Geometry of Polynomials Bootcamp	**Winter 2019**

Real-rooted Polynomials

Speaker: Jan Vondrak | *In-class exercise*

P1) Prove that the (complex) roots $\lambda_1, \lambda_2, \ldots, \lambda_n$ of a monic polynomial $p(z) = z^n + a_{n-1}z^{n-1} + \ldots + a_1 z + a_0$ depend continuously on the (complex) coefficients. There are several ways to do this. Think about the properties of the map between roots and coefficients, or about the properties of the function $p'(z)/p(z)$.

P2) Prove the Hermite-Sylvester theorem: $p(x) = \prod_{\ell=1}^{n}(x - \lambda_\ell)$ is real-rooted if and only if the following matrix is positive-semidefinite:

$$H = \left(\sum_{\ell=1}^{n} \lambda_\ell^{i+j-2} \right)_{i,j=1}^{n}.$$

First prove the $\Rightarrow$ implication. Then think about what kind of vector would violate the PSD property, in case the polynomial is not real-rooted.

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Real-rooted polynomials

Speaker: Jan Vondrak *Homeworks*

P1) Show that for every tree T, the matching polynomial

$$\mathcal{M}_T(x) = \sum_{\text{matching } M \subset T} (-1)^{|M|} x^{n-2|M|}$$

is equal to the characteristic polynomial of its adjacency matrix, $\chi_A(x) = \det(xI - A)$.