The KRW conjecture

Results and Open problems

Or Meir

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- Major frontier: Explicit f with $\mathsf{D}(f) = \omega(\log n)$.
- a.k.a. $\mathbf{P} \neq \mathbf{NC}^1$.

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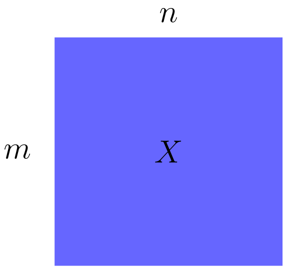

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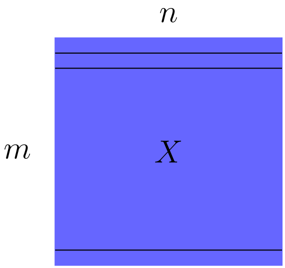

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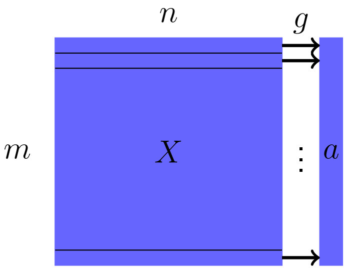

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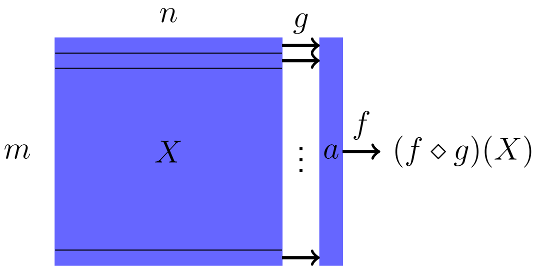

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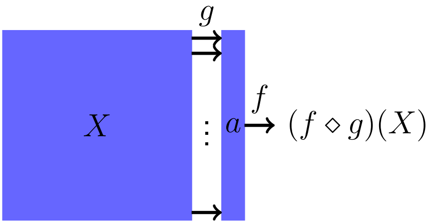

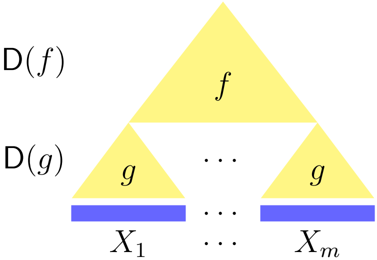

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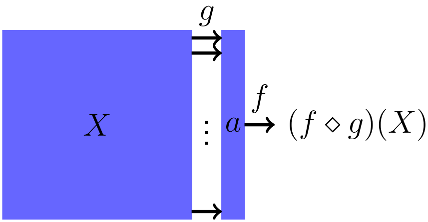

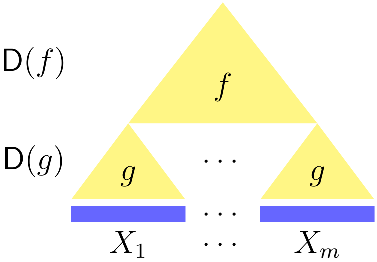

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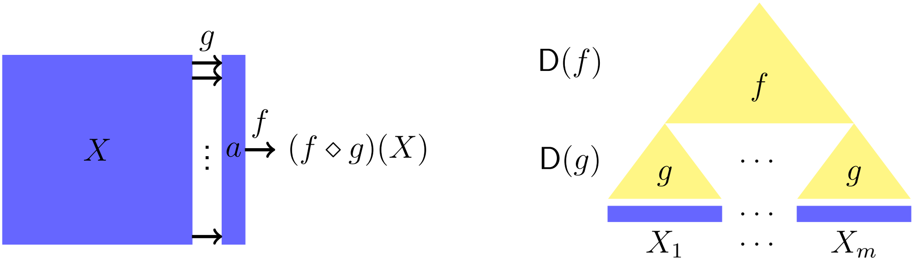

