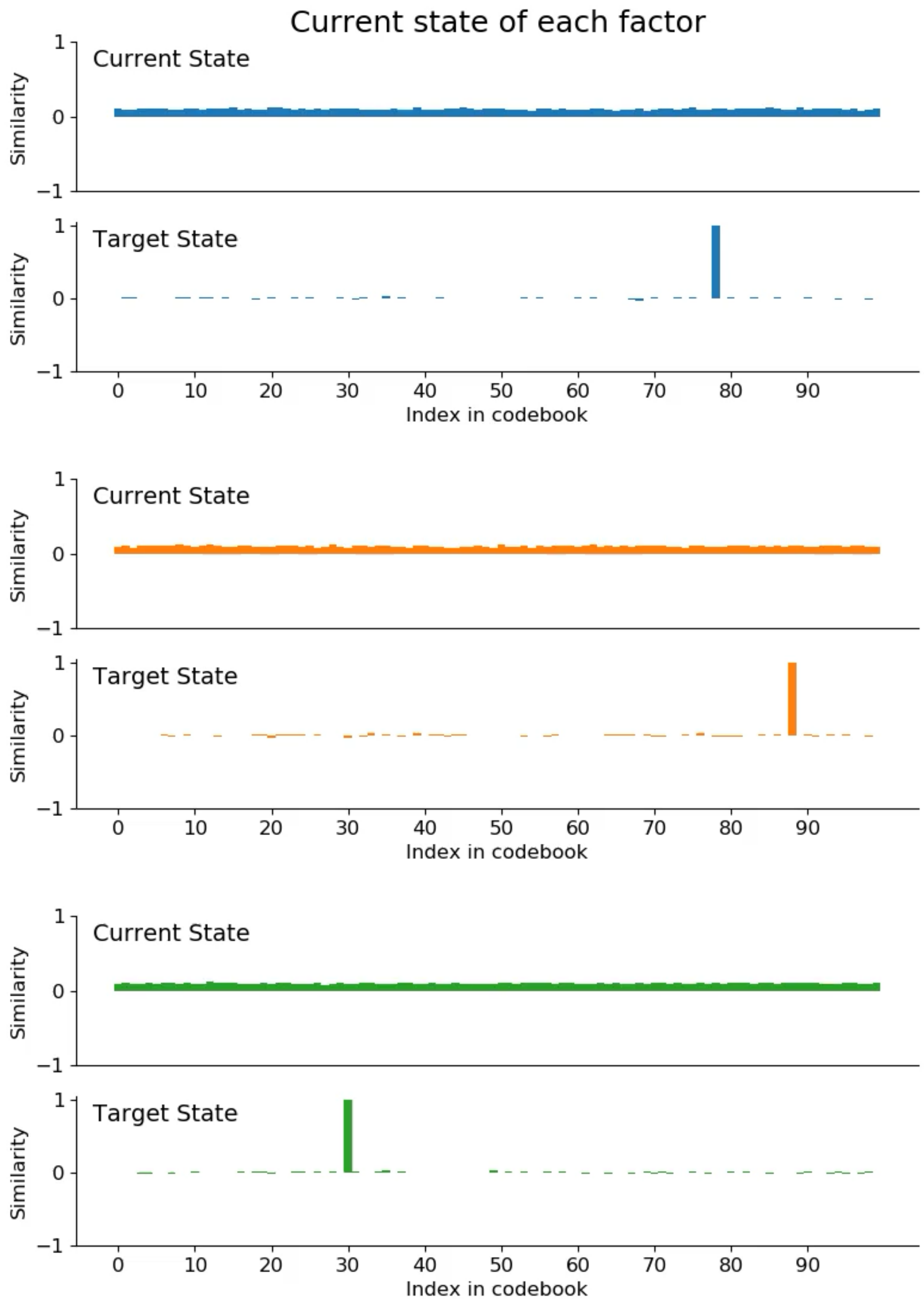
(c) Avg. cosine similarity vs. wall-clock time (only trials with accuracy 1.0)

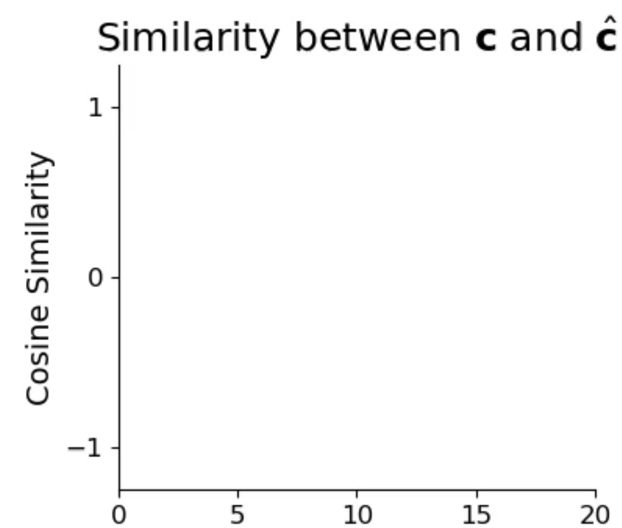
Multiplicative weights (N=1000)



