Quantum Proofs of Knowledge

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Post Quantum Crypto

- Post Quantum Crypto
  - Classical crypto
  - Secure against quantum computers

- Needs:
  - Quantum hard problems (e.g., lattice crypto)
  - New security proofs

This talk

Zero Knowledge Proofs

Prover

Graphs G and H are isomorphic

Verifier

Permute G

Permuted graph J

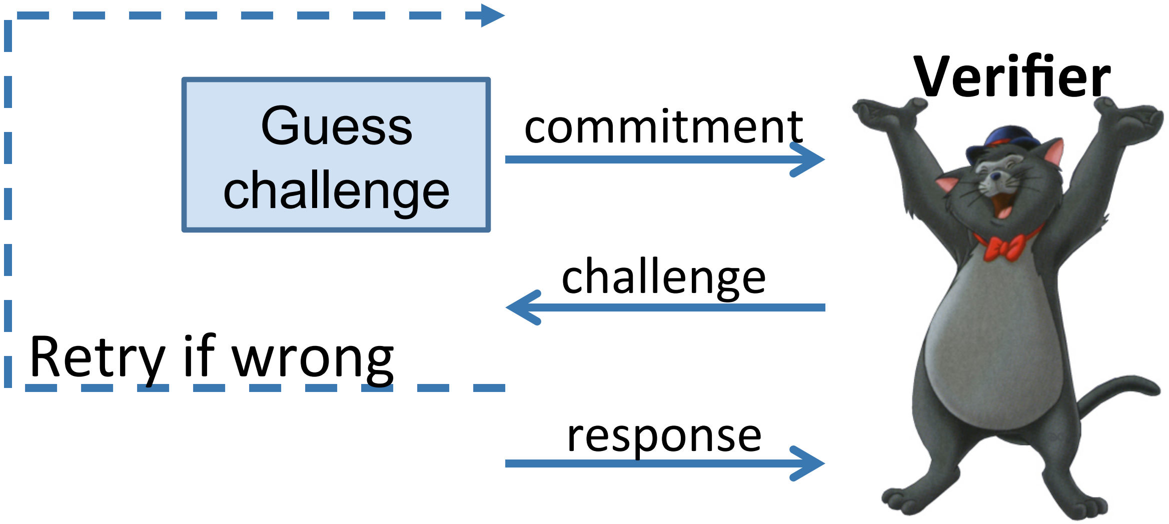
G or H

Pick G or H

Iso between $\boldsymbol{J}$ **and** $\boldsymbol{G}$ or $\boldsymbol{J}$ **and** $\boldsymbol{H}$

Zero-knowledge: how to show?

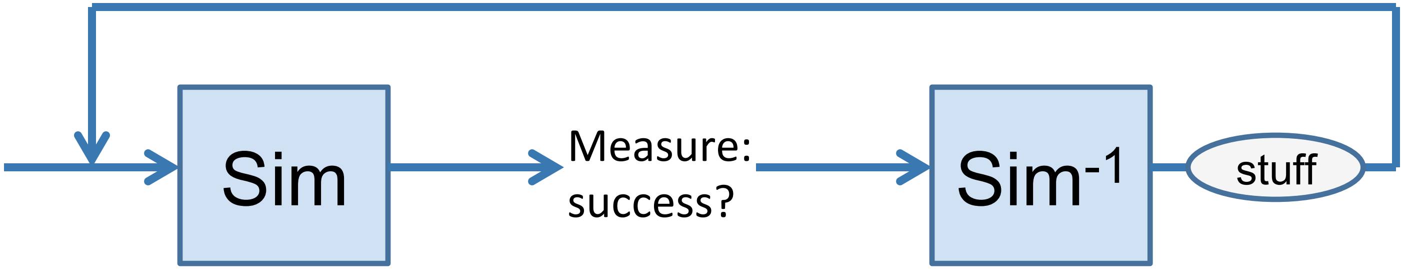
- Given only malicious verifier:
  simulate interaction ⇒ nothing learned



- Quantum case: Rewinding = state copying!