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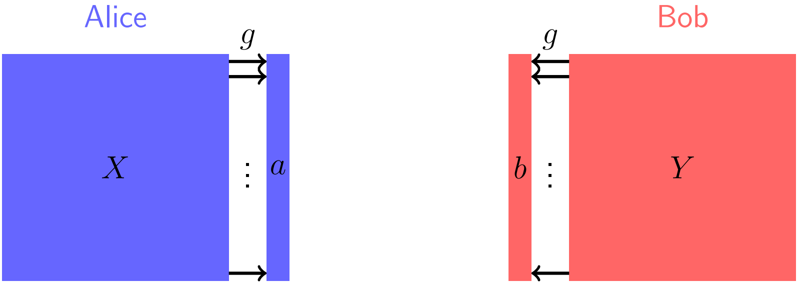

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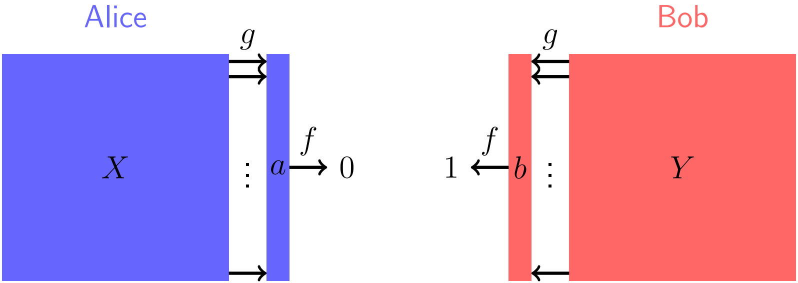

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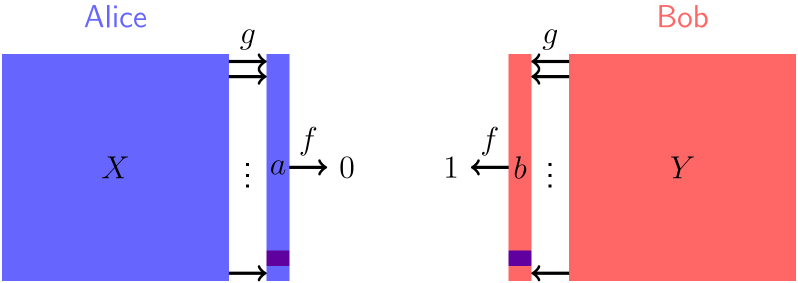

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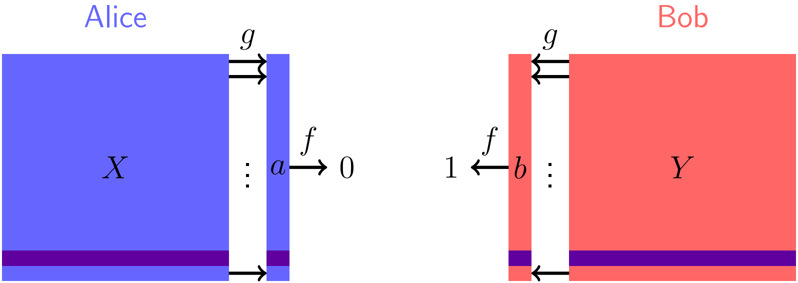

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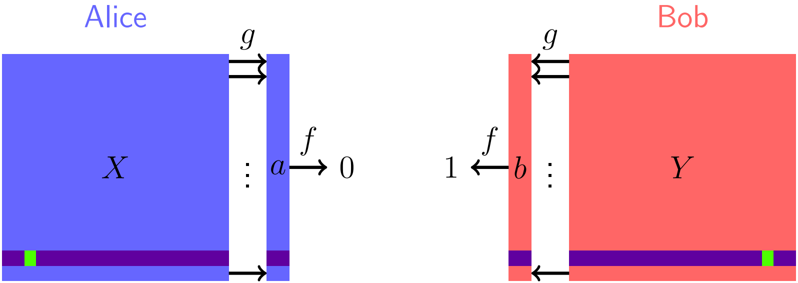