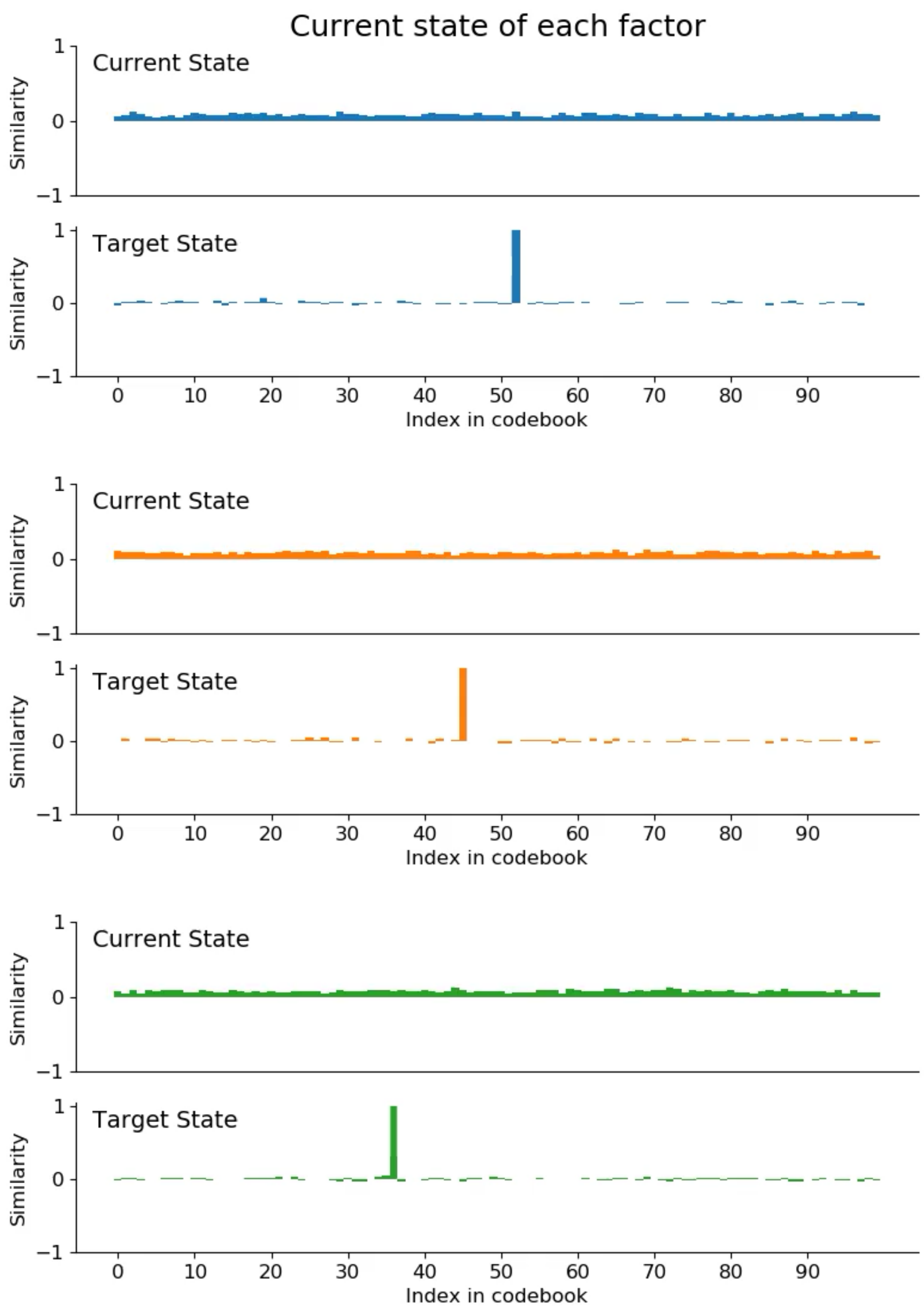


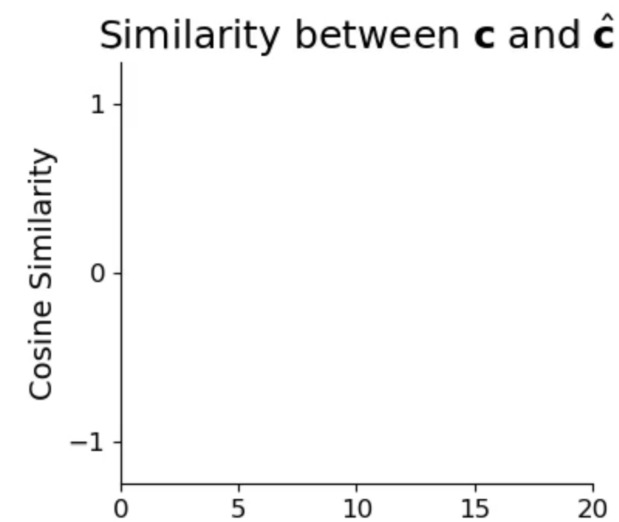
Multiplicative weights (N=5000)



