Geometry of Polynomials Bootcamp **Winter 2019**

Expected Characteristic Polynomials

Speaker: Nikhil Srivastava *In-class exercise*

P1) Suppose $r_1, \ldots, r_m$ are independent vector-valued random variables in $\mathbb{R}^n$, with $\mathbb{E} r_i r_i^T = A_i$. Show that

$$\mathbb{E} \det \left(xI - \sum_{i \leq m} r_i r_i^T \right) = \prod_{i \leq m} \left(1 + \frac{\partial}{\partial z_i} \right) \det \left(xI - \sum_{i \leq m} z_i A_i \right) |_{z_1 = \ldots = z_m = 0}.$$

P2) Show that if $M_G(x)$ is the matching polynomial of G then:

$$M_G'(x) = \sum_{r \in V} M_{G \setminus r}(x) \quad \text{and} \quad M_G(x) = x M_{G \setminus r}(x) - \sum_{r \sim v} M_{G \setminus rv}.$$

P3) Show that if T is a tree with maximum degree d, then the number of closed walks of length ℓ starting at a vertex $v \in T$ is at most $(2\sqrt{d-1} + o_\ell(1))^\ell$.

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P1) Show that if G is a $d-$regular graph with adjacency matrix A, then

$$\mathrm{tr}(A^\ell) \geq d^\ell + (2\sqrt{d-1} - o_n(1))^\ell.$$

This is a weak form of the Alon-Boppana theorem.