Watrous' quantum rewinding

- Cannot copy state → have to restore it



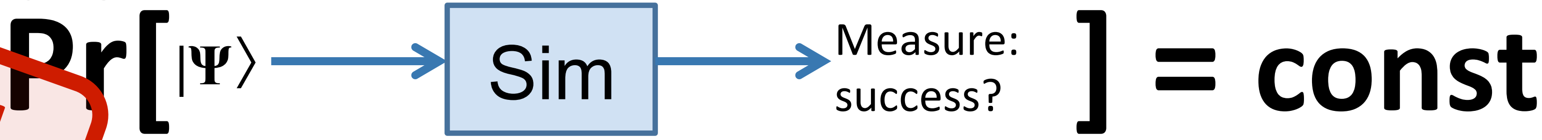
- Variant of amplitude amplification

[Watrous 09]

Limitations of Watrous' rewinding

- Oblivious rewinding:
  When simulator rewinds, he forgets everything

- Success probability independent of initial state



Pr[$|\Psi\rangle$ → Sim → Measure: success?] = const

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Intuition: success carries no info about $|\Psi\rangle$

Quantum ZK solved?

- Watrous' rewinding
  covers many important ZK proofs
- But not all…
  E.g., graph non-isomorphism.

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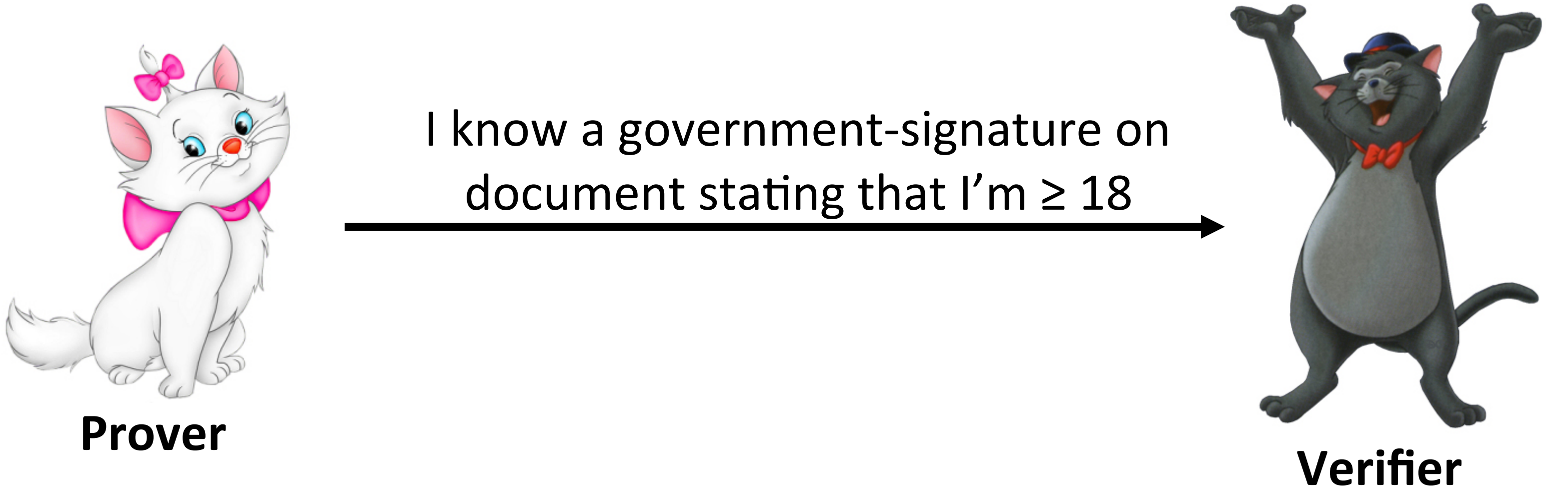
- And not: Proofs of knowledge

Proofs of knowledge

- Example: Want to prove age (e.g., e-passport)



I know a government-signature on document stating that I’m ≥ 18

Prover

Verifier

Proofs of knowledge – definition

If prover is successful:

~~prover knows witness~~

~~could output witness~~

there is an extractor that,

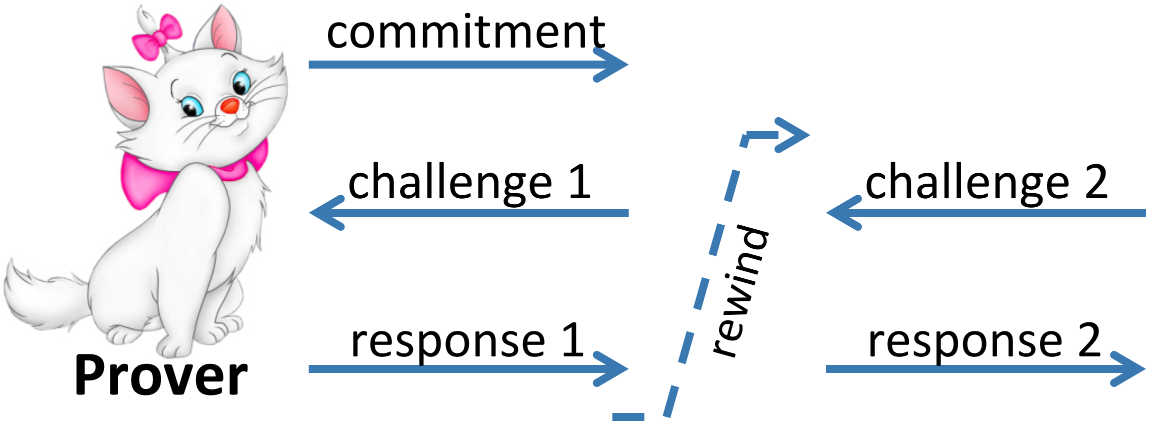
given provers state,

outputs witness

Definition – more formally

- There is a poly time extractor E
- Such that for any malicious prover P^*
- If P^* makes the verifier accept with probability α
- Then E^{P^*} outputs witness with probability $\Omega(poly(\alpha\text{-}const))$.

Constructing extractors



Prover

commitment

challenge 1

response 1

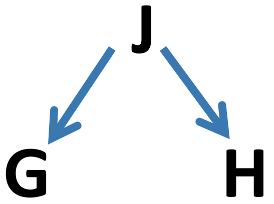
rewind

challenge 2

response 2

“Special soundness”: Two different responses allow to compute witness

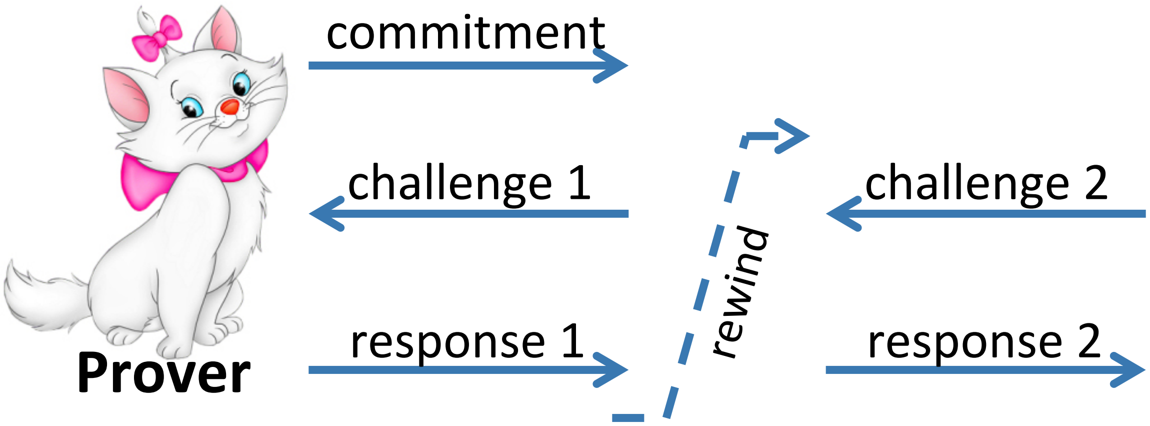
- E.g., isomorphisms from J to G and H give isomorphism between G and H



Quantum extractors?