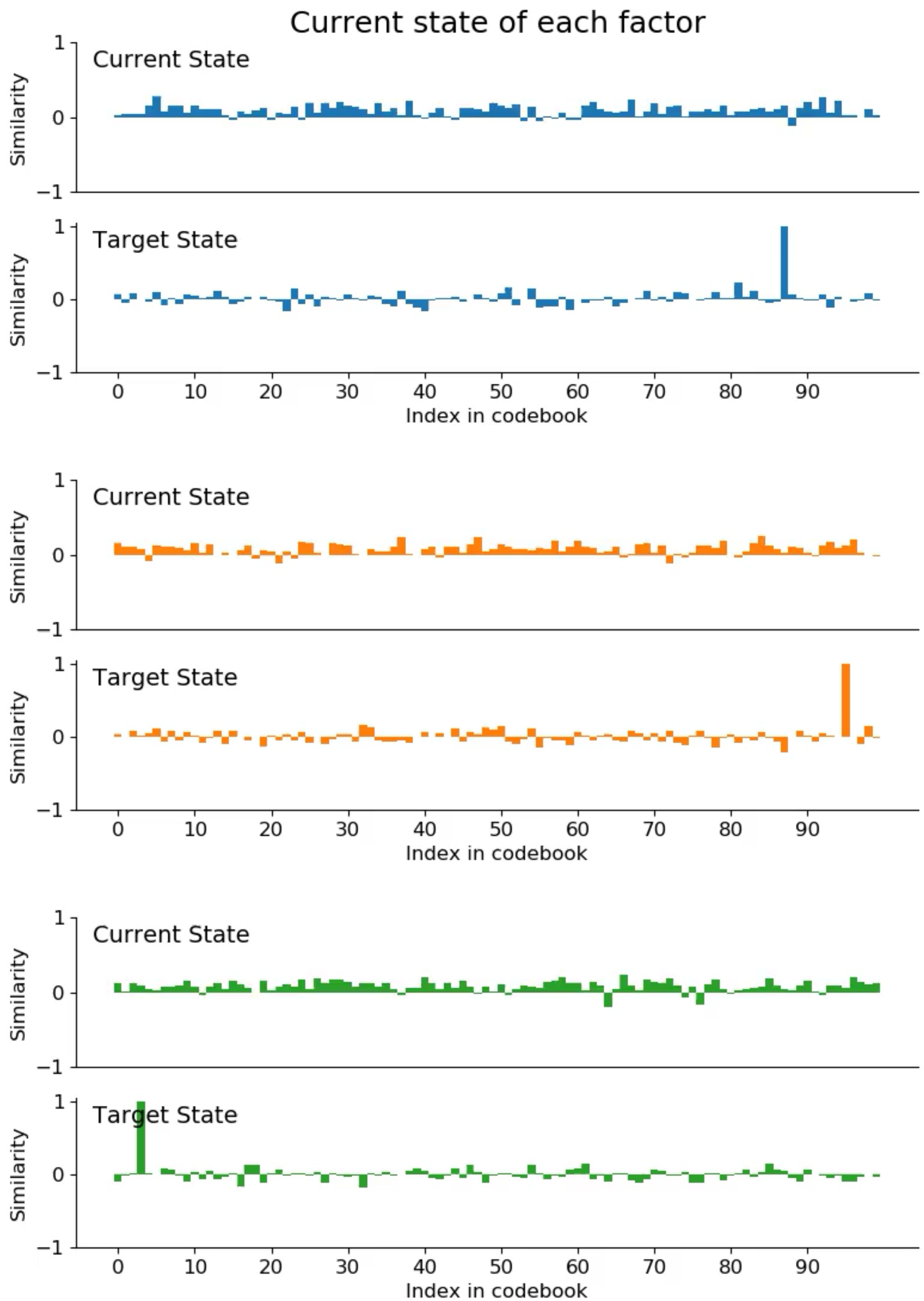


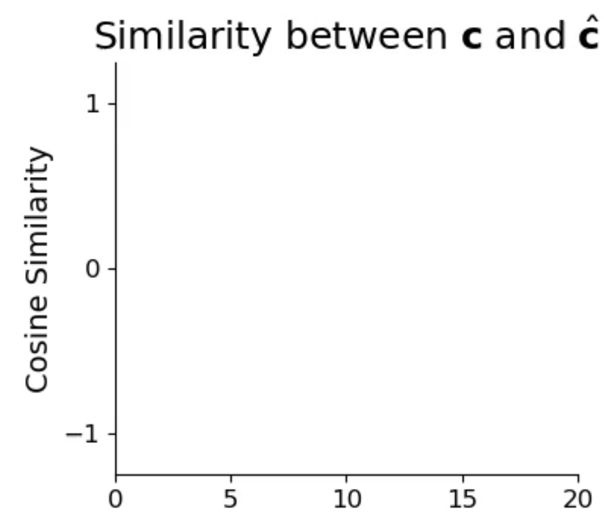
Resonator (N=1700)





