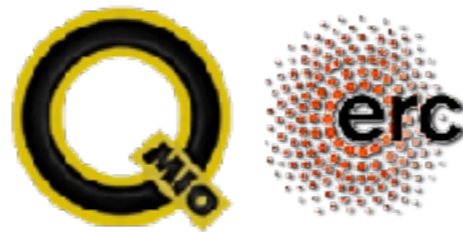

Analog quantum simulations

What we have, what we would like to have,
and the potential for seeing quantum advantages

Jens Eisert, Freie Universität Berlin



Challenges in Quantum Computation Workshop, Berkeley, June 2018

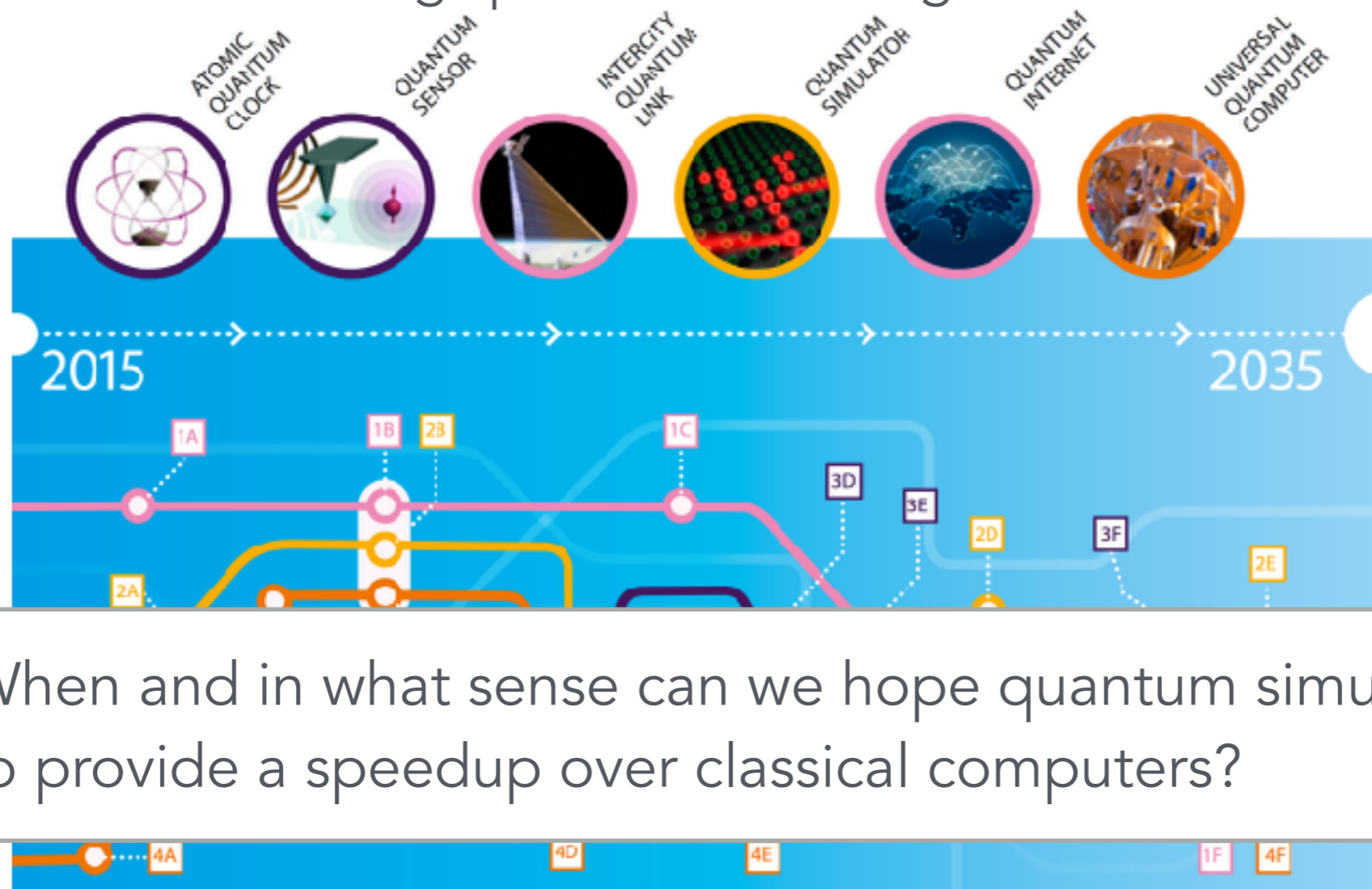
Analog quantum simulations

What we have, what we would like to have,
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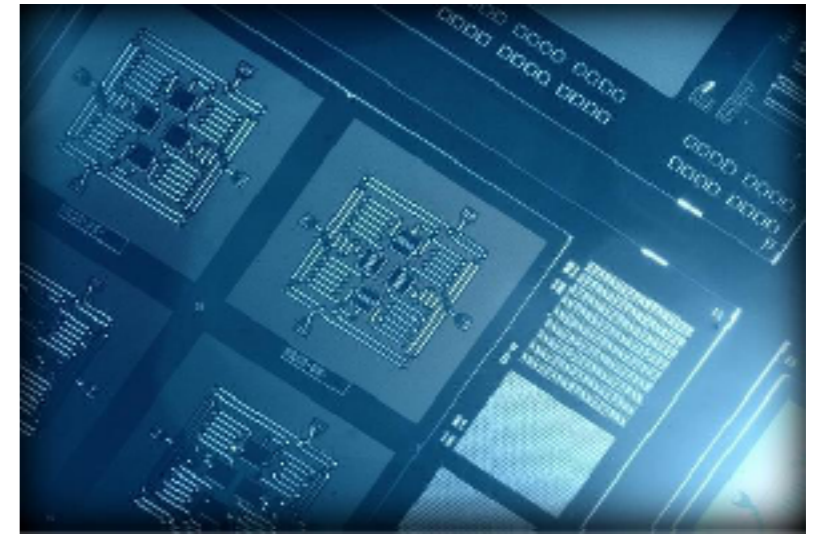
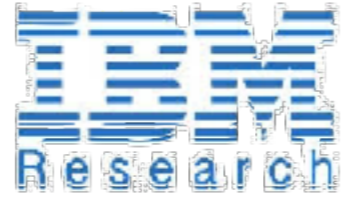
- What is an analog quantum simulator? What are relevant problems?

Analog quantum simulations

What we have, what we would like to have,
and the potential for seeing quantum advantages



- When and in what sense can we hope quantum simulators to provide a speedup over classical computers?



- 20-50 qubit quantum devices
- Noisy intermediate scale quantum computers

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COMPUTER

- When and in what sense can we hope quantum simulators to provide a speedup over classical computers

ERGENCY
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SIMULATOR

QUA
IN

- Analog(ue) quantum simulators
 - Address interesting physics problems
 - Not BQP-complete, what is computational power?
 - Error correction/fault tolerance unavailable
 - Robustness?

ERGENCY
QUANTUM
LINK

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SIMULATOR

QUA
IN



- When can it be claimed that a system has been successfully simulated?
- Testable advantage?

Analog quantum simulators

Analog quantum simulators



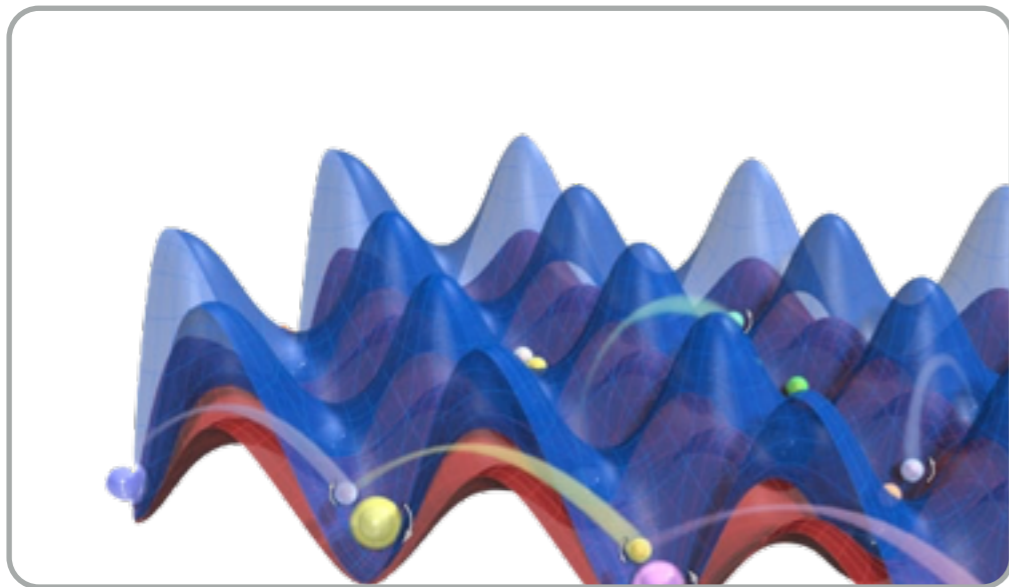
- “Analog”, rather than discrete
- Probing questions in physics (not so much quantum chemistry)

- System size n
- Local Hamiltonians with some levels of control
- Noise levels
- Classes of preparations and measurements

Analog quantum simulators



- Cold atoms in optical lattices most advanced



- Global control over $n \sim 10^5$ sites (1D-3D)
- Bosons and fermions
- Some tuneability
- Time-of-flight and in-situ measurements

Bloch, Dalibard, Nascimbene, Nature Physics 8, 267 (2012)

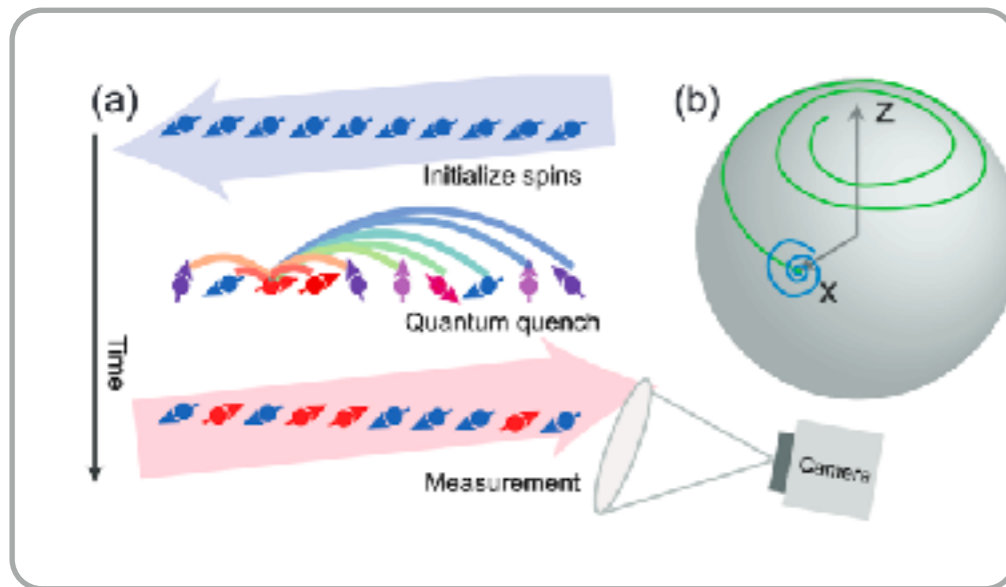
Parsons, Mazurenko, Chiu, Ji, Greif, Greiner, Science, 353, 1253 (2016)