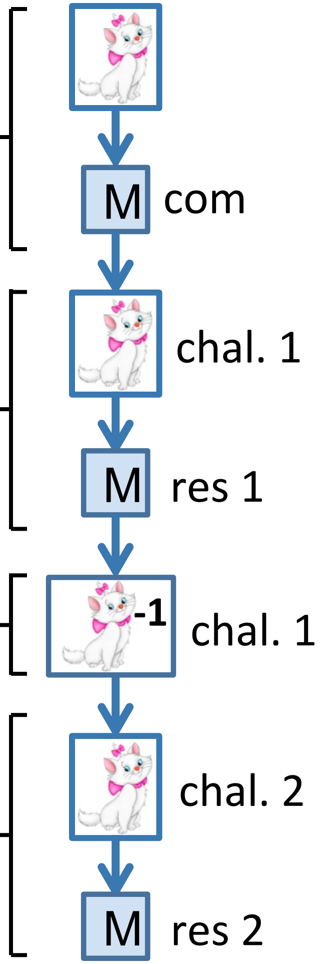
- Quantum case:
  Rewinding = copying. Not possible

- Watrous' "oblivious" rewinding does not work:
  Forgets response 1



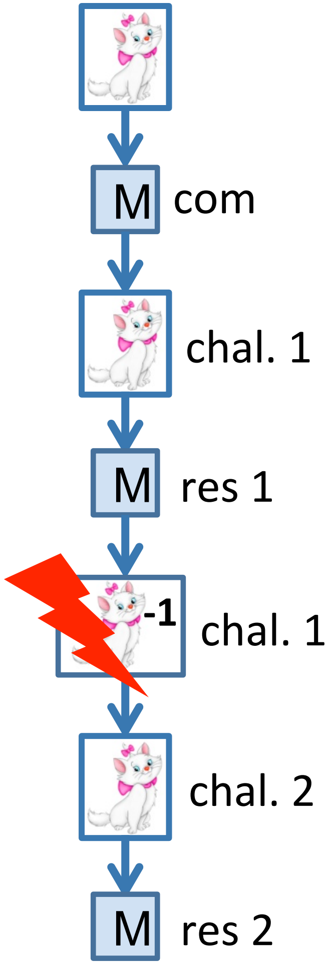
Canonical extractor

1. Run prover, measure commitment
2. Run prover on “challenge 1”, measure response 1
3. Run inverse prover
4. Run prover on “challenge 2”, measure response 2



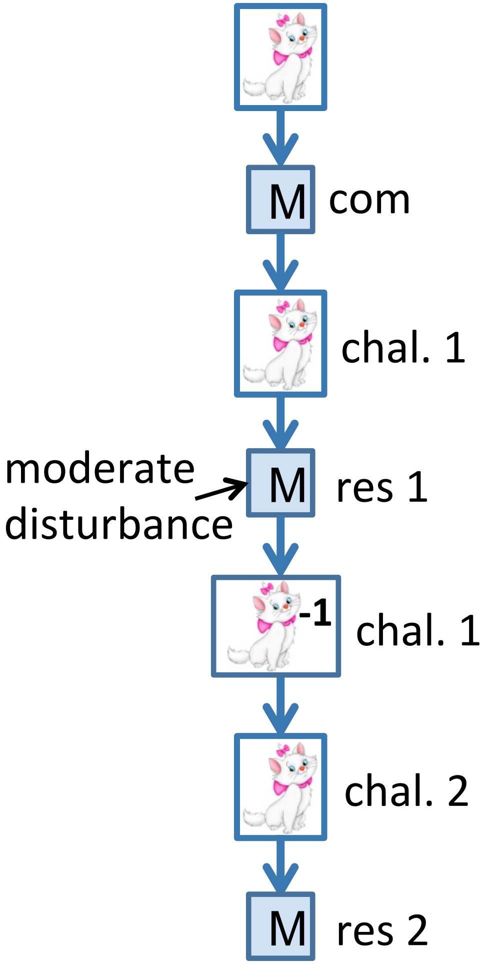
Canonical extractor (ctd.)

- Does it work?
- Measuring “response 1” disturbs state
- Rewinding fails…



Making extraction work

- Thought experiment: “response” was only 1 bit
- Then: measuring “res 1” disturbs only moderately
- Extraction would work



Making extraction work (ctd.)

- Idea: Make “response” effectively be 1 bit
- **“Strict soundness”:** For any challenge, exists at most 1 valid response
- Given strict soundness, canonical extractor works!