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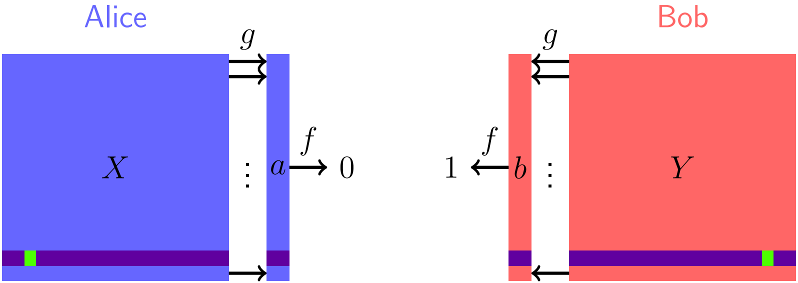

- Hence, $\mathsf{C}(KW_{f \diamond g}) \leq \mathsf{C}(KW_f) + \mathsf{C}(KW_g)$.
- KRW conjecture: the obvious protocol is essentially optimal.

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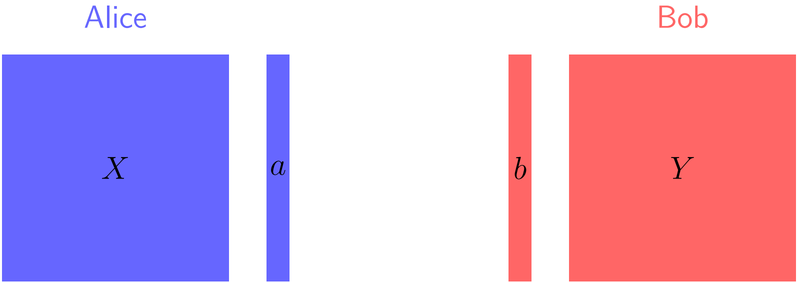

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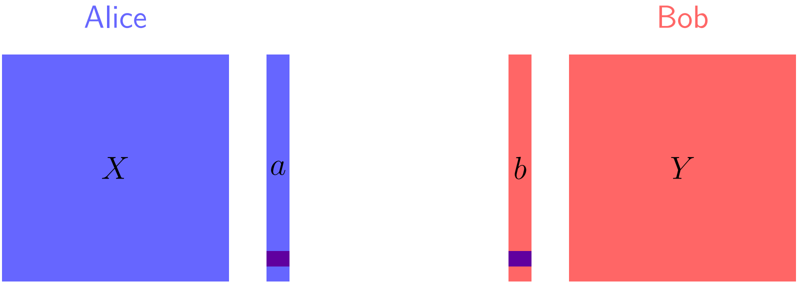

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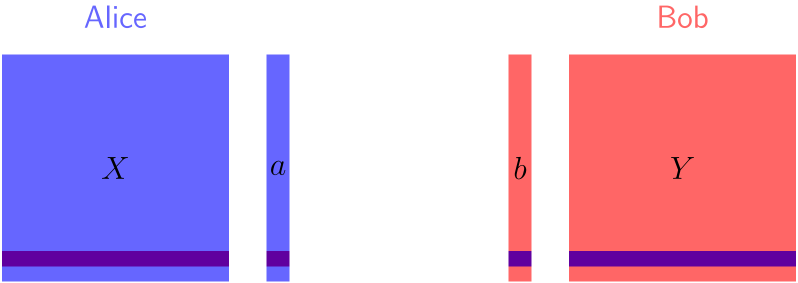

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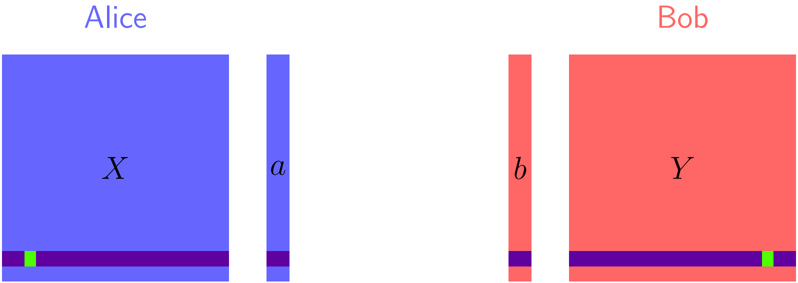