- Towards programmable potentials

Analog quantum simulators



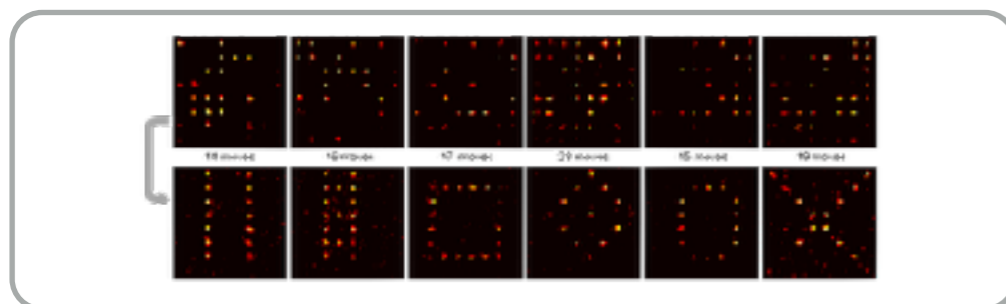
• Trapped ions



- $n \leq 53$
- Universal control
- Some global gates easier than others
- Tomographically complete measurements

Zhang, Pagano, Hess, Kyprianidis, Becker, Kaplan, Gorshkov, Gong, Monroe 551, 601 (2017)
Blatt, Roos, Nature Phys 8, 277 (2012)

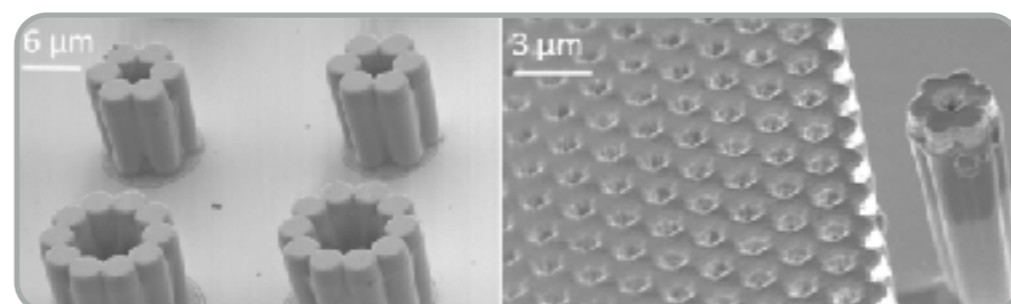
• Optical microtraps



Labuhn, Barredo, Ravets, Léséleuc, Macrì, Lahaye, Browaeys, Nature 534, 667 (2016)

- $n \sim 50 \times 50$, long-ranged Ising

• Polaritonic/photonic architectures



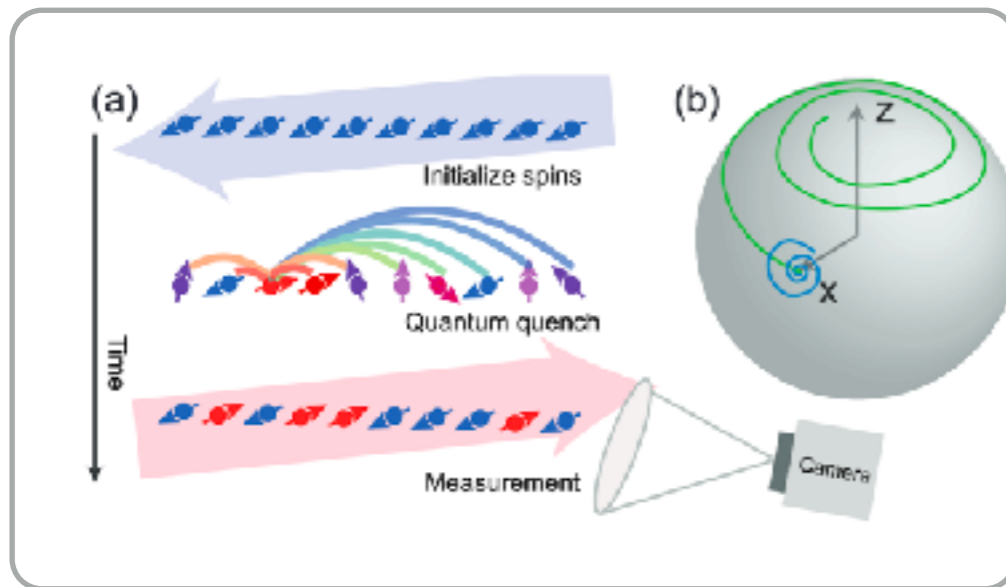
Wertz, Ferrier, Solnyshkov, Johne, Sanvitto, Lemaître, Sagnes, Grousson, Kavokin, Senellart, Malpuech, Bloch, Nature Phys 6, 860 (2010)

- Large, but intrinsically open and noisy

Analog quantum simulators



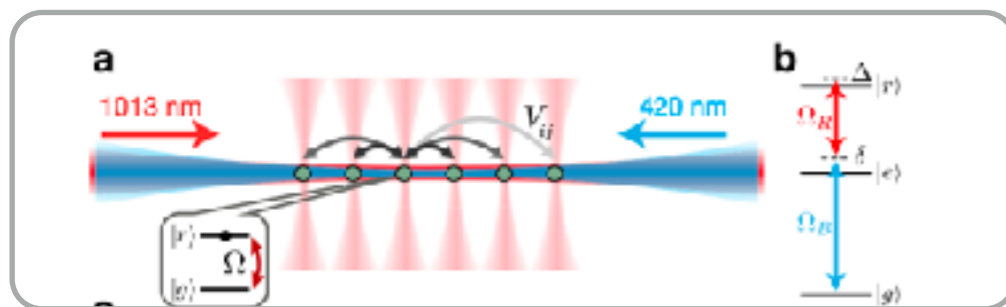
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Zhang, Pagano, Hess, Kyprianidis, Becker, Kaplan, Gorshkov, Gong, Monroe 551, 601 (2017)
Blatt, Roos, Nature Phys 8, 277 (2012)

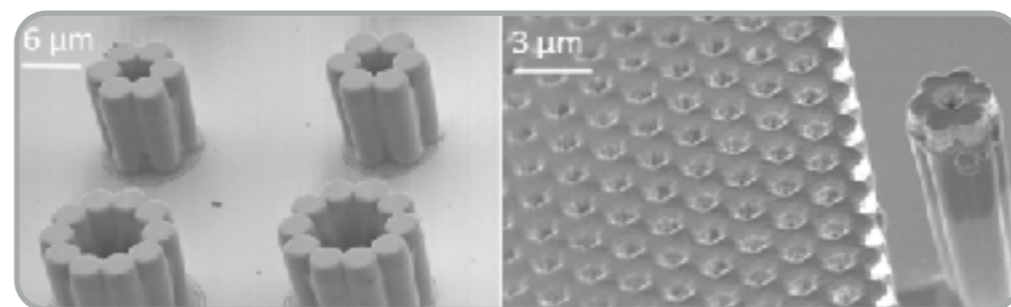
• Cold atoms in Rydberg states



Bernien, Schwartz, Keesling, Levine, Omran, Pichler, Choi, Zibrov, Endres, Greiner, Vuletic, Lukin, Nature 551, 579 (2017)

• Programmable

• Polaritonic/photonic architectures



Wertz, Ferrier, Solnyshkov, Johne, Sanvitto, Lemaître, Sagnes, Grousson, Kavokin, Senellart, Malpuech, Bloch, Nature Phys 6, 860 (2010)

• Large, but intrinsically open and noisy

What can they probe?

What can they probe?



- **Time-dependent** problems ("quenches")

$$\rho(t) = e^{-itH} \rho e^{itH}$$

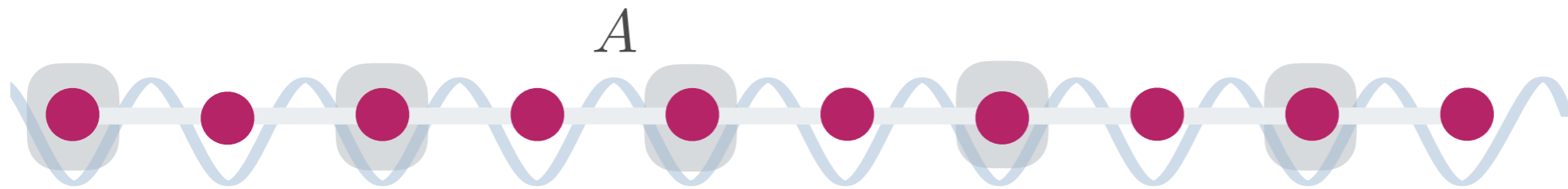
- E.g. probe **equilibration and thermalisation**

Eisert, Friesdorf, Gogolin, Nature Phys 11, 124 (2015)