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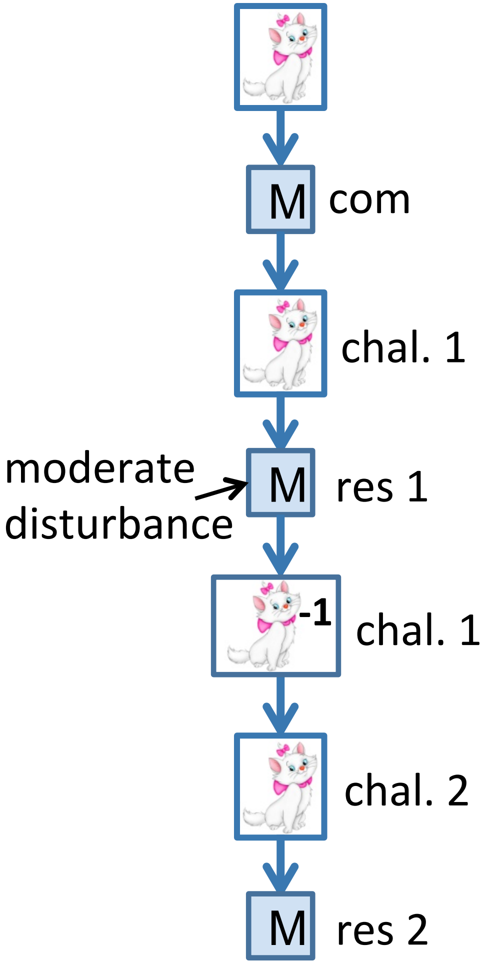

[Unruh 12]

Main result

Assume: Special soundness, strict soundness

Then

$$\Pr[extract] \geq \left(\Pr[verify] - 1/\sqrt{\#challenges}\right)^{3}$$

- Classical: no √, exponent 2.
- Computational security?

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Achieving strict soundness

- Graph Isomorphism proof does not have strict soundness
  - Unless graphs are "rigid"

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- Discrete log proof has (but uninteresting quantumly)

- Alternative trick (for #challenges poly):
  - Commit to all responses in advance
  - Need: "Strict binding" for unique unveil

Plugging things together

- Proof system for Hamiltonian cycles
- Commitments from injective OWFs

Assuming injective quantum OWFs,

quantum ZK proofs of knowledge

exist for all NP languages

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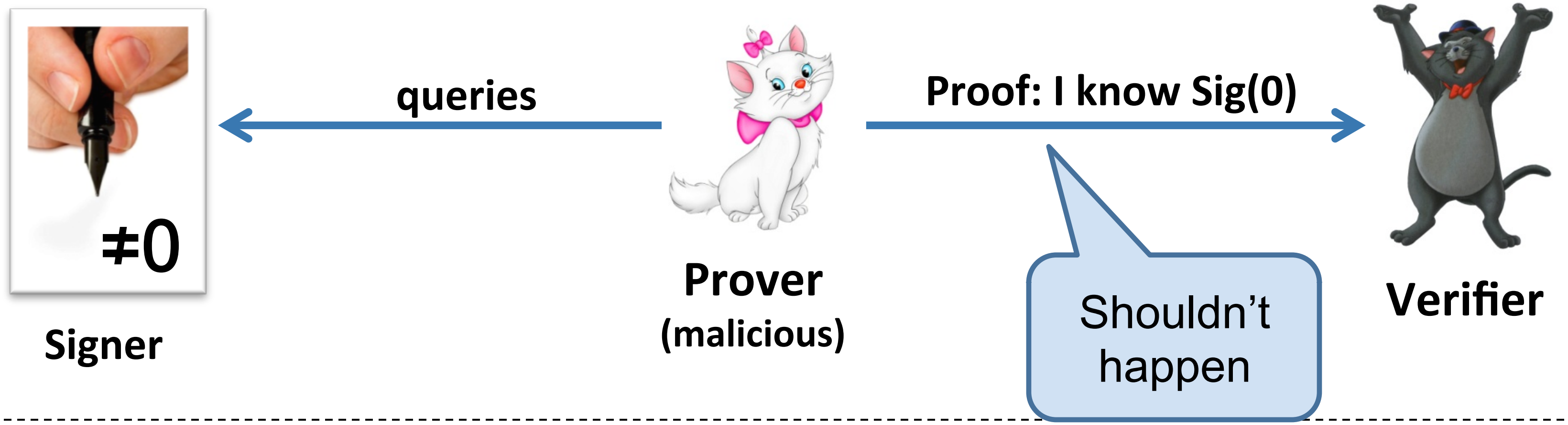
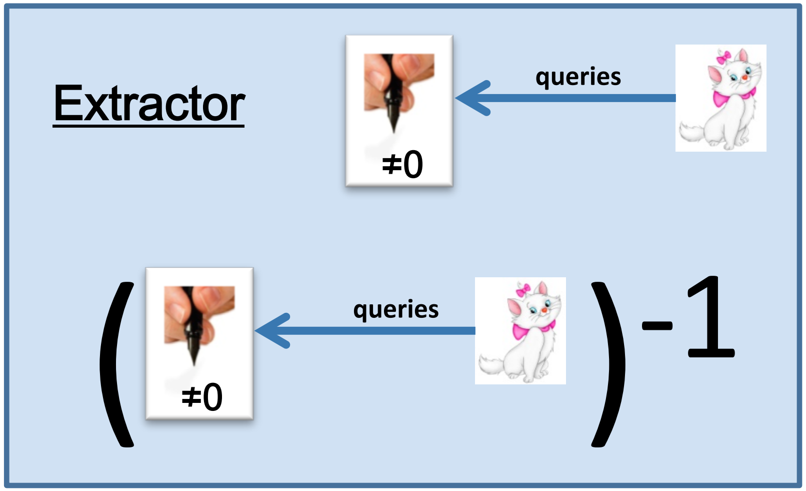
Caveat: No candidates for injective OWFs known.

