

# Viability of a Privacy-differentiated Market for Free Online Services



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Joint work with Chong Huang (ASU)

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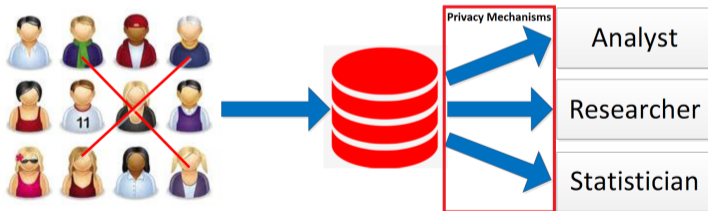
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  - Differential privacy
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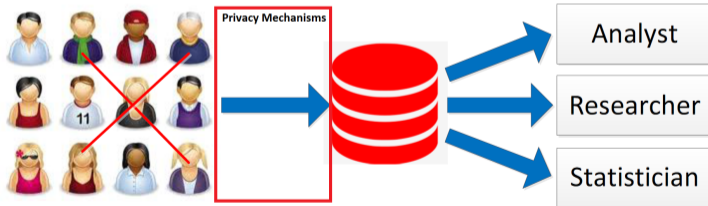


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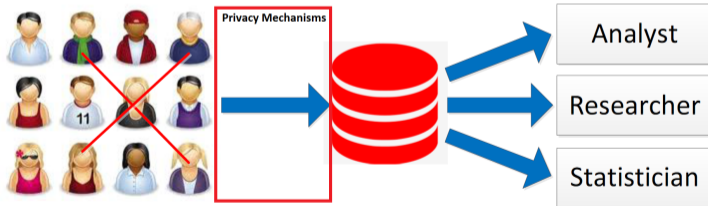


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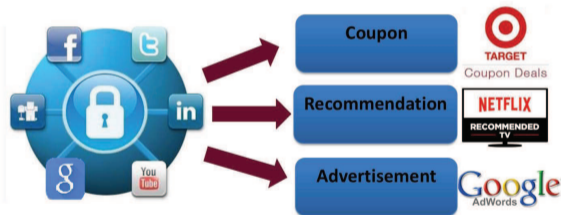


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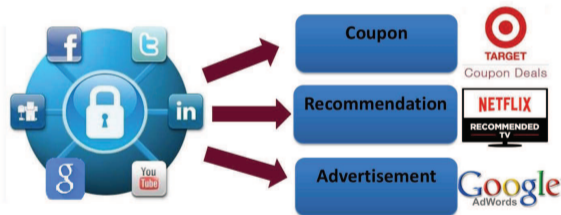


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- **Consumer-service provider interactions?**
  - Need models to study privacy-sensitive consumer-service provider interactions

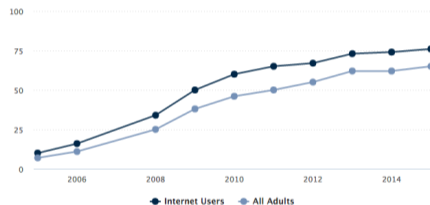




# Introduction

- Dramatic increase in online interactions between online service providers (SPs) and consumers

*% of all American adults and internet-using adults who use at least one social networking site*



Source: Pew Research Center surveys, 2005-2006, 2008-2015. No data are available for 2007.

- Often times online services are offered for free



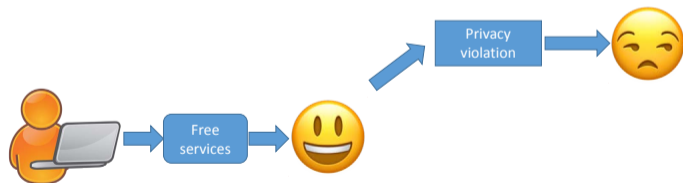
# Motivation

- Consumers enjoy free services



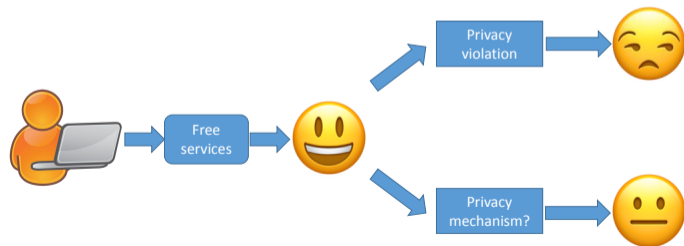
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- Consumers enjoy free services until they begin encountering privacy violations on a daily/frequent basis



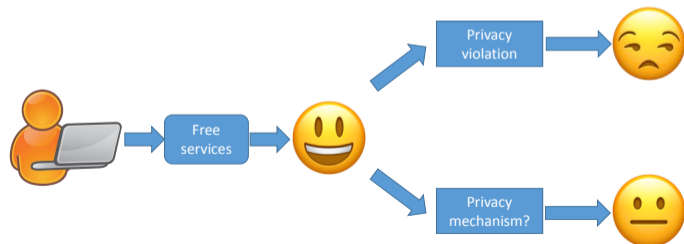
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- Consumers enjoy free services until they begin encountering privacy violations on a daily/frequent basis
- Service providers beginning to acknowledge consumers' sensitivity to privacy violations (e.g., Google RAPPOR)