- **Dynamical phase transitions**

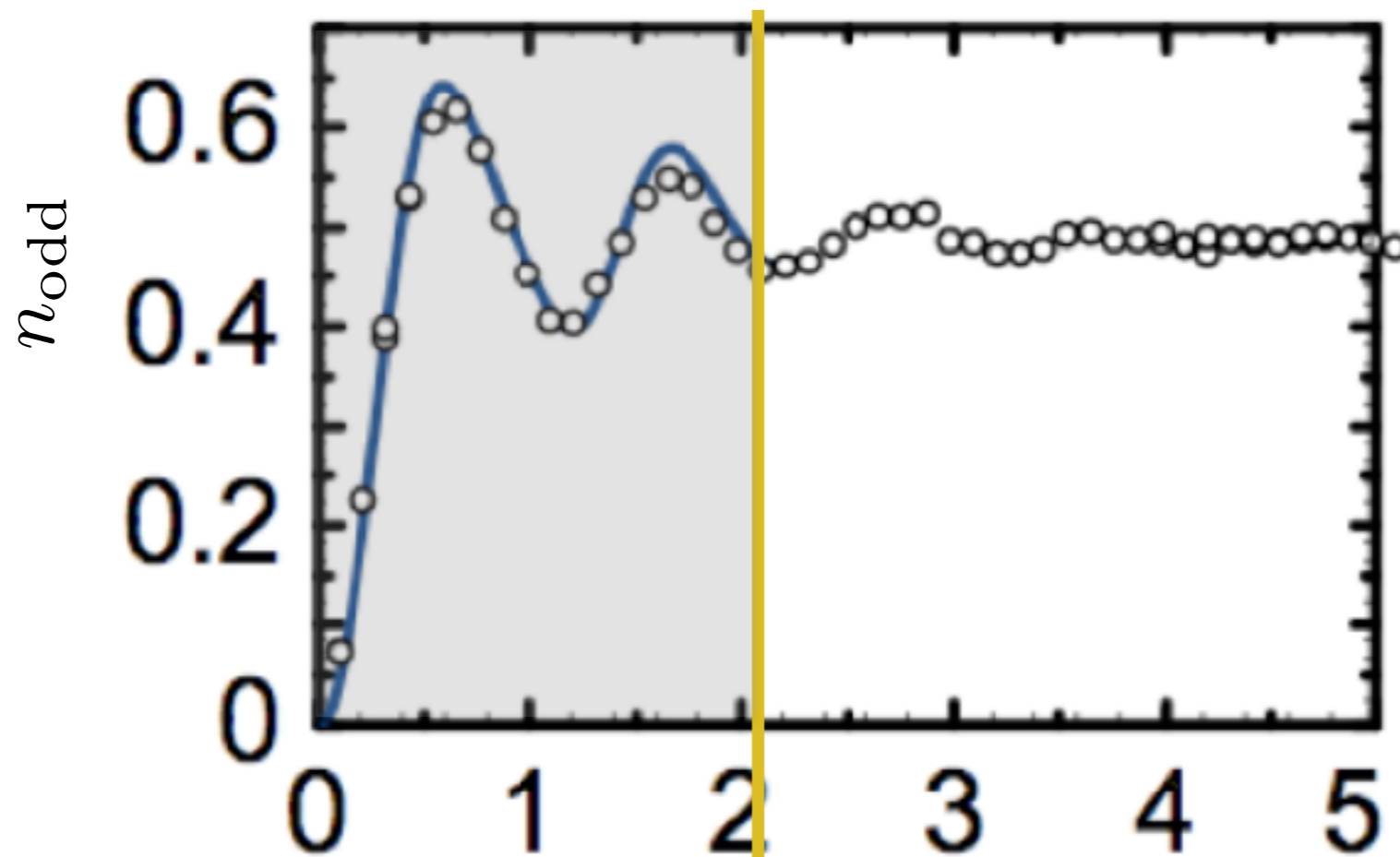
Zhang, Pagano, Hess, Kyprianidis, Becker, Kaplan, Gorshkov, Gong, Monroe, Nature 551, 601 (2017)

What can they probe?



- **Time-dependent** problems ("quenches")

- Imbalance as function of time for $|\psi(0)\rangle = |0, 1, \dots, 0, 1\rangle$ under Bose-Hubbard Hamiltonian (MPQ)



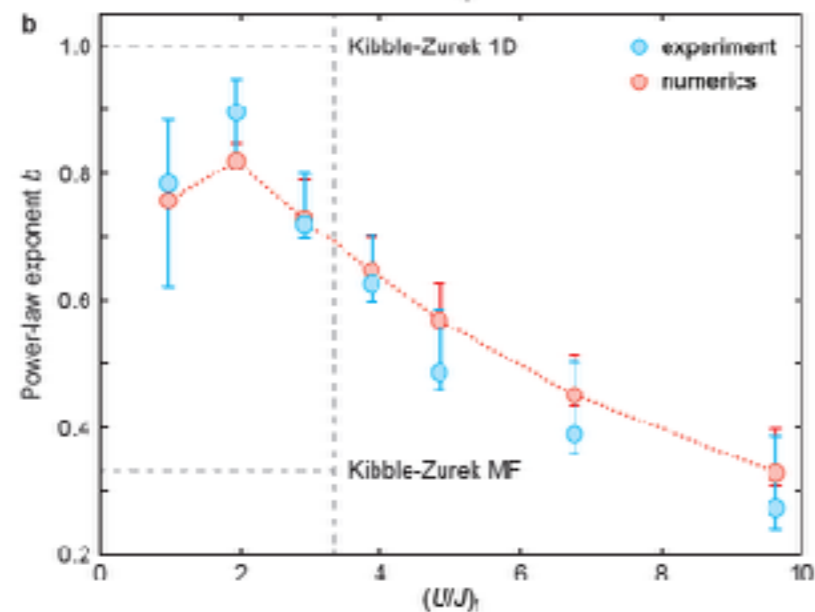
Best available classical tensor network simulation, bond dimension 5000

What can they probe?



- **Slow parameter variations** (reminiscent of adiabatic quantum algorithms)

- E.g., Kibble-Zurek dynamics (1D-2D)



- Probing scaling laws of correlations

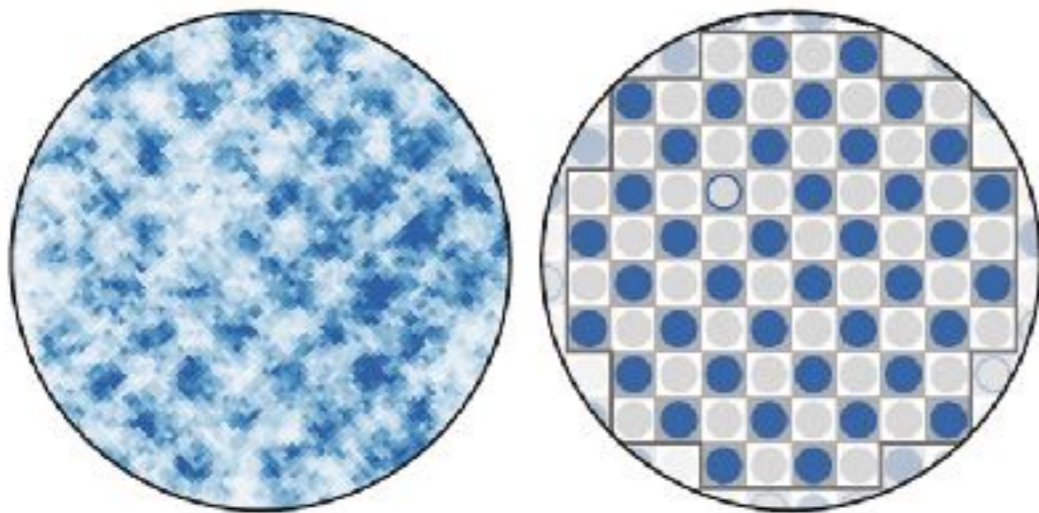
Braun, Friesdorf, Hodgman, Schreiber, Ronzheimer, Riera, del Rey, Bloch, Eisert, Schneider, Proc Natl Acad Sci 112, 3641 (2015)

What can they probe?



• Ground state problems

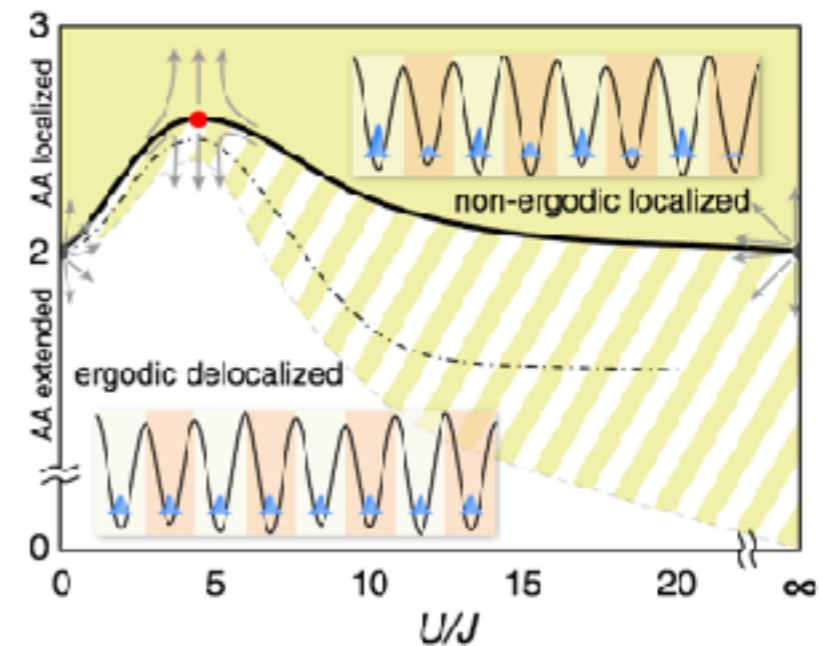
- Hubbard model, probing high- T_c superconductivity



- Cooled to create a magnetic state with long-range order

Mazurenko, Chiu, Ji, Parsons, Kanász-Nagy, Schmidt, Grusdt, Demler, Greif, Greiner, Nature 545, 462 (2017)
Esslinger, Ann Rev Cond Mat Phys 1, 129 2010

- Many-body localization (1D-2D)



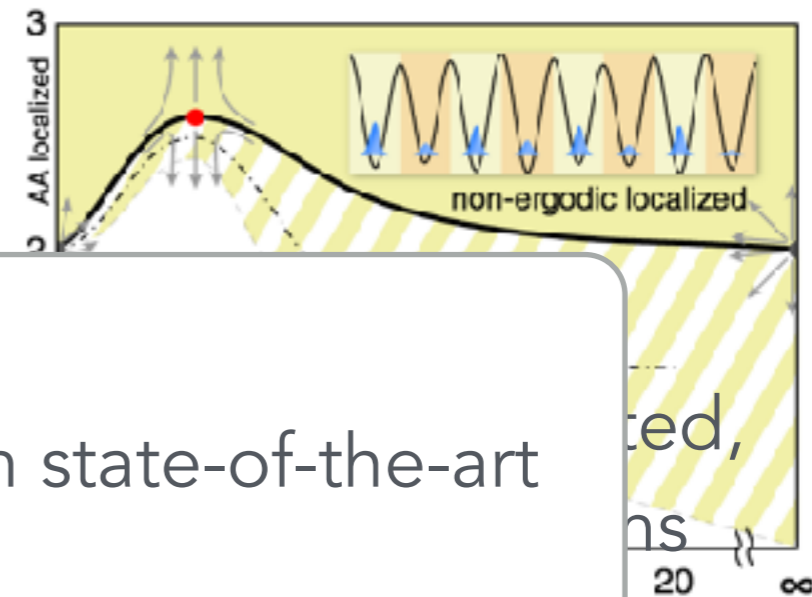
- Debated in 2D

Schreiber, Hodgman, Bordia, Lüschen, Fischer, Vosk, Altman, Schneider, Bloch, Science 349, 842 (2015)

What can they probe?