Using extractors (I)



Shouldn't happen

Proof:



→ Sig(0)

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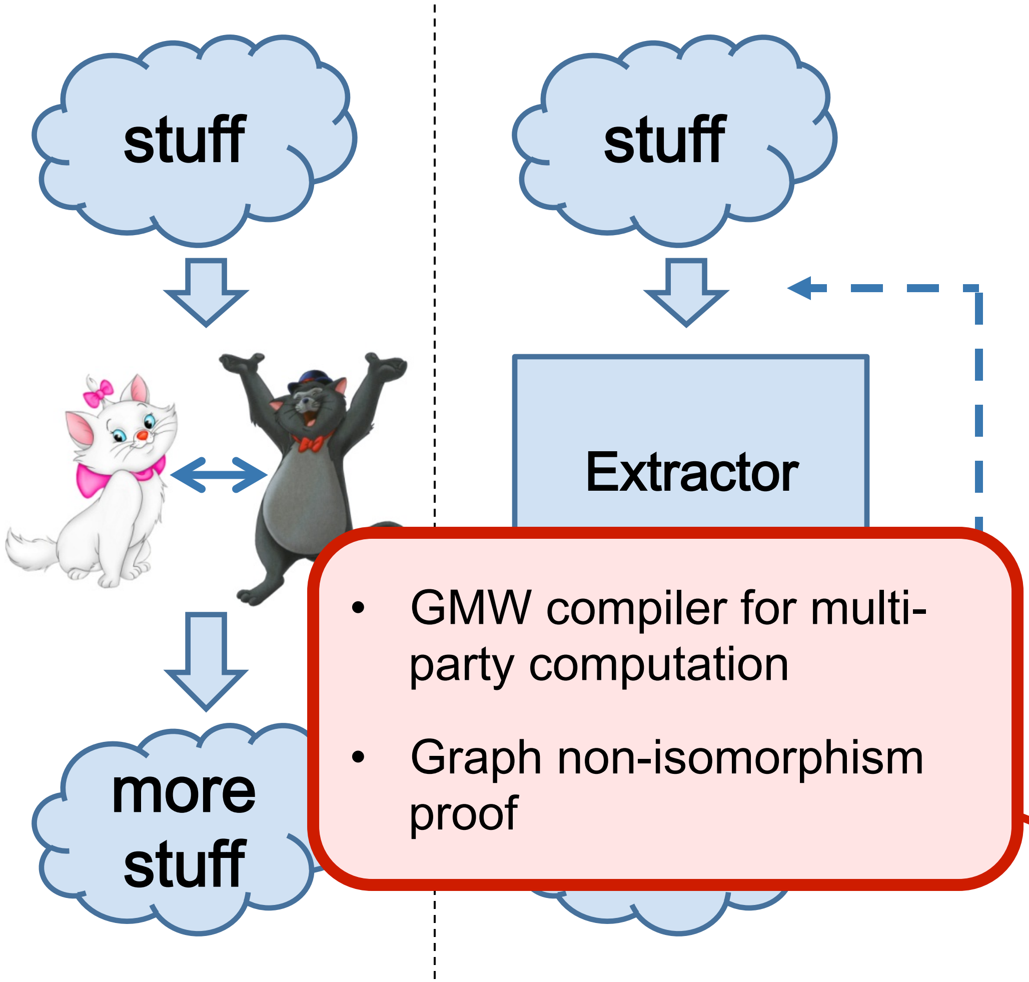
Extractor finds Sig(0),
given only Sig(≠0) and $(\mathrm{Sig}(\neq 0))^{-1}$
⇒
~~contradiction~~