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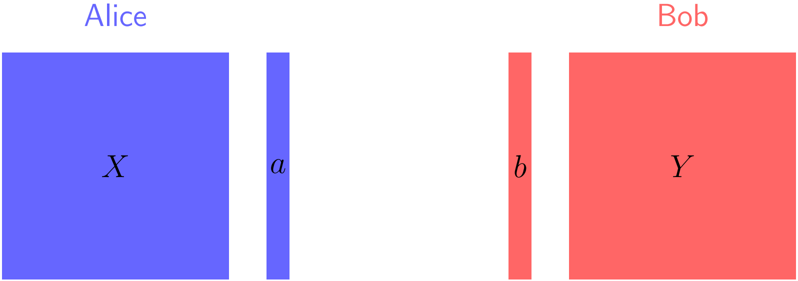

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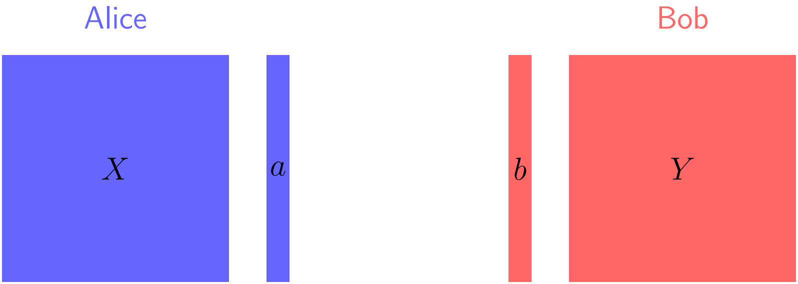

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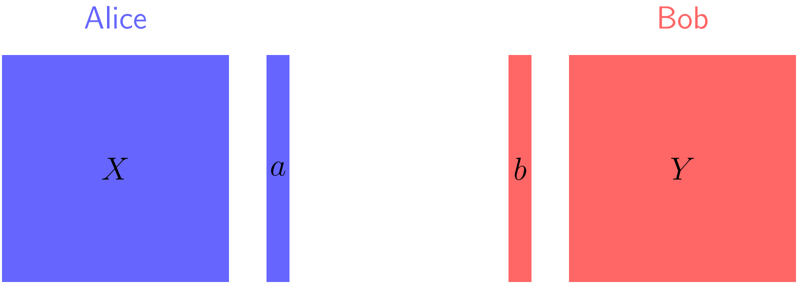
- Alternative proof obtained by [Håstad-Wigderson-93].



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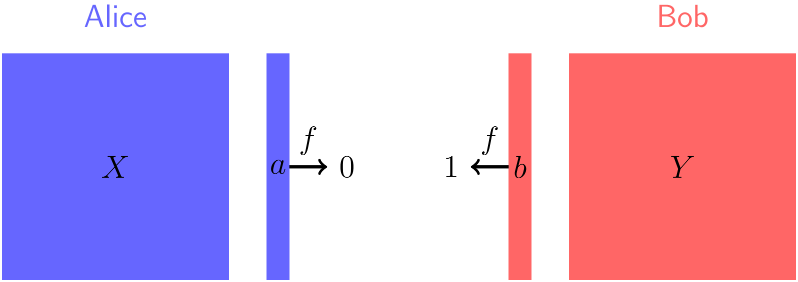

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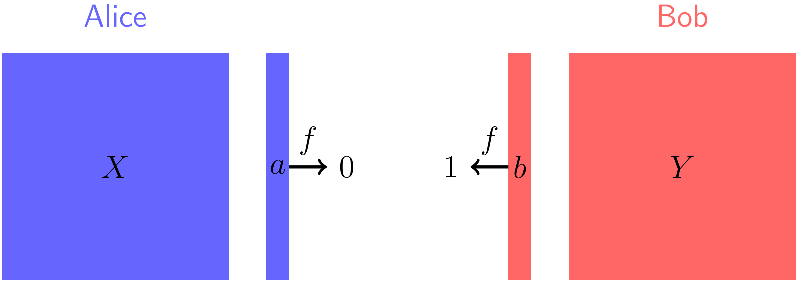