- Many-body localization (1D-2D)

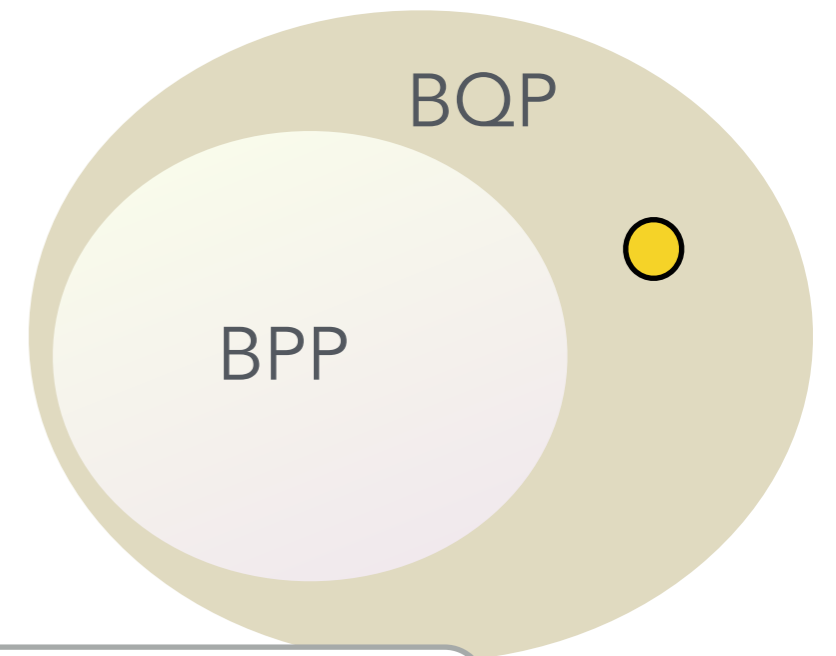


- **Quantum simulators**

Existing quantum simulators outperform state-of-the-art algorithms on classical supercomputers

- Debated in 2D

- Cleverer simulation method?



- **Intermediate problems**

To be safe against “lack of imagination”, we must prove the hardness of the task in a complexity-theoretic sense

Super-polynomial quantum advantages?

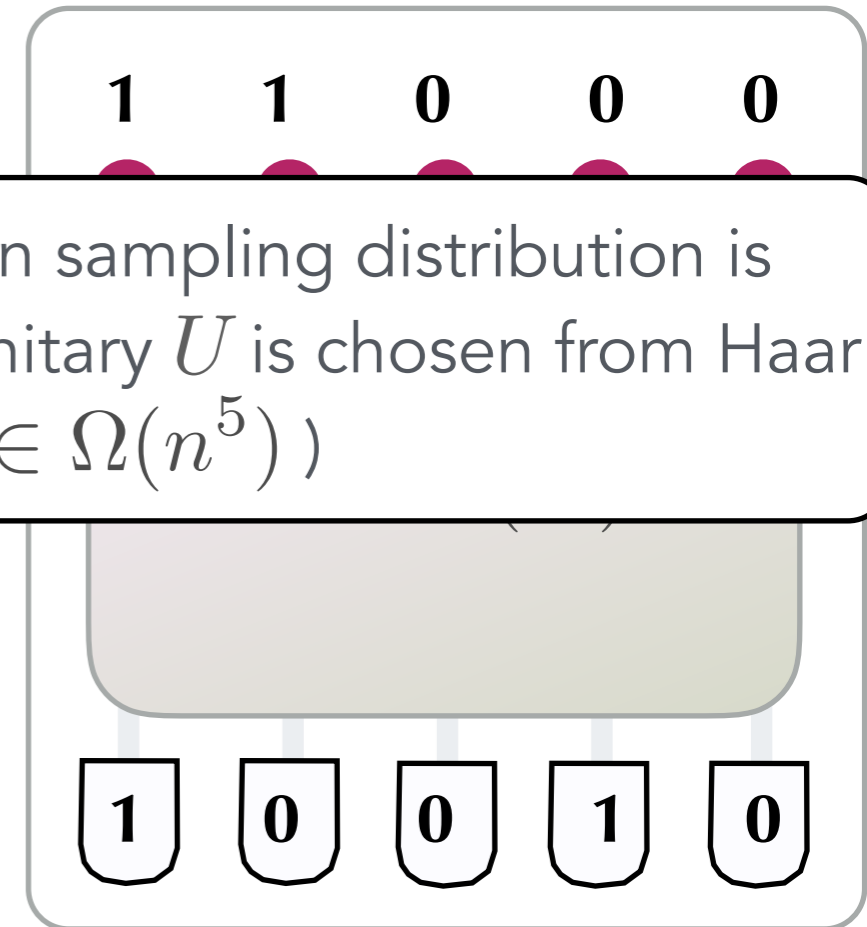
Complexity-theoretic quantum advantages

- **Aim:** Find some problem with strong evidence for quantum advantage

- **Boson sampling**

Aaronson, Arkhipov, Th Comp 9, 143 (2013)

Sampling from a distribution close in l_1 norm to boson sampling distribution is "computationally hard" with high probability if the unitary U is chosen from Haar measure and m increases sufficiently fast with n ($m \in \Omega(n^5)$)



- **IQP and random universal circuits**

Bremner, Montanaro, Shepherd: Phys Rev Lett 117, 080501 (2016)

Bremner, Jozsa, Shepherd, arXiv:1005.1407

Boixo, Isakov, Smelzanski, Babbush, Ding, Jiang, Bremner, Martinis, Neven, Nature Physics 14, 595-600 (2018)

- **Ising-type interactions (but, period 56 of unit cell)**

Gao, Wang, Duan, Phys Rev Lett, 118, 040502 (2017)

Complexity-theoretic quantum advantages

- **Aim:** Find **some** problem with strong evidence for quantum advantage

- Verification and testing? Black-box verification seems out of question

Hamiltonian quantum simulation architectures

- **Aim:** Find **some** problem with strong evidence for quantum advantage

- **Challenging prescription:** Is it possible to scale it up to provably hard regimes, in an architecture close to a quantum simulation?

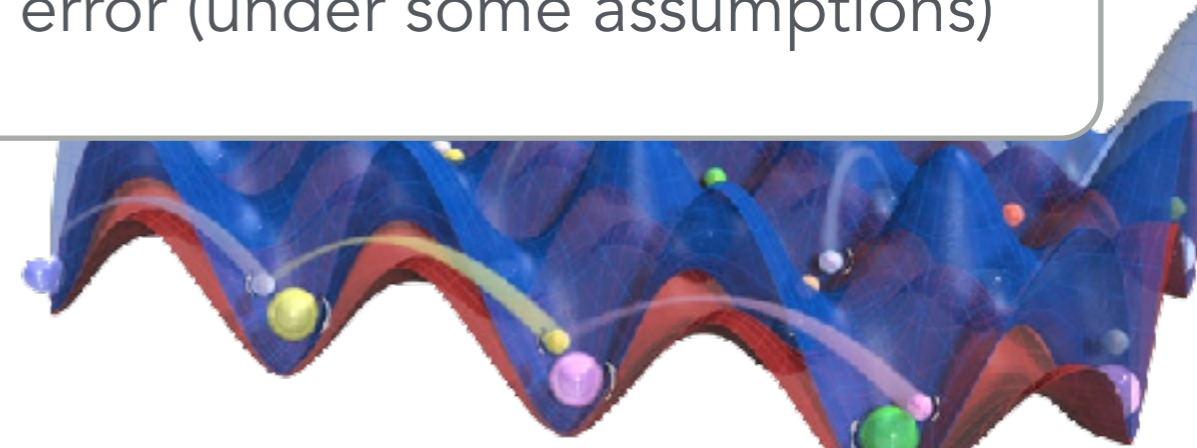
Hamiltonian quantum simulation architectures

- **Aim:** Find some problem with strong evidence for quantum advantage

Combine benefits of both worlds



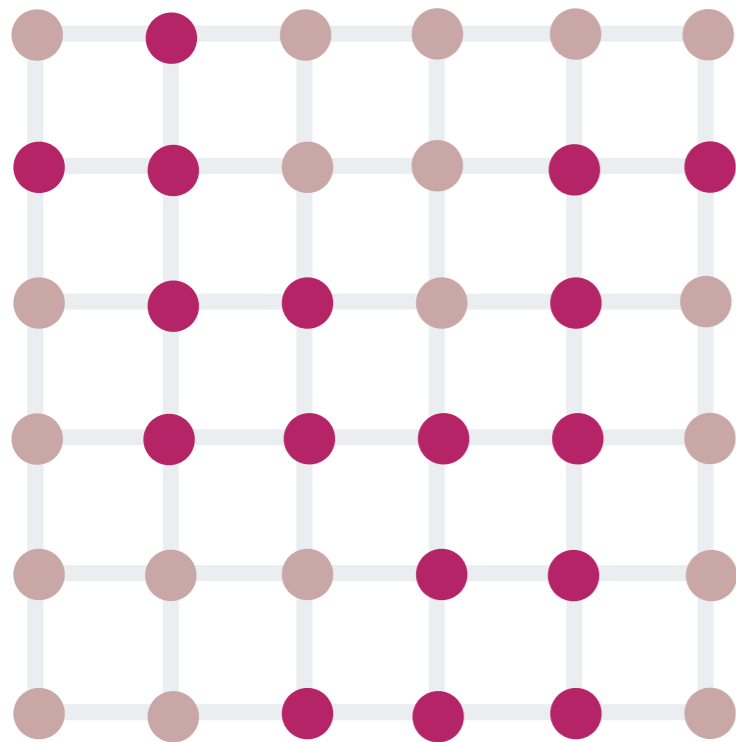
- **Hamiltonian quench architecture**
- **Low periodicity** of the interaction Hamiltonian (NN or NNN)
- **Hardness proofs** with l_1 -norm error (under some assumptions)