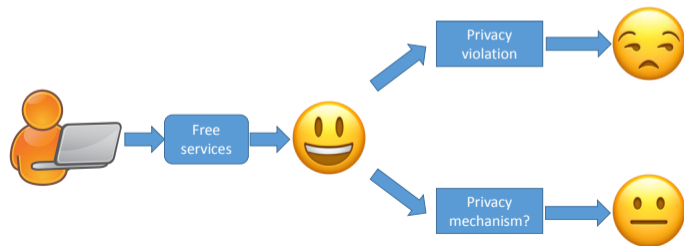
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- Consumers enjoy free services until they begin encountering privacy violations on a daily/frequent basis
- Service providers beginning to acknowledge consumers' sensitivity to privacy violations (e.g., Google RAPPOR)
  - The details of these privacy preserve mechanisms are opaque
  - Consumers may not have a choice



Can privacy-differentiated products lead to a sustainable marketplace?

# Motivation

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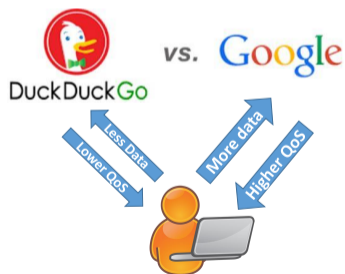
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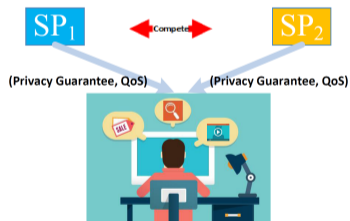
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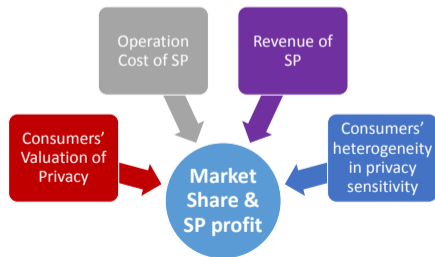
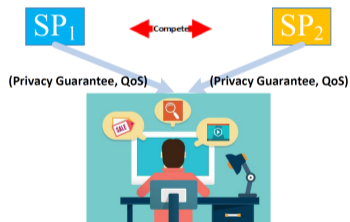
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- Shy and Stenbacka, 2015
  - Effects of varying degrees of privacy protection on industry profits, consumer welfare and total welfare in competition
- Chellappa and Shivendu, 2010
  - Monopolistic model for free services targeting under privacy concern
- Jentzsch, Preibusch, and Harasser, 2012
  - Price-based competitions between two service providers considering consumer's privacy preference
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Market segmentation of **free** services allowing for a **wide range** of privacy sensitivities has not yet been studied

# Our Approach

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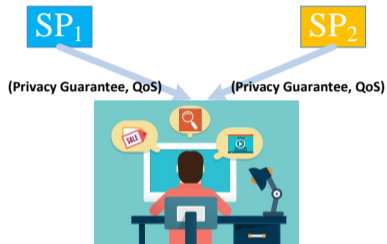
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Model consumer privacy preference as a distribution over a range

Modify Hotelling model to analyze market segmentation

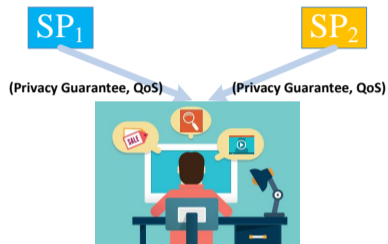


# System Model



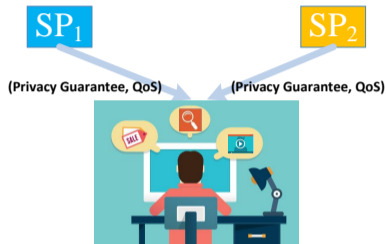
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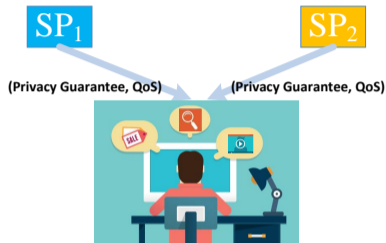


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- SPs can generate revenue by using the data obtained from their consumers
- Consumers choose the SP that optimally satisfies their privacy and QoS choices

# Two-SP Model

- Two rational (profit maximization) SPs:  $SP_1$  (e.g., Duckduckgo) and  $SP_2$  (e.g., Google)

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	$SP_1$	$SP_2$
QoS offered	$v_1$	$v_2$
Privacy risk guarantee	$\epsilon_1$	$\epsilon_2$