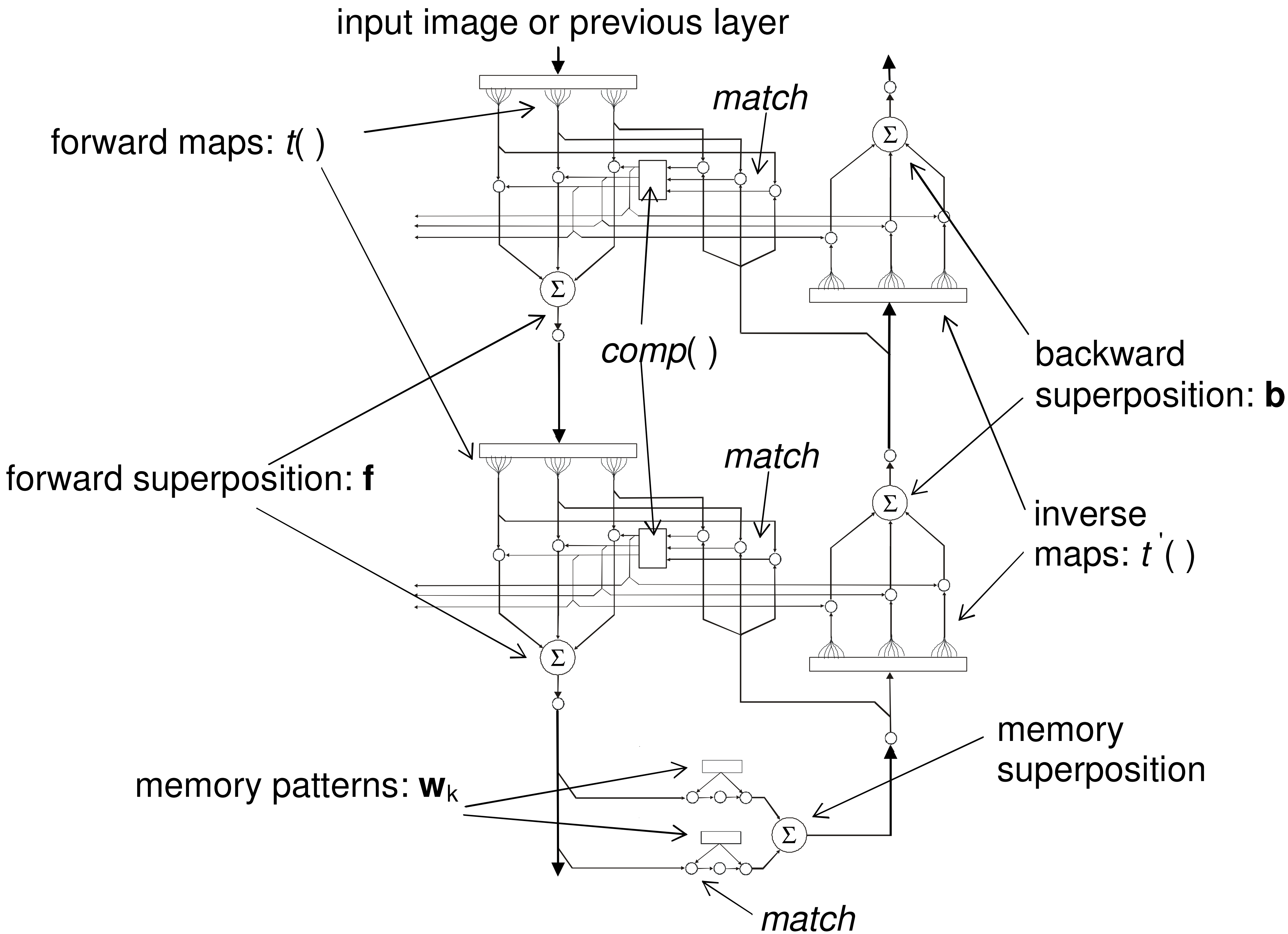
Resonator (N=200)





Map-Seeking Circuit

David Arathorn - www.giclab.com

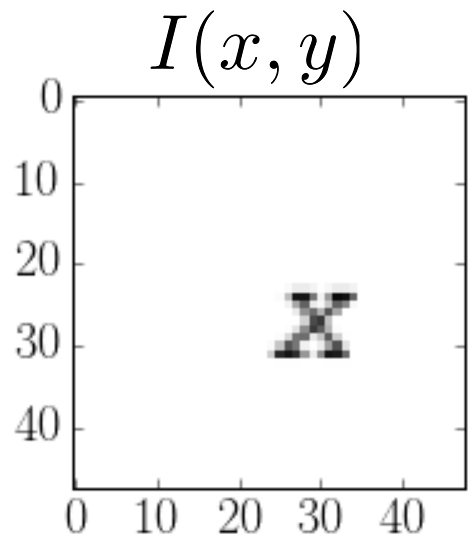


HD map-seeking circuit
(Paxon Frady)