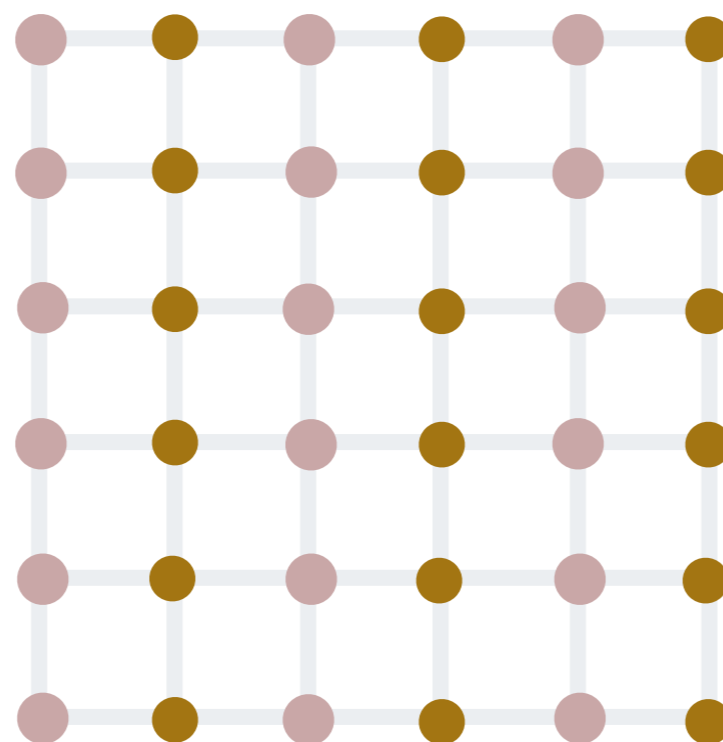
Hamiltonian quantum simulation architectures

- **Aim:** Find some problem with strong evidence for quantum advantage

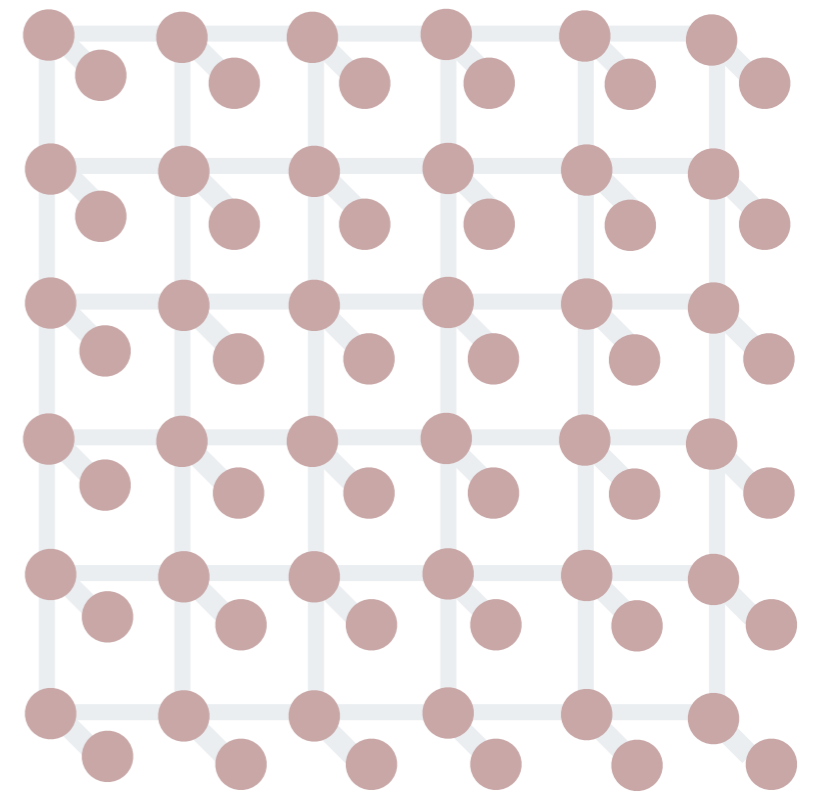
Combine benefits of both worlds



Random



Quasi-periodic



Translationally invariant

Simple Ising models

- Prepare N qubits in $n \times m$ square lattice in product

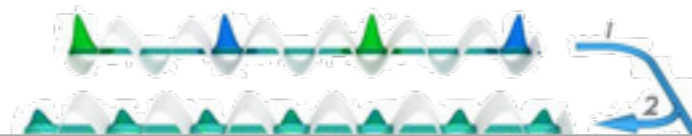
$$|\psi_\beta\rangle = \bigotimes_{i,j=1}^{n,m} (|0\rangle + e^{i\beta_{i,j}} |1\rangle)$$

with $\beta_{i,j} \in \{0, \pi/4\}$, $\{\bullet, \bullet\}$ i.i.d. randomly

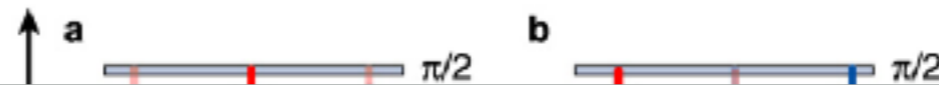
- Quench to $H = \sum_{(i,j)} \dots$

$\sum_{(i,j)}$

- Reminiscent of disordered optical lattices

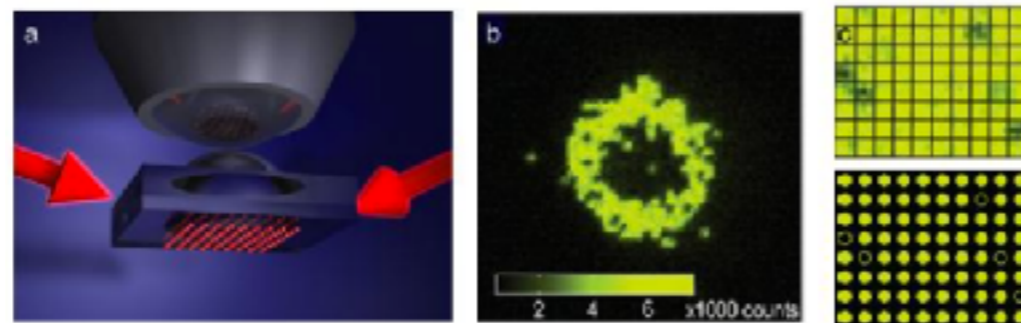


- Controlled coherent collisions long realized



- Measure all qubits in

- Single-site addressing possible (within limits)



Bakr, Gillen, Peng, Foelling, Greiner, Nature 462, 74–77 (2009)

Weitenberg, Endres, Sherson, Cheneau, Schauß, Fukuhara, Bloch, Kuhr, Nature 471, 319 (2011)

42 (2015)

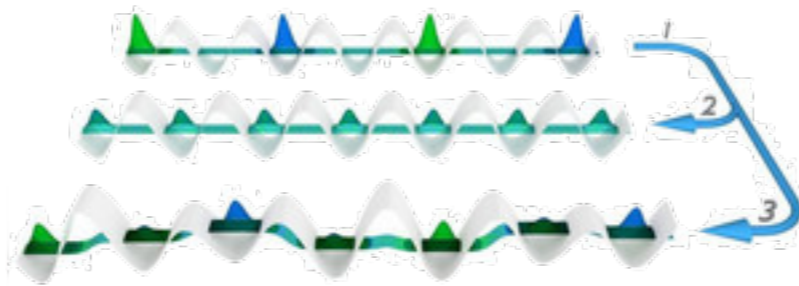
Simple Ising models

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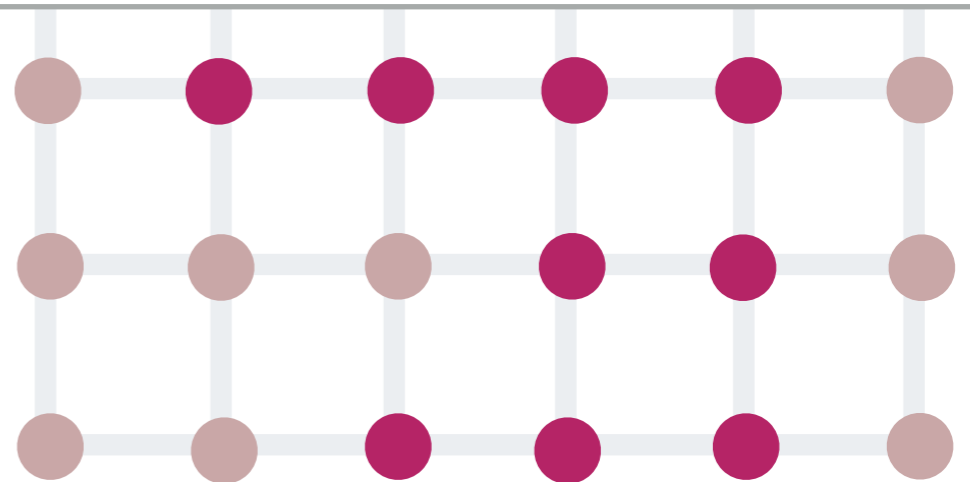
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Simple Ising models

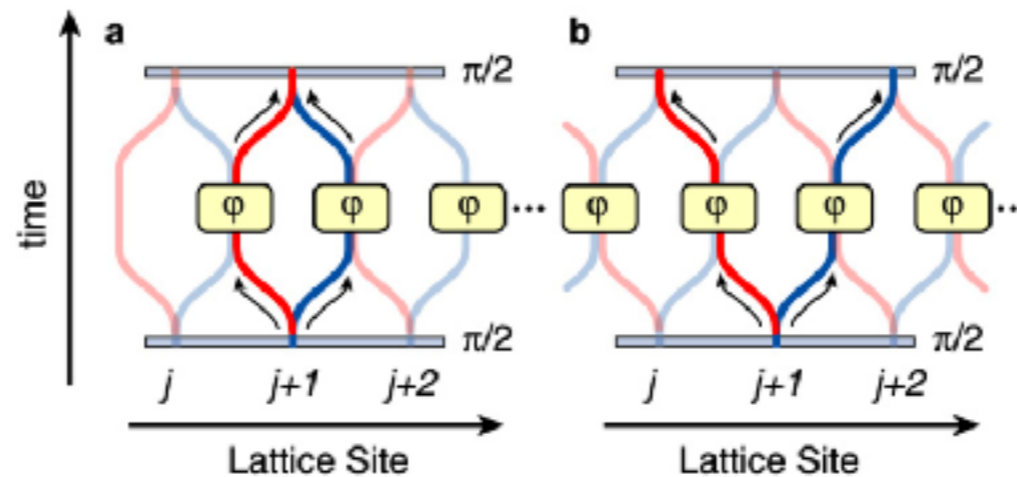
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- Quench to $H = \sum_{(i,j) \in E} Z_i Z_j + \frac{\pi}{4} \sum_{i \in V} Z_i$ and evolve under $U = e^{iH}$