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- Assume  $\epsilon_1 \leq \epsilon_2, \implies v_2 \geq v_1$ 
  - Otherwise  $SP_2$ 's strategy is strictly dominated

# Two-SP Model: Cost and Revenue

Total Cost to $SP_i$	Cost of providing services with QoS	Cost of exploiting private data
$C(v_i, \varepsilon_i) = C_{QoS}(v_i) + C_P(\varepsilon_i)$	$C_{QoS}(v_i)$	$C_P(\varepsilon_i)$



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Total Revenue of $SP_i$	Revenue from using consumer's private information	Revenue independent of consumer's private information
$R(\varepsilon_i) = R_P(\varepsilon_i) + R_{NP,i}$	$R_P(\varepsilon_i)$	$R_{NP,i}$

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How to model consumer-SP interaction?

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How to model consumer-SP interaction?

- **Modified** Hotelling model

# Modified Hotelling: Consumer Privacy Preferences and Retailers Risks

Hotelling model has been used to study market segmentation in a variety of contexts

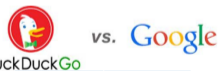
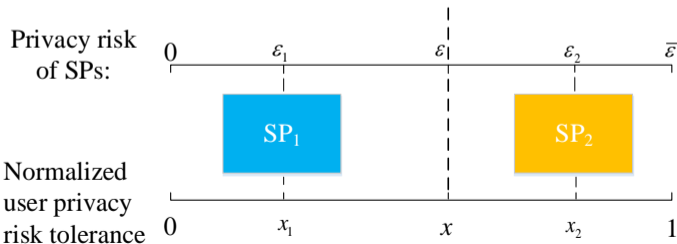


$SP_1$		$SP_2$
$v_1$	$\leq$	$v_2$
$\varepsilon_1$	$\leq$	$\varepsilon_2$

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- Map consumer privacy preference and SP risk offered from arbitrary range  $([0, \bar{\epsilon}])$  to  $[0, 1]$

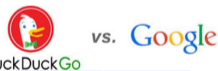
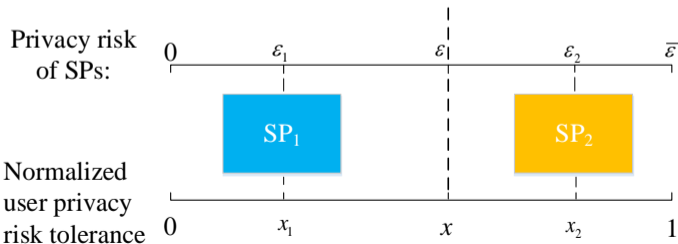


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- Heterogeneous privacy preference of consumers: random variable  $E \in [0, \bar{\varepsilon}]$  with CDF  $F_E(\varepsilon)$



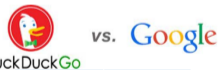
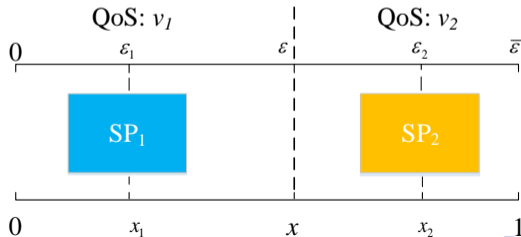
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# Modifying Hotelling Model For Consumer-Retailer Interaction

- Price captured by QoS

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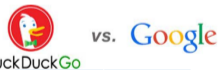
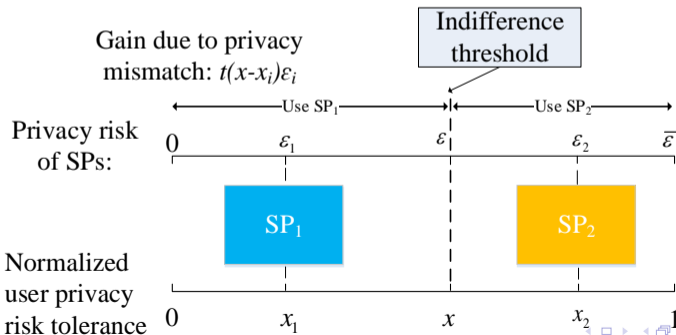
Normalized  
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risk tolerance



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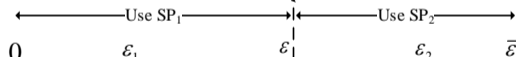
# Modifying Hotelling Model For Consumer-Retailer Interaction

- Price captured by QoS
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- Consumer perceived privacy gain:  $t(x - x_i)\varepsilon_i$  ( $t$ : consumer's valuation of privacy; factor mapping privacy mismatch to QoS)
  - Offered privacy risk < consumer's preference  $\implies$  positive utility (extra privacy protection)
  - Offered privacy risk > consumer's preference  $\implies$  negative utility (privacy violation)

Gain due to privacy mismatch:  $t(x - x_i)\varepsilon_i$

Indifference threshold

Privacy risk of SPs:



Normalized user privacy risk tolerance



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$v_1 \leq v_2$	
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Utility of consumer located at  $x$  for choosing  $SP_i$ :  $u_i(x) = v_i + t(x - x_i)\varepsilon_i$

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- Fraction of consumers who choose  $SP_i$ :  $n_i(v_i; \varepsilon_i; v_{-i}; \varepsilon_{-i})$

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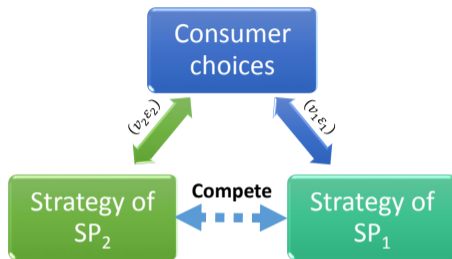
- **Assumption 1:** The services provided by both SPs have non-negative QoS
- **Assumption 2:** The model parameters are chosen such that they ensure the market is completely covered by  $SP_1$  and  $SP_2$

# Two-SP **Non-cooperative** Sequential Game



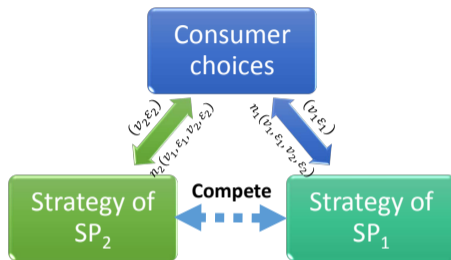
- SPs market segmentation is a two player non-cooperative sequential game

# Two-SP **Non-cooperative** Sequential Game



- Actions of each SP: (QoS, Privacy risk)

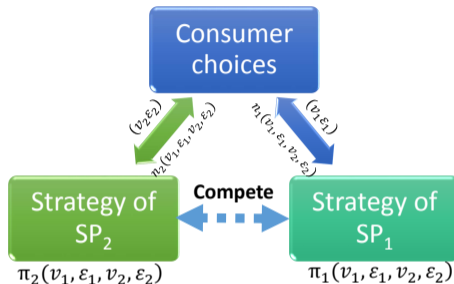
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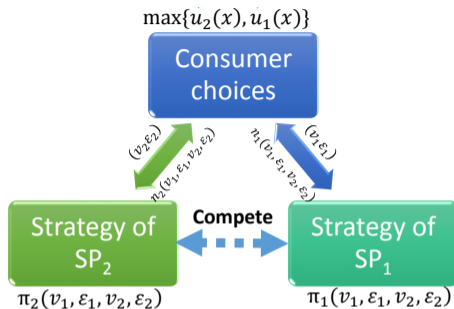


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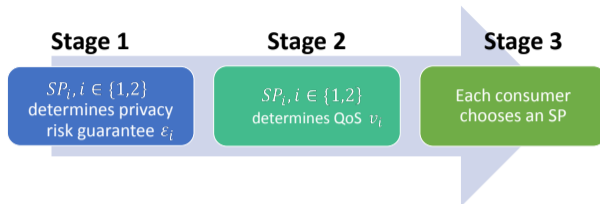
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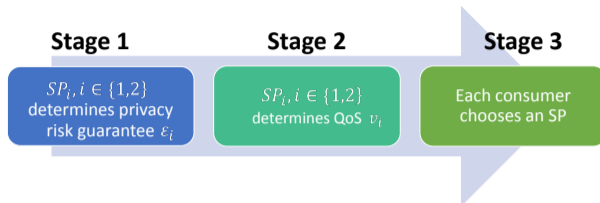
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- Rewards for  $SP_i$ : profit  $\pi_i(v_i; \epsilon_i; v_{-i}; \epsilon_{-i})$
- Rewards for consumer:  $\max\{u_1(x), u_2(x)\}$

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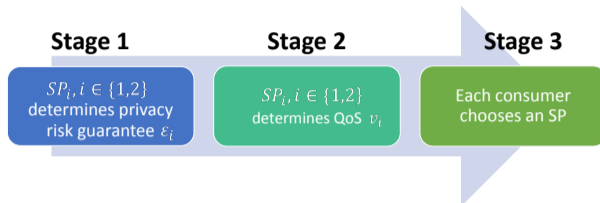
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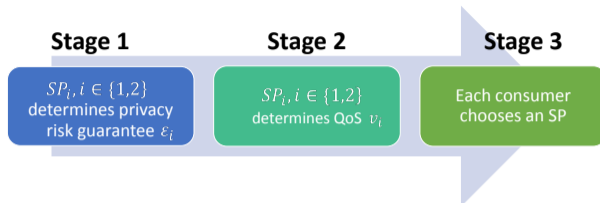
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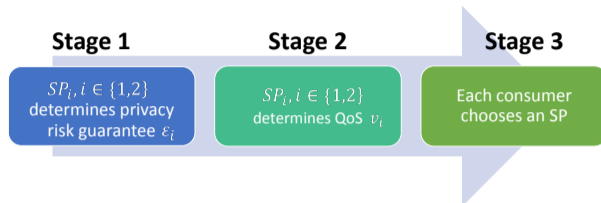
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## Theorem 1

There is no SPNE in which both SPs offer the same privacy risk.