$\mathbf{x}^{x_i}$ = horizontal position x_i

$\mathbf{y}^{y_j}$ = vertical position y_j

$\mathbf{p}_k$ = pattern k



$$\mathbf{v} = \sum_{i,j} I(x_i, y_j)\, \mathbf{x}^{x_i}\, \mathbf{y}^{y_j}$$

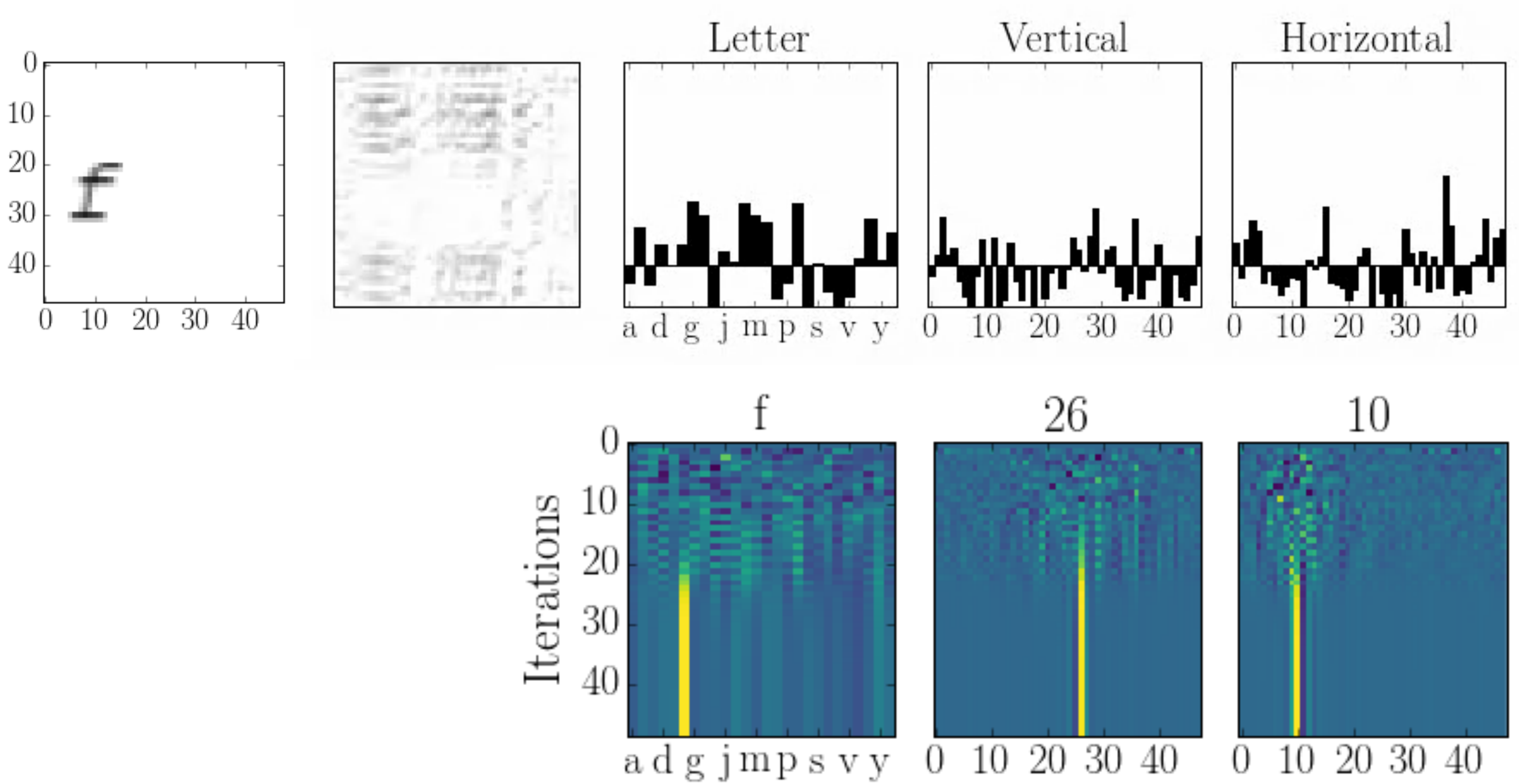
Given $\mathbf{v}$, find $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{p}$ via resonator:

$$\hat{\mathbf{x}}_{t+1} = g\big(\mathbf{X}\mathbf{X}^\top (\mathbf{v} \otimes \hat{\mathbf{y}}_t^{-1} \otimes \hat{\mathbf{p}}_t^{-1})\big)$$