- Controlled coherent collisions long realized



Mandel, Greiner, Widera, Rom, Hänsch, Bloch, Nature, 425, 937 (2003)

Simple Ising models

- Prepare N qubits in $n \times m$ square lattice in product

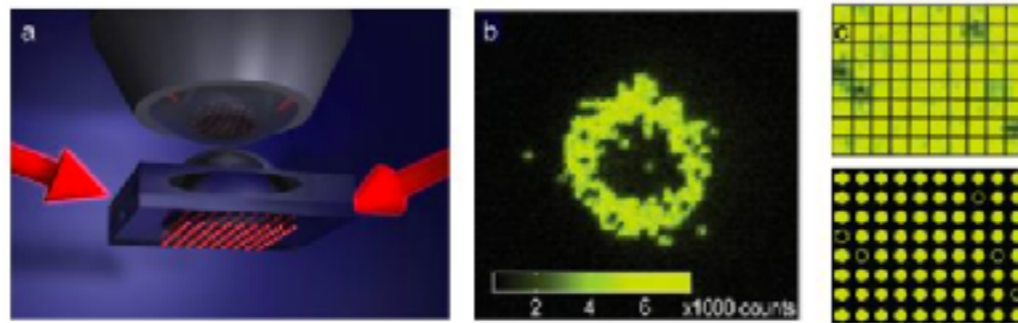
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- Measure all qubits in X -basis

- Single-site addressing possible (within limits)



Bakr, Gillen, Peng, Foelling, Greiner, Nature 462, 74–77 (2009)

Weitenberg, Endres, Sherson, Cheneau, Schauß, Fukuhara, Bloch, Kuhr, Nature 471, 319 (2011)

Simple Ising models

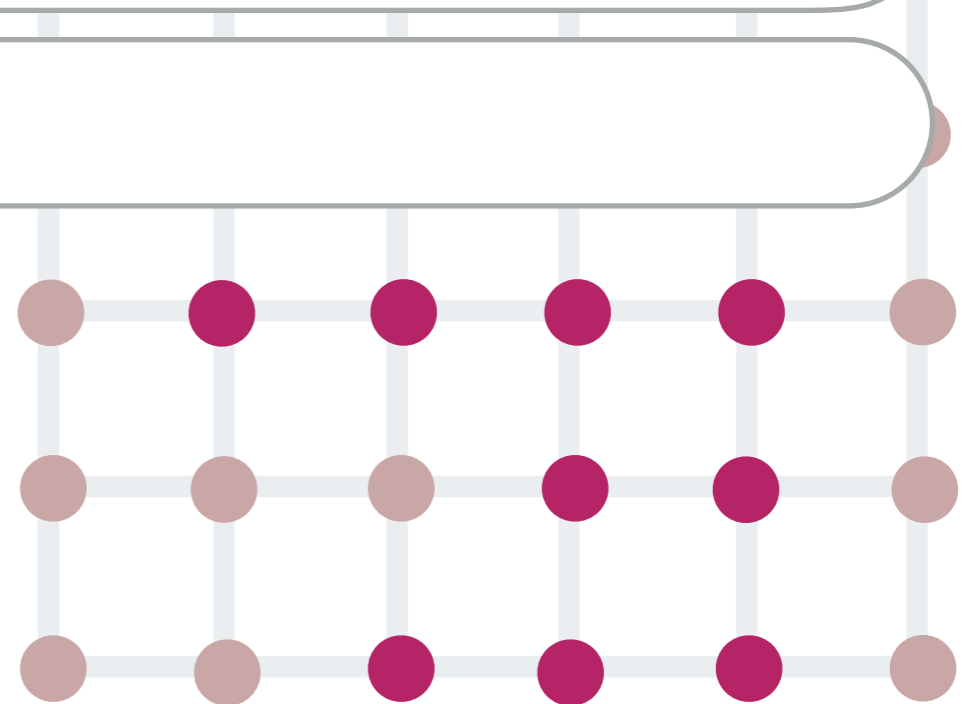
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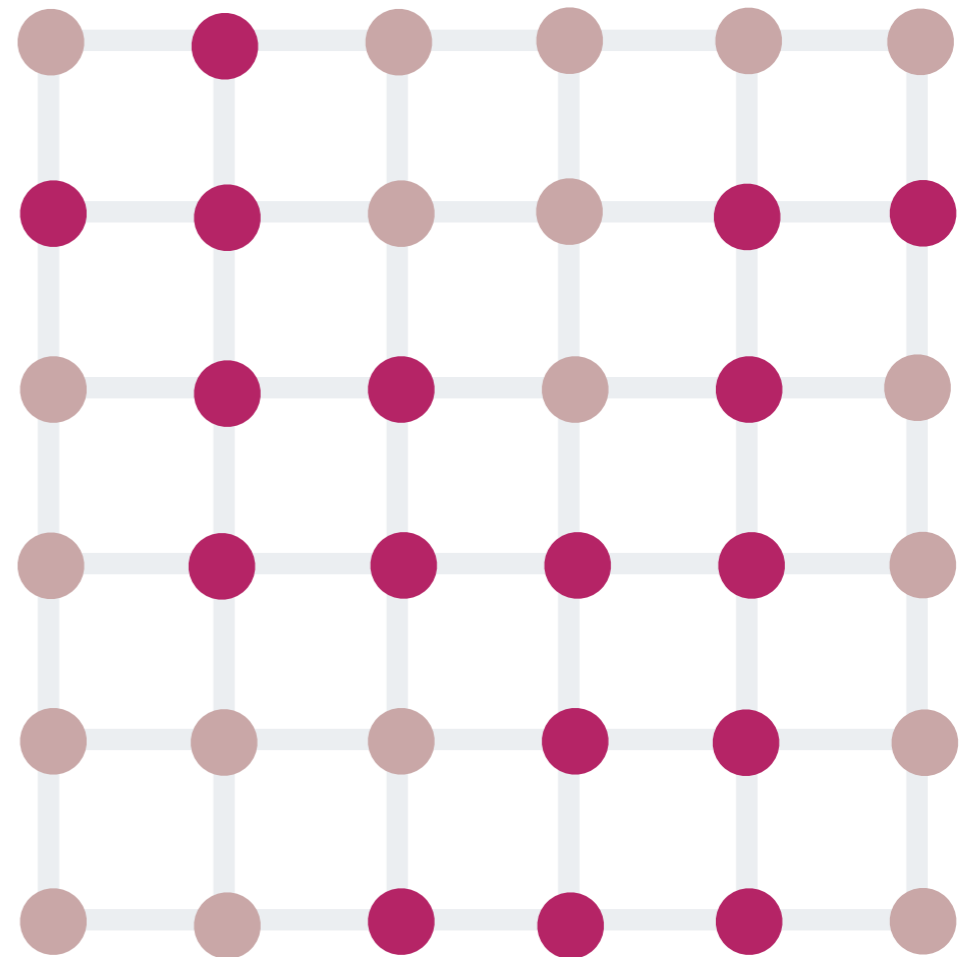
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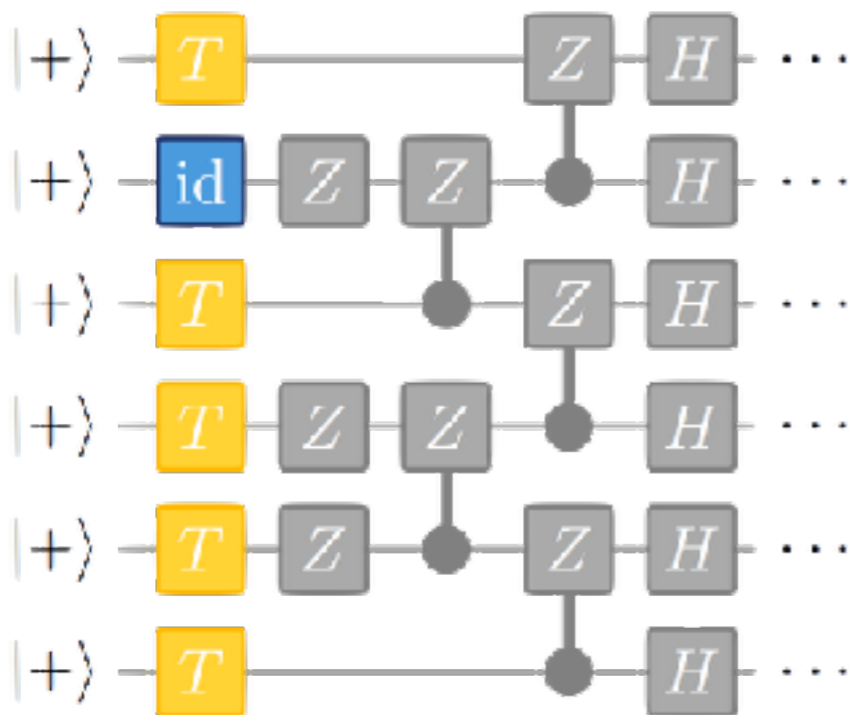
- **Theorem** (Hardness of classical sampling):

Assuming three highly plausible complexity-theoretic conjectures are true a classical computer cannot efficiently sample from the outcome distribution of our scheme up to constant error in l_1 distance



- Relate quench architecture to post-selected measurement-based quantum computing

- Universal quantum circuit for postBQP



- It is #P-hard to approximate the outcome distribution

- Polynomial hierarchy (similar $P \neq NP$)