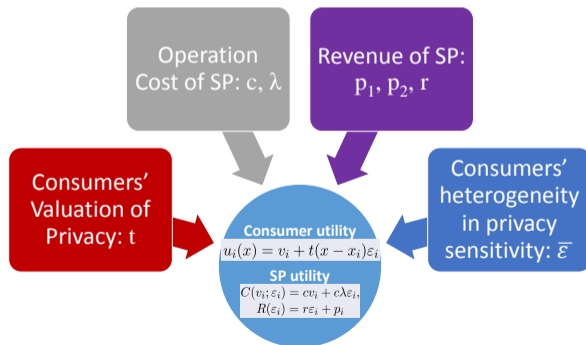
# Two-SP Market With Linear Cost and Revenue Functions

- Linear cost and revenue model for each  $SP_i, i \in \{1, 2\}$ :

$$C(v_i; \varepsilon_i) = cv_i + c\lambda\varepsilon_i,$$

$$R(\varepsilon_i) = r\varepsilon_i + p_i$$

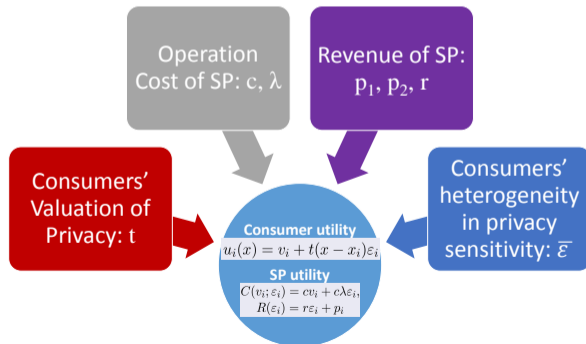
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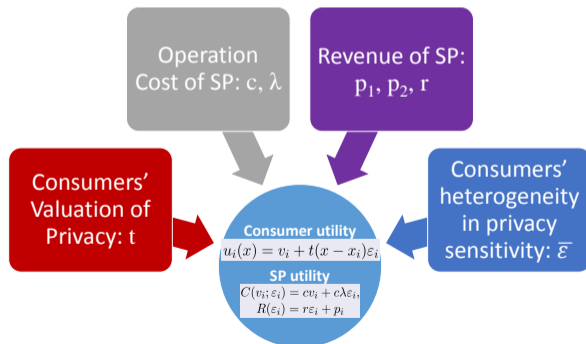
# Consumers with Uniformly Distributed Privacy Risk Preference

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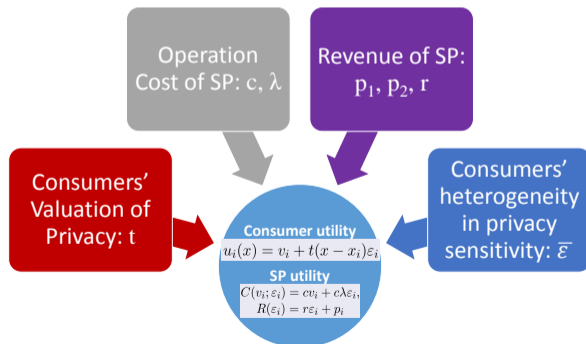
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- Normalized privacy risk of each SP:  $x_i = F_E(\varepsilon_i) = \frac{\varepsilon_i}{\bar{\varepsilon}}, i \in \{1, 2\}$



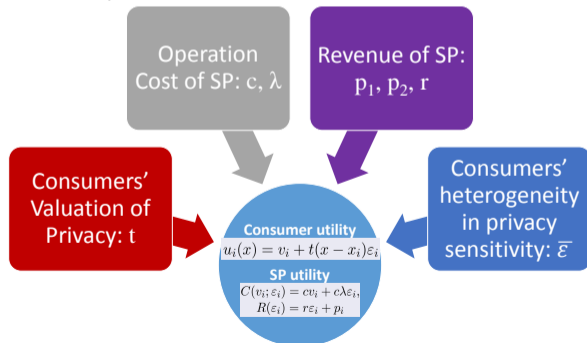
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## Theorem 2

There exists an SPNE for the two-SP non-cooperative game if the model parameters  $\{c, \alpha, t, \bar{\varepsilon}, p_1, p_2\}$  facilitate a competitive market.