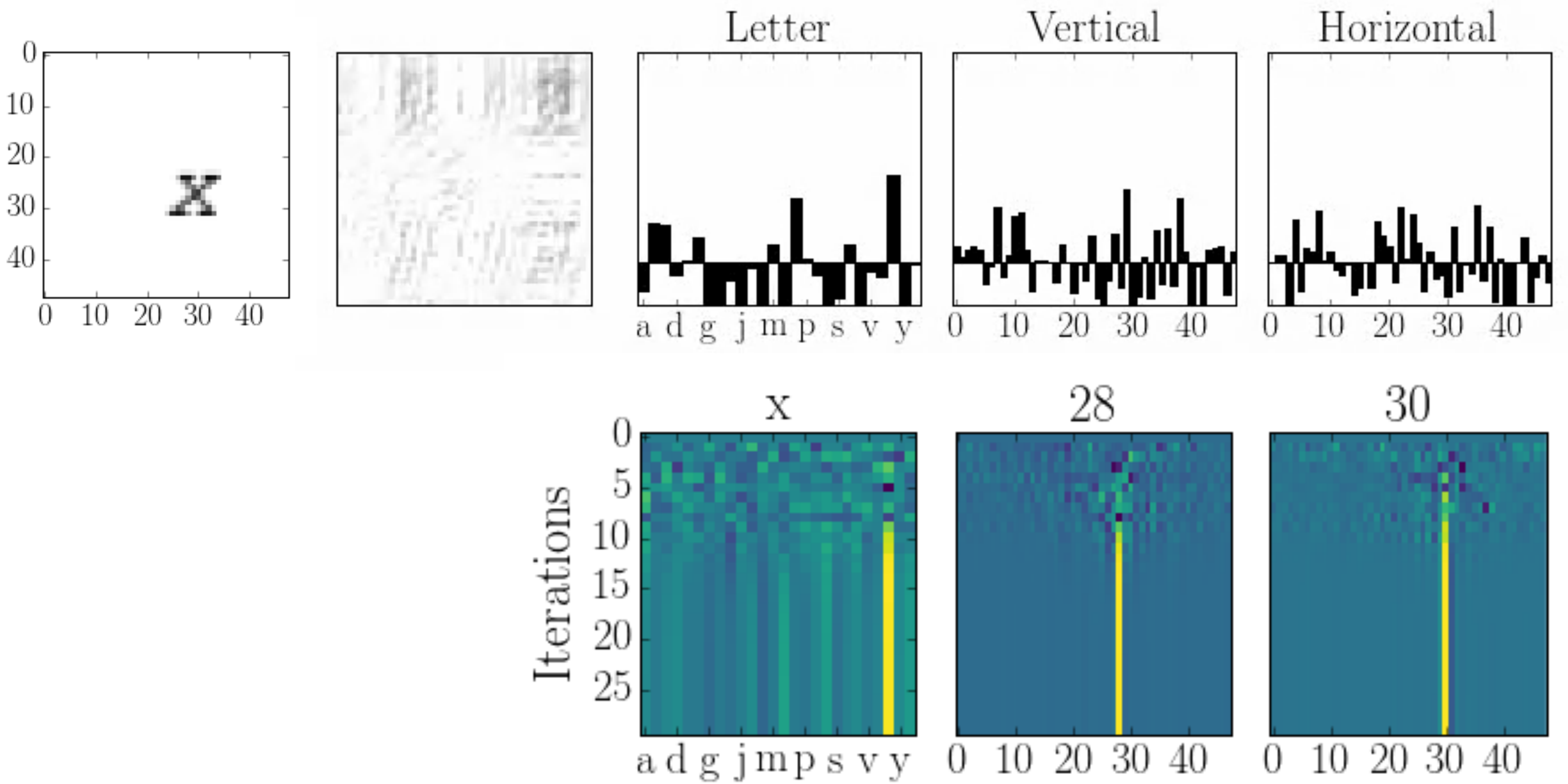
$$\hat{\mathbf{y}}_{t+1} = g\big(\mathbf{Y}\mathbf{Y}^\top (\mathbf{v} \otimes \hat{\mathbf{x}}_t^{-1} \otimes \hat{\mathbf{p}}_t^{-1})\big)$$

$$\hat{\mathbf{p}}_{t+1} = g\big(\mathbf{P}\mathbf{P}^\top (\mathbf{v} \otimes \hat{\mathbf{x}}_t^{-1} \otimes \hat{\mathbf{y}}_t^{-1})\big)$$