Using extractors (II)



- GMW compiler for multi-party computation
- Graph non-isomorphism proof

- Success prob. too low
- Repeat.

- Quantum?
- Watrous? No!
- Success prob. not indep. of state.

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Conclusions

- ZK and proof of knowledge
  → New challenges in quantum case
- Solved in basic settings, many unsolved issues
  (Challenge: Graph non-isomorphism is ZK)
- Same problems likely to occur in more complex settings
  (e.g., multiparty computation)

I thank for your attention

Logo soup

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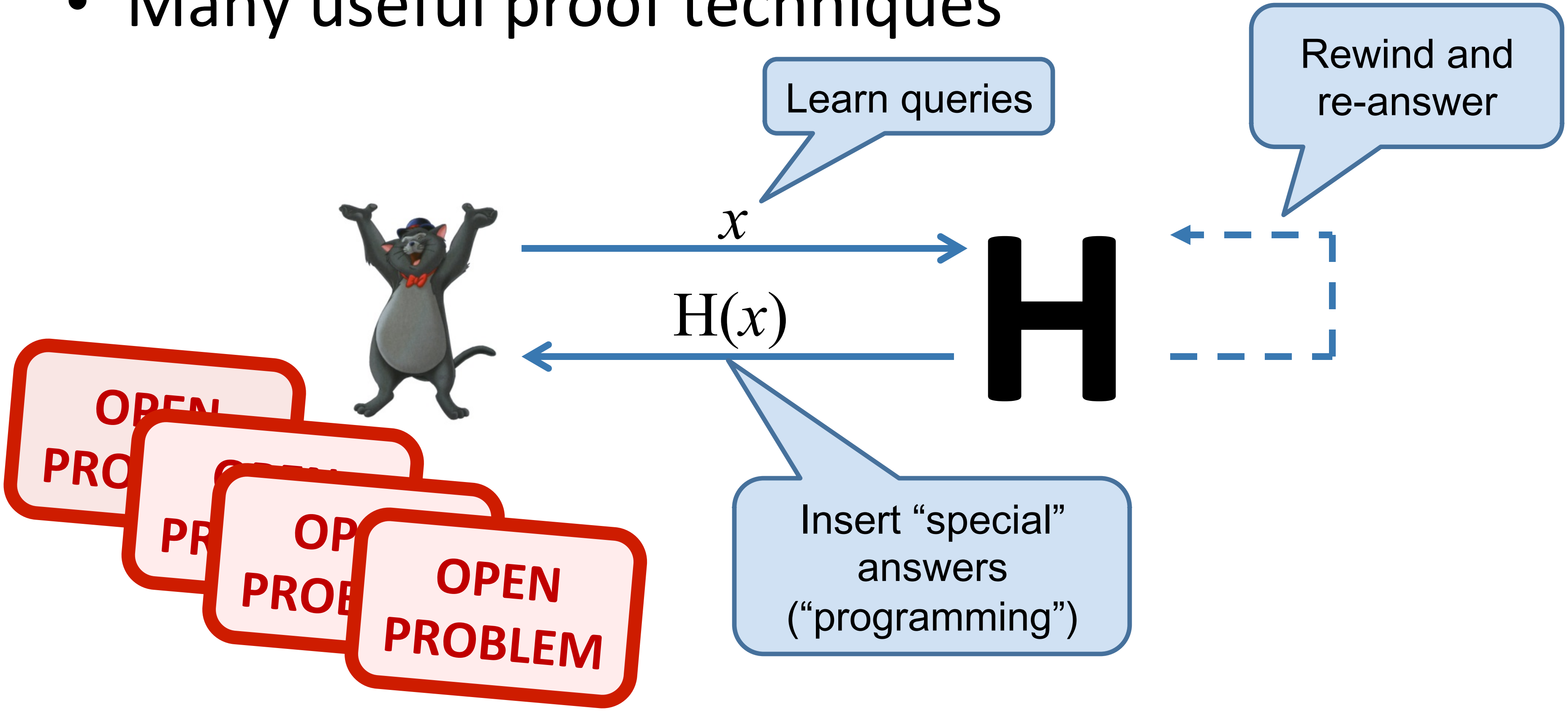
Random Oracles