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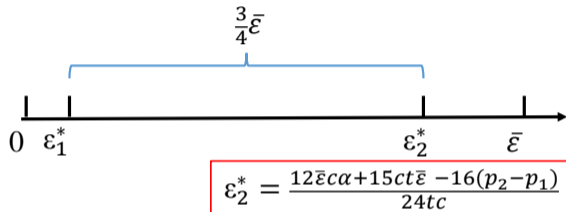
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$$\begin{aligned} -1 &\leq \frac{16(p_2 - p_1)}{9ct\bar{\varepsilon}} \leq 1, \\ \frac{4\alpha - 3t}{3t} &\leq \frac{16(p_2 - p_1)}{9ct\bar{\varepsilon}} \leq \frac{4\alpha - t}{3t}, \\ (12c\alpha\bar{\varepsilon})^2 - (15ct\bar{\varepsilon})^2 + 288ct\bar{\varepsilon}(p_2 + p_1) &\geq [16(p_2 - p_1)]^2 \end{aligned}$$

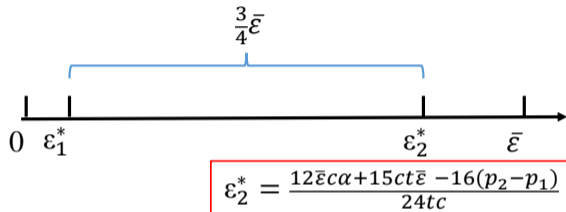
# Closed Form Solution of the SPNE

- Equilibrium privacy risk strategies:

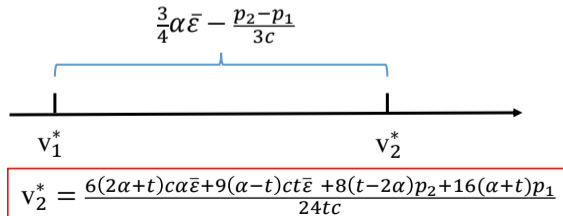


# Closed Form Solution of the SPNE

- Equilibrium privacy risk strategies:



- Equilibrium QoS strategies:





# Market Share of Each SP at Equilibrium

- $p_i$ :  $SP_i$ 's revenue independent of consumers' private data

$c$	Operation cost factor in units of cost/QoS
$p_i$	Revenue of $SP_i$ independent of consumers' private data
$t$	Consumer's valuation of privacy
$\bar{\varepsilon}$	Consumer's heterogeneity in privacy sensitivity



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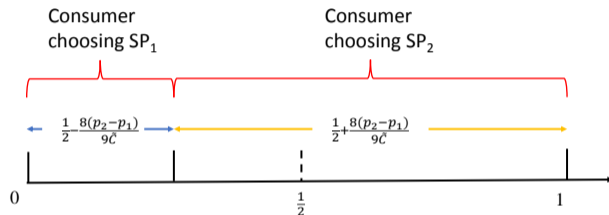


DuckDuckGo vs. Google

$SP_1$		$SP_2$
$v_1$	$\leq$	$v_2$
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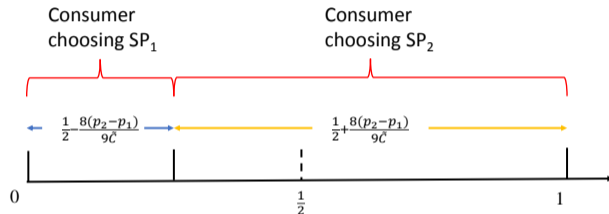


 vs. 

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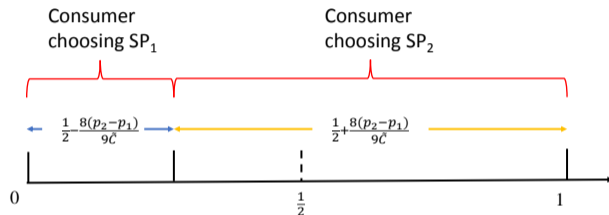
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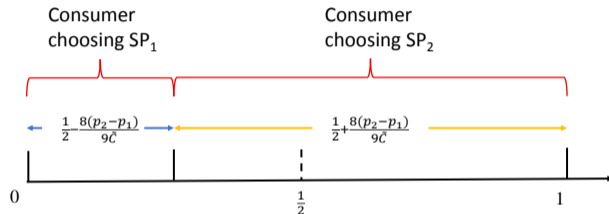
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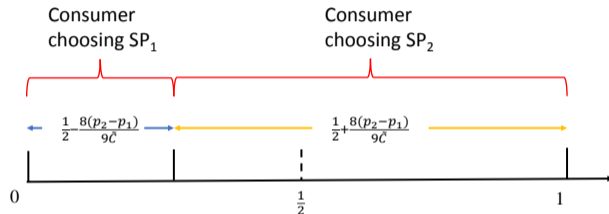
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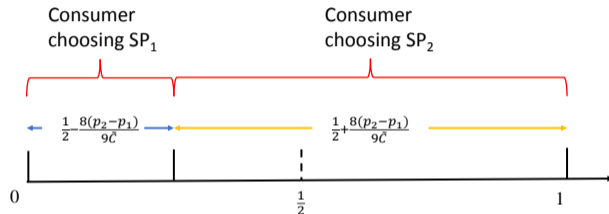