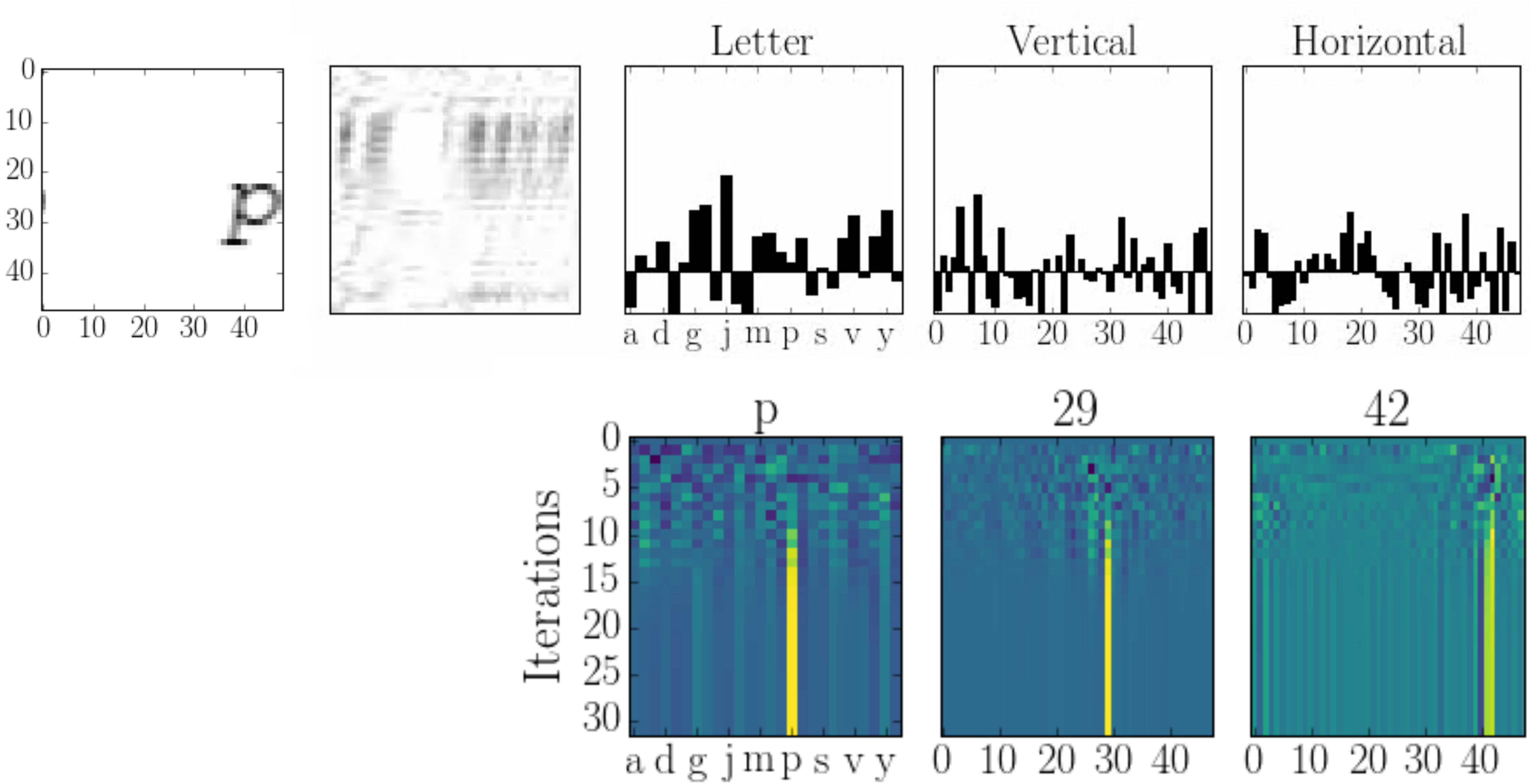
Some examples



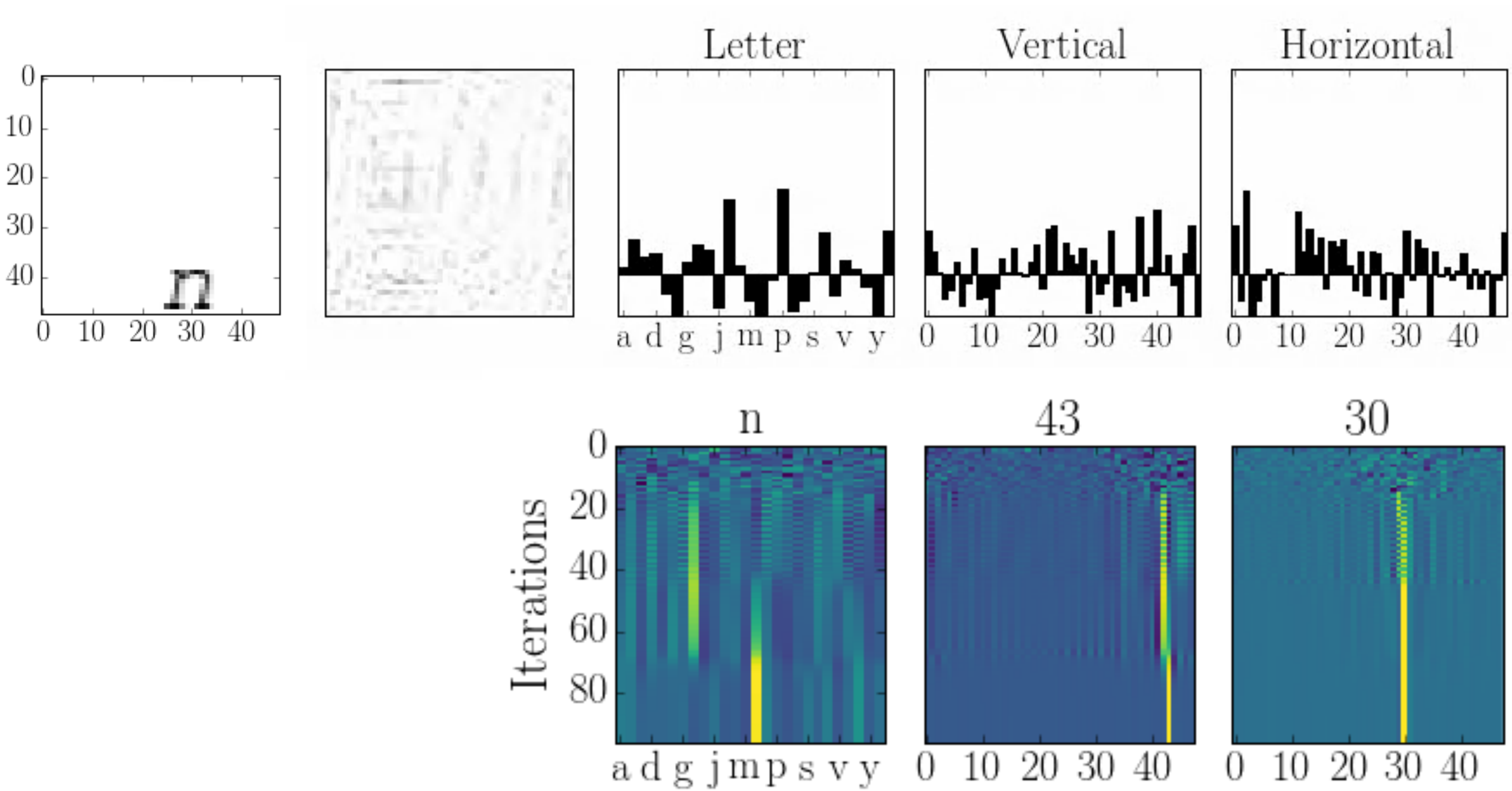
Some examples



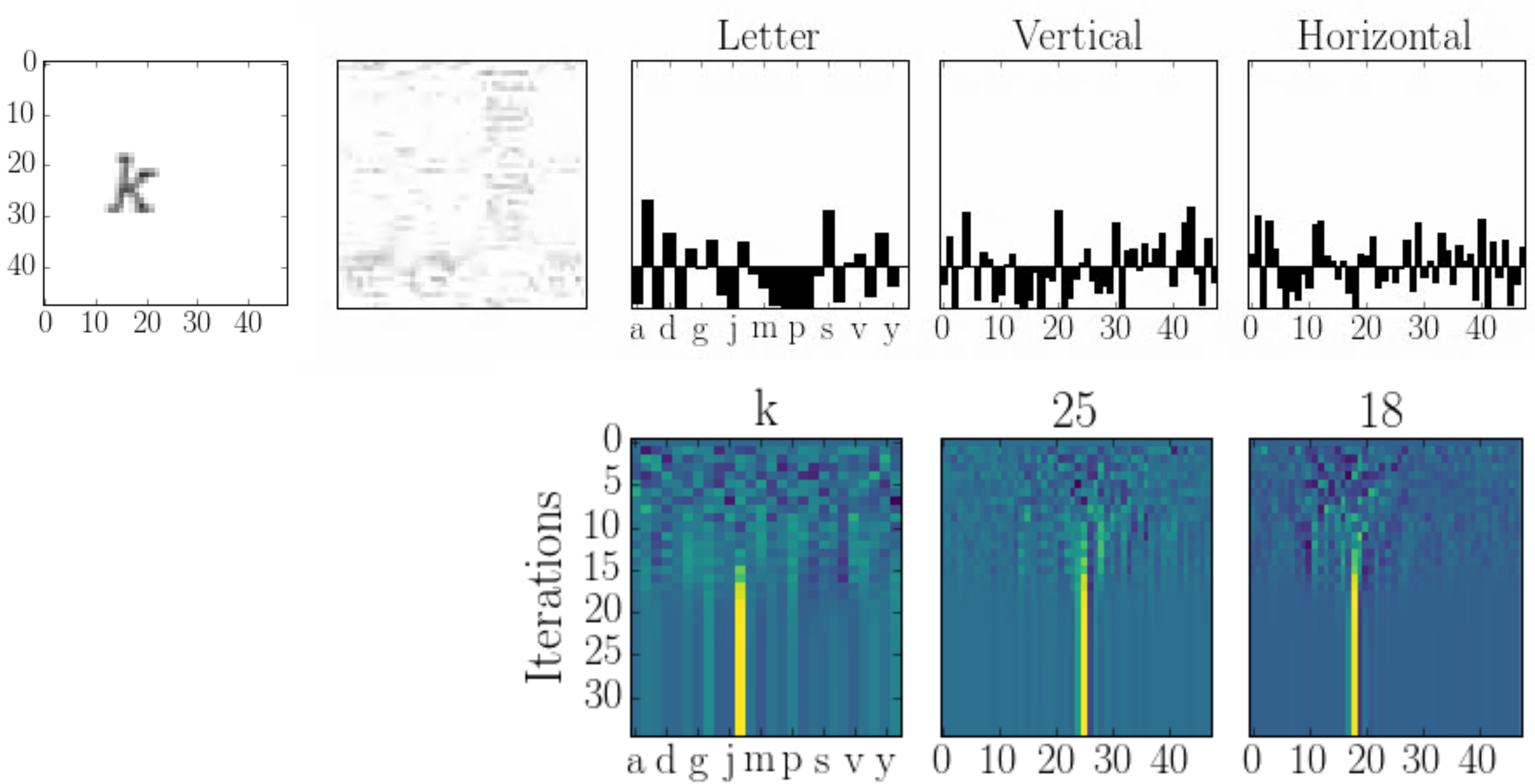
Some examples



Some examples



Some examples



Main points

- Visual perception requires solving inverse problems in the form of factorization.
- HD algebra allows for searching among many factorial combinations at once via superposition of vectors.
- This property may be exploited for disentangling properties such as object shape, color and position from a scene containing multiple objects.
- Tradeoff between “well behaved” gradient methods vs. more powerful dynamics for exploring solution space?