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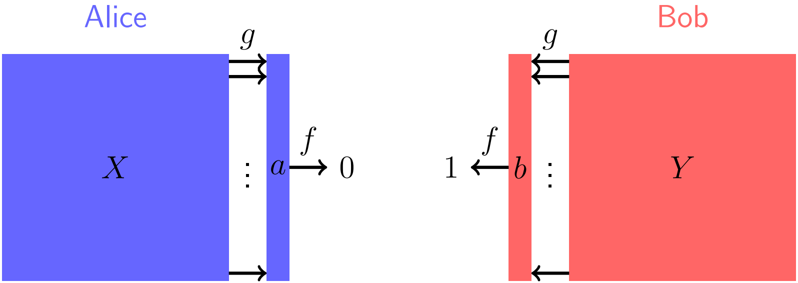
- However, our proof was very different, and more in line with the other works on the KRW conjecture.

Outline

Why should the obvious protocol be optimal?



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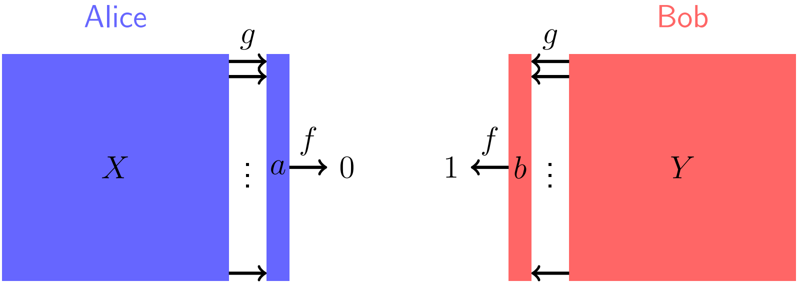

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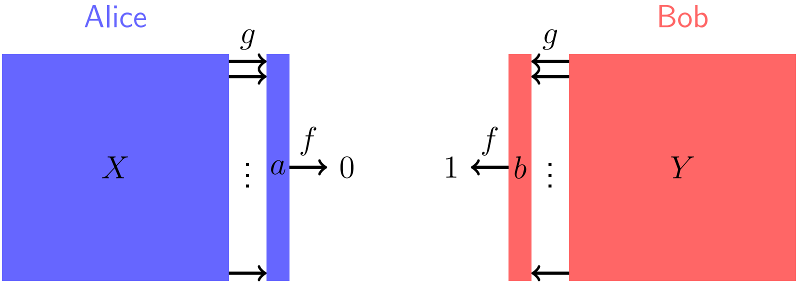

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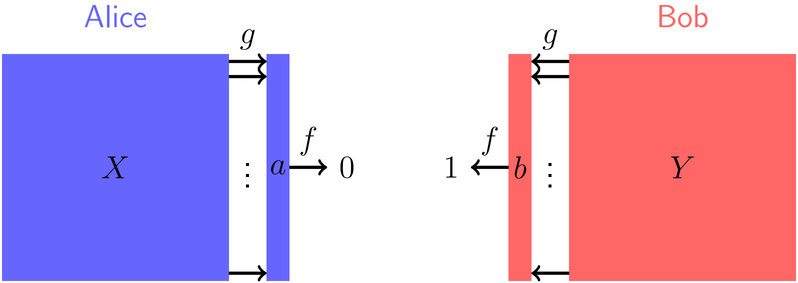

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- Therefore, the players must first solve KW_f and then solve KW_g.

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- This is how the proofs for universal relation and parity work.
- Call the adversary of KW_g an "information-theoretic adversary".

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- We have a candidate: the multiplexor relation.

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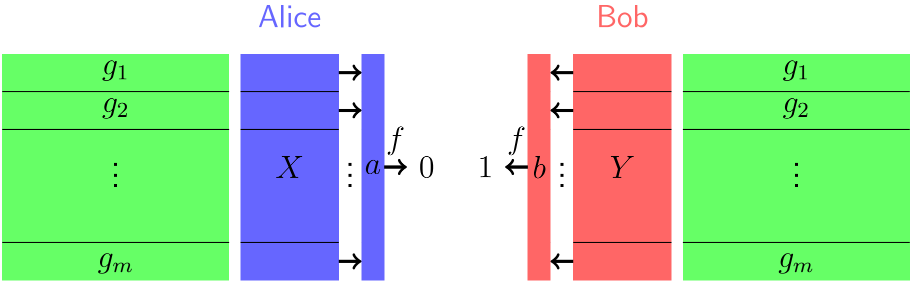
- Like KW relation of the address function, but with promise.