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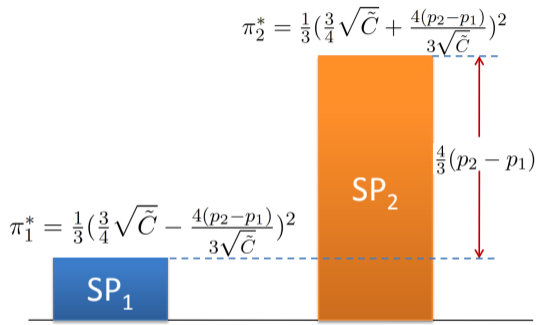
# Profit for Each SP



SP <sub>1</sub>	SP <sub>2</sub>
$v_1$	$v_2$
$\varepsilon_1$	$\varepsilon_2$

$$C(v_i; \varepsilon_i) = cv_i + c\lambda\varepsilon_i$$
$$R(\varepsilon_i) = r\varepsilon_i + p_i$$

## Total Profit



# Profit for Each SP

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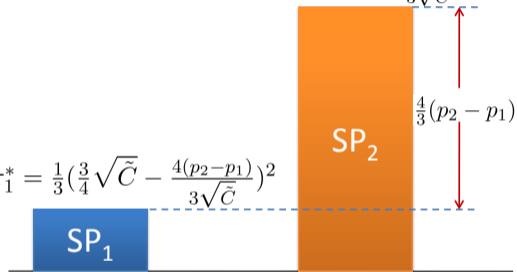
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$$\pi_2^* = \frac{1}{3} \left( \frac{3}{4} \sqrt{\tilde{C}} + \frac{4(p_2 - p_1)}{3\sqrt{\tilde{C}}} \right)^2$$

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# Profit for Each SP

- Both SPs will make more profits at the SPNE with a larger  $\tilde{C}$ 
  - Higher revenue from exploiting private data ( $\bar{\varepsilon} \uparrow$ )
  - Lower cost for offering low QoS and low privacy risk ( $t \uparrow$ )
  - Offer high privacy risk to increase profit ( $c \uparrow$ )



SP <sub>1</sub>	SP <sub>2</sub>
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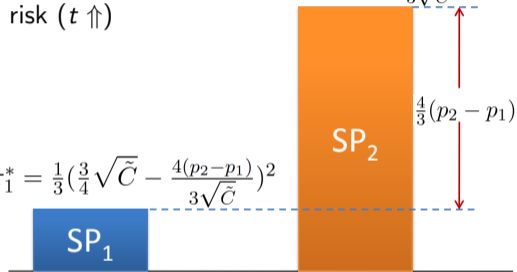
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SP <sub>1</sub>	SP <sub>2</sub>
$v_1 \leq v_2$	
$\varepsilon_1 \leq \varepsilon_2$	

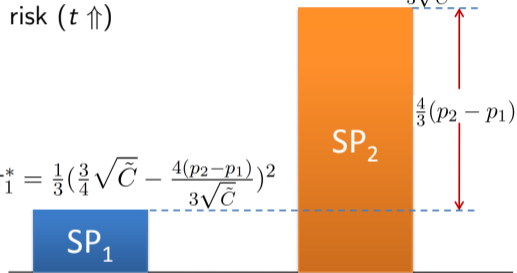
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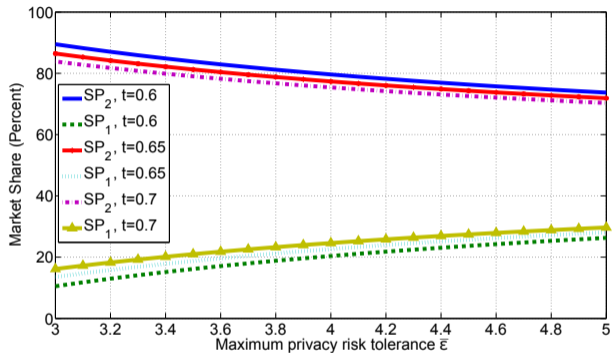
- Increase in difference of privacy independent revenue  $\implies$  increases in difference of profit between two SPs