- Average-case complexity

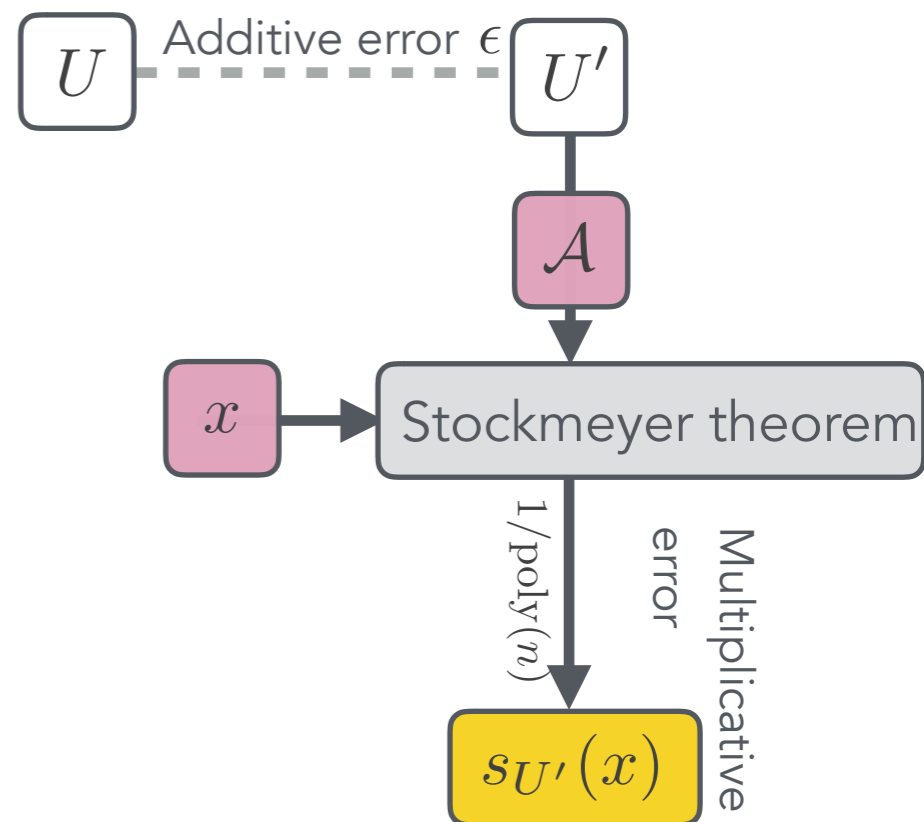
Bouland, Fefferman, Nirkhe, Vazirani, arXiv:1803.04402

- Anti-concentration

Hangleiter, Bermejo-Vega, Schwarz, Eisert, Quantum 2, 65 (2018)

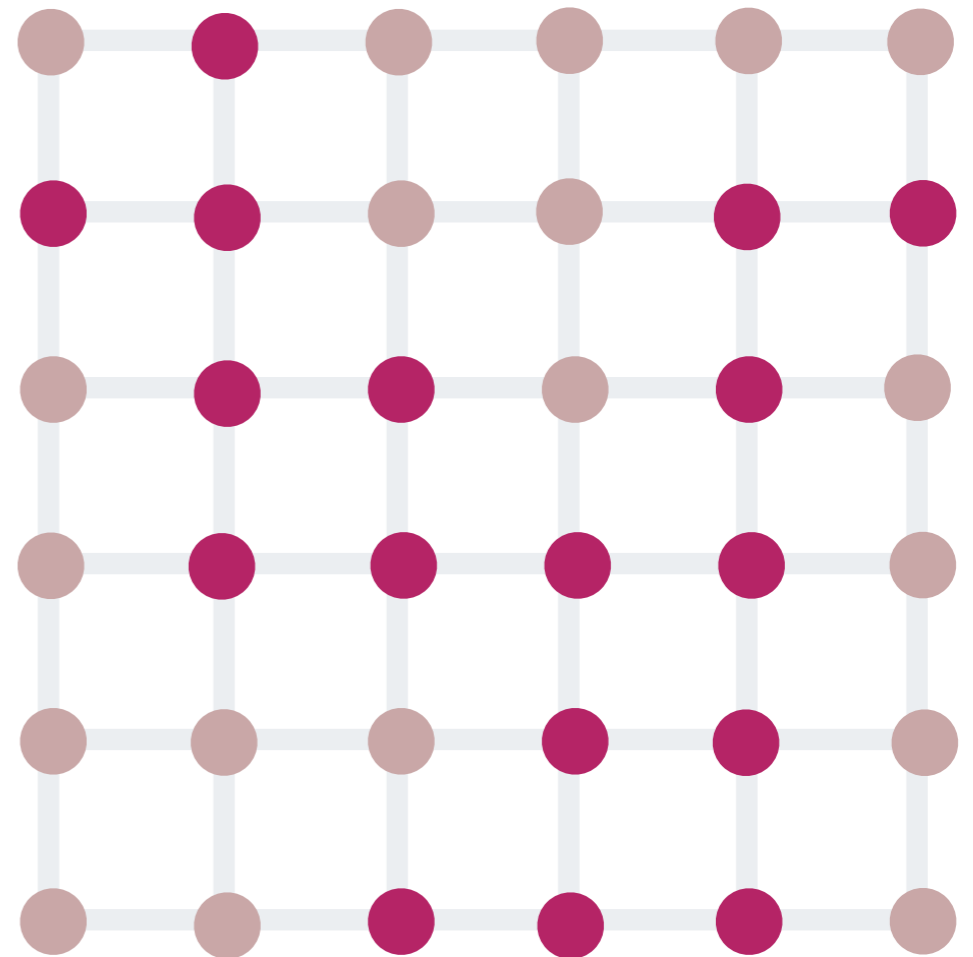
Mann, Bremner, arXiv:1711.00686

- Relate hardness of computing probabilities to hardness of sampling with additive errors



• **Theorem** (Hardness of classical sampling):

Assuming three highly plausible complexity-theoretic conjectures are true a classical computer cannot efficiently sample from the outcome distribution of our scheme up to constant error in l_1 distance



- This quantum simulation is intractable for classical computers



Verifiable quantum devices showing a quantum advantage

- One can with $\theta(N)$ many measurements detect closeness in l_1 -norm!
- Ground state of fictitious frustration-free Hamiltonian
- Much simpler than fault tolerance



Verifiable quantum devices showing a quantum advantage

- **Common prejudice:** In order to be able to verify a quantum simulation, one needs to be able to efficiently simulate it