The multiplexor relation [EIRS91]

- The function g becomes part of the input.
- Alice gets a function $g : \{0,1\}^n \to \{0,1\}$ and $x \in g^{-1}(0)$.
- Bob gets the same function g and $y \in g^{-1}(1)$.
- Want to find $i \in [n]$ such that $x_i \neq y_i$.
- Like KW relation of the address function, but with promise.
- Easy to prove: $\mathsf{C}(MUX_n) = \Omega(n)$.

Multiplexor composition

- Given f, define the composition $KW_f \diamond MUX_n$:



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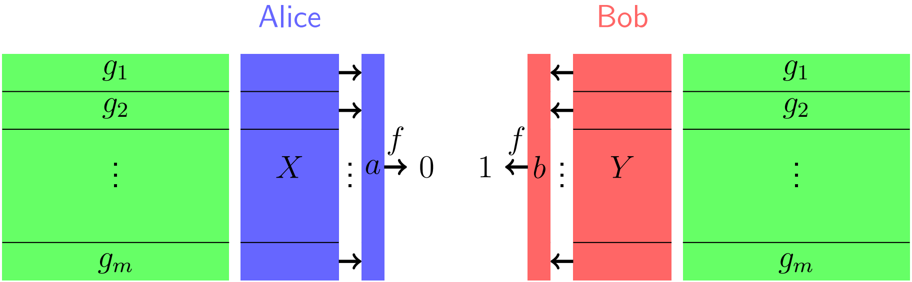

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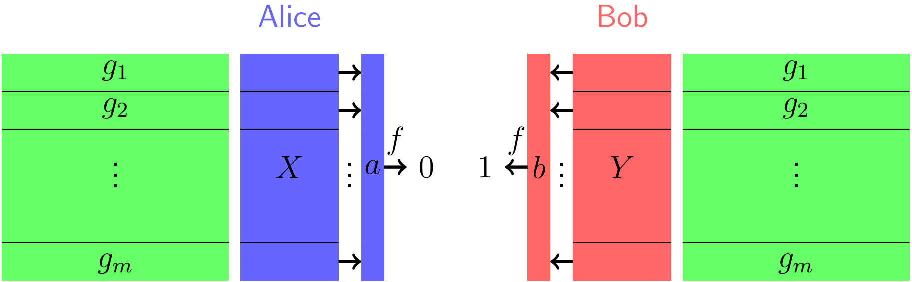

- Conjecture 1: $\mathsf{C}(KW_f \diamond MUX_n) \gtrapprox \mathsf{C}(KW_f) + \Omega(n)$.
- Conjecture 2: Conjecture 1 implies that $\mathbf{P} \neq \mathbf{NC}^1$.

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  - Prove KRW conjecture for $KW_f \diamond MUX$.
  - Separate $\mathbf{P}$ from $\mathbf{NC}^1$.
- Unfortunately, the adversary of MUX is very complicated.
- Very hard to incorporate in our proof strategy.

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- There are several nice communication problems with such adversaries.
- Let's try to prove composition results for them.

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- Can we prove the KRW conjecture for $KW_f \diamond \textsc{Fork}$, $KW_f \diamond st\textsc{Conn}$ or $KW_f \diamond \textsc{Clique}$?

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- Those are clean and (hopefuly) tractable open questions.

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- We know how to prove it when the inner function is
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  - Prove the KRW conjecture when the inner function is the multiplexor relation.
- Open problems: Prove the KRW conjecture when the inner function is
  - the $\textsc{Fork}$ relation.
  - the monotone $st\textsc{Conn}$ relation,
  - or the monotone $\textsc{Clique}$ relation.

Thank you!