# Illustration of Results (Market Share)

Parameter	$c$	$\lambda$	$r$	$p_1$	$p_2$
Value	0.5	0.75	0.7	0.4	0.8

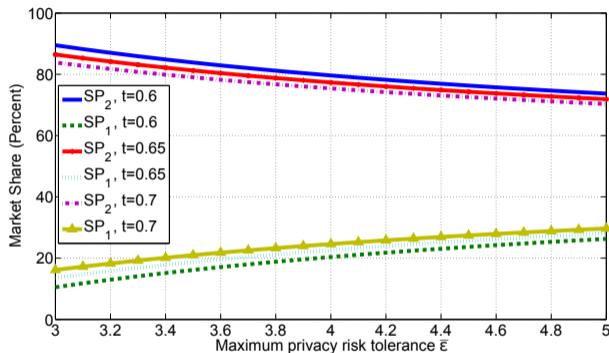


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$SP_1$		$SP_2$
$v_1$	$\leq$	$v_2$
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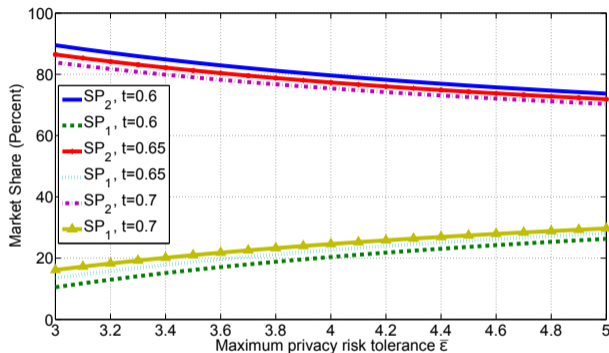
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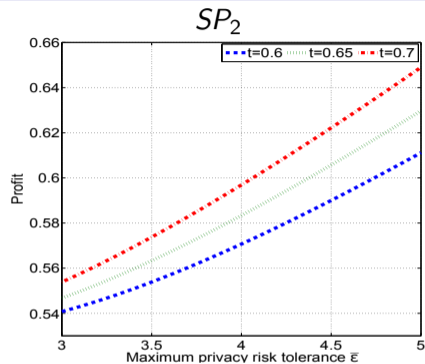
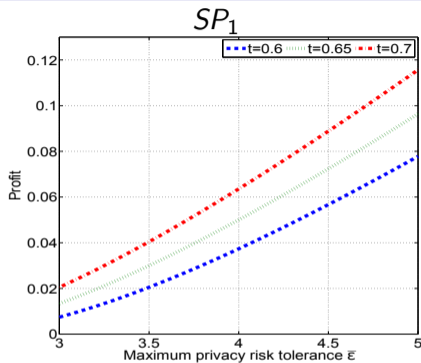


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- Higher valuation of privacy by consumer (larger  $t$ )  $\implies$  lower market share for  $SP_2$
- As  $\bar{\epsilon}$  decreases,  $SP_2$  offers high QoS and high privacy risk, thus its market share increases



# Illustration of Results (SP Profit)



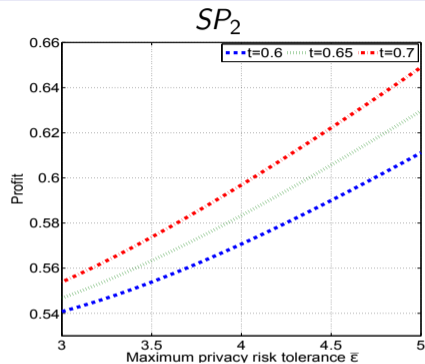
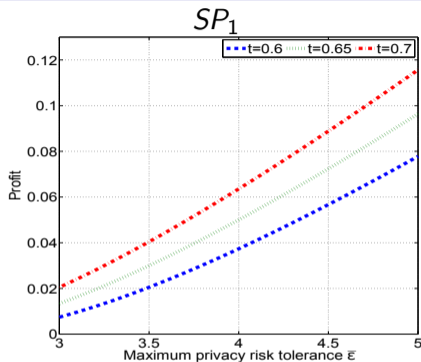
 vs.   
 DuckDuckGo

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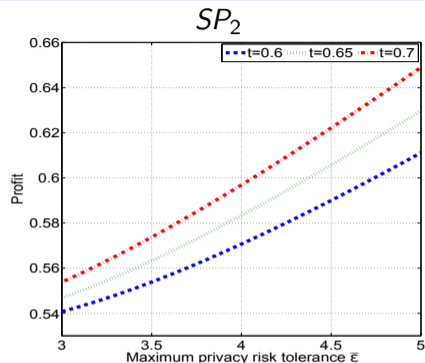
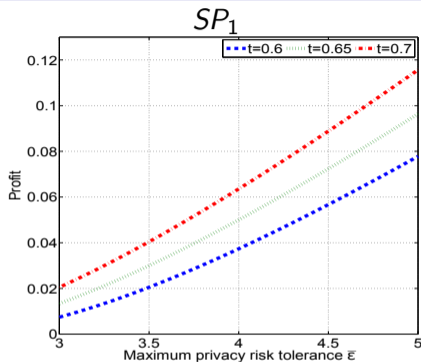
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- A larger  $\bar{\epsilon}$  indicates a larger range of consumer preferences for SPs to exploit private data
- Increasing  $t$ : SPs offer lower risk & lower QoS (Cost reduction more than revenue reduction)  $\implies$  higher profit for SPs

## Concluding Remarks

- Market segmentation for privacy differentiated “free” services is studied

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- Market segmentation for privacy differentiated “free” services is studied
- Investigated influences of consumers’ valuation and heterogeneity in privacy preference on market share and SP profit
  - High valuation of privacy by consumers “softens” competition
  - Offering privacy aware services can still be profitable

Thank you!