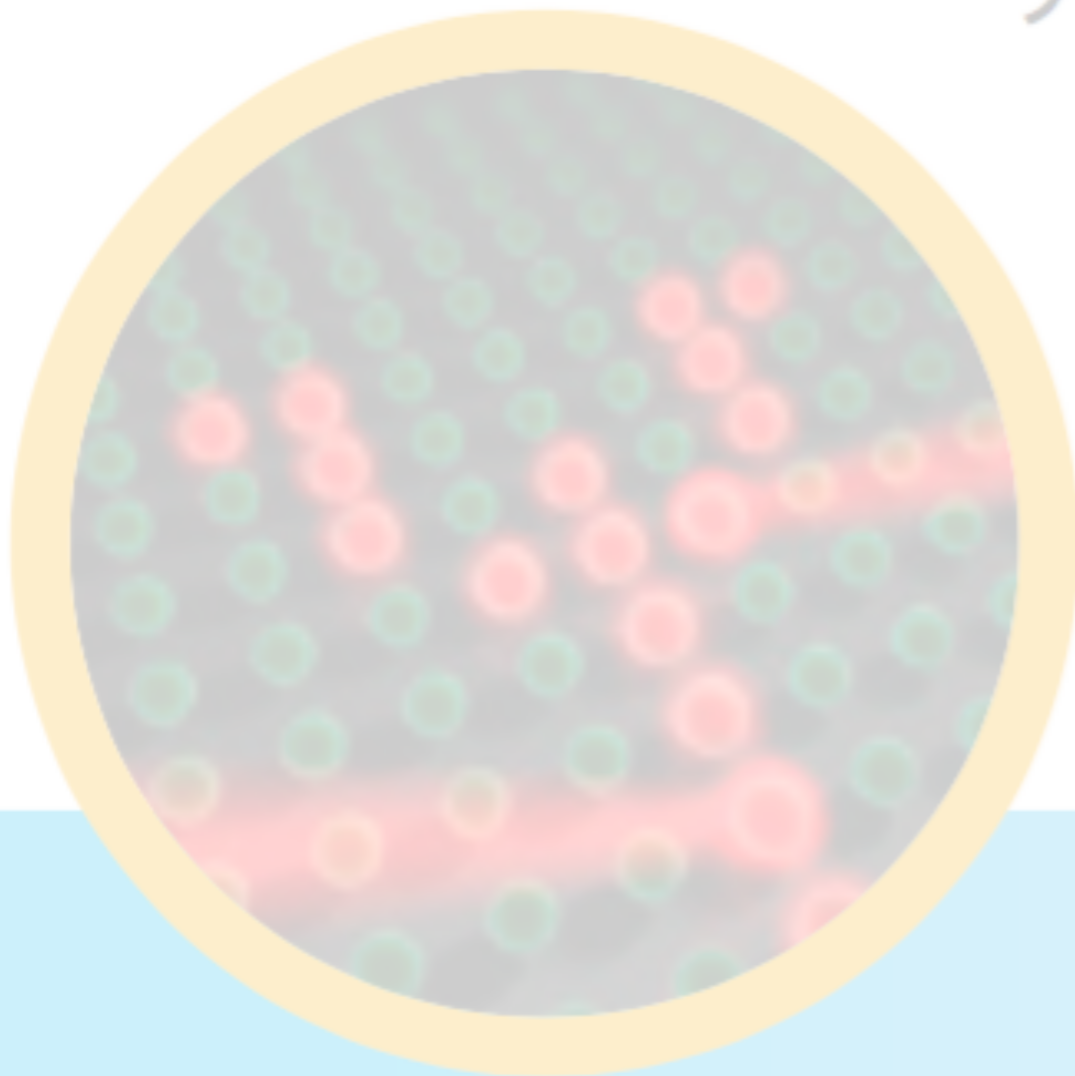
Summary, outlook and open questions

- Analog quantum simulators already outperform good classical algorithms

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LINK

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SIMULATOR

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INTEGRATION



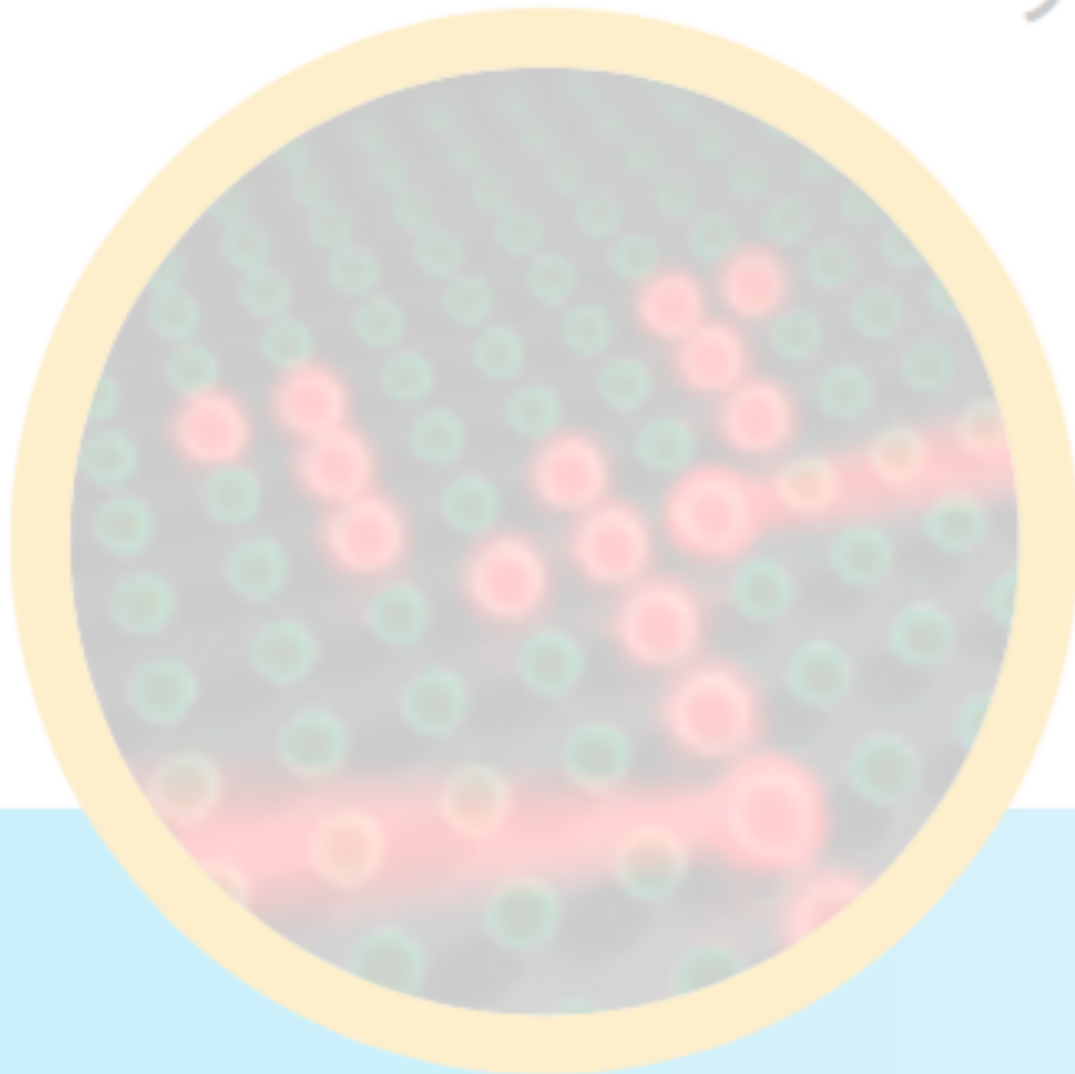
Summary, outlook and open questions

- Analog quantum simulators already outperform good classical algorithms
- Hope for **feasible** quantum simulators with **superpolynomial speedup**

QUANTUM
LINK

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SIMULATOR

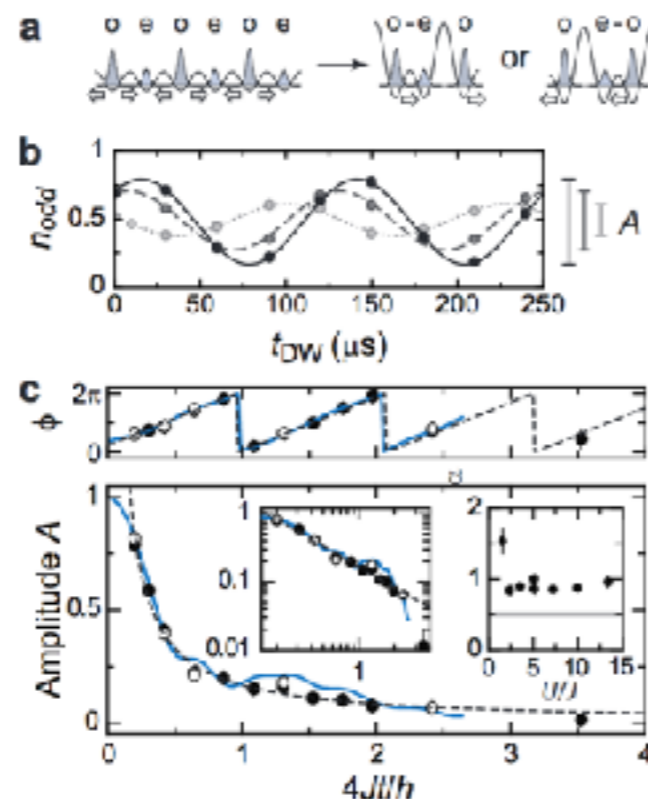
QUANTUM
INTEGRATION



Summary, outlook and open questions

- Analog quantum simulators already outperform good classical algorithms
- Hope for **feasible** quantum simulators with superpolynomial speedup
- Not fault tolerant, but can be certified: Bell test for quantum computing - even if simulators exhibit quantum computational speedup

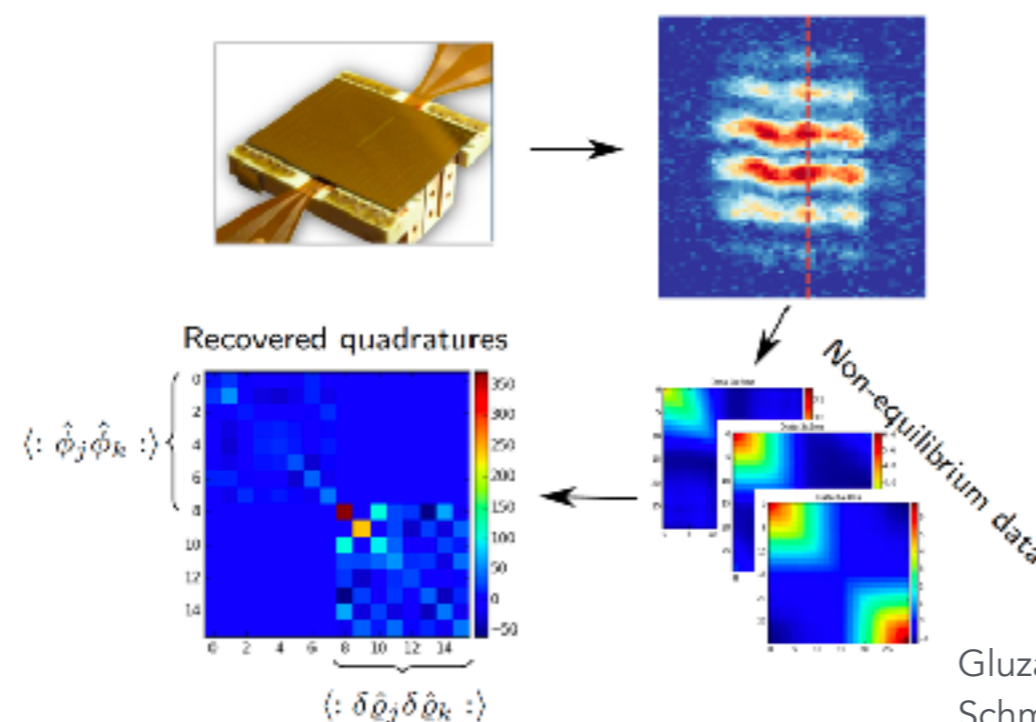
- Closer to physically more interesting schemes?
- More structured problems, optimization?



Summary, outlook and open questions

- Analog quantum simulators already outperform good classical algorithms
- Hope for **feasible** quantum simulators with **superpolynomial speedup**
- Not fault tolerant, but can be **certified**: Bell test for quantum computing - even if simulators exhibit quantum computational speedup

- Closer to physically more interesting schemes?
- More structured problems, optimization?
- Robustness of quantum simulators? Readout?

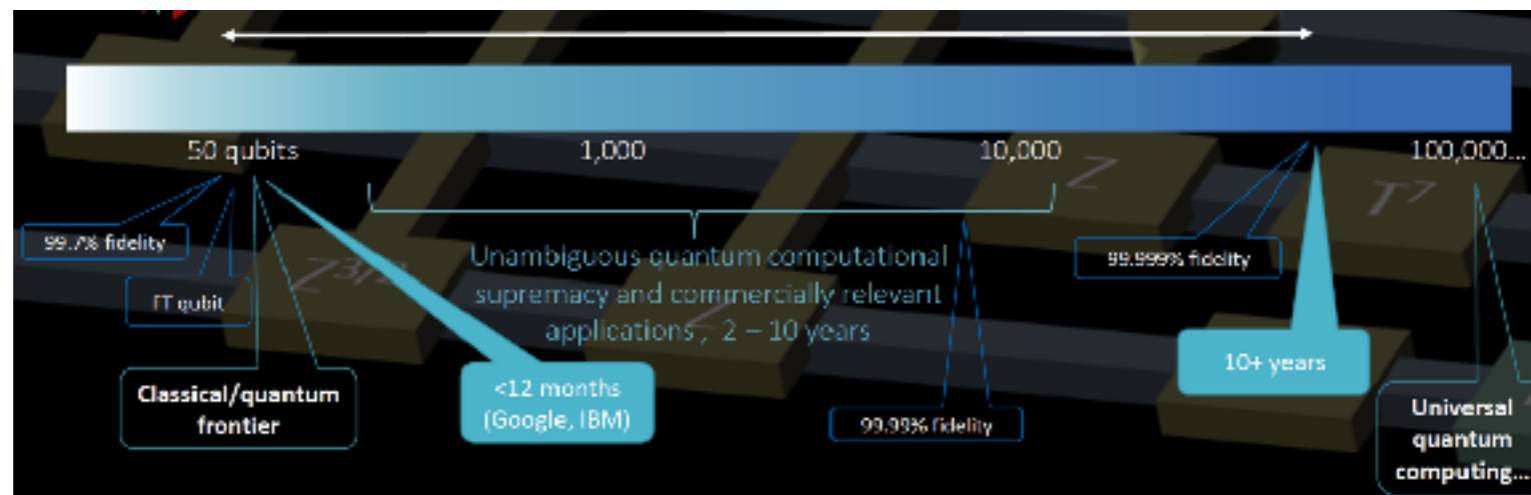


Gluza, Schweigler, Krumnow, Rauer, Schriedmayer, Eisert, in preparation

Summary, outlook and open questions

- Analog quantum simulators already outperform good classical algorithms
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- Space time trade offs?



(Mick Bremner)

Summary, outlook and open questions

- Analog quantum simulators already outperform good classical algorithms
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Thanks for your attention!