- Model hash function as **random function H**
- Many useful proof techniques



Learn queries

Rewind and re-answer

Insert “special” answers (“programming”)

Limited programming of RO

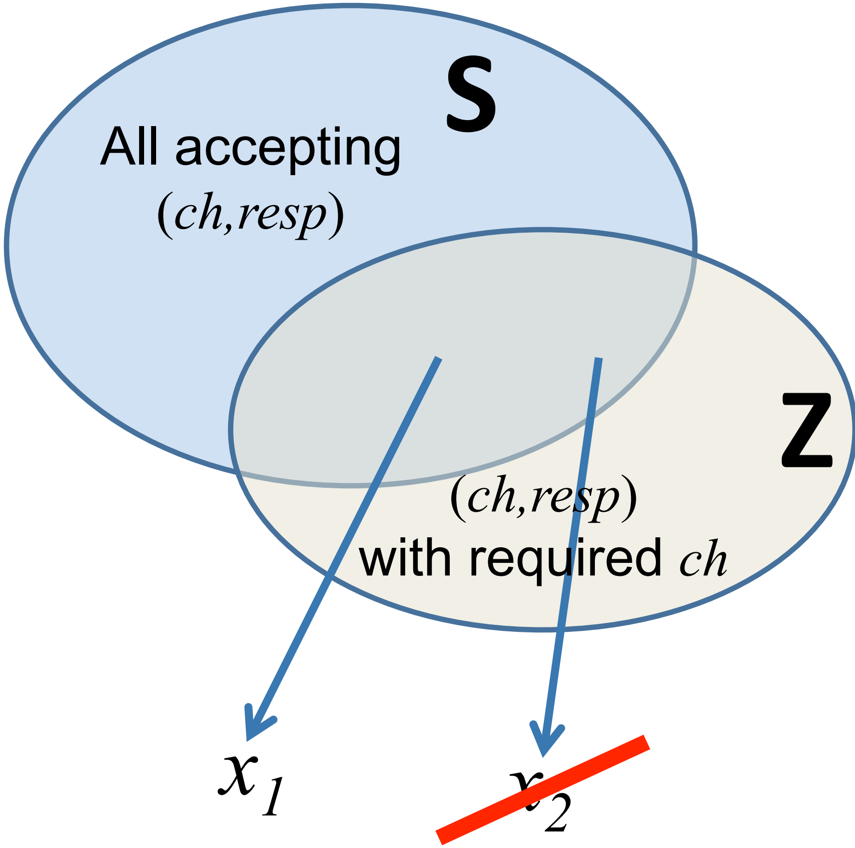
- Want to give answer $\mathrm{H}(x)=y_{special}$
- Don’t know which x is queried

- Solution: Put $y_{special}$ in many (not too many) images of H
- With noticeable probability:
  Exactly one query hits $y_{special}$
- Even works quantumly [Zhandry 12]

WORK IN PROGRESS!

Necessity of strict soundness

- Given a set S
- can encode it as a quantum state $|\Psi\rangle$
- s.t. for any set Z
- you find one $x_1 \in S \cap Z$
- but not two $x_1, x_2 \in S$


S
All accepting
$(ch,resp)$
Z
$(ch,resp)$
with required ch
x_1
~~x_2~~